Quiz 7 – Safe and useless edges

Due Date ....February 18
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.

- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.

- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).

- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.

- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 6- Safe and Useless Edges

2.1 Problem 1

Problem 1. Consider the following graph $G(V, E, w)$. Suppose we have the intermediate spanning forest $\mathcal{F}$ (indicated using thick edges) consisting of the edges $\{A, B\}$, $\{A, C\}$, and $\{D, E\}$. Clearly identify the safe, useless, and undecided edges. Justify your reasoning.
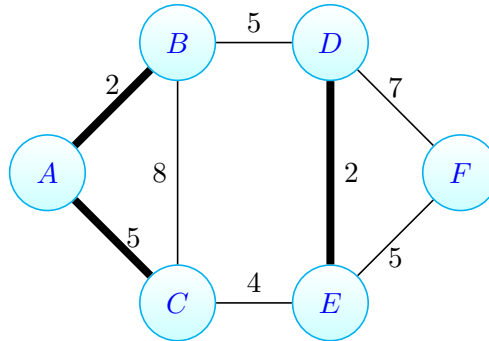
Answer. I use the **abridged cycle property**, the solution is the following:
An edge is useless when adding it will create a cycle on the given forest. Edge $\{B,C\}$ is **useless** edge because it will create a cycle with $\{A,B\}$ and $\{A,C\}$.

Cite: Adapted from HW3 solution
To determine the safe edges, we apply the following property as follows: Let Ti as tree in a weighted graph G(V, E, w). Fix a tree Ti, and let e ∈ E(G) be a light edge with exactly one endpoint in Ti. Then e is safe.

- We apply the property to the tree with edges, $\{\{A, B\}, \{A, C\}\}$. The minimum weight edge with exactly one endpoint in $\{\{A, B\}, \{B, C\}\}$ is the edge $\{C, E\}$. Therefore, $\{C, E\}$ is safe. We note that applying the property to the tree $\{D,E\}$ would also have yielded that $\{C, E\}$ is **safe**.

- We apply the property to the tree with only one vertex, $\{ F\}$. The minimum weight edge with exactly one endpoint in $\{F\}$ is the edge $\{E, F\}$. Therefore, $\{E, F\}$ is **safe**.

- The remaining edges are **undecided**: $\{B, D\}$, $\{D, F\}$.

In conclusion, **useless edges** is $\{B,C\}$, and **safe edges** are $\{C, E\}$, $\{E, F\}$ and **undecided edges** are $\{B, D\}$, $\{D, F\}$. $\qquad\square$