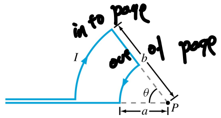
Q1

1) The loop of current I as shown in Fig. 5-36 consists of radial lines and segments of circles whose centers are at point P. Determine the magnetic field $\mathbf{H}$ at P in terms of a,b,θ,I



$\hat{R} = \frac{R}{R}$

$$d\mathbf{H} = \frac{I}{4\pi}\frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2}$$

$dl \leftarrow \quad \hat{R} \rightarrow$

For radial lines, $dl \times \hat{R} = 0$ due to same direction

For segments of circles

With radius a, the I is out of page by right hand rule

$dl \times \hat{R} = a \cdot d\theta \times \hat{R}$ Define $+\hat{z}$ as out of page

$$H = \frac{I}{4\pi}\int_0^{\theta}\frac{a \cdot d\theta}{a^2} = +\frac{I}{4\pi a}\cdot\theta \ \hat{z}$$

With radius b

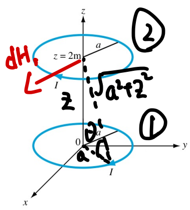
$$H = -\frac{I}{4\pi b}\hat{z}$$

$$H_{total} = \hat{z}\frac{I}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$$

Q2

2) Two parallel, circular loops carrying a current of 40 A each are arranged as shown in the figure below. The first loop is situated in the x-y plane with its center at the origin, and the second loop's center is at z=2m. If the two loops have the same radius a=3m, determine the magnetic field at

a. z=0
b. z=1m
c. z=2m



$I = 40\ A$

H has only $\hat{z}$ component

$$dH = \frac{I}{4\pi R^2} dl \times \hat{R} \quad ; \quad dH = \hat{z}\, dH_z = \hat{z}\, dH \cdot \cos\theta$$

so for Loop 1 $\qquad R^2 = a^2 + z^2$

$$dH_1 = -\frac{I\cos\theta}{4\pi R^2} dl\ \hat{z}$$

$$H_1 = -\frac{I\cos\theta}{4\pi(a^2+z^2)} \cdot 2\pi a \qquad \cos\theta = \frac{a}{\sqrt{a^2+z^2}}$$

$$= -\frac{I\, a^2 \cdot 2\pi}{4\pi(a^2+z^2)^{3/2}}\hat{z} = -\frac{I\, a^2}{2(a^2+z^2)^{\frac{3}{2}}}\hat{z}$$

For Loop 2

$$dH_2 = -\frac{I\cos\theta}{4\pi R^2}\hat{z} \qquad R^2 = a^2 + (z-2)^2$$

$$H_2 = -\frac{I\, a^2 \cdot 2\pi}{4\pi(a^2+(z-2)^2)^{\frac{3}{2}}}\hat{z} \qquad \cos\theta = \frac{a}{\sqrt{a^2+(z-2)^2}}$$

$$= \frac{-I\, a^2}{2\,(a^2+(z-2)^2)^{\frac{3}{2}}}\hat{z}$$

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a) $z = 0$

$I = 40A$
$a = 3$

$$H\,\text{total} = -\frac{I a^2}{2}\left[\frac{1}{(a^2+z^2)^{3/2}} + \frac{1}{(a^2+(z-2)^2)^{3/2}}\right]\hat{z} = -10.507\ \frac{A}{m}\ \hat{z}$$

$z = 1$

$$H\text{total} = -11.384\ \frac{A}{m}\ \hat{z}$$

$z = 2$

$$H\text{total} = -10.507\ \frac{A}{m}\ \hat{z}$$

Q3

3)

Current I flows along the positive z-direction in the inner conductor of a long coaxial cable and returns through the outer conductor. The inner conductor has radius a, and the inner and outer radii of the outer conductor are b and c, respectively.

(a) Determine the magnetic field in each of the following regions: $0 \le r \le a$, $a \le r \le b$, $b \le r \le c$, and $r \ge c$.

(b) Plot the magnitude of $\mathbf{H}$ as a function of r over the range from $r = 0$ to $r = 10$ cm, given that $I = 10$ A, $a = 2$ cm, $b = 4$ cm, and $c = 5$ cm.

by right hand rule $\hat{H}$ is in to the page

$$d\mathbf{H} = \frac{I}{4\pi} \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2}$$

$$dl \times \hat{R} = \hat{z} \times \hat{R} = \hat{\theta}$$

$$dl = r \sin\theta \, d\theta$$

$$\hat{H} = \int dH = \frac{Ir}{4\pi R^2} \int_0^{2\pi} \sin d\theta \, \hat{\theta} = \frac{Ir}{4\pi R^2} (-\cos\theta)\Big|_0^{2\pi} \hat{\theta} = -\frac{Ir}{2\pi R^2} \hat{\theta}$$

For $0 \le r \le a$

$$\boxed{\hat{H} = -\frac{Ir}{2\pi a^2} \hat{\theta}}$$

For $a \le r \le b$

$$\hat{H} = -\frac{Ir}{2\pi r^2} \hat{\theta} = \boxed{\frac{-I}{2\pi r} \hat{\theta}}$$

For $b \le r \le c$

I shared by outer conductor is $\pi(c^2 - b^2)$ ← return through

since $b \le r \le c$, the fraction is $\pi(r^2 - b^2)$

$$H = \frac{I}{2\pi r} \hat{\theta}$$

The remaining I is $= I - I \cdot \frac{r^2 - b^2}{c^2 - b^2}$

$$\hat{H} = \boxed{\frac{I}{2\pi r}\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right) \hat{\theta}}$$

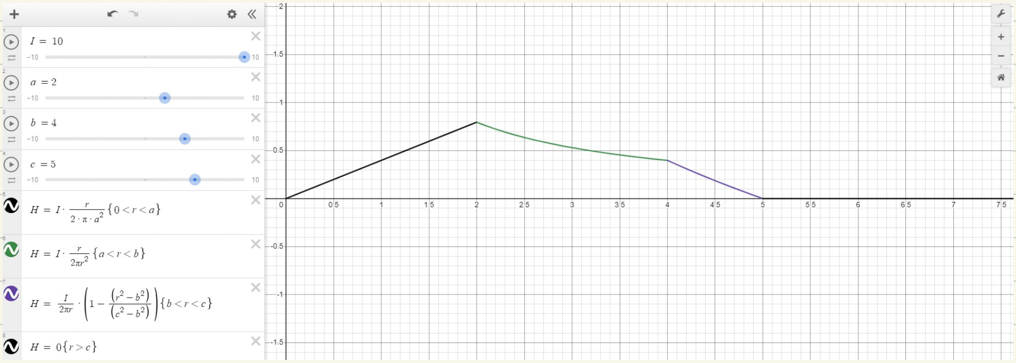
For $r \ge c$, H is in the opposite direction

Inside inner conductor $\hat{H}$ is into the page

Outter conductor $\hat{H}$ is out of the page

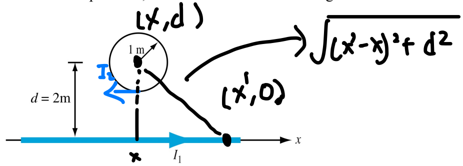
so $\boxed{\hat{H} = 0}$

b)


I = 10
a = 2
b = 4
c = 5
H = I · r / (2 · π · a^2) {0 < r < a}
H = I · r / (2πr^2) {a < r < b}
H = I / (2πr) · (1 − (r^2 − b^2) / (c^2 − b^2)) {b < r < c}
H = 0{r > c}

Q4

4) An infinitely long wire carrying a 25-A current in a the positive x-direction is places along the x-axis in the vicinity of a 20-turn circular loop located in the x-y plane. If the magnetic field at the center of the loop is zero, what is the direction and magnitude of the current flowing in the loop?



$$d\mathbf{H} = \frac{I}{4\pi}\frac{d\mathbf{l}\times\hat{\mathbf{R}}}{R^2}$$

$$\mathbf{H} = -\hat{\boldsymbol{\phi}}H = -\hat{\boldsymbol{\phi}}\frac{NI}{2\pi r} \quad \text{(for } a<r<b\text{).}$$

$dl = +\hat{x}$ $\quad \hat{R} = +\hat{y}$ $\quad \hat{H}$ = out of page $+\hat{z}$

And the net magotic field at the center of loop is 0. then the direction of loop is in the $\boxed{\text{opposite direction , clock wise}}$

Due to wire H

$$\int dH = \frac{I}{4\pi}\cdot\int_{-\infty}^{\infty}\hat{z}\,\frac{1}{(x'-x)^2+d^2}\,dx'$$

$$H_w = \frac{I}{4\pi}\cdot\frac{\pi}{d}\hat{z} = \frac{I_w}{4\pi d}\hat{z}$$

Due to the loop

$$H_L = -\hat{z}\frac{NI}{2\pi r} = \frac{20 I_L}{2\pi\cdot 1} - \hat{z}$$

$$H_w = H_L$$

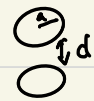
$$\frac{25}{4\pi\cdot 2} = \frac{20 I_L}{2\pi} \Rightarrow I_L = \boxed{0.3125A}$$

Q5

5) Consider two parallel disks of radius a separated by a small distance d. The space between the disks is filled with a conducting material of conductivity sigma and permittivity epsilon. The potential difference between the disks is V0.

a) Find the electric field in the space between the disks.
b) Find the corresponding volumetric current density.
c) Find the Magnetic field in the space between the disks.

$\sigma \quad \varepsilon$



a) $V = E \cdot l \Rightarrow E = \dfrac{V_0}{d}$

b) $\mathbf{J} = \sigma \mathbf{E}$ $\qquad J = \sigma \cdot \dfrac{V_0}{d}$

c) $\oint_C \mathbf{H} \cdot d\mathbf{l} = I$

For $r < a$

$$H \cdot 2\pi r = J \cdot A = \frac{\sigma V_0}{d} \cdot \pi r^2$$

$$H = \frac{\sigma V_0 r}{2d}$$

For $r > a$

$$H \cdot 2\pi r = J \cdot A = \frac{\sigma V_0}{d} \pi a^2$$

$$H = \frac{\sigma V_0 a^2}{2dr}$$