Quiz 9 - Prim's Algorithm

Due Date .... February 18
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Contents

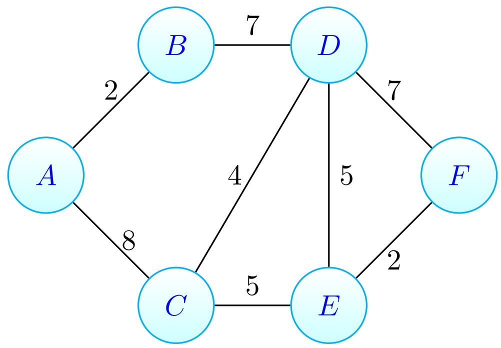
1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 9 - Prim's Algorithm

2.1 Problem 1

Problem 1. Consider the following graph $G(V, E, w)$. Clearly indicate the order in which Prim's algorithm adds the edges to the minimum-weight spanning tree **using F as the source vertex**. You may simply list the order of the edges; it is not necessary to exhibit the state of the algorithm at each iteration.



Answer. The algorithm first initialize an intermediate spanning tree $\mathcal{F}$ with all the vertex in weighted graph G, but no edges in it.
Then, algorithm initialize an priority queue Q (ascending order) to contains the edge incident to the source vertex F.
So: Q:[({E,F },2), ({D,F },7)]

- EF: Of the edges connected to F whose addition would not create a cycle, EF has the minimum weight and is added to the tree.
  So: Q:[({C,E },5), ({D,E },5),({D,F },7)]

- CE: Of the edges connected to F or E whose addition would not create a cycle, CE has the minimum weight and is added to the tree.
  So: Q:[({C,D },4), ({D,E },5),({D,F },7),({A,C },8)]

- CD: Of the edges connected to F or E or C whose addition would not create a cycle, CD has the minimum weight and is added to the tree.
  So: Q:[({D,E },5),({D,F },7),({B,D },7),({A,C },8)]

- DE: Of the edges connected to F or E or C or D whose addition would create a cycle, So DE is not added to the tree.
  So: Q:[({D,F },7),({B,D },7),({A,C },8)]

- DF: Of the edges connected to F or E or C or D whose addition would create a cycle, So DF is not added to the tree.
  So: Q:[({B,D },7),({A,C },8)]

- BD: Of the edges connected to F or E or C or D whose addition would not create a cycle, BD has the minimum weight and is added to the tree.
  So: Q:[({A,B },2),({A,C },8)]

- AB: Of the edges connected to F or E or C or D or B whose addition would not create a cycle, AB has the minimum weight and is added to the tree.
  So: Q:[({A,C },8)]

- AC: Of the edges connected to F or E or C or D or B or A whose addition would create a cycle, So AC is not added to the tree.
  So: Q:[]

- Algorithm terminates, because we have 6 vertex and $6 - 1 = 5$ edges in $\mathcal{F}$. Then algorithm will return $\mathcal{F}$ as result.
- So Prim's algorithm added the edges in order: {E, F}, {C, E}, {C, D}, {B, D}, {A, B}.

$\square$