CSCI 3104 Spring 2022
Instructors: Profs. Chen and Layer

Quiz 20 - DP: Write down recurrence

Due Date .... April 1
Name .... **Chengming Li**
Student ID **109251991**

Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 20 - DP: Write down Recurrence

Problem 1. Suppose we have an m-letter alphabet, $\Sigma = \{0, 1, \ldots, m-1\}$. Determine a recurrence for the number of strings ω of length n, such that no two consecutive characters in ω are the same. Clearly justify your recurrence.

Answer.

- if n = 0, it is an empty string, so 0 string.
- if n = 1, there is m combinations because it is impossible to have two characters in strings ω
- $f_2 = |W_2| = |\Sigma^2| = \{0, 1, \ldots, m-1\} \cdot \{1, \ldots, m-1\} = m \cdot (m-1) + (m-1)$. The "m-1" term is due to ω does not have 00 as a substring. For the first string,{0, 1, ..., m-1}, we have m combinations. And for each combination in the first string, the second string, $\{1, \ldots, m-1\}$, can give(m-1) combinations with that. That is why we have $m \cdot (m-1)$ term as the first term. The first term only eliminates the possibility of the first 0 occurrence. So we need to add another (m-1) term to eliminate the possibility of the second 0 occurrence.
- $f_3 = |W_3| = |\Sigma^3| = \{0, 1, \ldots, m-1\} \cdot \{1, \ldots, m-1\} \cdot \{1, \ldots, m-1\} = (m-1)f_2 + (m-1)f_1$. The "m-1" term is due to ω does not have 00 as a substring.
- Same thing apply to the general case. (m-1) is used to eliminate the possibility of 0 occurrence. And we multiply the result to f_{n-1} combinations, which we already known the solution of W that doesn't have substring 00. And same rule apply to the result to f_{n-2} combinations.

$$f_n = \begin{cases} 0 & : \text{n} = 0, \\ \text{m} & : \text{n} = 1, \\ f_1 \cdot (m-1) + (m-1) & : \text{n} = 2., \\ f_{n-1} \cdot (m-1) + f_{n-2} \cdot (m-1) & : n \geq 3., \end{cases}$$

□