Quiz 16 - Tree Method

Due Date .... March 11
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

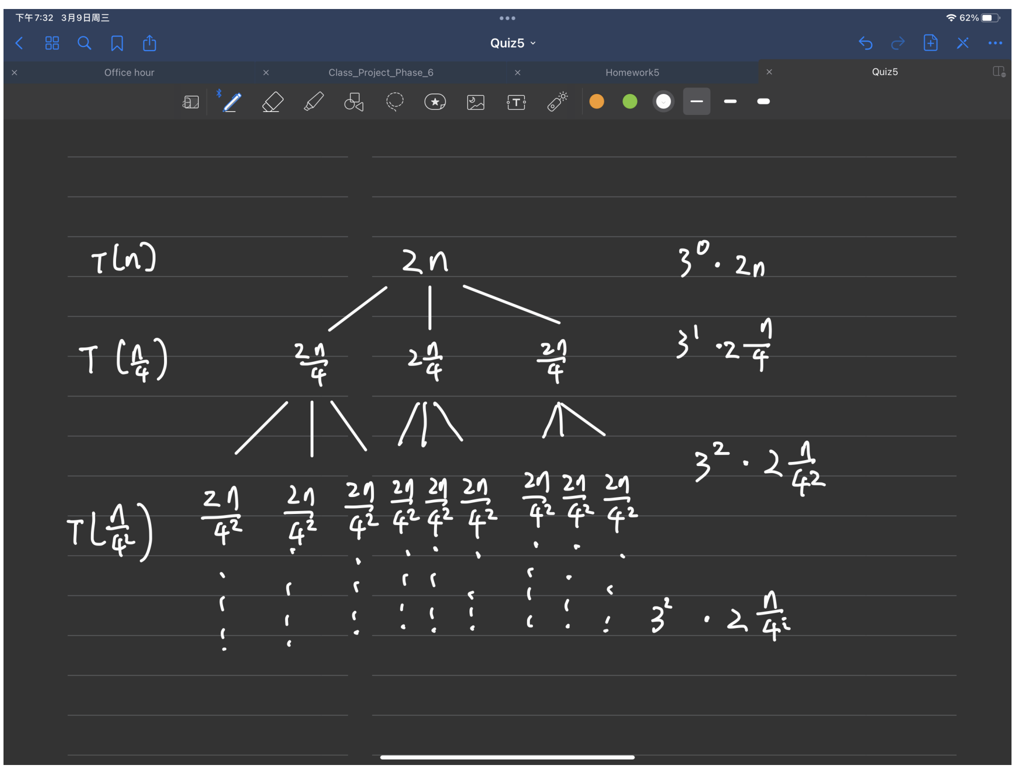
2 Standard 16 - Tree Method

Problem 1. Using the tree method, find a suitable function $f(n)$ such that $T(n) \in \Theta(f(n))$. Show all work. You may without loss of generality assume that n is a power of 5; that is, $n = 5^k$ for some integer $k \geq 0$. You may also find the following formula for sum of geometric sequence useful: $\sum_{i=0}^{k} a^i = \frac{1-a^{k+1}}{1-a}$.

$$T(n) = \begin{cases} 3 & : n < 4, \\ 3T(n/4) + 2n & : n \geq 4. \end{cases}$$

Answer. **Finding the depth of tree first**

$$\begin{aligned} \frac{n}{4^k} &< 4 \\ \frac{n}{4} &< 4^k \\ \ln n/4k \ln 4 \\ k &> log_4(\frac{n}{4}) \\ k &\approx log_4(\frac{n}{4}) \end{aligned}$$



Finding running time:

$$
\begin{aligned}
T(n) &= 2\sum\nolimits_{i=0}^{log_4 \frac{n}{4}} 3^i \cdot \left(\frac{n}{4^i}\right) + (3 \cdot 4^{log_4 \frac{n}{4}}) \\
&= 2n \sum\nolimits_{i=0}^{log_4 \frac{n}{4}} \left(\frac{3}{4}\right)^i + (3 \cdot 4^{log_4 \frac{n}{4}}) \\
&= 2n \sum_{i=0}^{log_4 \frac{n}{4}} \left(\frac{3}{4}\right)^i + (3 \cdot 4^{log_4 \frac{n}{4}}) \\
&= 2n \left(4\left(1 - \left(\frac{3}{4}\right)^{log_4 \frac{n}{4}+1}\right)\right) \; + \; 3 \cdot 4^{log_4 \frac{n}{4}} \\
&= 2n \left(4\left(1 - \frac{3}{4}\left(\frac{3}{4}\right)^{log_4 n - log_4 4}\right)\right) \; + \; 3 \cdot \; 4^{log_4 \frac{n}{4}} \\
&= 2n \left(4\left(1 - \left(\frac{3^{log_4 n}}{4^{log_4 n}}\right)\right)\right) \; + \; 3 \cdot \; \frac{n}{4} \\
&= 2n \left(4\left(1 - \left(\frac{3^{log_4 n}}{n}\right)\right)\right) \; + \; 3 \cdot \; \frac{n}{4}
\end{aligned}
$$

Higher order term dominates whole expression. So, $T(n) \in \Theta(2n\left(4\left(1 - \left(\frac{3^{log_4 n}}{n}\right)\right)\right) \; + \; 3 \cdot \; \frac{n}{4}) = \Theta(n)$ $\square$