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A Glance on the Estimation of COBB-DOUGLAS Production Functional Model

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Abstract: The main purpose of this research article is to describe the methods of estimation of most frequently used nonlinear production functional model in econometric analysis namely COBB-DOUGLAS production functional model by using nonlinear method of estimation and logarithmic linear transformation. B. Mahaboob et al. in 2017(see [1]). In their research paper, in 2017, estimated the parameters of generalized COBB-DOUGLAS production functional model. In 2017, B. Venkateswarlu et al (see [2]). In their research article discussed the fitting of full COBB-DOUGLAS by solving goal programming problem. Shaiara Husain et al.in 2016 (see [4]), in their research study, found that COBB-DOUGLAS production function is applicable in the context of manufacturing sector and revealed an interesting aspect that this sector exhibits increasing returns to scale.

INTRODUCTION

The most important inferential phase in nonlinear production model problem is the estimation of parameters of the relations among dependent and independent variables. In Econometric the frequently used nonlinear production function models are Cobb-Douglas; Constant Elasticity of Substitution (CES) and Transcendental logarithmic (Translog) production functions. One of the most common situations of statistical analysis in Business, Economics, Engineering, Physical Sciences is likely to lead to a nonlinear regression model. Generally the models to be obtained as solutions of differential equations arising in Physical Sciences, Engineering, Ecology and Business Economics are some examples of nonlinear models. Several nonlinear regression models may be classified into three types namely Univariate nonlinear regression models, multivariate nonlinear regression models and nonlinear simultaneous equations regression models. Among these nonlinear regression models the univariate nonlinear Regression models have a wide number of applications in practice than other types of nonlinear regression models.

NONLINEAR METHOD OF ESTIMATION OF COBB-DOUGLAS PRODUCTION FUNCTIONAL MODEL

Consider the Cobb-Douglas production function model as

$$Z = BY_1^{\gamma}Y_2^{\delta} \tag{1}$$

where β, γ, δ are parameters

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one can construct the following expression with the help of Taylor series expansion in the neighbourhood of $B-B_0$, $\gamma-\gamma_0, \delta-\delta_0$ by choosing some starting values of the unknown parameters.

$$Z_0 = B_0 Y_1^{\gamma_0} Y_2^{\delta_0} + \left(\frac{\partial z}{\partial B}\right)_{B=B_0} (B-B_0) + \left(\frac{\partial Z}{\partial \gamma}\right)_{\gamma=\gamma_0} (\gamma-\gamma_0) + \left(\frac{\partial z}{\partial \delta}\right)_{\delta=\delta_0} (\delta-\delta_0) \tag{2}$$

Adding an error term to this expression and minimize the error sum of squares

$$\psi' = \sum_{i=1}^{n} \left(Z_i - Z_{0i}\right)^2$$

One can obtain the estimates of parameters as $B_1, \gamma_1 \quad and \quad \delta_1$

Now we form a linearized expression as

$$Z_1 = [B_1 Y_1^{\gamma_1} Y_2^{\delta_1}] + \left(\frac{\partial Z}{\partial B}\right)_{B=B_1} \left(B-B_1\right) + \left(\frac{\partial Z}{\partial \gamma}\right)_{\gamma=\gamma_1} \left(\gamma-\gamma_1\right) + \left(\frac{\partial Z}{\partial \delta}\right)_{\delta=\delta_1} (\delta-\delta_1) \tag{3}$$

Adding an error term to this expression and the second iterative estimates of parameters namely $B_2, \gamma_2 . \delta_2$ can be obtained by minimizing error sum of squares

$$\psi'' = \sum_{i=1}^{n} (Z - Z_{1i})^2 \tag{4}$$

One can continue this process until the parameters converge.

Another method is to minimize the RSS

$\psi = \sum_{i=1}^{n} [Z_i - \hat{B} Y_{1i}^{\ \hat{\gamma}} Y_{2i}^{\ \hat{\delta}}]^2$ with respect to $\hat{B}, \hat{\gamma}, \hat{\delta}$ obtain the following system of three equations

$\dfrac{\partial \psi}{\partial \hat{B}} = 0, \ \dfrac{\partial \psi}{\partial \hat{\gamma}} = 0 \, and \, \dfrac{\partial \psi}{\partial \hat{\delta}} = 0$ are nonlinear simultaneous linear equations in three variables (parameters). Each of the these three equations can be expanded about assumed initial values of the parameters as follows

$$\frac{\partial \psi}{\partial B} = \left(\frac{\partial \psi}{\partial B}\right)_{B=B_0} + \left(\frac{\partial^2 \psi}{\partial B^2}\right)(B-B_0) + \left(\frac{\partial^2 \psi}{\partial B \partial \gamma}\right)_{B=B_0, \gamma=\gamma_0} (\gamma-\gamma_0) + \left(\frac{\partial^2 S}{\partial B \partial \delta}\right)_{B=B_0, \delta=\delta_0} (\delta-\delta_0) = 0 \tag{5}$$

The similar equations for $\dfrac{\partial \psi}{\partial \gamma}, \dfrac{\partial \psi}{\partial \delta}$ about B_0, γ_0, δ_0 give three simultaneous linear equations in three unknown parameters and they are solved for B_1, γ_1, δ_1 by iterative process. This technique guarantees the convergence to a local extremum but one can't certain that one found the global minimum. The number iterations required depends upon how accurate the initial values are taken.

ESTIMATION OF COBB-DOUGLAS PRODUCTION FUNCTION USING LOGARITHMIC LINEAR TRANSFORMATION

Consider the COBB-DOUGLAS production functional model given by

$$Z_j = B Y_{1i}^{\ \gamma} Y_{2j}^{\ \delta} e^{\varepsilon_j} \quad i = 1,2,\ldots m \tag{6}$$

Z=output of the firm

$Y_1 =$ Labour Input

$Y_2 =$ Capital input

B=Technological coefficient

$\gamma =$ Labour input elasticity parameter

$\delta =$ Capital input elasticity parameter

ε =Classical disturbance variable

By taking the logarithmic transformation on both sides of the model (6), we see

$$\log Z_j = \log B + \gamma \log Y_{1j} + \delta \log Y_{2j} + \varepsilon_j \tag{7}$$

$$Z_j^{\ *} = B^* + \gamma Y_{1j}^{\ *} + \delta Y_{2j}^{\ *} + \varepsilon_j \qquad \text{j=1, 2…n} \tag{8}$$

where $Z_j^{\ *} = \log Z_j$; $B^* = \log B$; $Y_{1j}^{\ *} = \log Y_{1j}$;$Y_{2j}^{\ *} = \log Y_{2j}$

Equation (8) is termed as three variables linear model. The least squares estimates of $\gamma, \delta \ \ and \ \ B^*$ can be obtained as

$$\begin{bmatrix} \hat{\gamma} \\ \hat{\delta} \end{bmatrix} = \begin{bmatrix} \sum y_1^{\ *2} & \sum y_1^{\ *} y_2^{\ *} \\ \sum y_1^{\ *} y_2^{\ *} & \sum y_2^{\ *2} \end{bmatrix}^{-1} \begin{bmatrix} \sum y_1^* z^* \\ \sum y_2^* z^* \end{bmatrix} \tag{9}$$

and $\hat{B}^* = \overline{Z}^* - \hat{\gamma}\overline{Y}_1^* - \hat{\delta}\overline{Y}_2^*$ (10)

where $\sum y_1^{\ *2} = \left[\sum Y_1^{*2} - \frac{\left(\sum Y_1^{\ *}\right)^2}{m} \right]$

$$\sum y_2^{\ *2} = \left[\sum Y_2^{*2} - \frac{\left(\sum X_2^{\ *}\right)^2}{m} \right]$$

$$\sum y_1^* y_2^* = \left[\sum Y_1^* Y_2^* - \frac{\left(\sum Y_1^*\right)\left(\sum Y_2^*\right)}{m} \right]$$

$$\sum y_1^* z^* = \left[\sum Y_1^* Z^* - \frac{\left(\sum Y_1^*\right)\left(\sum Z^*\right)}{m} \right]$$

$$\sum y_2^* z^* = \left[\sum Y_2^* Z^* - \frac{\left(\sum Y_2^*\right)\left(\sum Z^*\right)}{m} \right]$$

$$\hat{B} = Anti \log\,(\hat{B}^*)$$

The least squares estimated Cobb-Douglas production functional model is given by

$$\hat{Z}_i = \hat{B} Y_{1j}^{\hat{\gamma}} Y_{2j}^{\hat{\delta}} \;;\; j = 1,2,\ldots m \qquad (11)$$

CONCLUSIONS

In the above research article the methods of estimation of most important nonlinear production functional model in Econometric namely COBB-DOUGLAS production functional model by using nonlinear method of estimation and logarithmic linear transformation have been proposed. In the context of future research one can estimate the same one through cost function, multiplicative and additive errors and by removing multi collinearity.

REFERENCES

1. B. Mahaboob, B. Venkateswarlu, I.J of Pharmacy and Tech., ISSN: 0975-766X, CODEN: 1J1TF1, (2017).
2. B. Venkateswarlu, B. Mahaboob IOP Conference series: Materials Science and Engineering (2017).
3. Jesus Felipe, F. Gerard Adams, Eastern Economic Journal, **31**, (2005)
4. Shaiara Hussain, and Md. Shahidul Islam, I.J. of Business &Economic Research, **5,** 149-154 (2016).
5. Win Meeusen and Julien Van Den Broeck, International Economic Review, 18 (1977).
6. Andrews, D.W.K and Fair, R.C Review of Economic Studies, **55**, 615-640 (1988).
7. Bard, Y Academic Press, New York (1974).
8. Bodken, R.G and Klein, L.R Review of Economics and Statistics, **49**, 28-44, (1967).
9. Bera, A.K Samkhya, **46**, Series B, **3**,285-290 (1984).
10. Christensen, R J. of the American Statistical Association, **95**, 1290-1293 (2000).