

# Sensitivity to hypothesis sparsity in a category discrimination task

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## Abstract

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## Introduction

Hypothesis testing strategies are relevant to a wide range of naturalistic learning tasks where the learner has some control over what information will be received. Where learners are able to explore an environment, produce candidate examples of a target concept for validation, or request labels for some subset of the total data available to them, the question arises of how to do this efficiently (Settles, 2009; Gureckis & Markant, 2012). Where should exploration be directed? Which candidate examples should be produced? Which data should be labelled?

A number of possible sampling strategies have been discussed (Nelson, 2005). One such strategy drawn from the philosophy of science is falsification, whereby learners should choose tests that aim to falsify their current hypothesis, on the grounds that confirming evidence is always open to alternative explanations, but counterexamples always definitively rule out a hypothesis (Popper, 1959). Although widely accepted as a scientific norm, strict falsification is rarely followed by people faced with hypothesis-testing tasks (P. C. Wason, 1960; P. Wason, 1968). This failure to falsify, or ‘confirmation bias’ has been replicated in a wide range of tasks and contexts (Nickerson, 1998). In the process, its status as an irrational bias has been challenged. One important observation, due to Klayman and Ha (Klayman & Ha, 1987), is that tests that are apparently confirmatory, in that they are drawn from a candidate hypothesis, may yield a falsification in situations where the true hypothesis is not a superset of the candidate one. In choosing whether to probe from within the scope of a candidate hypothesis or outside it, the learner must consider the base rate probability that a member of the domain under consideration is also a member of the target set, the proportion of domain members that are covered by the candidate hypothesis, and (estimated) positive and negative error rates under the candidate hypothesis (Klayman & Ha, 1987). When the target set is a relatively small subset of the whole domain, and its size is approximately known, positive testing is a defensible strategy in terms of maximizing the chance of falsification of a candidate hypothesis. More recent work has extended this approach with a variety of methods for judging the utility of a given test, for example in terms

of expected probability gain (minimizing the expected probability of error on a randomly selected domain member after the seeing the results of the test) [CITE], or expected information gain (maximally reducing uncertainty defined in terms of Shannon information) [CITE]. Although differing in their predictions under some circumstances, these measures have in common a shared perspective of tests as gambles with uncertain rewards paid out in evidential value (Poletiek & Berndsen, 2000). Unlike strict falsification, a strategy of maximizing expected probability gain has been shown to account well for human responses in simplified hypothesis testing tasks (Nelson, McKenzie, Cottrell, & Sejnowski, 2010).

Recent work has been ambiguous as to whether these responses are due to sensitivity to the value of a given test under the specific structure of the task (Meder & Nelson, 2012), or if a positive test strategy is adopted in an indiscriminate way because the conditions under which it is a favourable strategy (Klayman & Ha, 1987; Austerweil & Griffiths, 2011) are common, making it a reasonable heuristic adopted even when more sophisticated information-sensitive strategies are available (Cherubini, Rusconi, Russo, Di Bari, & Sacchi, 2010). This study aims to directly test the extent to which people are sensitive to the evidential-reward structure of a particular hypothesis testing task by manipulating one of the key assumptions underlying the expected utility of positive tests: the sparsity of the hypothesis.

The sparsity of a hypothesis refers to the proportion of all members of a domain that are indexed by the hypothesis in question: sparse hypotheses index fewer than half of the members of the relevant domain (Navarro & Perfors, 2011). For example, in the domain ‘living species’ the category ‘DOGS’ is sparse, while ‘AEROBIC ORGANISM’ is a non-sparse, since most living things are not dogs, but do metabolise oxygen. Sparsity can vary in degree: while ‘dogs’ and ‘poodles’ are both sparse categories in the domain of living things, the category ‘poodles’ is more sparse.

Sparsity is one factor impacting the utility of the positive test strategy (Klayman & Ha, 1987; Navarro & Perfors, 2011). Where the target hypothesis is very sparse, the information value of negative results is greatly reduced, BECAUSE OF MATHS AND STUFF.

This paper examines human behaviour in one particular task ??from family X??, a sorting task asking participants to learn a category boundary in a stimulus space consisting of simple geometric shapes varying on three feature-dimensions. Participants were able to request labels for a randomly selected positive example of the target category or

a negative non-target example. Participants were aware of what proportion of all stimuli belonged to the target category, and between-subjects manipulations of this proportion showed both a sensitivity to the information value of each type of request and a preference for requesting positive examples.

Missing: emphasis on abstract non-spatial nature of hypothesis space, contrast battleships? Nice quote for this: “... recourse to asymmetric testing... probably depends on context-related motivations and prior knowledge. In abstract tasks, where that knowledge is not available, more simple strategies, such as positive testing, are prevalent” [Relationship between ‘asymmetry’ and information sensitivity is not simple... also, their specific abstract task is a monster, evidence consistent with ‘+’ve tests as fallback under bamboozlement”]

## Experiment

367 adults were recruited via Amazon Mechanical Turk. Of these, 301 completed the task, and 121 were excluded from further analysis for either failing to make any label requests at all (85 participants), making more than 60 requests (9 participants), or failing to sort labelled examples correctly (36 participants). Nine participants were excluded for a combination of these reasons. The remaining 180 participants contributed 360 trials, with between 104 and 131 trials falling in each of three sparsity conditions. These set the proportion of stimuli belonging to the target category at 25%, 50%, or 75%.

Ages ranged from 19 to 67 (mean: 34.4), 45.0% were female. 117 of the final participants were from the United States and 52 were from India. Those remaining were from 8 other countries in Africa, North and South America, Europe, and Asia. All participants were paid \$0.60US for the 15 minute experiment.

## Procedure

The cover story for the study described a fictitious company interested in harvesting a new substance called ‘selenoid’ from plankton. Participants were told selenoid-rich plankton were desirable for harvesting, and were given the percentage of all plankton expected to be selenoid-rich, either 25%, 50%, or 75% depending on the experimental condition. In each trial, participants were presented with two ‘bins’, each containing a random selection of half the possible plankton examples. Buttons below each bin allowed participants to request a label for either a selenoid-rich or a selenoid-poor plankton, which appeared as a persistent coloured highlight around the randomly selected example after a two-second delay. Plankton could be swapped between bins by clicking on them, and participants were asked to click a submit button after they had sorted each plankton into the correct bin. Once a sort was submitted, the true selenoid status for each plankton was revealed and a score displayed calculated as 10 points for each one correctly sorted and -10 for each incorrectly sorted. An inference-efficiency score defined as total score divided by number of requests made was also displayed.

All participants answered a series of multiple-choice questions to make sure they had read and understood the instructions. The main task was then presented three times, the first of which was labelled as a practice trial and required participants to try all the available actions and submit a plausible sort before continuing.

## Test stimuli

The stimuli were geometric shapes consisting of a ring and a number of radial arms. They varied in colour, size of the ring, and number of arms, with four levels in each dimension giving 64 combinations of feature values.



Figure 1: 64 different stimuli were used, corresponding to all unique combinations of four possible values on three dimensions. These were colour, ring size, and number of arms, shown here increasing from left to right.

The true selenoid status of the plankton in a given trial was determined by a threshold rule on one dimension of variation, randomly selected under the constraint that rules could not repeat across the three trials presented to any one participant. The location of this threshold was determined by sparsity condition, which varied between participants. Selenoid-rich plankton could be 25% (sparse target), 50% (neutral target), or 75% (non-sparse target) of all possible plankton. In the 25%:75% split conditions, members of the minority group shared one extreme value on one type of feature. In the 50%:50% split condition, members of the same group shared one of two adjacent values in the discriminating feature.

## Results

The comparisons of interest between conditions required that participants be engaged with the task. The average score across participants was 368, corresponding to 50.4 plankton correctly sorted. Score distributions were bimodal, with one peak at the expected score due to chance (0 for 50%:50% splits and 160 for 25%:75% splits) and the other peak at perfect performance. While 27% of trials scored at or below chance, many people were highly successful, defined as able to sort 60 or more plankton correctly on the basis of fewer than 6 labels (18% of all trials). The mean number of swapping actions (36.0) was close to the expected required number of swaps to correct a random sort to an ordered one (32), indicating that participants meeting the inclusion criteria above understood and engaged with the task. Score and number of label request distributions were similar across the first and second non-practice trials.

Where positive testing bias predicts a preference for requests labelling the ‘selenoid-rich’ plankton category regardless of the population proportions, sensitivity to the information value of requests implies a preference for requesting labels from the minority classification if this is possible. To

account for the fact that different participants made different numbers of requests, the different conditions were compared in terms of the proportion of positive requests made by each participant in a single trial. Mean proportion of positive requests in the 25% rich, 50% rich, and 75% rich conditions were 0.56, 0.53, and 0.45 respectively. An analysis of variance suggested that significant differences exist between conditions ( $F(2, 359) = 8.581, p < .001$ ). Post-hoc Tukey HSD intervals showed that the proportion of positive requests was significantly higher when 25% of the population were described as ‘rich’ than when 75% were. The proportion of positive requests was also significantly higher in the 50% rich condition than in 75% rich. Potential nuisance variables colour, trial number, and left/right order of the request buttons were not found to have a significant effect (did not improve model AIC).

The differences in means appear to be due to a qualitative shift between distinct request strategies, with participants’ responses tending to cluster at the special values of 1, 0.5, and 0 positive requests (see Figure 2).

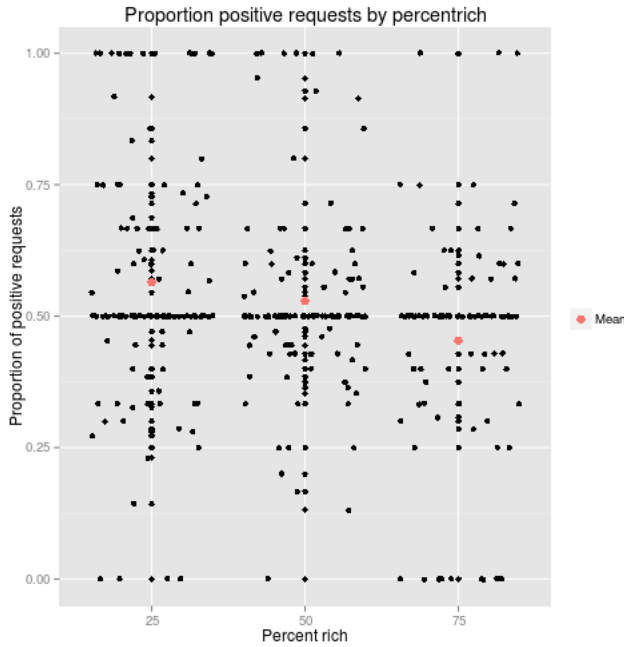


Figure 2: Proportion of positive requests appear to cluster at values 0, 0.5, and 1. The observed differences between means may be driven by participants choosing between a small number of distinct request strategies.

This clustering of request proportions motivates a categorization of responses by request strategy, ‘Prefers positive’, ‘Even’ and ‘Prefers negative’. An ‘Even’ request strategy was defined as a proportion of positive requests falling between 0.45 and 0.55, with ‘Prefers positive’ and ‘Prefers negative’ responses falling above and below these values (Figure 3).

All strategies were followed by some participants in all conditions, and the ‘Even’ request strategy balancing positive and negative requests was always the most popular, never lower than 44% of participants. The population percentage of

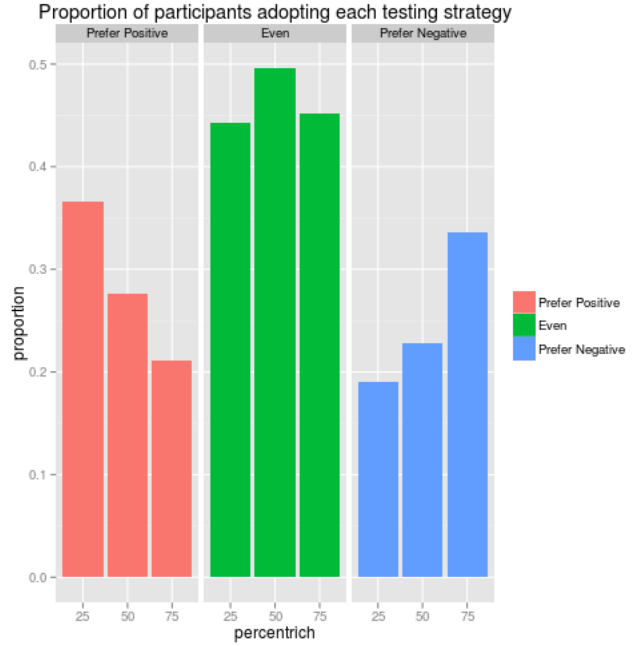


Figure 3: Proportion of participants using each testing strategy in the three conditions. Requesting equal amounts of information from both possible categorizations is always popular, however when population proportions are unequal, more people switch to preferring information about the minority group. This preference is somewhat asymmetrical, with people more readily switching to preferring positive requests

‘rich’ plankton did impact on the attractiveness of each testing strategy: the proportion of people preferring positive tests fell from 37% in the 25% rich condition to 21% in the 75% rich condition, while the proportion of people preferring negative tests rose from 19% to 33%, indicating a preference for whichever type of test corresponded to the minority classification in the whole population. When the population was evenly split, the ‘Even’ testing strategy was more popular than otherwise at 50% of all participants, and a preference for positive tests was more common than a preference for negative tests, at 27% and 23% of participants respectively.

## Discussion

The results show people adjusting their sampling strategies in response to the sparsity of the hypothesis testing task at hand, supporting an information sensitive account of natural hypothesis testing (Navarro & Perfors, 2011). This sensitivity to the relative size of the target category in the stimulus space obtains even though this is an abstract space defined over the similarity of a set of geometrical shapes. The effect appears more pronounced among participants who achieved high scores, but the predicted sparsity-based sampling differences are also apparent among participants with less-than-perfect scores, making it unlikely that the pattern observed is due to deductive problem-solving by a subset of participants.

Although consistent with a degree of sensitivity to the information value of requests, these data show a number of ways in which people’s requests deviate from information-

utility treatments of the task. Firstly, a positive testing bias is evident, with participants favouring the target 'rich' category asymmetrically despite the symmetry of the task under the sparsity manipulation. It's unclear if this is due to a form of matching [CITE!] or an expectation that the conditions that favour positive testing are generally ubiquitous, although in this artificial case they are not. The clustering of positive-test proportions at the special values 0, .5, and 1 in all conditions also suggests a kind of heuristic approach, albeit a heuristic that is to some extent context sensitive. It is unclear from these results if this clustering is reflective of granularity in the perception of request utility, an effort-saving heuristic in responding after accurate perception of request utility, or simply an artifact of the fact that participants were limited to two different request types, which to some extent naturally emphasises these values, especially for small numbers of requests.

- Lots of drop-out, high exclusion rate. Suspicious?
- Visual box-sizes for grouping plankton, undermines claim of abstract hypothesis space?
- How critical is being able to see all possible plankton examples? Knowledge of limits of variation critical for sparsity manipulation? (Rarely available in real life...?)

- How fine-grained are these judgements? Could a different design get smoother response variation with more smoothly varying information value, or (more) strategy-clusters in different places? (ie, battleships... so, yes?). Note everything here is discrete: features are discrete, selenoid status is binary, request type also binary, possibly this artificially encourages the clustering seen here? Testable: what if there are three categories, and three label-request buttons?

- Positivity bias attenuated in higher scoring group? (No relevant results reported in this draft). positivity bias possibly a fallback strategy, indicator of not understanding? Consistent with 'naturalistic Wason improves performance' (others...?)

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