Quantum vs Classical Local Algorithms for Local MaxCut

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Introduction

- The Quantum Approximate Optimization Algorithm (QAOA) has been shown to solve MaxCut with provable performance guarantees at low depth. [1, 2]
- Hastings constructs a family of classical local algorithms that outperform QAOA on MaxCut. [3]

Question

How do classical and quantum approaches perform on a local version of MaxCut?

Local Max Cut

Given a graph $G = (V, E)$, a *cut* is any subset of the vertices $S \subseteq V$. Given a cut S, a vertex $v \in V$ is *happy* if moving v does not increase the cut value. A cut S is *locally maximal* when all vertices in S are happy.

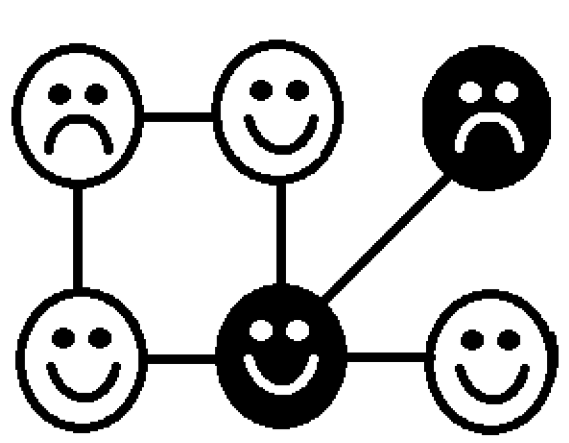

Figure 1: A cut that is not locally maximal.

Quantum Approach

We use the QAOA for the local MaxCut problem on n vertices. Each run of the QAOA utilizes two tuples of angles $\gamma \in [0, 2\pi]^p$ and $\beta \in [0, \pi]^p$ to construct gates resulting in a quantum circuit depth of $O(p)$. We construct diagonal Hamiltonian $C \in \mathbb{C}^{2^n \times 2^n}$ whose entries corresponds to the number of happy vertices for each of the 2^n cuts. The standard mixing operator $B = \Sigma_v \sigma_x^{(v)}$ is used. We use the angles $\gamma = (\gamma_1, \ldots, \gamma_p)$ and $\beta = (\beta_1, \ldots, \beta_p)$ to construct state $|\gamma, \beta\rangle = e^{-i\beta_p B} e^{-i\gamma_p C} \cdots e^{-i\beta_1 B} e^{-i\gamma_1 C} |+\rangle_n$, where $|+\rangle_n = H^{\otimes n} |0\rangle$ is the uniform superposition. The problem reduces to finding $M(C) := \max_{\gamma,\beta} \{\langle \gamma, \beta | C | \gamma, \beta \rangle\}$.

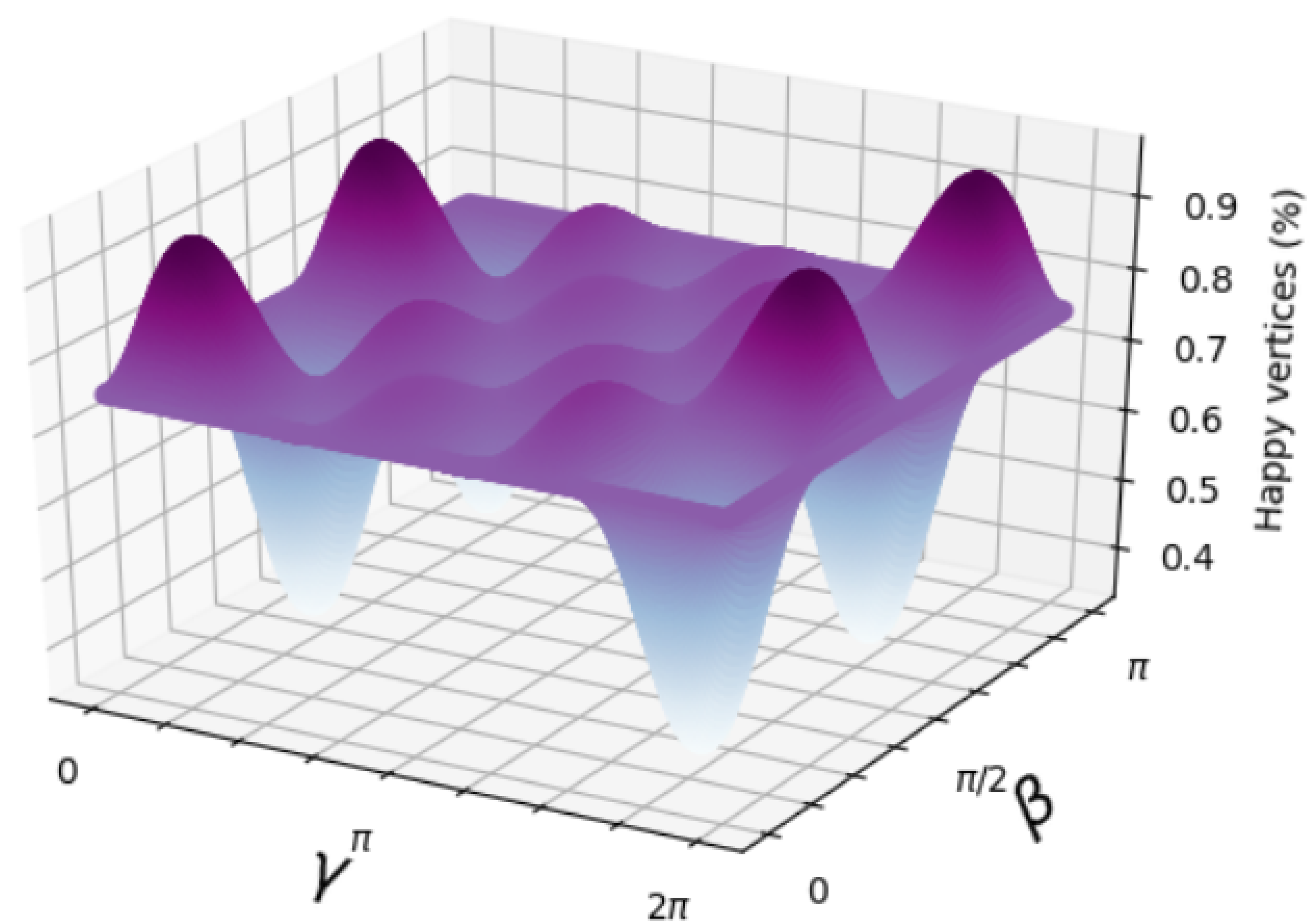

Figure 2: $M(C)$ for $p = 1$ on $2-$regular graphs

Classical Approach

We use a probabilistic algorithm with p steps that acts on each vertex using only local information.

Theorem

There exists a classical local algorithm for local MaxCut such that after $p > 0$ steps on a 2-regular graph, the expected fraction of happy vertices is at least $1 - (5/100)(1/2)^{p-1}$.

Results

p	0	1	2	3
Quantum	0.75	0.939	0.956	0.956
Classical	0.75	0.95	0.975	0.9875

Table 1: The expected percentage of happy vertices on graphs with degree 2

Open Questions

- How does the QAOA behave for large circuit depth on local MaxCut?
- What is the relationship between approaches as graph degree increases?
- Can we improve upon the the classical bounds with more in depth analysis or more complex algorithms?

References

[1] Edward Farhi, Jeffrey Goldstone, and Sam Gutmann.
A quantum approximate optimization algorithm applied to a bounded occurrence constraint problem, 2015.

[2] Edward Farhi and Aram W Harrow.
Quantum supremacy through the quantum approximate optimization algorithm, 2019.

[3] M. B. Hastings.
Classical and quantum bounded depth approximation algorithms, 2019.

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