Q1

2.23* Two half-wave dipole antennas, each with an impedance of 75 Ω, are connected in parallel through a pair of transmission lines, and the combination is connected to a feed transmission line, as shown in Fig. 2-39. All lines are 50 Ω and lossless.

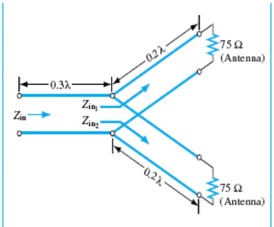


Figure 2-39: Circuit for Problem 2.23.

(a) Calculate Z_{in_1}, the input impedance of the antenna-terminated line, at the parallel juncture.

(b) Combine Z_{in_1} and Z_{in_2} in parallel to obtain Z'_L, the effective load impedance of the feedline.

(c) Calculate Z_{in} of the feedline.

$$Z_{in}(-l) = Z_0\left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right)$$

$\beta = \frac{2\pi}{\lambda}$

a)

$$Z_{in_1} = 50 \cdot \left(\frac{75 + j50\tan(\frac{2\pi}{\lambda}\cdot 0.2\lambda)}{50 + j75\tan(\frac{2\pi}{\lambda}\cdot 0.2\lambda)}\right)$$

$$= 50 \cdot \left(\frac{75 + j153.88}{50 + j230.83}\right)$$

$$= 50\left(\frac{(75 + j153.88)(50 - j230.83)}{50^2 + 230.83^2}\right)$$

$$= \boxed{35.1993 - j8.6212}$$

b)

$$Z'_L = \frac{Z_{in_1} \cdot Z_{in_2}}{Z_{in_1} + Z_{in_2}} \quad ; \quad Z_1 = Z_2$$

$$= \frac{Z_{in}^2}{2Z_{in_1}} = \boxed{17.5997 - j4.3106}$$

c)

$$Z_{in} = 50\left(\frac{(17.6 - j4.3) + j50\tan(\frac{2\pi}{\lambda}\cdot 0.3\lambda)}{50 + j(17.6 - j4.3)\cdot\tan(\frac{2\pi}{\lambda}\cdot 0.3\lambda)}\right)$$

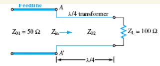
Solved by Matlab

$$= \boxed{107.57 - j56.706}$$

Q2

2.28 A 100-MHz FM broadcast station uses a 300-Ω transmission line between the transmitter and a tower-mounted half-wave dipole antenna. The antenna impedance is 73 Ω. You are asked to design a quarter-wave transformer to match the antenna to the line.

- $Z_{in} = \dfrac{Z_{02}^2}{Z_L}$
- Choose $Z_{02} = \sqrt{Z_{01} Z_L}$
- $\Rightarrow Z_{in} = Z_{01}$
- $\Rightarrow \Gamma = 0$ i.e. the quarter wave transformer can be used to eliminate reflections entirely!



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(a) Determine the electrical length and characteristic impedance of the quarter-wave section.

(b) If the quarter-wave section is a two-wire line with $d = 2.5$ cm, and the spacing between the wires is made of polystyrene with $\varepsilon_r = 2.6$, determine the physical length of the quarter-wave section and the radius of the two wire conductors.

a)

$$Z_{02} = \sqrt{Z_{01} Z_L} = \sqrt{300 \cdot 73} \approx \boxed{148}$$

$10\sqrt{219}$

$$l = \frac{\lambda}{4} = \frac{c}{f \cdot 4} = \frac{3 \cdot 10^8}{400 \cdot 10^6} = \boxed{0.75\,m}$$

b)

$$\lambda = \frac{u_p}{f} = \frac{c}{f} \frac{1}{\sqrt{\varepsilon_r}}$$

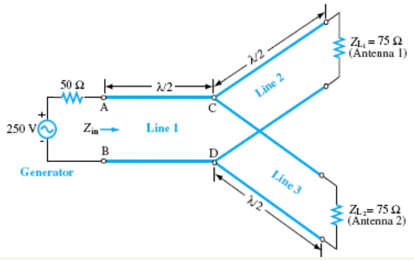
$$\lambda = 0.75 \cdot \frac{1}{\sqrt{2.6}} = \boxed{0.465\,m}$$

$$Z_0 = (120/\sqrt{\varepsilon_r}) \cdot \ln[(d/2a) + \sqrt{(d/2a)^2 - 1}]$$

$$148 = \left(\frac{120}{\sqrt{2.6}} \cdot \ln\left[\left(\frac{d}{2a}\right) + \sqrt{\left(\frac{d}{2a}\right)^2 - 1}\right]\right.$$

$$\boxed{a = 0.3359\text{ cm}}$$

2.32 If the two-antenna configuration shown in Fig. 2-41 is connected to a generator with $\tilde{V}_g = 250$ V and $Z_g = 50\ \Omega$, how much average power is delivered to each antenna?



$$Z_{in}(-l) = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right)$$

$$\beta = \frac{2\pi}{\lambda} \qquad l = \frac{\lambda}{2}$$

$$\tan \beta l = \tan \pi = 0$$

$$Z_{in,\,lin_2} = Z_0 \cdot \frac{Z_L}{Z_0} = Z_L = Z_{in,\,lin_3} = 75\ \Omega$$

Parallel: $$Z_{L,\,CD} = \frac{Z_{in \cdot lin_2}^{\,2}}{2 \cdot Z_{in \cdot lin2}} = \frac{1}{2} Z_{in\,lin_2} = 37.5\ \Omega$$

Since Between A C, $l = \frac{\lambda}{2}$

$$Z_{in} = Z_L = 37.5\ \Omega$$

$$Z_{tot} = 50 + 37.5 = 87.5\ \Omega$$

$$V_i = V_g \cdot \frac{Z_{in}}{Z_g + Z_{in}} = 107.14\,V$$

$$I_i = \frac{V_i}{Z_{in}} = \frac{107.14}{37.5} = 2.857\,A$$

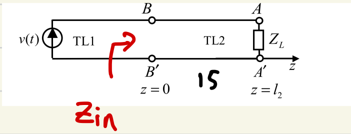
$$P_{avg} = \frac{1}{2} I V = 153.06\ W$$

$$P_{antenna} = \frac{P_{avg}}{2} = \boxed{76.53\ W}$$

lossless

4) (Undergraduate option of the ECE Comprehensive graduate exam, 2008)

Consider a system of two parallel plate transmission lines TL1 and TL2 as shown in Fig. 1. The transmission lines are made of perfect electrically conducting plates and the space between the plates is filled with vacuum. The width, separation between the plates, and length of TL1 are $w_1 = 1\text{cm}$, $d_1 = 0.125\text{cm}$, and $l_1 = 20\text{cm}$, respectively. The width, separation between the plates, and length of TL2 are different for different items below. The left end of TL1 is connected to a voltage source $v(t) = V_0 \cos(2\pi f t)$, where f is the linear frequency. The right end of TL2 is connected to a load resistive impedance $Z_L = 100\Omega$. The origin of the coordinate system $z = 0$ is at port BB'.



a) Consider the case where $f = 0$. Give the voltages at ports AA' and BB'.

a) $f=0 \qquad v = V_0 \cos(0) = V_0$

So $V_{BB'} = V_{AA'} = V_0$

---

b) Now, the frequency is non-vanishing and is given by $f = 1\text{GHz}$. The width, separation between the plates, and length of TL2 are $w_2 = 1\text{cm}$, $d_2 = 0.25\text{cm}$, and $l_2 = 15\text{cm}$, respectively. Give the characteristic impedances Z_1 and Z_2 of TL1 and TL2 in terms of the transmission line geometrical parameters. Find the input impedance Z_{in} that is seen from TL1 at the port BB'. (If you cannot find Z_1 and Z_2 in terms of the geometrical parameters, give Z_{in} and all the results in the following items in terms of Z_1 and Z_2 without calculating the numbers).

llel-plate line

Parameter	Coaxial	Two Wire	Parallel Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{R_s}{\pi a}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right]$	$\frac{\mu d}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right]}$	$\frac{\sigma w}{d}$	S/m
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{\pi\varepsilon}{\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right]}$	$\frac{\varepsilon w}{d}$	F/m

Notes: (1) Refer to Fig. 2-4 for definitions of dimensions. (2) μ, ε, and σ pertain to the insulating material between the conductors. (3) $R_s = \sqrt{\pi f \mu_c/\sigma_c}$. (4) μ_c and σ_c pertain to the conductors. (5) If $(d/2a)^2 \gg 1$, then $\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right] \simeq \ln(d/a)$.

b)

$$Z = \sqrt{\frac{L'}{C'}}$$

$\mu = \mu_0 = 4\pi \cdot 10^{-7}$

$\varepsilon = 8.85 \cdot 10^{-12}$

$$L_1' = \frac{\mu d}{w} = \frac{4\pi \cdot 10^{-7} \cdot 0.125}{1} = 1.57 \cdot 10^{-7} \ \frac{H}{m}$$

$$C_1' = \frac{\varepsilon w}{d} = \frac{8.85 \cdot 10^{-12} \cdot 1}{0.125} = 7.08 \cdot 10^{-11} \ \frac{F}{m}$$

$Z_1 = 47.1 \ \Omega$

$$L_2' = \frac{4\pi \cdot 10^{-7} \cdot 0.25}{1} = 3.14 \cdot 10^{-7} \ \frac{H}{m}$$

$$C_2' = \frac{8.85 \cdot 10^{-12} \cdot 1}{0.25} = 3.54 \cdot 10^{-11}$$

$Z_2 = 94.2 \ \Omega$

$$Z_{in} = Z_2 \frac{(Z_L + j Z_2 \tan\beta l_2)}{(Z_2 + j Z_L \tan\beta l_2)}$$

$$\lambda = \frac{c}{f} = \frac{3 \cdot 10^8}{1 \cdot 10^9} = \frac{3}{10} \ m = 30cm$$

$$l_2 = \frac{\lambda}{2} = 15cm$$

$$\beta = \frac{2\pi}{\lambda} \qquad \beta \cdot l_2 = \pi \qquad \tan\pi = 0$$

$$\therefore \quad Z_{in} = Z_2 \cdot \frac{Z_L}{Z_2}$$

$$Z_{in} = Z_L = 100\ \Omega$$

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c)

c) The frequency and the width of TL2 are the same as in item (b) ($f = 1\text{GHz}, w_2 = 1\text{cm}$). Find the impedance Z_2, length l_2, and the separation between the plates d_2 that lead to perfect matching between TL1 and the load impedance (i.e. no reflected wave is present in TL1).

$Z_L = Z_1$

To match $Z_1 = Z_L$ $\qquad Z_{in} = \frac{Z_0^2}{Z_L}$

We need $Z_1 = Z_{in}$

$$= \frac{Z_2^2}{Z_L}$$

And using the Quarter wave TL

$$Z_2 = \sqrt{Z_1 \cdot Z_L} = \boxed{68.63\ \Omega}$$

$$l = \frac{\lambda}{4} = \frac{c}{1G \cdot 4} = 0.075\text{m} = \boxed{7.5\text{cm}}$$

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless $(R' = G' = 0)$	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (60/\sqrt{\varepsilon_r})\ln(b/a)$
Lossless two wire	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120/\sqrt{\varepsilon_r}) \cdot \ln[(d/2a) + \sqrt{(d/2a)^2 - 1}]$ $Z_0 \simeq (120/\sqrt{\varepsilon_r})\ln(d/a)$, if $d \gg a$
Lossless parallel plate	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120\pi/\sqrt{\varepsilon_r})(d/w)$

Notes: (1) $\mu = \mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $c = 1/\sqrt{\mu_0\varepsilon_0}$, and $\sqrt{\mu_0/\varepsilon_0} \simeq (120\pi)\ \Omega$, where ε_r is the relative permittivity of insulating material. (2) For coaxial line, a and b are radii of inner and outer conductors. (3) For two-wire line, a = wire radius and d = separation between wire centers. (4) For parallel-plate line, w = width of plate and d = separation between the plates.

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$$Z_0 = \frac{120\pi}{\sqrt{1}}\left(\frac{d_2}{w_2}\right)$$

$$\frac{68.63}{120\pi} \cdot w_2 = d_2 = \boxed{0.182\ \text{cm}}$$

d) All the parameters are identical to item (c) and the parameters of TL2 are such that TL1 is matched to the load. Give expressions for the voltage and current in TL1 as a function of z in the instantaneous form and in the phasor form. Neglecting all edge effects and assuming the electric and magnetic field uniformity in the y direction, give (phasor) expressions for the electric field, magnetic field, volumetric charge density, and surface charge density in TL1. What are differences between the voltage distributions in this case (non-vanishing frequency) and in the case of $f=0$ in item (a)?

B A

Since it's perfect matched no reflected wave will be considered.

$$\tilde{V}(z) = V_0 e^{-j\beta z}$$

$$V(z,t) = V_0 \cos(\omega t - \beta z)$$

$$\tilde{I}(z) = \frac{\tilde{V}}{Z_1} = \frac{V_0 e^{-j\beta z}}{Z_1}$$

$$I(z,t) = \frac{V_0}{Z_1} \cos(\omega t - \beta z)$$

Electric field

$$\tilde{E}(z) = E_0 \cdot e^{-j\beta z} \cdot x$$

Magnetic field

$$\tilde{H}(z) = H_0 \cdot e^{-j\beta z} \cdot y$$

Volumetric charge density

$$Q = \varepsilon_0 \cdot E \cdot A = \varepsilon_0 \cdot \Phi_E$$

$$\rho_v = \frac{Q}{V} = \frac{E_0 \cdot A \cdot \varepsilon_0 \cdot x \cdot e^{-j\beta z}}{V}$$

Surface charge Density

$$\sigma = \frac{Q}{A} = E_0 \cdot \varepsilon_0 \cdot x \cdot e^{-j\beta z}$$

The Difference is

In a), the voltage is independent to the z

But in this case

the voltage is a function of z