Q1

1)

5.26 A uniform current density given by

$$\mathbf{J} = \hat{\mathbf{z}} J_0 \qquad (\text{A/m}^2)$$

gives rise to a vector magnetic potential

$$\mathbf{A} = -\hat{\mathbf{z}} \frac{\mu_0 J_0}{4}(x^2 + y^2) \qquad (\text{Wb/m})$$

(b) Use the expression for **A** to find **H**.

(c) Use the expression for **J** in conjunction with Ampère's law to find **H**. Compare your result with that obtained in part (b).

2)

$B = \mu_0 H$

$$\boxed{\mathbf{A} = \frac{\mu}{4\pi} \int_{V'} \frac{\mathbf{J}}{R'} \, dv'}$$

$$\boxed{\mathbf{B} = \nabla \times \mathbf{A}}$$

a)

$$\nabla \times A = \begin{vmatrix} x & y & z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & A_z \end{vmatrix} = \hat{x} \cdot \frac{\partial}{\partial y} A_z + \hat{y}\, 0 - \frac{\partial}{\partial x} A_z + \hat{z}\, 0$$

$$B = \hat{x} - \frac{\mu_0 J_0}{4} \cdot 2y - \left(\hat{y} - \frac{\mu_0 J_0}{4} \cdot 2x\right)$$

$$\boxed{H = -\hat{x} \frac{J_0}{2} \cdot y + \hat{y} \frac{J_0}{2} x}$$

b)

$$\oint \underline{H} \cdot dl = I = J \cdot A \qquad dl = r \cdot d\varphi$$

$$\underline{H} = \hat{\phi} H_\phi (r)$$

$$\hat{\phi} H_\varphi \cdot 2\pi r = J_0 \cdot \pi r^2$$

$$H = \frac{J_0 r}{2} \cdot \hat{\phi}$$

Convert cartesion to cylindrical

$$r = \sqrt[+]{x^2 + y^2}$$
$$\phi = \tan^{-1}(y/x)$$
$$z = z$$

① $r = \sqrt{x^2 + y^2}$

$$\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin \phi + \hat{\mathbf{y}} \cos \phi$$

$$= -\hat{x} \sin\left(\tan^{-1}\left(\frac{y}{x}\right)\right) + \hat{y}\left(\cos\left(\tan^{-1}\left(\frac{y}{x}\right)\right)\right)$$

$$② = \frac{-y}{\sqrt{x^2+y^2}} \hat{x} + \frac{x}{\sqrt{x^2+y^2}} \hat{y}$$

$$H = \frac{J_0}{2} \cdot ① \cdot ②$$

$$= \boxed{\frac{J_0}{2}\left(-y \hat{x} + x \hat{y}\right)}$$

Q2

2)

6.19* If the current density in a conducting medium is given by

$$\mathbf{J}(x, y, z; t) = (\hat{\mathbf{x}}z - \hat{\mathbf{y}}3y^2 + \hat{\mathbf{z}}2x)\cos\omega t$$

determine the corresponding charge distribution $\rho_v(x, y, z; t)$.

$$\nabla \cdot \mathbf{J}_v = -\frac{\partial}{\partial t}\rho_v$$

$$\nabla \cdot J = \frac{\partial}{\partial x}J_x + \frac{\partial}{\partial y}J_y + \frac{\partial}{\partial z}J_z$$

$$= \cos wt\,(0 - 6y + 0).$$

$$= -6y\cos wt$$

$$6y\cos wt = \frac{\partial}{\partial t}\rho_v$$

$$\int 6y\cos wt\,dt = \rho_v$$

$$\boxed{\frac{1}{w}6y\sin wt = \rho_v}$$

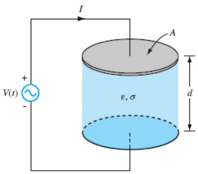
6.16 The parallel-plate capacitor shown in Fig. 6-25 is filled with a lossy dielectric material of relative permittivity ϵ_r and conductivity σ. The separation between the plates is d and each plate is of area A. The capacitor is connected to a time-varying voltage source $V(t)$.

(a) Obtain an expression for I_c, the conduction current flowing between the plates inside the capacitor, in terms of the given quantities.

(b) Obtain an expression for I_d, the displacement current flowing inside the capacitor.

(c) Based on your expressions for parts (a) and (b), give an equivalent-circuit representation for the capacitor.

(d) Evaluate the values of the circuit elements for $A = 4\ \text{cm}^2$, $d = 0.5$ cm, $\epsilon_r = 4$, $\sigma = 2.5$ (S/m), and $V(t) = 10\cos(3\pi \times 10^3 t)$ (V).



4)

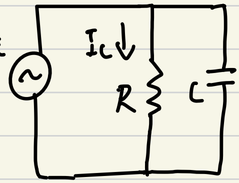
$\frac{\varepsilon}{\varepsilon_r} = \varepsilon_0$ $\quad D = \varepsilon E \cdot$

$R = \frac{d}{\sigma A}$

$$I_d = \iint_S \mathbf{J}_d \cdot d\mathbf{s} = \frac{d}{dt}\iint_S \mathbf{D} \cdot d\mathbf{s}$$

a) $I_c = \frac{V}{R} = \frac{V(t)\cdot \sigma A}{d}$

b) $I_d = \frac{d}{dt}\varepsilon E \cdot A = \frac{d}{dt}\varepsilon \frac{V(t)}{d}\cdot A = \frac{dV(t)}{dt}\cdot\frac{A}{d}\cdot\overbrace{\varepsilon_0\cdot\varepsilon_r}^{\varepsilon} = C\cdot\frac{dV}{dt}$

c) 

$R = \frac{d}{\sigma A}$ $\quad C = \frac{\varepsilon_0\varepsilon_r A}{d}$

d) $R = \frac{0.5\cdot 10^{-2}}{2.5\cdot 4\cdot 10^{-4}} = \boxed{5}$ $\quad C = \frac{8.85\cdot10^{-12}\cdot 4\cdot 4\cdot 10^{-4}}{0.5\cdot 10^{-2}}$

$= \boxed{2.832\cdot 10^{-12}\ F}$

$I_c = \boxed{2\cos(3\pi\cdot 10^3 t)}$ $\quad I_d = \frac{dV(t)}{dt}\cdot C$

$= \boxed{-10^4\cdot 3\pi\cdot\sin(3\pi\cdot 10^3 t)\cdot C}$

Q4

6.6 The square loop shown in Fig. 6-19 is coplanar with a long, straight wire carrying a current

$$I(t) = 5\cos 2\pi \times 10^4 t \quad \text{(A)}$$

(a) Determine the emf induced across a small gap created in the loop. $B = \mu_0 H$

(b) Determine the direction and magnitude of the current that would flow through a 4-Ω resistor connected across the gap. The loop has an internal resistance of 1 Ω.

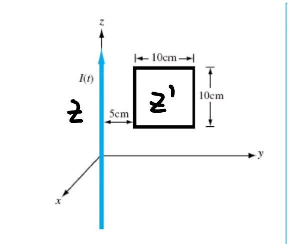
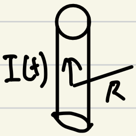


Figure 6-19: Loop coplanar with long wire (Problem 6.6).

$$V_{emf} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\iint_S \mathbf{B}\cdot d\mathbf{s}$$

$$\oint H \cdot dl = I$$

Treat $I(t)$ in cylindrical coordinates



$$\oint H_\varphi \cdot r\, d\varphi = I \qquad \hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$$

$$H_\varphi \cdot 2\pi r = I$$

$$H_\varphi = \frac{I}{2\pi r}\hat{\varphi} \qquad B_\varphi = \mu_0 \frac{I}{2\pi r}\hat{\varphi}$$

$$B_\varphi = \mu_0 \frac{I}{2\pi} \cdot \frac{1}{r} \cdot -\hat{x}\sin\varphi + \hat{y}\cos\phi$$

for the square loop $x = 0$

$$\frac{1}{\sqrt{x^2+y^2}} \cdot \frac{-y}{\sqrt{x^2+y^2}}\hat{x} + \frac{x}{\sqrt{x^2+y^2}}\hat{y}$$

$$B = \mu_0 \frac{I}{2\pi} \cdot \frac{-1}{y}\hat{x}$$

$$V_{emf} = -\frac{d}{dt}\iint B \cdot ds$$

$$= -\frac{d}{dt}\int_0^{10\cdot 10^{-2}}\int_{5\cdot 10^{-2}}^{15\cdot 10^{-2}} \mu_0 \frac{I}{2\pi} \cdot \frac{-1}{y}\hat{x} \cdot dy\,dz$$

$$= -\frac{d}{dt}\left(-\frac{\mu_0 I}{2\pi} \cdot \ln y \Big|_{5\cdot 10^{-2}}^{15\cdot 10^{-2}} \cdot 10 \cdot 10^{-2}\right)$$

$$= \frac{d}{dt}\left(\frac{\mu_0}{2\pi} I \cdot 10^{-1} \ln\left(\frac{15}{5}\right)\right)$$

$$= \frac{\mu_0}{2\pi} \cdot 10^{-1} \cdot \ln 3 \cdot \left[-5\sin(2\pi\cdot 10^4 t) \cdot 2\pi \cdot 10^4\right]$$

$\mu_0 = 4\pi \cdot 10^{-7}$

$$V_{emf} = \mu_0 \cdot 10^{-1} \cdot \ln 3 \cdot (-5) \cdot 10^{4} \cdot \sin(2\pi \cdot 10^{4} t)$$

$$= -6.9 \cdot 10^{-3} \sin(2\pi \cdot 10^{4} t)$$

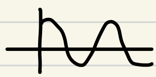
b)

$$I_{induce} = \frac{V_{emf}}{5} = -1.38 \cdot 10^{-3} \sin(2\pi \cdot 10^{4} t)$$

By Lenz' Law, the induced current will oppose the changing of H-field

If H-field ↑, induced current will generate H-field in the opposite direction to oppose the increase

If H-field ↓, induced current will generate H-field in the same direction to oppose the decrease

Since I is cos function  ⇒ decreasing behavior

The H-field is generated in $-\hat{x}$ by right-hand rule

So the induced current need to be $\boxed{CCW}$ to create a $-\hat{x}$ H-field