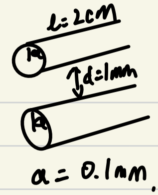
Q1

1) Two parallel wires of radius $a = 0.1\text{mm}$ and length 2cm separated by a distance $d = 1\text{mm}$ connect two elements on the motherboard of a computer. The wires are assumed to be made of perfect electric conductors and reside in a lossless medium with permittivity $\varepsilon = 2\varepsilon_0$ and permeability μ_0.

   a) Consider two signals: One of frequency 1 GHz and the other of frequency 20 GHz. Do we have to consider the two wires as a transmission line for the first signal? Do we have to consider the two wires as a transmission line for the second signal?

   b) In the case we have to consider the wires as a transmission line, what are the characteristic impedance and propagation constant of the line?



a) $l << CT$

$f = 1\,\text{GHz} \Rightarrow T = 10\,\text{ns}$
$f = 20\,\text{GHz} \Rightarrow T = 50\,\text{ps}$

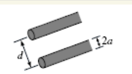
Signal 1 : $10\,\text{ns} \cdot 3 \cdot 10^8 \frac{m}{s} = 3\,m$
Signal 2 : $50\,\text{ps} \cdot 3 \cdot 10^8 \frac{m}{s} = 0.015\,m = 1.5\,cm$

For Signal 1 $l = 2cm << C \cdot T = 3m$, so we don't have to consider it as Transmission line

For Signal 2 $\overset{2cm}{l} > \overset{1.5cm}{C \cdot T}$ $\therefore$ we consider it as Transmission line

b) $\varepsilon = 2\varepsilon_0 = 8.85 \cdot 10^{-12} \cdot 2$

Since wire made of perfect electric conductors and reside in a lossless medium R' and G' is 0



(b) Two-wire line

$$R_s = \sqrt{\frac{\pi f \mu_c}{\sigma_c}} \quad (\Omega)$$

Two Wire

$$\frac{R_s}{\pi a} = R'$$

$$\frac{\mu}{\pi} \ln\left[(d/2a) + \sqrt{(d/2a)^2 - 1}\right] = L'$$

$$\frac{\pi \sigma}{\ln\left[(d/2a) + \sqrt{(d/2a)^2 - 1}\right]} = G'$$

$$\frac{\pi \varepsilon}{\ln\left[(d/2a) + \sqrt{(d/2a)^2 - 1}\right]} = C'$$

$$L' = \frac{\mu_0}{\pi} \ln\left[\left(\frac{1}{0.2}\right) + \sqrt{\left(\frac{1}{0.2}\right)^2 - 1}\right]$$

$$C' = \frac{\pi \varepsilon}{\ln\left[\left(\frac{1}{0.2}\right) + \sqrt{\left(\frac{1}{0.2}\right)^2 - 1}\right]}$$

$d = 1mm$

$a = 0.1mm$

Characteristic impedance $Z_0 = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$

R' and G' is 0

$$x = \ln\left[\frac{d}{2a} + \sqrt{\left(\frac{d}{2a}\right)^2 - 1}\right]$$

$$Z_0 = \sqrt{\frac{j\omega L'}{j\omega C'}} = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{u x}{\pi} \cdot \frac{x}{\pi \cdot 2\varepsilon_0}}$$

$$= \sqrt{\frac{u_0}{2\varepsilon_0} \cdot x^2 \cdot \frac{1}{\pi^2}}$$

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = 2$$

$$u_0 = 4\pi \cdot 10^{-7}$$

$$= \underbrace{\sqrt{\frac{u_0}{2\varepsilon_0}} \cdot \frac{1}{\pi}}_{84.814} \cdot \underbrace{\ln\left(\frac{d}{2a} + \sqrt{\left(\frac{d}{2a}\right)^2 - 1}\right)}_{2.292}$$

$$= \boxed{194.43\ \Omega}$$

Propagation Constant   Characteristic Impedance.

Lossless two wire	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120/\sqrt{\varepsilon_r}) \cdot \ln[(d/2a) + \sqrt{(d/2a)^2 - 1}]$ $Z_0 \simeq (120/\sqrt{\varepsilon_r}) \ln(d/a)$, if $d \gg a$

$$\gamma = 0 + j\,\frac{\omega\sqrt{\varepsilon_r}}{c} \qquad Z_0 \simeq \frac{120}{\sqrt{\varepsilon_r}}$$

$$\omega = 2\pi \cdot 20G = 1.26 \cdot 10^{11}\ Hz$$

$$\beta = \frac{\omega \cdot \sqrt{2}}{3 \cdot 10^8} = 592.38$$

$$\boxed{\gamma = 0 + j\,592.38}$$

Q2

2)

2.10 Using a slotted line, the voltage on a lossless transmission line was found to have a maximum magnitude of 1.5 V and a minimum magnitude of 0.6 V. Find the magnitude of the load's reflection coefficient.

Reflection coefficient is the ratio between

$$\boxed{\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} \quad \text{(dimensionless)}.}$$

Voltage standing wave ratio:

$$S = \frac{|\tilde{V}|_{max}}{|\tilde{V}|_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad \text{(dimensionless)}$$

$$\frac{1.5}{0.6} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$\Rightarrow \frac{5}{2} - \frac{5}{2}|\Gamma| = 1 + |\Gamma|$$

$$\frac{3}{2} = \frac{7}{2}|\Gamma|$$

$$\boxed{|\Gamma| = \frac{3}{7}}$$

3)

2.12 A 50-Ω lossless transmission line is terminated in a load with impedance $Z_L = (30 - j50)$ Ω. The wavelength is 8 cm. Find the following:

(a) The reflection coefficient at the load.

(b) The standing-wave ratio on the line.

(c) The position of the voltage maximum nearest the load.

(d) The position of the current maximum nearest the load.

$Z_0 = 50\ \Omega$

$Z_L = 30 - j50$

a)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 - j50 - 50}{80 - j50} = 0.1011 - j0.5618$$

$$= \boxed{0.5708 \angle -79.796^\circ}$$

b)

Voltage standing wave ratio:

$$S = \frac{|\tilde{V}|_{max}}{|\tilde{V}|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|} \quad \text{(dimensionless)}$$

$$S = \frac{1 + 0.5708}{1 - 0.5708} \approx \boxed{3.66}$$

c)

First maximum

$\theta_r \geq 0, \quad \Rightarrow \quad l_{max} = \theta_r\lambda/4\pi \quad (\text{for } n = 0)$

$\theta_r < 0, \quad \Rightarrow \quad l_{max} = (\theta_r\lambda/4\pi) + \lambda/2, \quad (\text{for } n = 1)$

$$l_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{\lambda}{2}$$

$$= \frac{\frac{-79.796\pi}{180} \cdot 0.08}{4\pi} + \frac{0.08}{2}$$

$$= -8.8662 \cdot 10^{-3} + 0.04$$

$$\boxed{l_{max} \cong 0.031\ m = 3.1\ cm}$$

d)

First minimum

$$l_{min} = \begin{cases} l_{max} + \lambda/4, & \text{if } l_{max} < \lambda/4, \\ l_{max} - \lambda/4, & \text{if } l_{max} \geq \lambda/4. \end{cases}$$

$$\frac{\lambda}{4} = 0.02\ m \leq l_{max}$$

$$l_{min} = l_{max} - \frac{\lambda}{4}$$

$$= 0.011\ m$$

$$= \boxed{1.1\ cm}$$

Q4

4)

2.13* On a 150-Ω lossless transmission line, the following observations were noted: distance of first voltage minimum from the load = 3 cm; distance of first voltage maximum from the load = 9 cm; $S = 3$. Find Z_L.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_0 = 150\ \Omega$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 3$$

$$3 - 3|\Gamma| = 1 + |\Gamma|$$
$$2 = 4|\Gamma|$$
$$|\Gamma| = 0.5$$

$$l_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{\lambda}{2}$$

$$l_{min} = l_{max} - \frac{\lambda}{4}$$

$$l_{min} = l_{max} - \frac{\lambda}{4}$$
$$3 = 9 - \frac{\lambda}{4}$$
$$\frac{\lambda}{4} = 6$$
$$\lambda = 24\ \text{cm}$$

$$\frac{l_{max} \cdot 4\pi}{\lambda} = \theta_r$$

$$\frac{0.09 \cdot 4\pi}{0.24} = \theta_r$$

$$\theta_r = \frac{3}{2}\pi$$

$$\Gamma = 0.5 \angle \frac{3}{2}\pi$$
$$= 0 - j0.5$$

$$\Gamma = \frac{Z_L - 150}{Z_L + 150}$$

$$-0.5i\,x - 75j = x - 150$$
$$x(-1 - 0.5j) = -150 + 75j$$
$$x = \frac{-150 + 75j}{-1 - 0.5j}$$
$$= \frac{112.5 - 150j}{1.25}$$

$$(-150 + 75j)(-1 + 0.5j)$$
$$150 - 75j - 75j - 37.5$$
$$112.5 - 150j$$

$$\boxed{Z_L = 90 - 120i}$$

5) (Quiz 1, Winter 2008) You are given a coaxial transmission line made of perfect electrically conducting cylinders of radii $r_1 = 1\,\text{cm}$ and $r_2 = 1.5\,\text{cm}$. The space between the cylinders is filled with a material having the permittivity $\varepsilon = 4\varepsilon_0$, permeability μ_0, and conductivity $\sigma = 0$. The transmission line is terminated by a load Z_L at $z = 0$. The transmission line is excited by a source of frequency $f = 1.8\,\text{GHz}$.

a) Find parameters characterizing the transmission line, i.e. $R', L', G', C', \beta, Z_0, \lambda, u_p$.

b) The magnitude and phase of the voltage measured at $z_1 = 0$ and $z_2 = -3\,\text{cm}$ are $|\tilde{V}(0)| = 3V$, $\arg\{\tilde{V}(0)\} = 0$ and $|\tilde{V}(z_2)| = 2V$, $\arg\{\tilde{V}(z_2)\} = \pi/3$, respectively.

- Give expressions describing the voltage and current in the transmission line in the form of phasors and in the time domain representation.
- Find the reflection coefficient from the load Γ.

late line

Parameter	Coaxial
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$
L'	$\frac{\mu}{2\pi}\ln(b/a)$
G'	$\frac{2\pi\sigma}{\ln(b/a)}$
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$

a) The transmission line made of perfect electrically conducting

So $\boxed{R' = 0}$

$$L' = \frac{u_0}{2\pi} \ln\frac{r_2}{r_1}$$

$u_0 = 4\pi \cdot 10^{-7}\ NA^{-2}$

$$= \boxed{8.109 \cdot 10^{-8}\ \frac{H}{m}}$$

$$\boxed{G' = \frac{2\pi 6}{\ln(\frac{b}{a})} = 0}$$

$6 = 0$

$$C' = \frac{2\pi\varepsilon}{\ln(\frac{b}{a})}$$

$\varepsilon_0 = 8.85 \cdot 10^{-12}\ Fm^{-1}$

$\varepsilon = 3.54 \cdot 10^{-11}\ Fm^{-1}$

$$= \boxed{5.486 \cdot 10^{-10}\ \frac{F}{m}}$$

$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless ($R' = G' = 0$)	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \sqrt{L'/C'}$

$\omega = 2\pi f = 1.13 \cdot 10^{10}\,Hz$

$$\beta = \omega\frac{\sqrt{\varepsilon_r}}{c} = \omega\sqrt{L'C'} = \omega\sqrt{u\varepsilon}$$

$$= \boxed{75.433}$$

$$Z_0 = \sqrt{\frac{L'}{C'}} = \boxed{12.16\ \Omega}$$

$$\lambda = \frac{c}{f} = \boxed{\frac{1}{6}\ m}$$

$$u_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{L'C'}} = \boxed{1.5 \cdot 10^8\ \frac{m}{s}}$$

3. The voltage and current in phasor forms: $\tilde{V}(z)=\tilde{V}^{+}e^{-j\beta z}+\tilde{V}^{-}e^{j\beta z}$ and $\tilde{I}(z)=(\tilde{V}(z)=\tilde{V}^{+}e^{-j\beta z}-\tilde{V}^{-}e^{j\beta z})/Z_0$. In the time domain, $V(z,t)=|V^{+}|\cos(\omega t-\beta z+\phi^{+})+|V^{-}|\cos(\omega t+\beta z+\phi^{-})$ and $I(z,t)=\left(|V^{+}|\cos(\omega t-\beta z+\phi^{+})-|V^{-}|\cos(\omega t+\beta z+\phi^{-})\right)/Z_0$

b) The magnitude and phase of the voltage measured at $z_1=0$ and $z_2=-3\text{cm}$ are $|\tilde{V}(0)|=3V$, $\arg\{\tilde{V}(0)\}=0$ and $|\tilde{V}(z_2)|=2V$, $\arg\{\tilde{V}(z_2)\}=\pi/3$, respectively.

- Give expressions describing the voltage and current in the transmission line in the form of phasors and in the time domain representation.
- Find the reflection coefficient from the load Γ.

- Find the load impedance Z_L.

b)

at $z_1=0$

$$\tilde{V}(0)=V^{+}e^{0}+V^{-}e^{0}$$

$$3 = V^{+}+V^{-} \Rightarrow V^{+}=3-V^{-}$$

at $z_2=-3$

$$\tilde{V}(-3)=V^{+}e^{-j(-3)}+V^{-}e^{j(-3)}$$

$$\tilde{V}(z_2)=(3-V^{-})e^{j3}+V^{-}e^{-j3}$$

$$\tilde{V}_2=3e^{j3}-V^{-}(e^{j3}-e^{-j3})$$

$$-\frac{\tilde{V}_2-3e^{j3}}{e^{j3}-e^{-j3}}=V^{-}=\frac{2e^{j\frac{\pi}{3}}-3e^{j3}}{e^{j3}-e^{-j3}}$$

Phasor

$$V^{-}=-4.6368+j14.066$$

$$V^{+}=3-V^{-}$$

$$=3-(-4.6368+j14.066)=7.6368-j14.066$$

$$\tilde{V}=\boxed{(7.6368-j14.066)e^{-j\beta z}+(-4.6368+j14.066)e^{j\beta z}}$$

$$\tilde{I}=\frac{V^{+}}{Z_0}e^{-j\beta z}+\frac{-V^{-}}{Z_0}e^{j\beta z}$$

$$=\boxed{(0.628-j1.1567)e^{-j\beta z}-(-0.3813+j1.1567)e^{j\beta z}}$$

Time Domain

$$\boxed{\begin{aligned} v(z,t)&=16\cos(\omega t-\beta z+(-1.0734))+14.81\cos(\omega t-\beta z+1.8892)\\ I(z,t)&=1.3162\cos(\omega t-\beta z-1.0734)+1.218\cos(\omega t-\beta z+1.8892)\end{aligned}}$$

$$\Gamma = \frac{\tilde{V}_0^-}{\tilde{V}_0^+} = \frac{-4.6368+j14.066}{7.6368-j14.066} = \boxed{-0.9106+j0.1647}$$

$$Z_L = \frac{\tilde{V}(0)}{\tilde{I}(0)} = \frac{3}{0.628-j1.1567+0.3813-j1.1567}$$

$$= \boxed{0.4753+j1.0894}$$

c) Assume that the line is terminated by a load that is comprised of a resistor R_L, capacitor C_L, and inductor L_L connected in series. The values of R_L, C_L, and L_L are finite. Find a combinations of R_L, C_L, and L_L that leads to a matched load for the frequency $f = 1.8\,\text{GHz}$ and to the VSWR=2 for the frequency $f = 1.85\,\text{GHz}$ (please, give expressions/explanations before proceeding with calculating the numerical values of the result).

c)

$$Z_L = Z_0 = 12.16\,\Omega$$

$$Z_L = R_L + j\omega L_L + \frac{1}{j\omega C_L} \qquad \omega = 2\pi \cdot 1.8\,G$$

$f = 1.8\,\text{GHz}$

Matched Load:

$$Z_L = R_L = Z_0 \qquad \text{and} \qquad j\omega L_L + \frac{1}{j\omega C_L} = 0$$

$$\boxed{R_L = 12.16\,\Omega}$$

$$(j\omega)^2 \cdot L_L C_L + 1 = 0$$

$$-\omega^2 L_L C_L = -1$$

$$\omega^2 L_L C_L = 1$$

Perfect matching $\leftarrow$ $L_L C_L = \frac{1}{\omega^2}$

$$L_L C_L = 7.818 \cdot 10^{-21}$$

$f = 1.85\,\text{GHz}$

VSWR=2

$$S = \frac{|\tilde{V}|_{max}}{|\tilde{V}|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|} = 2$$

$$2 - 2|\Gamma| = 1 + |\Gamma|$$

$$1 = 3|\Gamma|$$

$$|\Gamma| = \frac{1}{3}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{R_L + j\omega L_L + \frac{1}{j\omega C_L} - Z_0}{R_L + j\omega L_L + \frac{1}{j\omega C_L} + Z_0}$$

$$= \frac{j\left(\omega_2 L_L - \frac{1}{\omega_2 C_L}\right)}{2Z_0 + j\left(\omega_2 L_L - \frac{1}{\omega_2 C_L}\right)}$$

Given from the Discussion Session

$$|\Gamma| = \frac{\left|\omega_2 L_L - \frac{1}{\omega_2 C_L}\right|}{\sqrt{(2Z_0)^2 + \left(\omega_2 L_L - \frac{1}{\omega_2 C_L}\right)^2}} = \frac{1}{3}$$

C_L and L_L

Solved by the Matlab

$$\boxed{\begin{aligned} L_L &= 13.87\,nH \\ C_L &= 0.56\,pF \end{aligned}}$$