CSCI 3104 Spring 2022
Instructor: Profs. Chen and Layer

Quiz 3 - Exchange Arguments

Due Date .... February 4
Name .... **Your Name**
Student ID .... **Your Student ID**

Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

Problem 1.

- My submission is in my own words and reflects my understanding of the material.
- I have not collaborated with any other person.
- I have not posted to external services including, but not limited to Chegg, Discord, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (signature here). □

3 Standard 3- Exchange Arguments

3.1 Problem 2

Problem 2. Suppose that there are n homework assignments, where the ith homework assignment has difficulty $d_i > 0$. All of the assignments are released on the first day of class, and you may turn in one assignment per week. If you turn in assignment i on week k, then you receive $(n-k)e^{d_i}$ points. Do the following.

(a) Consider a solution in which you turn in assignment j before assignment i, even though $d_i > d_j$. Show that you can increase the number of points earned by turning in assignment i before assignment j.

Proof. Let k be the week in which assignment j is turned in. By assumption, j is turned in before assignment i, so let $k+h$ be the week in which assignment i is turned in, where $h > 0$. In this scenario, these two assignments earn you

$$(n-k)e^{d_j} + (n-(k+h))e^{d_i}$$

points.

Consider the alternative scenario in which we swap the two weeks in which these two assignments are turned in; that is, assignment i is turned in in week k and assignment j is turned in in week $k+h$. In this scenario, these two assignments earn you

$$(n-k)e^{d_i} + (n-(k+h))e^{d_j}$$

points.

The difference between the points earned in the second scenario and those earned in the first is:

$$\begin{aligned}
[(n-k)e^{d_j} + (n-(k+h))e^{d_i}] - [(n-k)e^{d_i} + (n-(k+h))e^{d_j}] &= (e^{d_i} - e^{d_j})(n-k) + (e^{d_i} - e^{d_j})(n-(k+h)) \\
&= (e^{d_i} - e^{d_j})(n-k) - (e^{d_i} - e^{d_j})(n-(k+h)) \\
&= (e^{d_i} - e^{d_j})((n-k) - (n-(k+h))) \\
&= (e^{d_i} - e^{d_j})h
\end{aligned}$$

Now, since we have assumed $d_i > d_j$ and $h > 0$, this final expression is strictly positive. Thus, the second scenario (turning in assignment i first) yields more points than the first scenario. □

(b) Using part (a), describe a greedy algorithm to order the assignments in order to maximize the number of points earned. Pseudo-code is not required, but you should provide enough detail that a CSCI 2270 student could reasonably be expected to implement the solution from your description.

Answer. 1) Sort the assignments in decreasing order of difficulty; this ensures the algorithm will take the highest-difficult (and thus highest-points) assignment first. Call these $d_1 \geq d_2 \geq \cdots \geq d_n$.

2) For each week, pop off the top of the list.

3) Use the difficulty to calculate the points: $(n-k)e^{d_k}$

4) The solution will be the sum of all the calculated points.

□