ECE 107 Project

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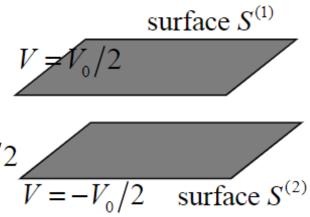
Part 1

Formulas are given for completeness of the project report

- Integral equation method
  - Given
    - Two conductors
    - Voltage on the conductors is $V_0/2$ and $-V_0/2$
    - Find the capacitance C



  - Formulation
    - Potential on the surfaces is given by $V_0/2$ and $-V_0/2$
    - Potential everywhere is calculated as $V(\mathbf{r})=\int_S \underbrace{\frac{1}{4\pi\varepsilon_0\,|\mathbf{r}-\mathbf{r}'|}}_{\text{Green's function}} \underbrace{\rho_s(\mathbf{r}')}_{\text{surface charge distribution}}\, dS'$

      where ρ_s is an unknown surface charge
    - Equate the potentials on the surfaces

$$\Rightarrow \boxed{\iint_{S^{(1)}+S^{(2)}} \underbrace{\frac{1}{4\pi\varepsilon_0\,|\mathbf{r}-\mathbf{r}'|}}_{\text{Green's function}} \underbrace{\rho_s(\mathbf{r}')}_{\substack{\text{surface charge}\\ \text{distribution}\\ \text{(unknown)}}}\, dS' = \begin{cases} V_0/2;\ \mathbf{r}\in S_1 \\ -V_0/2;\ \mathbf{r}\in S_2 \end{cases}} \leftarrow \text{integral equation}$$

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Numerical capacitance extraction (3)

- Matrix equation and solution
  - Matrix equation $\underline{\underline{Z}}\,\underline{Q}=\underline{V}$

$$\underline{\underline{Z}}: N\times N \text{ matrix};\ Z_{mn}=\frac{1}{\Delta s_n}\iint_{S_n}\frac{ds'}{4\pi\varepsilon_0\,|\mathbf{r}_m-\mathbf{r}'|} \Rightarrow \begin{cases} Z_{mm}\approx \dfrac{1}{2\varepsilon_0\sqrt{\pi\,\Delta s_n}};\ \text{self-patch} \\ Z_{mn}\approx \dfrac{1}{2\varepsilon_0\Delta s_n}\left(-d+\sqrt{\dfrac{\Delta s_n}{\pi}+d^2}\right);\ x_m=x_n\ \&\ y_m=y_n \\ Z_{mn}\approx \dfrac{1}{4\pi\varepsilon_0\,|\mathbf{r}_m-\mathbf{r}_n|};\ \text{otherwise} \end{cases}$$

For the case of parallel plates

$$\underline{Q}: N \text{ vector};\ Q_n=\rho_s(\mathbf{r}_n)\Delta s_n$$

$$\underline{V}: N \text{ vector};\ V_m=\begin{cases} V_0/2;\ \mathbf{r}_m\in S_1 \\ -V_0/2;\ \mathbf{r}_m\in S_2 \end{cases}$$

  - Solution $\underline{Q}=\underline{\underline{Z}}^{-1}\underline{V}$

- Extracted capacitance

$$C=\frac{\overbrace{Q_{S_1}}^{\substack{\text{total}\\ \text{charge on}\\ \text{surface 1}}}}{V_0} \Rightarrow \boxed{C=\frac{\sum_{n=1}^{N_{S_1}} Q_n}{V_0}}$$

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Part 2

```
function [C,Q1,Q2] = part2(Npatch_in_width,w,d)

    % Variable initilization
    epsilon0 = 8.85e-12;
    % w = 1e-2; % value from part3
    % d = 3e-3; % value from part3
    Ns1 = Npatch_in_width * Npatch_in_width; % Patch for S1
    Ns2 = Npatch_in_width * Npatch_in_width; % Pathch for S2
    N = Ns1 + Ns2; % Total Patch
    dw = w/Npatch_in_width; % width of single patch
    V0 = 1; % Initial Voltage

    % Construct the rm and rn
    d_bottom = 0;
    d_top = d_bottom + d;
    rm_top = zeros(Npatch_in_width*Npatch_in_width,3); % (x = 1,y = 2,z = 3) for top plates
    rn_bottom = rm_top; % for bottom plates
    rm_top(:,3) = d_top;
    rn_bottom(:,3) = d_bottom;
    loop_index = 1;
    for x = 1:Npatch_in_width
        for y = 1:Npatch_in_width
            rn_bottom(loop_index,1) = dw*x;
            rm_top(loop_index,1) = dw*x;
            rn_bottom(loop_index,2) = dw*y;
            rm_top(loop_index,2) = dw*y;
            loop_index = loop_index +1;
        end
    end
    rm = [rm_top;rn_bottom];
```

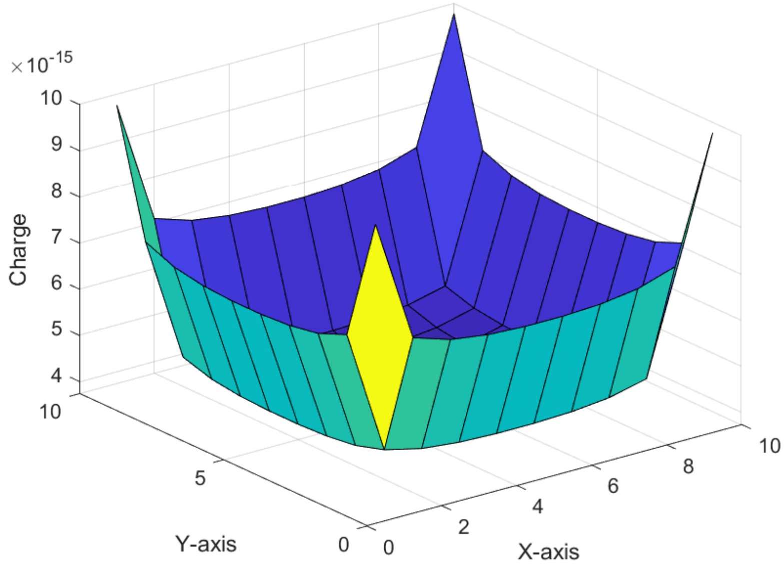
```
    % Calculate Z vector
    Z = zeros(N,N);
    deltaSn = dw*dw;
    for m = 1:N
        for n = 1:N
            if m ~= n
                Z(m,n) = 1/(4*pi*epsilon0*norm(rm(m,1:3) - rm(n,1:3)));
            elseif ((rm(m,1) == rm(n,1)) && (rm(m,2) == rm(n,2)))
                Z(m,n) = 1/(2*epsilon0*deltaSn)*(-1*d+sqrt(deltaSn/pi + d*d));
            else
                Z(m,n) = 1/(2*epsilon0*sqrt(pi*deltaSn));
            end
        end
    end
    % Q vector
    Q = zeros(N,1);
    % V vector
    V = zeros(N,1);
    V(1:N/2) = V0/2; % Top plate
    V(N/2+1:N) = -V0/2; % Bottom Plate

    % Solution:
    Q = Z\V;
    Q1 = Q(1:N/2); % Top charge
    Q2 = Q(N/2+1:N); % botoom charge
    Q1_sum = sum(Q1(1:N/2));
    C = Q1_sum/V0;
end
```

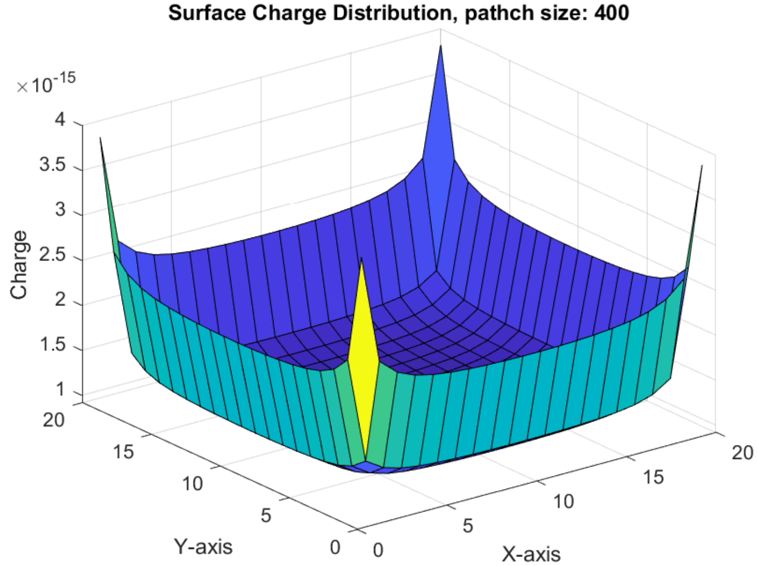
Part3

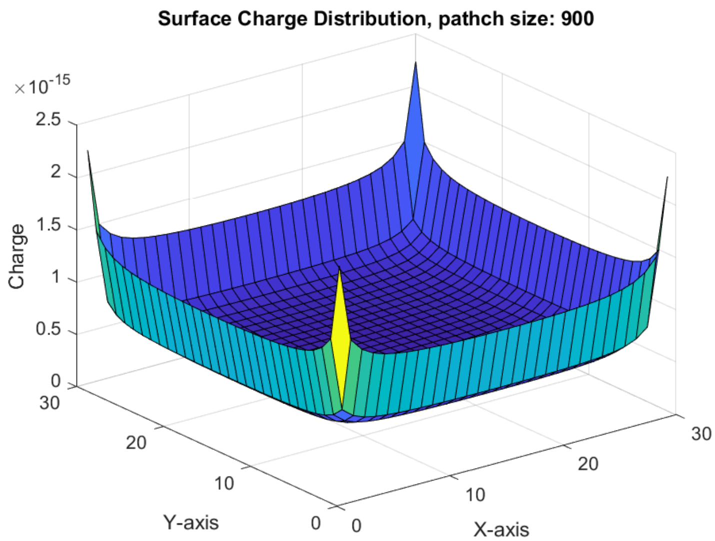
Before the new addition

Surface Charge Distribution, pathch size: 100

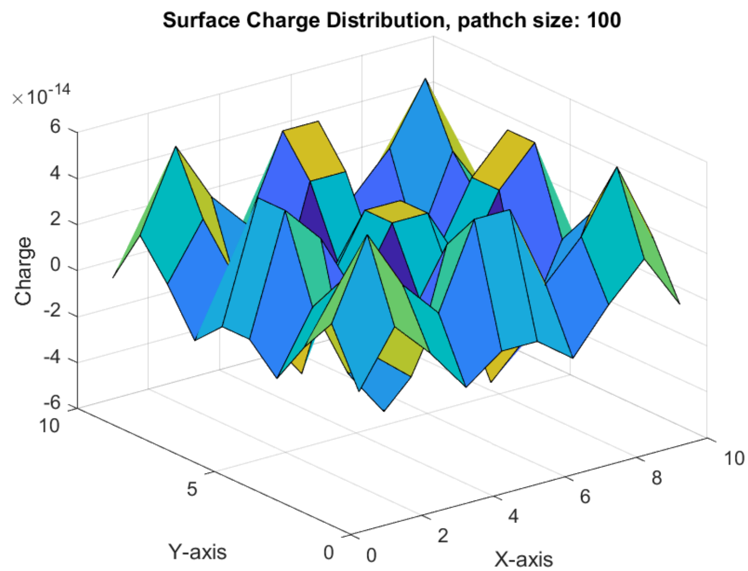


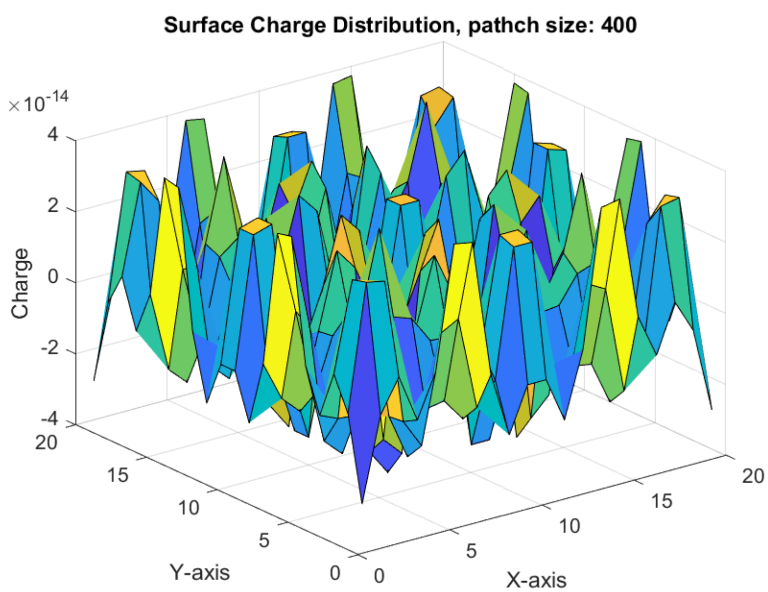
Surface Charge Distribution, pathch size: 400

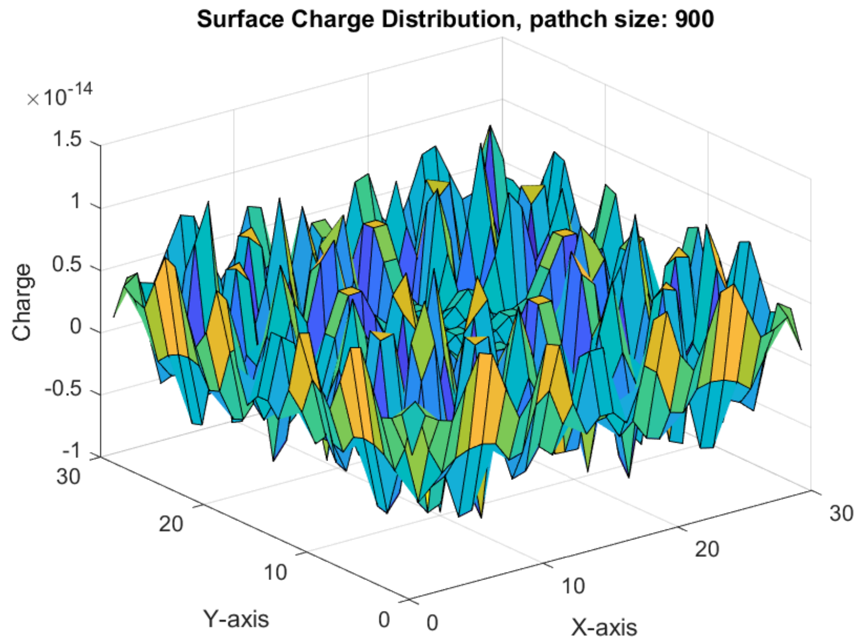




After the new condition:

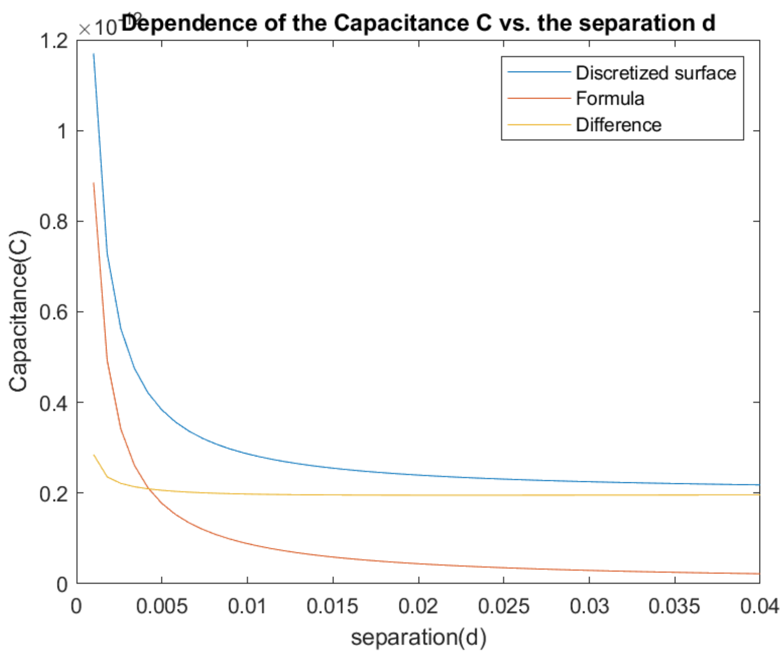

Surface Charge Distribution, pathch size: 100
×10^-14
6
4
2
0
-2
-4
-6
Charge
10
5
0
Y-axis
0
2
4
6
8
10
X-axis


Surface Charge Distribution, pathch size: 400
×10^-14
4
2
0
-2
-4
Charge
20
15
10
5
0
Y-axis
0
5
10
15
20
X-axis

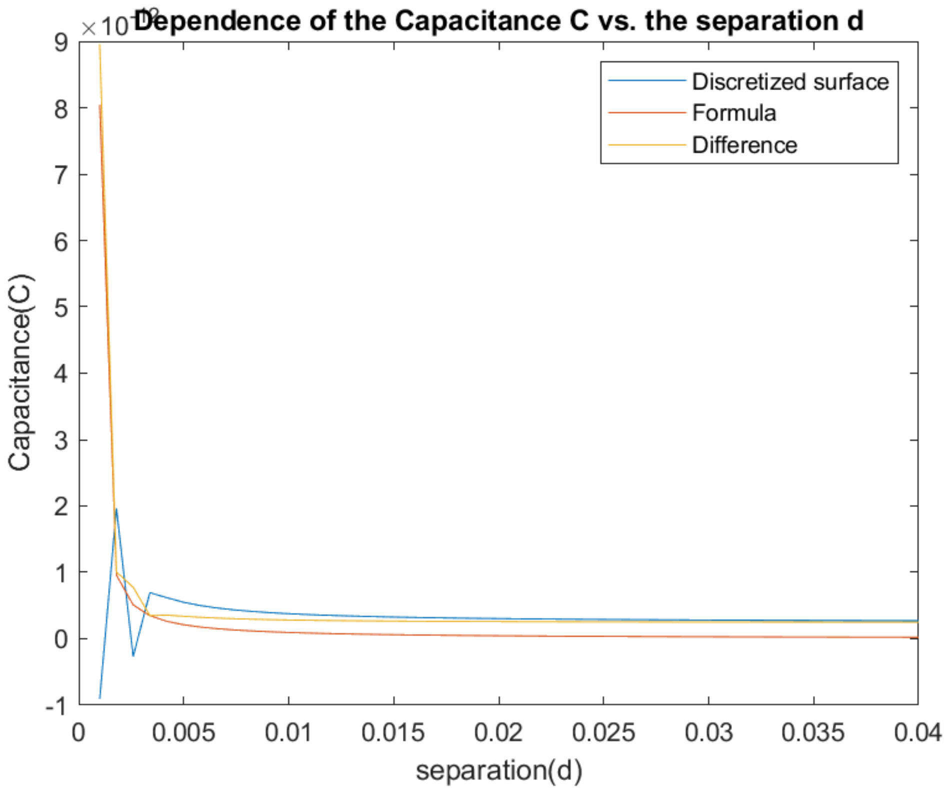


Part4

Before the new condition



after the new condition



Part 4

```
function part4(Npatch)
    w = 1e-2; % value from part4
    d = 4e-2; % value from part4
    loop_index = 1;
    epsilon0 = 8.85e-12;
    totalpoints = 50;
    for x = linspace(0.1e-3,d,totalpoints)
        [C,Q1,Q2] = part2(Npatch,w,x);
        C_discret_array(loop_index) = C;
        C_formula_array(loop_index) = epsilon0 * w^2/x;
        C_difference(loop_index) = abs(C-C_formula_array(loop_index));
        loop_index = loop_index + 1;
    end
    separation = linspace(1e-3,d,totalpoints);
    plot(separation,C_discret_array);
    hold on
    plot(separation, C_formula_array);
    plot(separation, C_difference);
    xlabel('separation(d)');
    ylabel('Capacitance(C)');
    title('Dependence of the Capacitance C vs. the separation d');
    legend('Discretized surface','Formula','Difference');
    saveas(gcf, 'Dependence of the Capacitance C vs the separation d.png');
end
```

50 points are taken for the above plot.

The result of the formula is closer to the results of the numerical solution when the separation distance is really small. Once the separation gets bigger, the difference of the results is distinguishable