Question #1 [Chapter #1]: Link Budget Analysis

- Verizon is setting up base stations (BS) for 5G-FR1 in B2 (E & F) with an output power of +46dBm using directional antennas with a gain of 18dBi. The base stations are mounted at 35m height from the ground, average building height is 5m and the user terminal (UT) can be assumed to be 1.5m from the ground. Assume that the pathloss follows RMa (Rural Macro) Line of Sight (LOS) model described in section 7.4.1 in "3GPP TR 38.901 version 17.0.0 Release 17". Assume that the NF of the user equipment (UE) receiver is 6dB (Front-end Duplexer + Receiver).
  - Part A: What is the maximum distance (d_{2D}) a user can be located from the base station while successfully receiving a voice call? Note: Voice call requires one subcarrier (SC) with SNR > 0dB.
  - Part B: What is the maximum distance (d_{2D}) a user can be located from the base station while successfully downloading 1080p video using 64QAM at 10^{-3} BER with an allocation of 100 SCs.
- Note 1: Assume each subcarrier (SC) in 5G-FR1 has a bandwidth of 15kHz
- Note 2: Total bandwidth of 5G-FR1 B2 (E & F) = 10MHz (9MHz/Signal + 1MHz/Guardband)

3GPP Band	Name	Downlink (MHz)	Uplink (MHz)
B2/B25	1900 PCS	1930-1990	1850-1910
AT&T	A	1930-1945	1850-1865
T-Mobile	B	1950-1965	1870-1875
Sprint	C	1975-1990	1895-1910
AT&T	D	1945-1950	1865-1870
Verizon	E	1965-1970	1875-1880
Verizon	F	1970-1975	1880-1885
Sprint	G	1990-1995	1910-1915

9MHz Available band 1965-1975 1M for guard Band

note 5.6

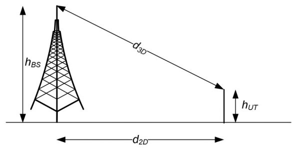


Figure7.4.1-1: Definition of d_{2D} and d_{3D} for outdoor UTs

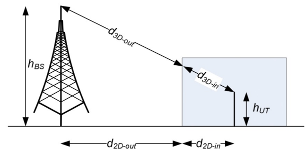


Figure 7.4.1-2: Definition of $d_{2D\text{-}out}$, $d_{2D\text{-}in}$ and $d_{3D\text{-}out}$, $d_{3D\text{-}in}$ for indoor UTs.

Note that

$$d_{3D-out} + d_{3D-in} = \sqrt{(d_{2D-out} + d_{2D-in})^2 + (h_{BS} - h_{UT})^2} \tag{7.4-1}$$

base noise
-174 dBm (reference)
KT noise

sub carrier
15KHz

PART A

Step1

BS: 46dBm + 18dBi = 64 dBm

① # of carriers

② $\dfrac{\text{the Power}}{\text{\# of carriers}}$ → to log

dBm 0 dBm → 1mW
↑ absolute

dB is not absolute
relative

BS Transmit Power = 46 dBm + $\underbrace{18\text{ dBi}}_{\text{gain}}$ = 64 dBm

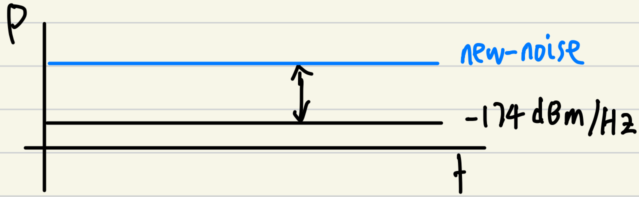
And 10MHz BW - 1 MHz Guard Band = 9 M

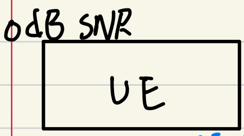
# of subcarrier: 9M ÷ 15kHz = 600 subcarrier

64 dBm ⇒ $10 \log_{10}(x) = 64$ ⇒ $10^{\frac{64}{10}}$

Transmit Power to each Sc : $10 \log_{10}\left(\dfrac{10^{\frac{64}{10}}}{600}\right) = \boxed{36.219 \text{ dBm}}$

Step 2 $(-174\,dBm + 6\,dB) \cdot 15\,kHz$





① Find the noise $(-174\,dBm + 6) \cdot 15\,kHz$
② Then find the received signal

$$NF = \frac{\overset{\text{kT noise}}{N_{th}} + N_{sys}}{N_{th}} \rightarrow N_{UE}$$

Thermal Noise floor = $-174\,dBm/Hz$
$NF\_UE = 6\,dB$
$NF\_Total = -168\,dBm/Hz$
Per sub-carrier = $-168\,dBm \cdot 15\,kHz$

$$= 10\log\left(10^{\frac{-168}{10}} \cdot 15\,kHz\right) = \boxed{-126.24\,dBm}$$

Step 3 Minimum Received Power Level in One SC to achieve SNR of 0dB :

$$S - N = 0dB \Rightarrow S = N$$

$$S = -126.24\,dBm$$

$$10\log_{10}\left(\frac{S}{N}\right) = S(dBm) - N(dBm)$$

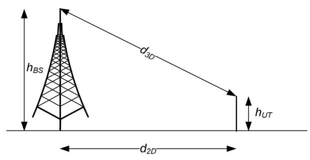


Figure7.4.1-1: Definition of d_{2D} and d_{3D} for outdoor UTs

Note that

$h_{BS} = 35m$

$h_{UT} = 1.5\ m$

$d_{3D} = \sqrt{d_{2D}^2 + (h_{BS} - h_{UT})^2}$

$h = 5\ m$

Step4 Pathloss = Transmit Power - Received Power

$= 36.219 - (-126.24) = 162.459\ dBm$

Step5 $f_c = 1970MHz$

$d_{BP} = 2\pi h_{BS} \cdot h_{UT} \cdot \frac{f_c}{c} = 2\pi \cdot 35 \cdot 1.5 \cdot \frac{\frac{1970M}{1G}}{3 \cdot 10^8} = 2166\ m$

$= 2.166\ km$

$$PL_1 = 20\log_{10}(40\pi d_{3D} f_c / 3) + \min(0.03h^{1.72}, 10)\log_{10}(d_{3D}) - \min(0.044h^{1.72}, 14.77) + 0.002\log_{10}(h)d_{3D}$$

$$PL_2 = PL_1(d_{BP}) + 40\log_{10}(d_{3D} / d_{BP})$$

$PL_1(d_{BP}) = 20\log_{10}\left(40\pi \cdot 2166 \cdot \frac{1.97}{3}\right) + \min(0.03 \cdot 5^{1.72}, 10)\log_{10}(2166)$

$105.044dB + 0.478 \cdot \log_{10}(2166) = 1.59dBm$

$- \min(0.044 \cdot 5^{1.72}, 14.77) + 0.002\log_{10}(5) \cdot 2166$

$- 0.7 + 3.028dBm$

$= 108.96\ dBm$

$PL_2 = 108.96\ dBm + 40\log_{10}\left(\frac{d_{3D}}{2166}\right)$

$162.459dBm = 108.96\ dBm + 40\log_{10}\left(\frac{d_{3D}}{2166}\right)$

$\frac{d_{3D}}{2166} = 10^{\frac{53.499}{40}} \Rightarrow d_{3D}\ 47112.2m = 47.1\ KM$

$$d_{2D} = \sqrt{d_{3D}^2 - (h_{BS} - h_{UT})^2}$$

$$= 47112.2 \text{ m} = \boxed{47.1 \text{ km}}$$

Part B

Step 1: Total Pixels in 1080P = $1920 \cdot 1080$

$= \boxed{2073600}$

Step 2: 24 bits / pixel

Total bits = $2073600 \cdot 24$ bit = $\boxed{49766400 \text{ bits}}$

Step 3: $\frac{49766400}{100} = 497664$ bits $\approx \boxed{0.497664 \text{ Mb}}$

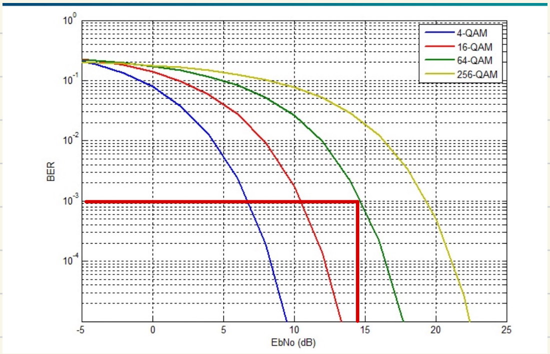
Step 4: For each frame, there is 0.497664 Mb

At 30 frame per second

The Data rate is $30 \cdot 0.497664 \text{ Mb} \approx 14.93 \frac{\text{Mb}}{\text{s}}$

$\boxed{R_b = 14.93 \text{ Mbps}}$

Step 5



$E_b N_0 = 14.5 \text{ dB}$

$$\frac{E_b}{N_0} = SNR \cdot \left(\frac{BW}{R_b}\right)$$

R_b = Bit rate per sec

BW = Bandwidth

E_b = Energy per bit

N_0 = Noise Power Density

$$14.5\,dB = SNR + 10\log\left(\frac{BW}{R_b}\right) \quad 15k\cdot 100$$

$$14.5\,dB = SNR - 9.97$$

$$\boxed{SNR = 24.48\,dB}$$

Step 6 100 SCs

Thermal Noise floor = -174 dBm / Hz

NF_UE = 6 dB

NF_Total = -168 dBm / Hz

Per sub-carrier = -168 dBm $\cdot$ 15 kHz $\cdot$ 100

$$= 10\log\left(10^{\frac{-168}{10}} \cdot 15\,kHz \cdot 100\right) = \boxed{-106.24\,dBm}$$

Step 7 SNR in dBm = $S - N = 24.48$ dB

$\therefore$ Signal = -106.24 dBm + 24.48 dB = $\boxed{-81.76\,dBm}$

Step 8 600 SCs.

Transmit Power to 100 SCs : $10\log_{10}\left(\frac{10^{\frac{64}{10}}}{6}\right) = \boxed{56.22\ dBm}$

$PL = P_{TX} - P_{RX}$

$$= 56.22\,dBm - (-81.76\,dBm) = \boxed{137.98\,dBm}$$

Step9

$$PL_1 = 20\log_{10}(40\pi d_{3D} f_c / 3) + \min(0.03h^{1.72}, 10)\log_{10}(d_{3D}) - \min(0.044h^{1.72}, 14.77) + 0.002\log_{10}(h) d_{3D}$$

$$PL_2 = PL_1(d_{BP}) + 40\log_{10}(d_{3D}/d_{BP})$$

$$PL_1(d_{BP}) = 20\log_{10}\left(40\pi \cdot 2166 \cdot \frac{1.97}{3}\right) + \min(0.03 \cdot 5^{1.72}, 10)\log_{10}(2166)$$

$$105.044\text{dB} \quad + 0.478 \cdot \log_{10}(2166) = 1.59\text{dBm}$$

$$- \min(0.044 \cdot 5^{1.72}, 14.77) + 0.002\log_{10}(5) \cdot 2166$$

$$- 0.7 \quad + 3.028\text{dBm}$$

$$= 108.96\text{ dBm}$$

$$PL_2 = 108.96\text{ dBm} + 40\log_{10}\left(\frac{d_{3D}}{2166}\right)$$

$$137.98\text{ dBm} = 108.96\text{ dBm} + 40\log_{10}\left(\frac{d_{3D}}{2166}\right)$$

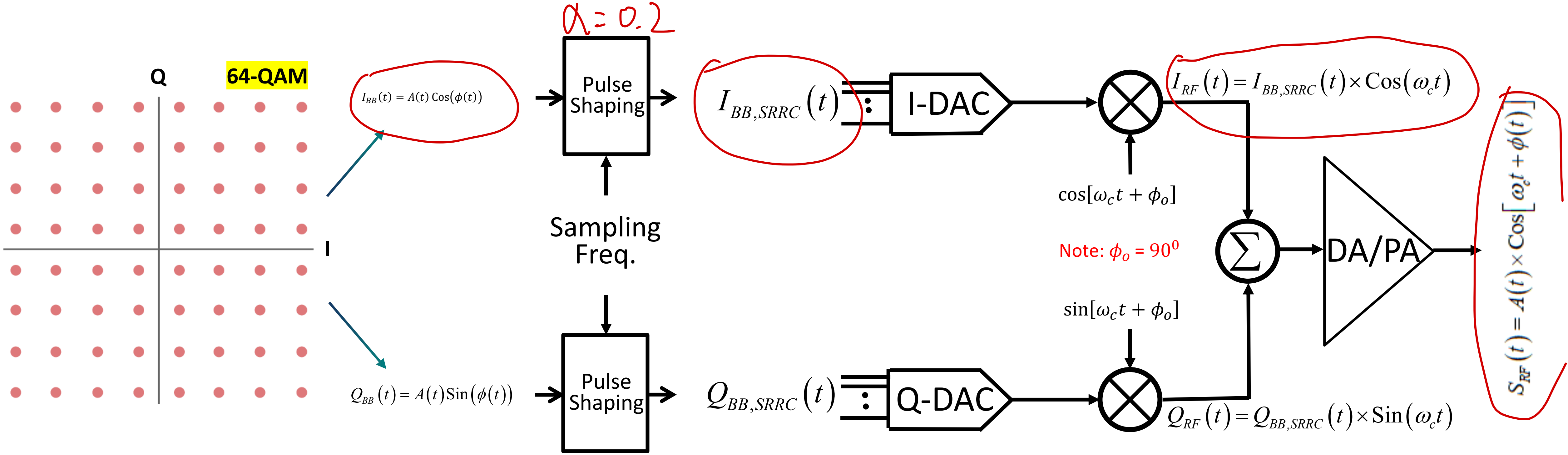
$$\frac{d_{3D}}{2166} = 10^{\frac{53.499}{40}} \Rightarrow d_{3D}\ 11512.2\text{ m} = 11.512\text{ KM}$$

$$d_{2D} = \sqrt{d_{3D}^2 - (h_{BS} - h_{UT})^2}$$

$$= 11512.15\text{ m} = \boxed{11.512\text{ KM}}$$

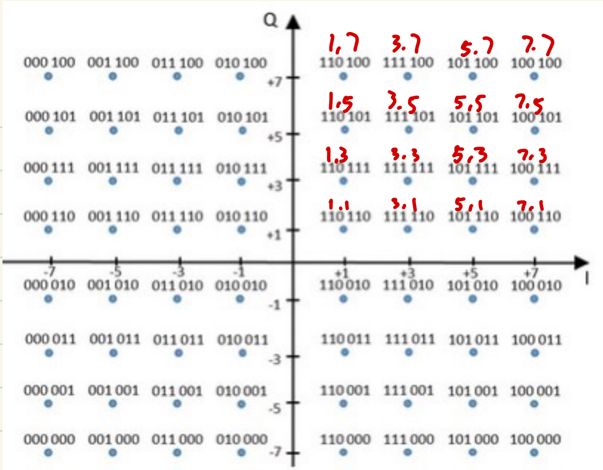
Question #2, Part A [Chapter #2]: PAPR in 3G Transmitter

- Following the steps in the lecture notes determine the Peak-to-Average Ratio (PAPR) at various points along the signal path chain (I + jQ constellation, I-channel input before SRRC, I-channel output after SRRC, I-channel output after UpC and at the PA output) of a 3G/WCDMA transmitter using 64-QAM modulation and SRRC pulse shaping with a roll-off factor of 0.2 (refer to chapter 2 lecture notes for plot of PAPR vs. roll off factor for SRRC). Show your calculations for each step in determining the PAPR along the signal path.



Q2

PART A



50	58	74	98
26	34	50	74
10	18	34	58
2	10	26	50

Step 1 Find the scaling factor

$$E_{64} = \frac{\sum_{i=1}^{64} E_i}{64} = \frac{4\times[(1^2+1^2)+(1^2+3^2)+(1^2+5)^2+(1^2+7)^2)\cdots+(7^2+7^2)]}{64} = \frac{2688}{64} = 42$$

Scaling factor $\sqrt{42}$

Step 2 Find Peak for I-channel

$$\text{PEAK} = \sqrt{\left(\frac{7}{\sqrt{42}}\right)^2} = \frac{7}{\sqrt{42}}$$

Step 3 Find RMS   RMS $[I_{BB}(t)]$

$$\text{RMS} = \sqrt{\frac{4\times\left[\left(\frac{1}{\sqrt{42}}\right)^2\cdot 4+\left(\frac{3}{\sqrt{42}}\right)^2\cdot 4+\left(\frac{5}{\sqrt{42}}\right)^2\cdot 4+\left(\frac{7}{\sqrt{42}}\right)^2\cdot 4\right]}{64}}$$

$$= \frac{\sqrt{2}}{2}$$

$$\text{PAPR} = 20\log\left[\frac{\text{Peak}}{\text{RMS}}\right] = 3.68\ \text{dB}$$

PAPR $[I_{BB}$ before SRRC$]$ = 3.68 dB

I_{BB}

For the I + jQ constellation

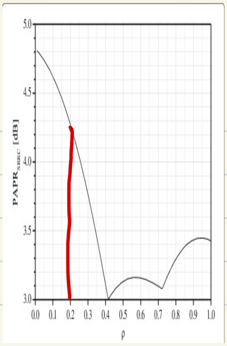
$$\text{PEAK} = \sqrt{\left(\frac{7}{\sqrt{42}}\right)^2 + \left(\frac{7}{\sqrt{42}}\right)^2} = \frac{\sqrt{98}}{\sqrt{42}}$$

Step3 Find RMS RMS $[S_{BB}(t)]$

$$\text{RMS} = \sqrt{\frac{4 \times \left[\left(\left(\frac{1}{\sqrt{42}}\right)^2 + \left(\frac{1}{\sqrt{42}}\right)^2\right) + \left(\left(\frac{1}{\sqrt{42}}\right)^2 + \left(\frac{3}{\sqrt{42}}\right)^2\right) + \left(\left(\frac{1}{\sqrt{42}}\right)^2 + \left(\frac{5}{\sqrt{42}}\right)^2\right) + \dots + \left(\left(\frac{1}{\sqrt{42}}\right)^2 + \left(\frac{7}{\sqrt{42}}\right)^2\right)}{64}}$$

$$= 1$$

$$20\log\frac{\text{PEAK}}{\text{RMS}} = 20\log\left(\frac{\sqrt{98}}{\sqrt{42}}\right) = \boxed{3.68\,\text{dB}}$$



Step 4 I-channel after SRRC

$\alpha = 0.2$

$PAPR = 4.25$ dB

$\therefore PAPR[I \text{ after SRRC}] = 3.68 + 4.25$

$I_{BB,SRRC} \qquad = \boxed{7.93 \text{ dB}}$

Step5 I-channel output after UpC

$$PEAK = I_{BB,SRRC} = 7.93 \text{ dB}$$

$$RMS[I_{RF}(t)] = \sqrt{\lim_{T\to\infty} \frac{1}{T} \int_0^T [I_{BB,SRRC}(t) \cdot \cos(\omega_c t)]^2}$$

$$= \sqrt{\lim_{T\to\infty} \frac{1}{T} \int_0^T I^2_{BB,SRRC}(t) \cdot \left[\frac{1+\cos(2\omega_c t)}{2}\right] dt}$$

$$= \sqrt{\frac{1}{2} \lim_{T\to\infty} \int_0^T [I_{BB,SRRC}(t)]^2 dt + \underbrace{\frac{1}{2} \lim_{T\to\infty} \int_0^T [I_{BB,SRRC}(t)]^2 \cdot \cos(2\omega_c t)\, dt}_{0}}$$

$$= \frac{1}{\sqrt{2}} \cdot \sqrt{\lim_{T\to\infty} \frac{1}{T} \int_0^T [I_{BB,SRRC}(t)]^2 dt} = \frac{1}{\sqrt{2}} \cdot RMS[I_{BB,SRRC}(t)]$$

$$PAPR[I_{Rf}(t)] = 20\log\left(\frac{PEAK[I_{Rf}(t)]}{RMS[I_{Rf}(t)]}\right) = 20\log\left(\frac{[I_{BB,SRRC}]}{RMS[I_{BB,SRRC}]} \cdot \sqrt{2}\right)$$

$$= PAPR[I_{BB,SRRC}] + 20\log(\sqrt{2})$$

$$= 7.93 + 3.01$$

$$= \boxed{10.94 \text{ dB}}$$

Step 6 PA Output

$$PEAK[S_{RF}(t)] = PEAK[I_{BB,SRRC}(t) \cdot \cos(\omega_c t) + Q_{BB,SRRC}(t) \cdot \sin(\omega_c t)]$$

$$= \sqrt{2}\, PEAK[I_{BB,SRRC}(t)]$$

$$RMS[S_{RF}(t)] = \frac{\sqrt{\{RMS[I_{BB,SRRC}(t)]\}^2 + \{RMS[Q_{BB,SRRC}(t)]\}^2}}{\sqrt{2}}$$

$$= RMS[I_{BB,SRRC}(t)]$$

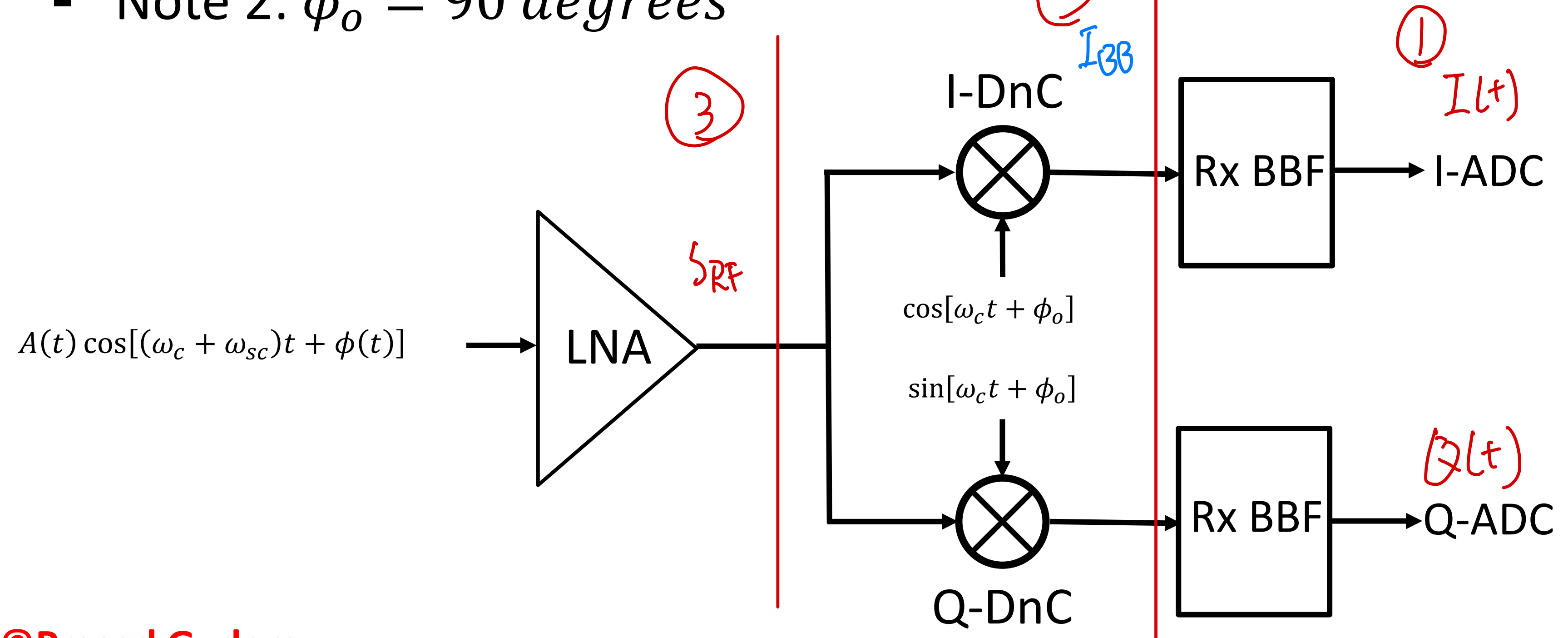
$$PAPR[S_{RF}(t)] = 20\log\left(\frac{PEAK[I_{BB,SRRC}(t)]}{RMS[I_{BB,SRRC}(t)]}\right) + 20\log\sqrt{2}$$

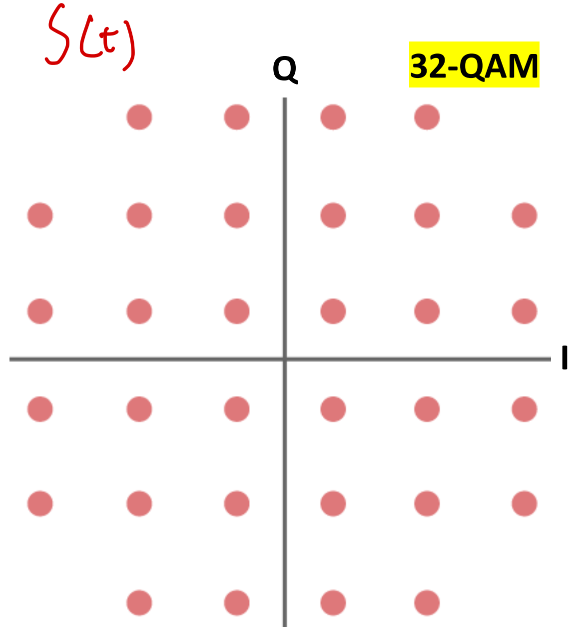
$$= 7.93 + 3.01$$

$$= \boxed{10.94\ B}$$

Question #2, Part B [Chapter #2]: PAPR in 4G Receivers

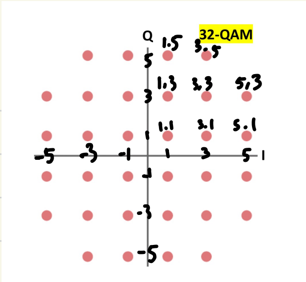
- Following the steps in the lecture notes determine the Peak-to-Average Ratio (PAPR) at various points along the signal path chain (LNA Input/Output, Output of I-DnC / Input of Rx BBF, Output of Rx BBF / Input of I-ADC) of a 4G/LTE receiver using one subcarrier with 32-QAM modulation for the following cases (a) null subcarrier $\omega_{sc}=0$ (b) non-null subcarrier $\omega_{sc}\neq 0$. Assume the pole frequency of the Rx BBF is much lower than the carrier frequency and higher than the subcarrier frequency.
- Note 1: Start the PAPR analysis from the ADC and work backwards
- Note 2: $\phi_o = 90\ degrees$





PART B

null-sub



26	34	
10	18	34
2	10	26

s(t)

$$E_{64} = \frac{\sum_{i=1}^{32} E_i}{\quad} = \frac{4\times\left[(1^2+1^2)+(1^2+3^2)+(1^2+5)^2+(1^2+3^2)\cdots+(3^2+5^2)\right]}{32} = \frac{640}{32}$$

$$= 20$$

$$\text{Scaling factor} = \sqrt{20}$$

$$\text{Peak}[s(t)] = \sqrt{\left(\frac{3}{\sqrt{20}}\right)^2+\left(\frac{5}{\sqrt{20}}\right)^2} = 1.304$$

$$\text{RMS}[s(t)] = \sqrt{\frac{\left[\left(\frac{1}{\sqrt{20}}\right)^2+\left(\frac{1}{\sqrt{20}}\right)^2\right]+\left[\left(\frac{1}{\sqrt{20}}\right)^2+\left(\frac{3}{\sqrt{20}}\right)^2\right]+\cdots+\left[\left(\frac{3}{\sqrt{20}}\right)^2+\left(\frac{5}{\sqrt{20}}\right)^2\right]\cdot 4}{32}}$$

$$= \sqrt{\frac{\frac{160}{20}}{8}} = 1$$

$$\text{PAPR}[s(t)] = 20\log\frac{1.304}{1} = \boxed{2.305\ dB}$$

1

$I(t)$

Peak$[I(t)]$

$$= \sqrt{\left(\frac{5}{\sqrt{20}}\right)^2} = \frac{5}{\sqrt{20}} = 1.118$$

RMS$[I(t)]$

$$= \sqrt{\frac{\frac{(1^2+3^2+5^2)\cdot 2 + (1^2+3^2)}{20}}{8}} = \frac{\sqrt{2}}{2}$$

$$\text{PAPR}(I(t)) = 20\log\left(\frac{1.118}{\frac{\sqrt{2}}{2}}\right) = \boxed{3.98\,dB}$$

2

$$I_{BB}(t) = A(t)\cos[w_c t + \psi(t)] \cdot \cos[w_c t + \psi_0]$$

$$= \frac{1}{2}A(t)\left(\underbrace{\cos(w_c t \cdot 2 + \psi(t))}_{\frac{1}{2}A[\cos(2w_c t)\cdot\cos(\psi t) - \sin(2w_c t)\cdot\sin(\psi(t))]} + \cos(\psi(t))\right) \rightarrow \text{Peak } I[t]$$

$\approx$ Peak of Constellation

$$\text{Peak}[I_{BB}(t)] = \text{PEAK}[A(t)] = 1.118 + 1.304 = 2.422\,dB$$

$$\text{RMS}[I_{BB}(t)] = \sqrt{\lim_{T\to\infty}\frac{1}{T}\int_0^T (I_{BB}(t))^2\,dt}$$

$$= \sqrt{\lim_{T\to\infty}\frac{1}{T}\int_0^T A^2(t)\left[\underbrace{\cos^2(2w_c t + \psi(t))}_{\frac{1}{2}(1+\underbrace{\cos 2(2w_c t + \psi(t))}_{\text{goes to } 0})} + \underbrace{2\cdot(\cos(2w_c t + 2\psi(t)) + \cos(2w_c t))}_{\text{goes to } 0 \text{ for } 1T = 2\pi \text{ integral}} + \underbrace{\cos^2(\psi(t))}_{\frac{1}{2}(1+\underbrace{\cos 2(\psi)}_{\text{goes to } 0 \text{ for } 1T \text{ integral}})}\right]}$$

$$= \sqrt{\lim_{T\to\infty}\frac{1}{T}\int_0^T A^2(t)\cdot\left(\frac{1}{2}+\frac{1}{2}\right)dt}$$

$$= \sqrt{\lim_{T\to\infty}\frac{1}{T}\int_0^T A^2(t)\,dt} = \sqrt{\lim_{T\to\infty}\int_0^1 A^2(t)\,dt} = \text{RMS}[A(t)] = 1$$

$$\mathrm{PAPR}[I_{BB}(t)] = 20\log\left(\frac{\mathrm{PEAK}(A(t))}{\mathrm{RMS}(A(t))}\right)$$

$$= 20\log\left(\frac{2.422}{1}\right) = \boxed{7.683\ \mathrm{dB}}$$

③

$$A(t)\left[\cos(w_c t)\cdot\cos(\psi(t)) - \sin(w_c t)\cdot\sin(\psi(t))\right]$$

$$S_{RF}(t) = A(t)\cos\left(w_c t + \psi(t)\right)$$

$$\mathrm{PEAK}(S_{RF}(t)) = \sqrt{\left(\frac{3}{\sqrt{20}}\right)^2 + \left(\frac{5}{\sqrt{20}}\right)^2} = 1.3$$

$$\mathrm{RMS}(S_{RF}(t)) = \sqrt{\lim_{T\to\infty}\frac{1}{T}\int_0^T A^2(t)\underbrace{\cos^2(w_c t + \psi(t))}_{\frac{1}{2}(1+\cos 2(w_c t+\psi(t)))}dt}$$

$$= \frac{1}{\sqrt{2}}\sqrt{\lim_{T\to\infty}\frac{1}{T}\int_0^T A^2(t)\,dt} = \frac{1}{\sqrt{2}}\mathrm{RMS}\,A(t)$$

$$= \frac{1}{\sqrt{2}}$$

$$\underset{(S_{RF}(t))}{\mathrm{PAPR}} = 20\log\left(\frac{1.3}{\frac{1}{\sqrt{2}}}\right) = \boxed{5.289\ \mathrm{dB}}$$

Case 2
Non null

For non-null SC, signal is transmitted

$w_{sc} \neq 0$

$$I_{RF} = A(t)\cos\left[(w_c + w_{sc})t + \psi(t)\right]$$

$$I_{BB}(t) = A(t)\cos\left[(w_c + w_{sc})t + \psi(t)\right] \cdot \cos\left[w_c t + \psi_0\right]$$

$$= \frac{1}{2}A(t)\left[\cos(2w_c t + w_{sc}t + \psi(t) + \psi_0) + \cos(w_{sc}t + \psi(t) - \psi_0)\right]$$

$$I(t) = A(t)\cos(w_{sc}t + \psi(t) - \psi_0)$$

$I(t)$

$$I(t) = A(t)\cos(w_{sc}t + \psi(t) - \psi_0)$$

$$= A(t)\left[\cos(w_{sc}t)\cdot\cos(\psi(t) - \psi_0) - \sin(w_{sc}t)\cdot\sin(\psi(t) - \psi_0)\right]$$

$$\text{PEAK}\left[I(t)\right] = \sqrt{\left(\frac{3}{\sqrt{20}}\right)^2 \left(\frac{5}{\sqrt{20}}\right)^2} = 1.3$$

$$\text{RMS}\left[I(t)\right]$$

$$= \sqrt{\frac{\frac{(1^2+3^2+5^2)\cdot 2 + (1^2+3^2)}{20}}{8}} = \frac{\sqrt{2}}{2}$$

$$\text{PAPR}\left(I(t)\right) = 20\log\left(\frac{1.118}{\frac{\sqrt{2}}{2}}\right) = \boxed{5.289\ dB}$$

$I_{BB}(t)$

$$I_{BB}(t) = A(t)\cos\left[(\omega_c + \omega_{sc})t + \psi(t)\right] \cdot \cos\left[\omega_c t + \phi_0\right]$$

$$= \frac{1}{2}A(t)\Big[\cos(\underbrace{2\omega_c t + \omega_{sc} t + \psi(t)}_{A}) + \cos(\underbrace{\omega_{sc} t + \psi(t) - \psi_0}_{B})\Big]$$

Ignore

$$= \frac{1}{2}A(t)\Big[\cos(2\omega_c t + \omega_{sc} t)\cdot\cos(\psi(t)) - \sin(2\omega_c t + \omega_{sc} t)\sin(\psi(t))$$
$$+ \cos(\omega_{sc} t)\cdot\cos(\psi(t) - \phi_0) - \sin(\omega_{sc} t)\cdot\sin(\psi(t) - \psi_0)\Big]$$

$$\text{Peak}(I_{BB}(t)) = 2\cdot\sqrt{\left(\frac{3}{\sqrt{20}}\right)^2 + \left(\frac{5}{\sqrt{20}}\right)^2} = 2.6$$

$$RMS(I_{BB}(t)) = \sqrt{\lim_{T\to\infty}\frac{1}{T}\int_0^T A^2(t)\,(\underbrace{\cos^2 A}_{\frac{1}{2}(1+\underbrace{\cos 2A}_{0})} + \underbrace{2\cos A\cos B}_{0} + \underbrace{\cos^2 B}_{\frac{1}{2}(1+\underbrace{\cos 2B}_{0})})\,dt}$$

over a integral of T period

$$= \sqrt{\lim_{T\to\infty}\frac{1}{T}\int_0^T A^2(t)\cdot 1\,dt} = RMS(A(t))$$
$$= 1 \quad = \text{constellation}$$

$$\boxed{PAPR(I_{BB}(t)) = 20\log\left(\frac{2.6}{1}\right) = 8.3\ \text{dB}}$$

S_{RF}

$$A(t)\cos((w_c+w_{sc})t)\cdot\cos(\psi(t)) - \sin((w_c+w_{sc})\cdot t)\sin(\psi(t)))$$

$$S_{RF}(t) = A(t)\underbrace{\cos\left((w_c+w_{sc})\,t+\psi(t)\right)}$$

$$\mathrm{PEAK}(S_{RF}(t)) = \sqrt{\left(\frac{3}{\sqrt{20}}\right)^2+\left(\frac{5}{\sqrt{20}}\right)^2} = 1.3$$

$$\mathrm{RMS}(S_{RF}(t)) = \sqrt{\lim_{T\to\infty}\frac{1}{T}\int_0^T A^2(t)\cos^2\underbrace{((w_c+w_{sc})t+\psi(t))}_{\frac{1}{2}(1+\cos(2(w_c+w_{sc})t+\psi(t)))}\,dt}$$

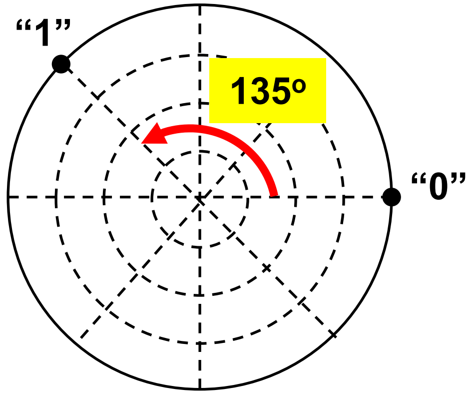
$$= \frac{1}{\sqrt{2}}\sqrt{\lim_{T\to\infty}\frac{1}{T}\int_0^T A^2(t)\,dt} = \frac{1}{\sqrt{2}}\,\mathrm{RMS}\,A(t)$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{2}$$

$$\mathrm{PAPR}(S_{RF}(t)) = 20\log\left(\frac{1.3}{\frac{1}{2}}\right) = \boxed{5.289\ dB}$$

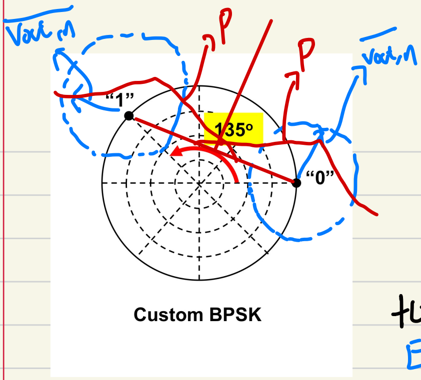
Question #3 [Chapter #3]: Bit-Error-Rate Derivation

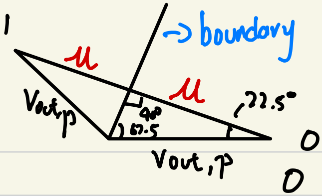
- Derive an approximate expression for the probability of bit error (also referred to as BER) in terms of Eb/No for a "custom" BPSK modulation shown in the figure below. Determine how much additional dB in Eb/No is required to support the proposed custom BPSK modulation relative to BPSK modulation to achieve the same BER of 10^{-3}. Note that decision boundary is based on demodulated output from both I and Q channels to minimize the BER.



Custom BPSK

Q3





σ: Standard deviation

μ: mean

General form of Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Equally Divide "0" and "1", so the boundary is no longer at 0, but at line shown above

Want to find μ: How much deviate from the average

In the regular BPSK, we find μ by $V_{out,p} \cdot \cos(\phi)$ $A(t)\cos[\phi(t)]$

So in our case $\mu = V_{out,p} \cdot \cos(180° - 90° - 67.5)°$

$= V_{out,p} \cdot \cos(22.5°)$

$$P_{e,bit0} = \int_0^\infty P_{e,bit0}(V_{out,n})dV_{out,n} = \int_0^\infty \left\{\left[\frac{1}{\sqrt{2\pi}V_{out,n}}\right] e^{-\frac{1}{2}\left(\frac{V_{out,n}+\overset{\mu}{V_{out,P}}}{V_{out,n}}\right)^2}\right\} dV_{out,n} = \int_{u=\frac{V_{out,P}}{V_{out,n}}}^{u=\infty} \left\{\left[\frac{1}{\sqrt{2\pi}}\right] e^{-\frac{u^2}{2}}\right\} du = Q\left(\frac{V_{out,P}}{V_{out,n}}\right) = Q\left(\frac{V_{in,P}\sqrt{T_b}}{V_{in,noise,1Hz}}\right) = Q\left(\sqrt{\frac{V_{in,P}^2 T_b}{V_{in,noise,1Hz}^2}}\right)$$

$$P_{e,bit0} = P_{e,bit1} \qquad P_{e,BPSK} = p_{bit0}P_{e,bit0} + p_{bit1}P_{e,bit1} = \left(\frac{1}{2}\right)P_{e,bit0} + \left(\frac{1}{2}\right)P_{e,bit1} = \boxed{Q\left(\sqrt{\frac{V_{in,P}^2 T_b}{V_{in,noise,1Hz}^2}}\right)}$$

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In our case:

$$\mu = \frac{V_{out,p}\cos(22.5°)}{V_{out,n}} = \frac{V_{out,p}T_b \cos(22.5°)}{\underbrace{V_{in,noise,1Hz} \cdot \sqrt{T_b}}_{V_{out,n}}} = \frac{V_{out,p}\sqrt{T_b}\cos(22.5°)}{V_{in,noise,1Hz}}$$

$$P_{e,bit1} = P_{e,bit0} = Q\left(\frac{V_{in,p}\sqrt{T_b}\cdot\cos(22.5°)}{V_{in,noise,1Hz}}\right) = Q\left(\sqrt{\frac{V_{in,p}^2 \cdot T_b}{V_{in,noise,1Hz}^2}} \cdot \cos(22.5°)\right)$$

$$P_{e,BPSK} = \frac{1}{2}P_{e\,bit0} + \frac{1}{2}P_{e\,bit1} = Q\left(\underbrace{\sqrt{\frac{V_{in,p}^2 \cdot T_b}{V_{in,noise,1Hz}^2}}}_{\sqrt{\frac{E_b}{N_0/2}}} \cdot \cos(22.5°)\right)$$

$N_0/2 = \overline{V_{in,noise,1Hz}^2}$

$E_b = \int_0^{T_b} P(t)dt = \int_0^{T_b} V_{in}^2(t)dt = V_{in,p}^2 \cdot T_b$

$$P_{e,\,custom\_BPSK} = Q\left(\sqrt{\frac{E_b \cdot 2}{N_0}} \cdot \overbrace{\cos(22.5^\circ)}^{0.924}\right)$$

$$P_{e\ BPSK} = Q\left(\sqrt{\frac{E_b \cdot 2}{N_0}}\right)$$

In dB scale, custom_BPSK is $\overbrace{20\log(\cos(22.5^\circ))}^{-0.688\,dB}$ less than the regular BPSK

So, custom BPSK need 0.688dB addition in $\frac{E_b}{N_0}$ to achieve the same BER of 10^{-3}

Question #4 [20 points]: 4G/LTE MIMO & CA Data Rate

- Verizon plans to deploy MIMO and CA technologies and support 64 QAM in all bands to enhance the data rate for its users. What is the maximum achievable downlink (DL) data rate for the following 3xCA DL configuration: B13/block C (SIMO) + B2/blocks E & F (2x2 MIMO) + B4/blocks B, E & F (4x4 MIMO) assuming the use of 3/5 code rate for all cases?

Band	Carrier	UE DL (MHz)	UE UL (MHz)
B2	VRZ, ATT, TMO	1930-1990	1850-1910
B4	VRZ, ATT, TMO	2110-2155	1710-1755
B5	VRZ, ATT, TMO	869-894	824-849
B12	ATT, TMO	729-746	699-716
B13	VRZ	746-756	777-787
B14	ATT	758-768	788-798
B17	ATT	734-746	704-716
B25	Sprint	1930-1995	1850-1915
B26	Sprint	859-894	814-849
B29	ATT	717-728	DL Only
B30	ATT	2350-2360	2305-2315
B41	Sprint	TDD: 2496-2690	
B66	VRZ, ATT, TMO	2110-2200	1710-1780
B71	TMO	617-652	663-698

Company	Carrier Aggregation
Verizon	B13 + B2 + B4
AT&T	B17 + B66 + B2 + B30
T-Mobile	B12 + B66 + B2
Sprint	B25 + B26 + B41

3GPP Block Definitions (A – J)

3GPP Band	Name	Downlink (MHz)	Uplink (MHz)
B2/B25	1900 PCS	1930-1990	1850-1910
AT&T	A	1930-1945	1850-1865
T-Mobile	B	1950-1965	1870-1875
Sprint	C	1975-1990	1895-1910
AT&T	D	1945-1950	1865-1870
Verizon	E	1965-1970	1875-1880
Verizon	F	1970-1975	1880-1885
Sprint	G	1990-1995	1910-1915
?	H	1995-2000	1915-1920

3GPP Band	Name	Downlink (MHz)	Uplink (MHz)
B4/B66	AWS-3	2110-2200	1710-1780
T-Mobile	A	2110-2120	1710-1720
Verizon	B	2120-2130	1720-1730
AT&T	C	2130-2135	1730-1735
AT&T	D	2135-2140	1735-1740
Verizon	E	2140-2145	1740-1745
Verizon	F	2145-2155	1745-1755
Verizon	G	2155-2160	1755-1760
AT&T	H	2160-2165	1760-1765
AT&T	I	2165-2170	1765-1770
Verizon	J	2170-2180	1770-1780

3GPP Band	Name	Downlink (MHz)	Uplink (MHz)
B5/B26	850	869-894	824-849
Verizon	A	869-880	824-835
Verizon	A	890-891.5	845-846.5
Verizon	B	880-890	835-845
Verizon	B	891.5-894	846.5-849

3GPP Band	Name	Downlink (MHz)	Uplink (MHz)
B12	700 a	729-746	699-716
AT&T	B	734-740	704-710
AT&T	C	740-746	710-716

3GPP Band	Name	Downlink (MHz)	Uplink (MHz)
B13	700 c	746-756	777-787
Verizon	C	746-756	777-787

3GPP Band	Name	Downlink (MHz)	Uplink (MHz)
B14	700 PS	758-768	788-798

3GPP Band	Name	Downlink (MHz)	Uplink (MHz)
B17	700 b	734-746	704-716
AT&T	B	734-740	704-710
AT&T	C	740-746	710-716

3GPP Band	Name	Downlink (MHz)	Uplink (MHz)
B29	700 d	717-728	DL Only
AT&T	E	722-728	-

3GPP Band	Name	Downlink (MHz)	Uplink (MHz)
B30	2300 WCS	2350-2360	2305-2315
AT&T	A	2350-2355	2305-2310
AT&T	B	2355-2360	2310-2315

3GPP Band	Name	Downlink (MHz)	Uplink (MHz)
B41	TD 2600+	2496-2690	TDD
Sprint	B2-D3	2524-2572	TDD
Sprint	D4-G4	2590-2596	TDD
Sprint	E1-G3	2624-2690	TDD

3GPP Band	Name	Downlink (MHz)	Uplink (MHz)
B71	600	617-652	663-698
T-Mobile	C	627-632	673-678
T-Mobile	D	632-637	678-683
T-Mobile	E	637-642	683-688
Dish	F	642-647	688-693
Dish	G	647-652	693-698

Company	Bands	Frequencies
Verizon	2, 4, 5, 13, 66	1900, 1700 f, 700 c
AT&T	2, 4, 5, 12, 14, 17, 29, 30, 66	1900, 1700 abcde, 700 bc
T-Mobile	2, 4, 5, 12, 66, 71	1900, 1700 def, 700 a, 600
Sprint	25, 26, 41	1900 g, 850, 2500

Company	Carrier Aggregation
Verizon	B13 + B2 + B4
AT&T	B17 + B66 + B2 + B30
T-Mobile	B12 + B66 + B2
Sprint	B25 + B26 + B41

Note: Information may not be 100% up to date.

Step1

Config 1 B13 / block C (SIMO) DL

DL: 746-756 MHz BW= 10 MHz

Config2 B2 / block E & F (2x2 MIMO)

DL E 1965-1970 MHz BW,E = 5 MHz

DL F 1970-1975 MHz BW,F = 5 MHz

Total = 10 MHz

Config3 B4 / blocks B, E and F (4x4 MIMO)

DL B 2120-2130 MHz BW,B = 10 MHz

DL E 2140-2145 MHz BW,E = 5 MHz

DL F 2145-2155 MHz BW,F = 10 MHz

$10ms \qquad 1ms$
1 frame = 10 subframe
$= 20 \overset{0.5ms}{\text{slots}}$
1 subframe = 2 slots

STEP2

Step 2: Calculate the throughput for SISO for a bandwidth of 10M:
- (i) Determine the number of frames per second. Determine the number of subframes per second. Determine the number of slots per second.
- (ii) Determine the number of symbols per slot.
- (iii) Determine the number of symbols per second per subcarrier.
- (iv) Determine the number of subcarriers.
- (v) Determine the number of symbols for all the subcarriers.
- (vi) Determine the number of bits per symbol.
- (vii) Determine the number of bits for all the subcarriers. Convert this to Mbps for maximum data rate per SISO channel.

(i) In 4G 1 frame = 10ms ; 1 subframe = 1ms

1 s ÷ 10 ms / frame = 100 frame ; # subframes / sec = 1 s ÷ 1ms / subframe
= 1000 subframe

1 slot = 0.5ms
1 s ÷ 0.5ms / slot = 2000 slots

(ii) In 4G 7 symbols per slot

(iii) $2000 \cdot 7 = 14000$ sym/s per subcarrier

(iv) In 4G, 600 subcarriers (9 MHz ÷ 15 kHz/scs) = 600

(v) $14000 \cdot 600 = 8{,}400{,}000$ symbols

(vi) 64 QAM = 2^6 => 6 bits per symbol

(vii) $8{,}400{,}000 \cdot 6 = 50.4$ Mbps

- Step 3:
  - (i) Determine the data rate for SIMO for B13 using the result in step 2 (vii)
  - (ii) Determine the data rate for 2x2 MIMO for B2 using the result in step 2 (vii)
  - (iii) Determine the data rate for 4x4 MIMO for B4 using the result in step 2 (vii)
  - (iv) Using results from step 3 (i)-(iii), determine the maximum achievable DL throughput (remember to factor in the coding rate).
  - (v) Determine minimum SNR required in each band to achieve the data rates determined in (i), (ii) and (iii)

i) SIMO ⇒ spatial stream = 1

BW = 10 MHz

Data rate = 50.4 Mbps · 1 = $\boxed{50.4 \text{ Mbps}}$

ii) 2x2 MIMO ⇒ spatial stream = 2

BW = 10 MHz

Data rate = 50.4 Mbps · 2 = $\boxed{100.8 \text{ Mbps}}$

iii) 4x4 MIMO ⇒ Spatial stream = 4

BW = 25 MHz (10M + 15 MHz)

$$\text{Data rate}_{,tot} = \underset{10MHz}{\text{Data rate}_{B}} + \underset{15MHz}{\text{Data rate}_{,EF}}$$

$$= 50.4 \cdot 4 \qquad 50.4 \cdot \frac{15}{10} \cdot 4$$

$$= \boxed{504 \text{ Mbps}}$$

iv)

$$\text{Throughput} = (50.4 + 100.8 + 504) \cdot \frac{3}{5}$$

$$= 655.2 \cdot \frac{3}{5}$$

$$= \boxed{393.12 \text{ Mbps}}$$

v)

$$C \approx W \cdot n \cdot \log_2(1 + SNR)$$

For B13

$W = 10$ MHz

$n = 1$

$C = 50.4$ Mbps

$$SNR = 2^{\frac{C}{W \cdot n}} - 1$$

$$= 2^{\frac{50.4}{10M \cdot 1}} - 1 = 31.9 = \boxed{15.04 dB}$$

For B2

$W = 10$ MHz

$n = 2$

$C = 100.8$ Mbps

$$SNR = 2^{\frac{100.8}{10M \cdot 2}} - 1$$

$$= 31.9$$

$$= \boxed{15.04 dB}$$

For B4

$W = 25$ MHz

$n = 4$

$C = 504$

$$SNR = 2^{\frac{504}{25 \cdot 4}} - 1$$

$$= 31.4$$

$$= \boxed{15.04 dB}$$