

Quantum vs Classical Local Algorithms for Local MaxCut

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Introduction

- The Quantum Approximate Optimization Algorithm (QAOA) has been shown to solve several optimization problems with provable performance guarantees at low depth. [1, 2]
- For MAXCUT, QAOA provides a nontrivial performance guarantee [1], but its performance is eclipsed by classical SDP approaches. [3]
- More recently QAOA was shown to be outperformed by classical local algorithms. [4]

Question

How do classical and quantum approaches compare on a local version of MAXCUT?

Local Max Cut

Given a graph $G = (V, E)$, a *cut* is any subset of the vertices $S \subseteq V$. Given a cut S , a vertex $v \in V$ is *happy* if moving v does not increase the cut value. A cut S is *locally maximal* when all vertices in S are happy.

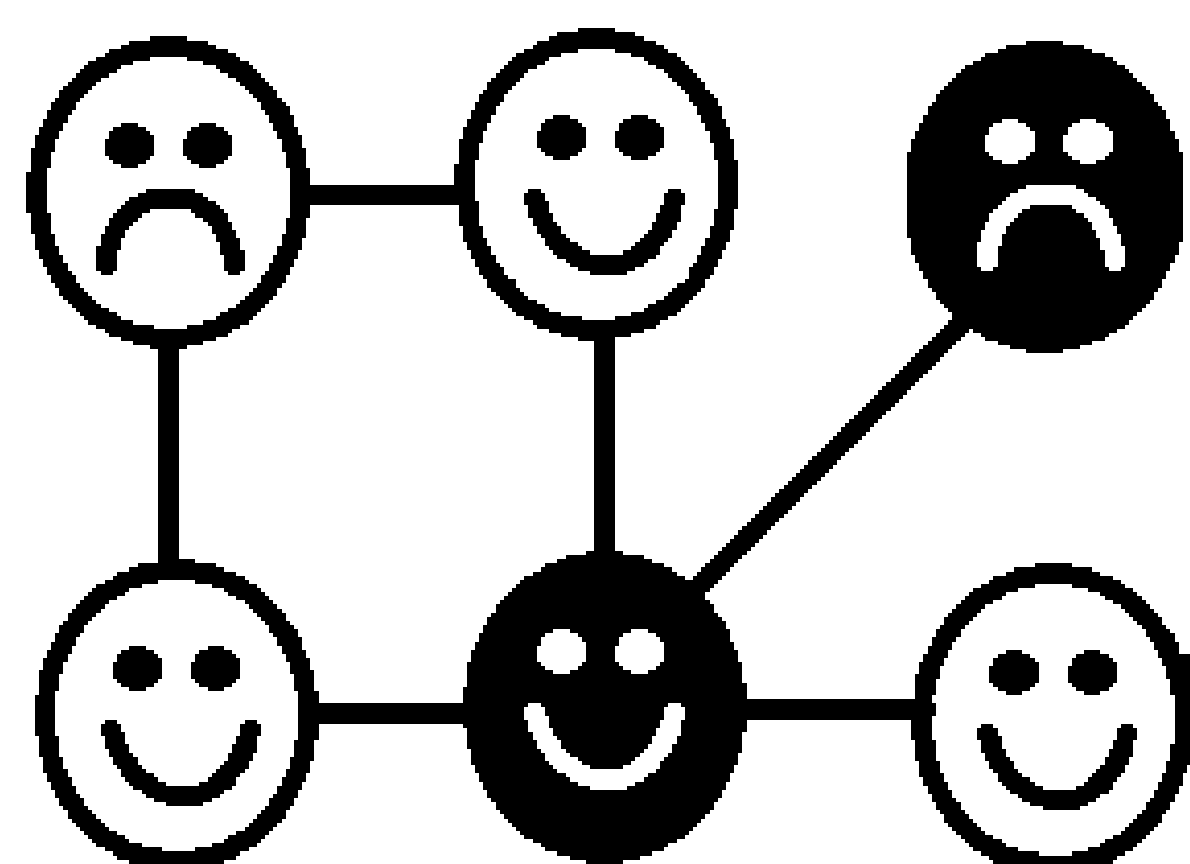


Figure 1: A cut that is not locally maximal.

Quantum Approach

We use the QAOA for the local MAXCUT problem on n vertices. Each run of the QAOA utilizes two tuples of angles $\gamma \in [0, 2\pi]^p$ and $\beta \in [0, \pi]^p$ to construct gates resulting in a quantum circuit depth of $O(p)$. We construct diagonal Hamiltonian $C \in \mathbb{C}^{2^n \times 2^n}$ whose entries correspond to the number of happy vertices for each of the 2^n cuts. The standard mixing operator $B = \sum_v \sigma_x^{(v)}$ is used. We use the angles $\gamma = (\gamma_1, \dots, \gamma_p)$ and $\beta = (\beta_1, \dots, \beta_p)$ to construct state $|\gamma, \beta\rangle = e^{-i\beta_p B} e^{-i\gamma_p C} \dots e^{-i\beta_1 B} e^{-i\gamma_1 C} |+\rangle_n$, where $|+\rangle_n = H^{\otimes n} |0\rangle$ is the uniform superposition. The problem reduces to finding $M(C) := \max_{\gamma, \beta} \{\langle \gamma, \beta | C | \gamma, \beta \rangle\}$.

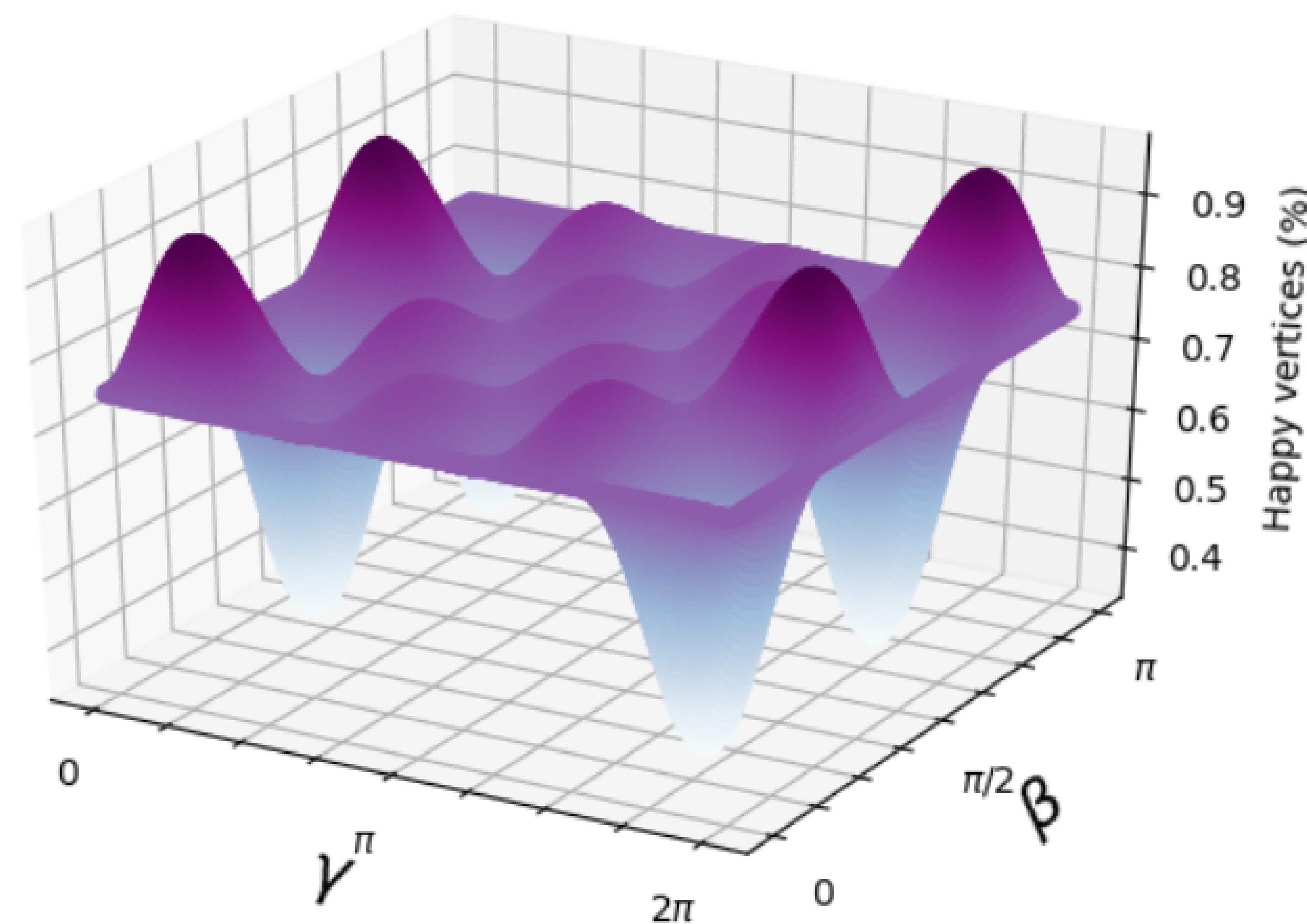


Figure 2: $M(C)$ for $p = 1$ on 2-regular graphs

Classical Approach

We use a probabilistic algorithm with p steps that acts on each vertex using only local information.

Theorem

There exists a classical local algorithm for local MAXCUT such that after $p > 0$ steps on a 2-regular graph, the expected fraction of happy vertices is at least $1 - (5/100)(1/2)^{p-1}$.

Results

p	0	1	2	3
Quantum	0.75	0.939	0.956	0.956
Classical	0.75	0.95	0.975	0.9875

Table 1: The expected percentage of happy vertices on graphs with degree 2. The QAOA is non-decreasing as p increases but our simulations have not found γ, β resulting in improving expectation values for $p = 3$.

Open Questions

- How does the QAOA behave for large circuit depth on local MAXCUT?
- What is the relationship between approaches as graph degree increases?
- Can we improve upon the the classical bounds with more in depth analysis or more complex algorithms?

References

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- [4] M. B. Hastings. Classical and quantum bounded depth approximation algorithms, 2019.

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