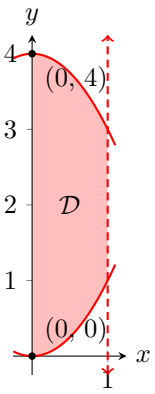
1. Sketch the domain

$$\mathcal{D} : 0 \le x \le 1,\ x^2 \le y \le 4 - x^2$$

and evaluate $\iint_{\mathcal{D}} y \, dA$ as an iterated interval.

Solution

$$\begin{aligned}
&\iint_{\mathcal{D}} y \, dA \\
&= \int_0^1 \int_{x^2}^{4-x^2} y \, dy \, dx \\
&= \int_0^1 \left[\frac{1}{2} y^2 \right]_{x^2}^{4-x^2} dx \\
&= \int_0^1 \frac{1}{2} \left[(4-x^2)^2 - (x^2)^2 \right] dx \\
&= \int_0^1 \frac{1}{2} \left(16 - 8x^2 + x^4 - x^4 \right) dx \\
&= \int_0^1 8 - 4x^2 \, dx \\
&= \left[8x - \frac{4}{3} x^3 \right]_0^1 \\
&= 8 - \frac{4}{3} \\
&\boxed{= \frac{20}{3}}
\end{aligned}$$



2. Sketch the domain $\mathcal{D}$ defined by $x + y \le 12$, $x \ge 4$, $y \ge 4$ and compute $\iint_{\mathcal{D}} e^{x+y} \, dA$.

Solution

$$\begin{aligned}
&\iint_{\mathcal{D}} e^{x+y} \, dA \\
&= \int_4^8 \int_4^{12-x} e^{x+y} \, dy \, dx \\
&= \int_4^8 \left[e^{x+y} \right]_4^{12-x} dx \\
&= \int_4^8 e^{12} - e^{x+4} \, dx \\
&= \left[x e^{12} - e^{x+4} \right]_4^8 \\
&= 8e^{12} - 4e^{12} - \left(e^{12} - e^8 \right) \\
&\boxed{= 3e^{12} + e^8}
\end{aligned}$$