1)

Evaluate each of the following complex numbers and express the result in rectangular form:

(a) $z_1 = 4e^{j\pi/3}$

(b) $z_2 = \sqrt{3}\, e^{j3\pi/4}$

(c) $z_3 = 6e^{-j\pi/2}$

(d) $z_4 = j^3$

(e) $z_5 = j^{-4}$

(f) $z_6 = (1 - j)^3$

(g) $z_7 = (1 - j)^{1/2}$

$a + jb$

$a = r\cos(\theta)$

$b = r\sin(\theta)$

$e^{j\theta} = \cos\theta + j\sin\theta$

a) $z_1 = 4e^{j\pi/3}$

$a = 4\cos(\frac{\pi}{3})$
$= 2$

$b = 4\sin(\frac{\pi}{3})$
$= 2\sqrt{3}$

$z_1 = \boxed{2 + 2\sqrt{3}j}$

b) $z_2 = \sqrt{3}e^{j\frac{3\pi}{4}}$

$a = \sqrt{3}\cos(\frac{3\pi}{4})$
$= \frac{-\sqrt{6}}{2}$

$b = \sqrt{3}\sin(\frac{3\pi}{4})$
$= \frac{\sqrt{6}}{2}$

$\boxed{z_2 = \frac{-\sqrt{6}}{2} + \frac{\sqrt{6}}{2}j}$

c) $z_3 = 6e^{-j\pi/2}$

$a = 6\cos(\frac{-\pi}{2})$
$= 0$

$b = 6\sin(\frac{-\pi}{2})$
$= -6$

$\boxed{z_3 = 0 - 6j}$

d)

$z_4 = j^3 = j^2 \cdot j = (\sqrt{-1})^2 \cdot j = \boxed{0-j}$

e) $z_5 = j^{-4} = \frac{1}{j^4} = \frac{1}{(\sqrt{-1})^2 \cdot (\sqrt{-1})^2} = \boxed{1+0j}$

f) $z_6 = (1-j)^3$

$= 1^3 - 3 \cdot 1^2 \cdot j + 3 \cdot 1 \cdot j^2 - j^3$

$= 1 - 3j - 3 + j$

$= \boxed{-2-2j}$

g) $z_7 = (1-j)^{\frac{1}{2}}$

$r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$ $\quad \theta = \tan^{-1}\left(\frac{-1}{1}\right) = -\frac{1}{4}\pi$

$= \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + j\sin\left(-\frac{\pi}{4}\right)\right)$

Raise to power of $\frac{1}{2}$

$= 2^{\frac{1}{4}}\left(\cos\left(-\frac{\pi}{8}\right) + j\sin\left(-\frac{\pi}{8}\right)\right)$

$= \boxed{1.0987 - 0.455j}$

Q2

2)

Complex numbers z_1 and z_2 are given by the following:

$$z_1 = 3 - j2$$
$$z_2 = -4 + j3$$

(a) Express z_1 and z_2 in polar form.

(b) Find $|z_1|$ by first applying Eq. (1.41) and then by applying Eq. (1.43).

(c) Determine the product $z_1 z_2$ in polar form.

(d) Determine the ratio z_1/z_2 in polar form.

(e) Determine z_1^3 in polar form.

$$\frac{\pi}{4} = 45^0$$

$$\frac{\pi}{180} = \frac{1.35}{x}$$

a) $Z_1 \Rightarrow r = \sqrt{3^2 + (-2)^2} = \sqrt{13}$ $\quad \theta = \tan^{-1}\left(\frac{-2}{3}\right) = -0.588 = -33.69°$

$$\boxed{Z_1 = \sqrt{13}\left(\cos(-33.69°) + j\sin(-33.69°)\right)}$$

$Z_2 \Rightarrow r = \sqrt{(-4)^2 + 3^2} = 5$ $\quad \theta = \tan^{-1}\left(\frac{3}{-4}\right) = -36.87^0$

$$\boxed{Z_2 = 5\left(\cos(-36.87°) + j\sin(-36.87°)\right)}$$

b) Eqn (1.41) = $|z| = +\sqrt{x^2 + y^2} = \sqrt{3^2 + (-2)^2} = \boxed{\sqrt{13}}$

$$\theta = \tan^{-1}\left(\frac{-2}{3}\right) = \boxed{-33.69°}$$

$$z^* = 3 + j2$$

$$|z| = +\sqrt{z \cdot z^*} = \sqrt{(3-j2)(3+j2)}$$

$$= \sqrt{9 + 6j - 6j + 4} = \boxed{\sqrt{13}}$$

$$\frac{\pi}{180} = \frac{x}{70.56}$$

c) $Z = z_1 \cdot z_2 = (3 - j2)(-4 + j3)$

$$= -12 + 9j + 8j + 6$$
$$= -6 + 17j$$

$r = \sqrt{(-6)^2 + (17)^2}$ $\quad \theta = \tan^{-1}\left(\frac{17}{-6}\right)$

$= 5\sqrt{13}$ $\quad = -70.56°$

$$\boxed{Z = 5\sqrt{13}\left(\cos(-70.56°) + j\sin(-70.56°)\right)}$$

d) $Z = \frac{Z_1}{Z_2}$

$= \frac{3 - j2}{-4 + j3} \cdot \frac{-4 - j3}{-4 - j3} = \frac{-12 - 9j + 8j - 6}{16 + 9} = \frac{-18 - j}{25}$

$= -0.72 - 0.04j$

$r = \sqrt{0.72^2 + 0.04^2} = 0.7211$

$\theta = \tan^{-1}\left(\frac{-0.04}{-0.72}\right) = 3.18^\circ$

$\boxed{Z = 0.7211\left(\cos(3.18^\circ) + j\sin(3.18^\circ)\right)}$

e) $z_1^3 = (3 - j2)^3 = 27 - 3 \cdot 9 \cdot 2j + 3 \cdot 3 \cdot (2j)^2 - (2j)^3$

$= -9 - 46i$

$r = \sqrt{9^2 + 46^2} = 13\sqrt{13}$

$\theta = \tan^{-1}\left(\frac{-46}{-9}\right) = 78.93^\circ$

$\boxed{Z_1^3 = 13\sqrt{13}\left(\cos(78.93^\circ) + j\sin(78.93^\circ)\right)}$

Q3

3)

If $z = -2 + j4$, determine the following quantities in polar form:

(a) $1/z$

(b) z^3

(c) $|z|^2$

(d) $\mathfrak{Im}\{z\}$

(e) $\mathfrak{Im}\{z^*\}$

a) $\frac{1}{z} = \frac{1}{-2+j4} \Rightarrow \frac{(-2-j4)}{(-2)^2-(j4)^2} = \frac{-2-j4}{20} = -0.1 - 0.2j$

$r = \sqrt{0.1^2+0.2^2} = 0.2236$

$\theta = \tan^{-1}\left(\frac{-0.2}{-0.1}\right) = 63.435^0$

$$\boxed{\frac{1}{z} = 0.2236\left(\cos(63.435^\circ) + j\sin(63.435^\circ)\right)}$$

b) $z^3 = 88 - 16j$

$r = \sqrt{88^2+16^2} = 89.4427$

$\theta = \tan^{-1}\left(\frac{-16}{88}\right) = -10.3048^\circ$

$$\boxed{z^3 = 89.4427\left(\cos(-10.3048^\circ) + j\sin(-10.3048^\circ)\right)}$$

c) $= |z|^2 = \left(\sqrt{2^2+4^2}\right)^2 = \left(\sqrt{20}\right)^2 = 20$

$r = 20$

$\theta = \tan^{-1}\left(\frac{0}{20}\right) = 0$

$$\boxed{|z|^2 = 20\left(\cos 0^\circ + j\sin 0^\circ\right)}$$

d) $Im(z) = j4$

$r = 4$

$\theta = \tan^{-1}\left(\frac{4}{0}\right) \Rightarrow \frac{\pi}{2}$

$= \boxed{4\left(\cos\frac{\pi}{2} + j\sin\frac{\pi}{2}\right)}$

e) $Im(z^*) = -j4$

$r = 4$

$\theta = \tan^{-1}\left(\frac{-4}{0}\right) \Rightarrow \frac{-\pi}{2}$

$= \boxed{4\left(\cos\left(\frac{-\pi}{2}\right) + j\sin\left(\frac{-\pi}{2}\right)\right)}$

Q4

4)

Find the phasors of the following time functio

(a) $v(t) = 3\cos(\omega t - \pi/3)$ (V)

(b) $v(t) = 12\sin(\omega t + \pi/4)$ (V)

(c) $i(x,t) = 2e^{-3x}\sin(\omega t + \pi/6)$ (A)

(d) $i(t) = -2\cos(\omega t + 3\pi/4)$ (A)

(e) $i(t) = 4\sin(\omega t + \pi/3) + 3\cos(\omega t - \pi/6)$ (A)

$A\cos(\omega t + \phi)$

Phasor $= Ae^{j\phi}$

a) $\boxed{\text{Phasor: } 3e^{-j\frac{\pi}{3}}}$

b) Phasor: $\boxed{12e^{j(\frac{\pi}{4}-\frac{\pi}{2})} = 12e^{j(-\frac{\pi}{4})}}$

c) Phasor: $\boxed{2e^{-3x}e^{j(\frac{\pi}{6}-\frac{\pi}{2})} = 2e^{-3x}e^{j(-\frac{\pi}{3})}}$

d) Phasor: $\boxed{-2e^{j\frac{3\pi}{4}}}$

e) Phasor: $4e^{j(\frac{\pi}{3}-\frac{\pi}{2})} + 3e^{j\frac{-\pi}{6}}$

$= 4e^{-j\frac{\pi}{6}} + 3e^{-j\frac{\pi}{6}}$

$= \boxed{7e^{-j\frac{\pi}{6}}}$

5)

5)

Find the instantaneous time sinusoidal functions corresponding to the following phasors:

(a) $\widetilde{V} = -5e^{j\pi/3}$ (V)

(b) $\widetilde{V} = j6e^{-j\pi/4}$ (V)

(c) $\widetilde{I} = (6 + j8)$ (A)

(d) $\widetilde{I} = -3 + j2$ (A)

(e) $\widetilde{I} = j$ (A)

(f) $\widetilde{I} = 2e^{j\pi/6}$ (A)

a) $\boxed{v(t) = -5\cos\left(wt + \frac{\pi}{3}\right)}$

b) $\boxed{v(t) = j6\cos\left(wt - \frac{\pi}{4}\right)}$

c) $6+j8 \Rightarrow$ $r = \sqrt{6^2+8^2} = 10$

$\theta = \tan^{-1}\left(\frac{8}{6}\right) = 53.13°$

-33.69

$\boxed{i(t) = 10\cos(wt + 53.13°)}$

d) $= -3+j2 \Rightarrow$ $r = \sqrt{3^2+2^2} = 3.6056$

$\theta = 146.3099 = \tan^{-1}\left(\frac{2}{-3}\right)$

$= -33.69°$

$\boxed{i(t) = 3.6056\cos(wt - 33.69°)}$

e) $= j$ $r = 1$

$\theta = \tan^{-1}\left(\frac{1}{0}\right) = \frac{\pi}{2}$

$\boxed{i(t) = 1\cos\left(wt + \frac{\pi}{2}\right)}$

f) $= 2e^{j\frac{\pi}{6}} \Rightarrow$ $\boxed{i(t) = 2\cos\left(wt + \frac{\pi}{6}\right)}$

6) Consider a traveling wave having the amplitude and initial phase of $A_1 = 10$, $\phi_1 = \pi/4$ at $x_1 = 2$ and $A_2 = 5$, $\phi_2 = \pi/3$ at $x_2 = 3$. Find the attenuation constant (α), wavenumber (β), and reference phase (ϕ_0).

$$y(x,t) = A_0 e^{-ax} \cos(\omega t \underbrace{-\beta x + \phi_0}_{\phi})$$

$A_1 = 10 \quad \phi_1 = \frac{\pi}{4}$ when $x = 2$

$$10 = A_0 e^{-a \cdot 2} \cdot e^{\phi_1}$$

$$10 = A_0 e^{-a \cdot 2}$$

$A_2 = 5 \quad \phi_2 = \frac{\pi}{3}$, when $x = 3$

$$5 = A_0 e^{-a \cdot 3} \cdot e^{\phi_2}$$

$$5 = A_0 e^{-a \cdot 3}$$

① $\frac{10}{5} = \frac{e^{-a \cdot 2}}{e^{-a \cdot 3}} = 2 \Rightarrow e^{-a \cdot 2 - (-a \cdot 3)} = 2$

$$e^{a} = 2$$

$$\boxed{a = \ln(2)}$$

$$\boxed{\begin{aligned} A_0 &= A e^{a x_2} \\ &= 5 e^{(\ln 2) \cdot 3} \\ &= 40 \end{aligned}}$$

② $-2 \cdot \beta + \phi_0 = \frac{\pi}{4}$ ⓐ $\qquad -3\beta + \phi_0 = \frac{\pi}{3}$ ⓑ

b − a

$$-\beta = \frac{\pi}{3} - \frac{\pi}{4} = \frac{4\pi - 3\pi}{12} \Rightarrow \boxed{\beta = \frac{-\pi}{12}}$$

$$\phi_0 = \phi_1 + \beta \cdot 2$$

$$\phi_0 = \frac{\pi}{4} + \frac{-\pi}{12} \cdot 2$$

$$\phi_0 = \frac{3\pi - 2\pi}{12}$$

$$\boxed{\phi_0 = \frac{\pi}{12}}$$

7) (a) Write the phasor representation of the wave in Problem 6

(b) Write the time domain representation of the wave in Problem (6).

$Ae^{-\alpha x} e^{-j(\beta x+\phi_0)}$

a) Phasor : $\boxed{40 e^{-\ln 2 \cdot x} \cdot e^{-j \frac{\pi}{12} x} \cdot e^{j\frac{\pi}{12}}}$

b) $\boxed{40 \, e^{-\ln 2 x} \cos\left(\omega t - \frac{\pi}{12} x + \frac{\pi}{12}\right)}$