1) Show that general expressions for a plane wave propagating in the direction $\hat{\mathbf{k}}$

$$\tilde{\mathbf{E}} = \mathbf{E}_0 e^{-jk\hat{\mathbf{k}}\cdot\mathbf{r}}, \tilde{\mathbf{H}} = \mathbf{H}_0 e^{-jk\hat{\mathbf{k}}\cdot\mathbf{r}}$$

$$\mathbf{H}_0 = \frac{1}{\eta}\hat{\mathbf{k}}\times\mathbf{E}_0, \mathbf{E}_0 = -\eta\hat{\mathbf{k}}\times\mathbf{H}_0$$

satisfy source-free Maxwell's equations.

Q1

$u\varepsilon$

① $\nabla \times E = -j\omega u H$ Faraday's law

② $\nabla \times (E_0 e^{-jk\hat{k}\cdot r}) = -j\omega u H_0 e^{-jk\hat{k}\cdot r}$

gradient ③ $(\nabla e^{-jk\hat{k}\cdot r}) \times E_0 = -j\omega u H_0 e^{-jk\hat{k}\cdot r}$

$(-jk\hat{k} \times E_0) e^{-jk\hat{k}\cdot r} = -j\omega u H_0 e^{-jk\hat{k}\cdot r}$

$k = \omega\sqrt{\varepsilon/u}$

$$H_0 = \frac{k}{\omega u} E_0 \cdot \hat{k}$$

$$= \sqrt{\frac{\varepsilon}{u}} E_0 \cdot \hat{k}$$

$\eta = \sqrt{\frac{u}{\varepsilon}}$

$$= \boxed{\frac{1}{\eta} E_0 \cdot \hat{k}}$$

$$E_0 = \frac{\omega u}{k} H_0 \cdot \hat{k}$$

$$= \frac{u}{\sqrt{\frac{\varepsilon}{u}}} H_0 \cdot \hat{k}$$

$$= \sqrt{\frac{u}{\varepsilon}} H_0 \cdot \hat{k}$$

$$= \boxed{\eta H_0 \cdot \hat{k}}$$

Q2

2) Show that expressions for the fields radiated by a short electric dipole satisfy source-free Maxwell's equations for $R > 0$.

$$\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$$

$$\hat{A} = \hat{z}\frac{\mu_0}{4\pi} I_0 l \left(\frac{e^{-jkR}}{R}\right)$$

$$\tilde{\mathbf{H}} = \frac{1}{\mu_0}\nabla\times\tilde{\mathbf{A}}, \qquad \tilde{\mathbf{E}} = \frac{1}{j\omega\varepsilon_0}\nabla\times\tilde{\mathbf{H}}$$

$$R^{-1} = -1R^{-2}$$

$$\begin{vmatrix} R & \Theta & \varphi \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi} \\ A_R & \hat{A}_\Theta & \hat{A}_\varphi \end{vmatrix}$$

$$\tilde{A} = \hat{R}\,\frac{\mu_0}{4\pi} I_0 l \cos\Theta\left(\frac{e^{-jkR}}{R}\right) - \hat{\Theta}\,\frac{\mu_0}{4\pi} I_0 l \sin\theta\left(\frac{e^{-jkR}}{R}\right)$$

$$\hat{H} = \frac{1}{\mu_0}\nabla\times\tilde{A} = \hat{R}\left(0 - \frac{\partial}{\partial\varphi}\overset{0}{A_\theta}\right) + \hat{\Theta}\left(\frac{\partial}{\partial\varphi}\overset{0}{A_\Theta} - \frac{\partial}{\partial\theta}\overset{0}{A_\varphi}\right) + \hat{\varphi}\left(\frac{\partial}{\partial R}A_\theta - \frac{\partial}{\partial\theta}A_R\right)$$

$$\hat{H}_\varphi = \hat{\varphi}\,-\frac{I_0 l}{4\pi}\sin\Theta\left(\frac{-jke^{-jkR}\cdot R - e^{-jkR}}{R^2}\right) + \quad 0$$

$$= \hat{\varphi}\,\frac{I_0 l}{4\pi}\sin\theta\left(\frac{jk}{R} + \frac{1}{R^2}\right)e^{-jkR}$$

$$\tilde{E} = \frac{1}{j\omega\varepsilon_0} = \hat{R}\left(\frac{\partial}{\partial\Theta}H_\varphi - \frac{\partial}{\partial\varphi}\overset{0}{H_\theta}\right) + \left(\frac{\partial}{\partial\varphi}\overset{0}{H_R} - \frac{\partial}{\partial R}H_\varphi\right)\hat{\Theta} + \hat{\varphi}\overbrace{\left(\frac{\partial}{\partial R}A_\Theta - \frac{\partial}{\partial\theta}H_R\right)}^{0}$$

$$=$$

3)

7.5* A wave radiated by a source in air is incident upon a soil surface, whereupon a part of the wave is transmitted into the soil medium. If the wavelength of the wave is 60 cm in air and 20 cm in the soil medium, what is the soil's relative permittivity? Assume the soil to be a very low-loss medium.

$$20 = \frac{60}{\sqrt{\varepsilon_r}}$$

$$\boxed{\varepsilon_r = 9}$$

$$V = \frac{c}{\sqrt{\varepsilon_r}}$$

$$\frac{\lambda_{soil}}{t} = \frac{\frac{\lambda_{vaccum}}{t}}{\sqrt{\varepsilon_r}}$$

4)

7.1* The magnetic field of a wave propagating through a certain nonmagnetic material is given by

$$\mathbf{H} = \hat{\mathbf{z}}\, 30 \cos(10^8 t - 0.5y) \qquad \text{(mA/m)}$$

Find the following:

(a) The direction of wave propagation.

(b) The phase velocity.

(c) The wavelength in the material.

(d) The relative permittivity of the material.

(e) The electric field phasor.

Phase velocity $v_p = 1/\sqrt{\mu\varepsilon} = \omega/k$ $(1/\sqrt{\mu_0\varepsilon_0} = 2\times10^8)$

Wavelength $\lambda = 2\pi/k = v_p/f$

Characteristic (intrinsic) impedance $\eta = \sqrt{\mu/\varepsilon}$ $(\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 120\pi\,\Omega)$

$H = 30 e^{-j0.5y}$

a) y-direction

b) $u_p = \frac{\omega}{k} = \frac{10^8}{0.5} = 2 \cdot 10^8$ m/s

c) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.5} = 4\pi$ m

d) $\varepsilon_r = \left(\frac{c}{u_p}\right)^2 = \frac{3 \cdot 10^8}{2 \cdot 10^8} = (1.5)^2 = 2.25$

e) $\mathbf{E}_0 = -\eta \hat{\mathbf{k}} \times \mathbf{H}_0$ $\qquad \eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} = \frac{120\pi}{\sqrt{\varepsilon_r}} = \frac{120\pi}{1.5}$

$\hat{k} = \hat{y} \qquad \hat{y} \times \hat{z} = \hat{x}$

$=$

$= -\hat{x}\, \frac{120\pi}{1.5} \cdot 30 e^{-0.5y} = 2400\pi e^{-0.5y}$ $\frac{mV}{m}$

$= -2400\, \hat{x} \cos(10^8 t - 0.5y)$

5)

7.3* The electric field phasor of a uniform plane wave is given by $\tilde{\mathbf{E}} = \hat{\mathbf{y}}\, 10 e^{j0.2z}$ (V/m). If the phase velocity of the wave is 1.5×10^8 m/s and the relative permeability of the medium is $\mu_r = 2.4$, find the following:

(a) The wavelength.

(b) The frequency f of the wave.

(c) (c) the relative permittivity of the medium.

(d) The magnetic field $\mathbf{H}(z, t)$.

$\frac{\lambda}{u_p} = t$

$u_p = 1.5 \cdot 10^8$

$u_r = 2.4$

$\bar{E} = \hat{y}\, 10\,(\omega t - 0.2z)$

a) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2} = 10\pi \text{ m}$

b) $f = \frac{u_p}{\lambda} = \frac{1.5 \cdot 10^8}{10\pi} = 4.77 \text{ MHz}$

c) $u_p = \frac{c}{\sqrt{\varepsilon_r \cdot u_r}} \Rightarrow \varepsilon_r = \frac{\left(\frac{c}{u_p}\right)^2}{u_r} = \frac{\left(\frac{3 \cdot 10^8}{1.5 \cdot 10^8}\right)^2}{2.4} = \frac{2^2}{2.4} = 1.67$

d) $H = \frac{1}{\eta} \hat{k} \times E \qquad \hat{k} = \hat{z}$

$\frac{1}{\eta} = \sqrt{\frac{\varepsilon}{u}} = \frac{1}{120\pi}\sqrt{\frac{\varepsilon_r}{u_r}} = \sqrt{\frac{1.67}{2.4}} \cdot \frac{1}{120\pi} = 2.2 \cdot 10^{-3}$

$= \hat{z} \times \hat{y} = \hat{x} \cdot 2.2 \cdot 10^{-3} \cdot 10 e^{j0.2z}$

$= \boxed{2.2 \cdot 10^{-2} \cos(2\pi \cdot 4.77\text{M} + 0.2z)}$

6)

7.15 Dry soil is characterized by $\varepsilon_r = 2.5$, $\mu_r = 1$, and $\sigma = 10^{-4}$ (S/m). At each of the following frequencies, determine if dry soil may be considered a good conductor, a quasi-conductor, or a low-loss dielectric, and then calculate α, β, λ, μ_p, and η_c:

(a) 60 Hz

(b) 1 kHz

(c) 1 MHz

(d) 1 GHz

$\sigma/(\omega\varepsilon)$ Conductivity

a) $\dfrac{10^{-4}}{60 \cdot 2\pi \cdot 2.5 \cdot 8.85 \cdot 10^{-8}} = 11989$

$u_p = \dfrac{c}{\sqrt{\varepsilon_r u_r}}$

7)

7.30 At microwave frequencies, the power density considered safe for human exposure is 1 (mW/cm^2). A radar radiates a wave with an electric field amplitude E that decays with distance as $E(R) = (3,000/R)$ (V/m), where R is the distance in meters. What is the radius of the unsafe region?

8) **(A part of final 2007)** Consider the layered medium depicted in Fig. 1. There are 4 regions with medium parameters ε_i, μ_i, i=1,..,4. The first and last regions are half-spaces, whereas the second and third regions are slabs of thickness d_2 and d_3, respectively. A uniform plane wave is incident from region 1 normally. It is given that $d_2 = \lambda_2/2$, where λ_2 is the wavelength in the second medium. Assume that ε_1, μ_1 and ε_4, μ_4 are given as well. What are the conditions on $\varepsilon_3, \mu_3, d_3$ that lead to no reflection?

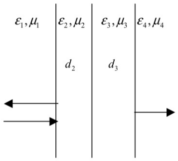

Fig. 1: A two slab system under normal incidence (a) and under oblique incidence (b).