



Q1

**Problem 1: Equivalent Resistance**

Consider NMOS device with  $W = 240\text{nm}$  and  $L = 100\text{nm}$  (effective  $L = 70\text{nm}$ ). Using the resistor averaging technique discussed in class, and parameters below, calculate  $R_{on}$  as  $V_{DD}$  changes from  $0.4\text{V}$  to  $1\text{V}$  in steps of  $0.2\text{V}$ . Compare the results of your hand calculations with simulation results in Table 2.

Use following parameters for hand calculations:

$$\begin{aligned}\lambda &= 0.795 \text{ V}^{-1} \\ k' &= 129.5 \mu\text{A/V}^2 \\ L_{eff} &= 70 \text{ nm} \\ V_{th} &= 0.168 \text{ V} \\ V_{dsat} &= 0.3 \text{ V}\end{aligned}$$

Table 2:  $R_{on}$  (Analytical vs. Simulation)

$R_{on}$	$I_{DSAT} (\mu\text{A})$	Analytical Model ( $R_{eq}$ )	Simulation 2-point avg
$V_{DD} = 0.4\text{V}$	10.9224	20.188 kΩ	27.8 kΩ
$V_{DD} = 0.6\text{V}$	37.56	7.218 kΩ	9.0 kΩ
$V_{DD} = 0.8\text{V}$	64.2	5.631 kΩ	6.5 kΩ
$V_{DD} = 1.0\text{V}$	90.84	2.786 kΩ	5.0 kΩ

$$I_{DSAT} = k' \frac{W}{L} \left( [V_{DD} - V_{th}] V_{DSAT} - \frac{V_0^2 \gamma \lambda}{2} \right)$$

$$R_{on} = \frac{3}{4} \frac{V_0}{I_{DSAT}} \left( 1 - \frac{5}{6} \lambda V_{DD} \right)$$

@  $V_{DD} = 0.4\text{V}$

$$I_{DSAT} = 129.5 \mu\text{A} \cdot \frac{240\text{n}}{70\text{n}} \left( [0.4 - 0.168] \cdot 0.3 - \frac{0.3^2}{2} \right) = 10.9224 \mu\text{A}$$

$$R_{on} = \frac{3 \cdot 0.4}{4 \cdot 10.9224 \mu\text{A}} \cdot \left( 1 - \frac{5}{6} \cdot 0.795 \cdot 0.4 \right) = 20.188 \text{ kΩ}$$

@  $V_{DD} = 0.6\text{V}$

$$I_{DSAT} = 129.5 \mu\text{A} \cdot \frac{240\text{n}}{70\text{n}} \left( [0.6 - 0.168] \cdot 0.3 - \frac{0.3^2}{2} \right) = 37.56 \mu\text{A}$$

$$R_{on} = \frac{3 \cdot 0.6}{4 \cdot 37.56 \mu\text{A}} \cdot \left( 1 - \frac{5}{6} \cdot 0.795 \cdot 0.6 \right) = 7.218 \text{ kΩ}$$

@  $V_{DD} = 0.8\text{V}$

$$I_{DSAT} = 129.5 \mu\text{A} \cdot \frac{240\text{n}}{70\text{n}} \left( [0.8 - 0.168] \cdot 0.3 - \frac{0.3^2}{2} \right) = 64.2 \mu\text{A}$$

$$R_{on} = \frac{3 \cdot 0.8}{4 \cdot 64.2 \mu\text{A}} \cdot \left( 1 - \frac{5}{6} \cdot 0.795 \cdot 0.8 \right) = 5.631 \text{ kΩ}$$

$$\textcircled{2} V_{DD} = 1 \text{ V}$$

$$I_{DSAT} = 129.5 \mu \cdot \frac{240 \text{ n}}{7 \text{ m}} \left( [1 - 0.168] \cdot 0.3 - \frac{0.3^2}{2} \right) = 90.84 \mu \text{A}$$

$$R_{on} = \frac{3 \cdot 0.6}{4 \cdot 37.56 \mu} \ln \left( 1 - \frac{5}{6} \cdot 0.795 \cdot 0.6 \right) = 2.786 \text{ k}\Omega$$

There is a big difference between simulation results and hand calculations

B2

**Problem 2: Inverter in Subthreshold**

The inverter below, operates with  $V_{DD} = 0.175V$  and is composed of  $V_{Th} = |V_{Tp}| = 0.20V$  devices. They can be described by the subthreshold voltage-current relation. The devices have identical  $I_S$  and  $n$ , ( $n=1.5$  and  $kT/q=26mV$ ).

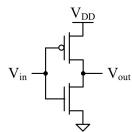


Figure 2: Inverter in Subthreshold.

- 2A** Calculate the switching threshold ( $V_M$ ) of this inverter.

If the equation is difficult to solve, plot both sides and find the intersection point for a numerical answer.

$$I_0 = I_S e^{\frac{V_{DS}}{nKT/q}} \left( 1 - e^{\frac{V_{DS}}{nKT/q}} \right)$$

$$\text{Set } I_{0,N} = I_0, p$$

$$I_S e^{\frac{V_M}{1.5 \cdot 26m}} \left( 1 - e^{\frac{V_M}{26m}} \right) = I_S e^{\frac{0.175 - V_M}{1.5 \cdot 26m}} \left( 1 - e^{\frac{0.175 - V_M}{26m}} \right)$$

Solved by symbolab  
 $V_M = 0.0875V$

- 2B** Calculate  $V_{IL}$  and  $V_{IH}$  of the inverter.

You may find piecewise-linear approximation helpful.

$$V_{IH} = V_M + V_M/g$$

$$V_{IL} = V_M - \frac{V_{DD} - V_M}{1/g}$$

$$g = -\frac{1}{n} \cdot \left( e^{\frac{0.175}{2.26m}} - 1 \right) = -18.63$$

$$V_{IH} = 0.0875 - \frac{0.0875}{-18.63} = 0.0922 \text{ V}$$

$$V_{IL} = 0.0875 - \frac{0.175 - 0.0875}{-18.63} = 0.0828 \text{ V}$$

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$$V_{GS,op} = 0.175 - V_M < |V_{TP}|$$

$$V_{GS,N} = V_M < V_{Th}$$

Due to  $V_{DD} < |V_{TN}|$   
 or  
 $|V_{TP}|$

B3

**Problem 3: MOS Capacitance**

Use the following table.

**Assumptions:** $|V_{TH}| = 0.2V$  for both NMOS and PMOS

For the purpose of capacitance calculation (only), treat velocity saturation as saturation

Param.	$C_{ox}$ [fF/ $\mu\text{m}^2$ ]	$C_o$ [fF/ $\mu\text{m}$ ]
NMOS	15	0.27
PMOS	14	0.25

$$\frac{1}{\text{cm}} \cdot \frac{10^4 \mu\text{m}}{\text{cm}}$$

**3A** What is the  $t_{ox}$  (nm) of the NMOS transistor?

$$C_{ox} = \frac{\epsilon_0 \kappa}{t_{ox}}$$

$$t_{ox} = \frac{\epsilon_0 \kappa}{C_{ox}} = \frac{3.5 \cdot 10^{-13} \frac{\text{F}}{\text{cm}} \cdot \frac{1 \text{ cm}}{10^9 \mu\text{m}}}{15 \frac{\text{fF}}{\mu\text{m}^2} \cdot \frac{10^{15} \text{ F}}{1 \text{ fF}}} = \frac{3.5 \cdot 10^{-13} \frac{\text{F}}{\text{cm}}}{15 \cdot 10^4 \frac{\text{F}}{\mu\text{m}}} \frac{\text{F}}{\mu\text{m}}$$

$$= 15 \cdot 10^{-15} \frac{\text{F}}{\mu\text{m}^2}$$

$$= 0.0023 \mu\text{m} \frac{10^3 \text{ nm}}{1 \mu\text{m}} = 2.3 \text{ nm}$$

**3B** Consider a PMOS biased with  $V_G = V_D = V_S = V_B = 0V$ Assume  $W = 480 \text{ nm}$ ,  $L = 120 \text{ nm}$ ,  $L_D = L_S = 240 \text{ nm}$ 

Calculate the following capacitances:

(B1) Gate-to-Channel capacitance ( $C_{GC}$ )(B2) Gate-to-Source capacitance ( $C_{GS}$ )Since there is 0V drop in PMOS, it's in **cutoff region**

$$\text{B1)} C_{GC} = C_{ox} W_{\text{Left}} = 14 \cdot 10^{-15} \frac{\text{F}}{\mu\text{m}^2} \cdot 480 \cdot 10^3 \mu\text{m} \cdot 120 \cdot 10^3 \mu\text{m}$$

$$= 8.064 \cdot 10^{-16} \text{ F}$$

$$\begin{aligned} \text{B2)} C_{GS} &= C_{GSD} + C_{GSO} \\ &= 0 + C_D \cdot W \\ &\approx 0.27 \cdot 10^{-15} \frac{\text{F}}{\mu\text{m}} \cdot 480 \cdot 10^3 \mu\text{m} \\ &= 1.296 \cdot 10^{-16} \text{ F} \end{aligned}$$

- 3C Consider an NMOS biased with  $V_G = V_D = 0.8V$ ,  $V_S = V_B = 0V$   
 Assume  $W = 240\text{ nm}$ ,  $L = 120\text{ nm}$ ,  $L_D = L_S = 240\text{ nm}$   
 Calculate the following capacitances:  
 (C1) Gate-to-Channel capacitance ( $C_{GC}$ )  
 (C2) Gate-to-Source capacitance ( $C_{GS}$ )

$$V_{DS} = 0.8V$$

$$V_{GS} = 0.8V$$

$$V_{GT} = V_{GS} - V_{TN}$$

$$= 0.8 - 0.2 = 0.6V$$

$$V_{DSAT,N} < V_{GT} < V_{DS}$$

so it's in V-sat-

NMOS

$$C_{GC} = \frac{2}{3} \cdot C_{ox} \cdot W \cdot L_{eff}$$

$$(1) \quad = \frac{2}{3} \cdot 15 \cdot 10^{-15} \frac{F}{\mu m^2} \cdot 0.24 \mu m \cdot 0.12 = \boxed{2.88 \cdot 10^{-16} F}$$

$$(2) \quad C_{GS} = C_{GCS} + C_{GSO}$$

$$= 2.88 \cdot 10^{-16} F + C_o \cdot W$$

$$= 2.88 \cdot 10^{-16} F + 0.27 \cdot 10^{-15} \frac{F}{\mu m} \cdot 0.24$$

$$= 3.528 \cdot 10^{-16} F$$

**Problem 4: Computing Capacitance**

Consider the circuit in Figure 4. Calculate the total equivalent capacitance on node X as it charges from 0 to  $V_{DD}/2$ .

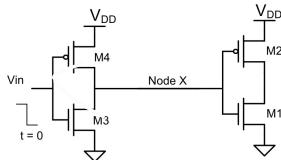


Figure 4

You can use the following capacitance values in your calculations:

$$\begin{aligned} C_{db\_NMOS} &= 0.06 \text{ fF}, & C_{db\_PMOS} &= 0.10 \text{ fF} \\ C_{gd\_NMOS} &= 0.14 \text{ fF}, & C_{gd\_PMOS} &= 0.26 \text{ fF} \\ C_{g1} &= 0.16 \text{ fF}, & C_{g2} &= 0.30 \text{ fF} \end{aligned}$$

At  $t=0$   $M_4, M_2$  is On.

$$\begin{aligned} C &= C_{db\_PMOS} + C_{gd\_PMOS} + C_{g2} \\ &= 0.10 + 0.26 + 0.30 = 0.66 \text{ fF} \end{aligned}$$

when it charges to  $V_{DD}/2$ ,  $M_1, M_3$  are also on

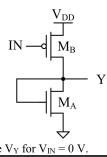
$$\begin{aligned} C &= C_{db\_NMOS} + C_{gd\_NMOS} + C_{g1} \\ &= 0.06 + 0.14 + 0.16 = 0.36 \text{ fF} \end{aligned}$$

$$C_{\text{Total}} = 0.66 \text{ fF} + 0.36 \text{ fF} = \boxed{1.02 \text{ fF}}$$

Q5

**Problem 5: VTC**

Consider the inverter below, with the input at the gate of the PMOS and a diode-connected NMOS.

(a) Calculate  $V_Y$  for  $V_{IN} = 0 \text{ V}$ .

Device parameters:

- $V_{TN} = 0.2 \text{ V}$ ,  $V_{TP} = -0.2 \text{ V}$
- $|V_{DSATP}| = 0.3 \text{ V}$
- $K_n = 130 \mu\text{A}/\text{V}^2$
- $K_p = -100 \mu\text{A}/\text{V}^2$
- $(W/L)_n = 1$
- $(W/L)_p = 4$
- $\lambda = 0$ ,  $y = 0$  for both transistors
- $V_{DD} = 1 \text{ V}$

$$\begin{aligned} V_{GT} &= |V_{SG} - V_{T,p}| \\ &= |1 - 0.2| : 0.8 \end{aligned}$$

$$V_{in} = 0 \text{ V}.$$

(a) Calculate  $V_Y$  for  $V_{IN} = 0 \text{ V}$ .

$$V_Y = 0.926 \text{ V}$$

$$I_{d, \text{NMOS}} = I_{d, \text{PMOS}}$$

$V_B$  is Linear region, because it turns on.

Assuming  $V_A$  is in V-sat

$$130 \mu\text{A} \cdot 1 \cdot \left( (V_Y - 0.2) \cdot 0.3 - \frac{0.3^2}{2} \right) = |-130 \mu\text{A}| \cdot 4 \cdot \left( 0.8 \cdot (1 - V_Y) - \frac{(1 - V_Y)^2}{2} \right)$$

Solved By Symbolab  
 $V_Y = 0.926 \text{ V}$

$$\text{NMOS: } V_{DS} = 0.926 \text{ V} \quad V_{GT} = V_{GS} - 0.2 = 0.726 \text{ V}$$

$\therefore V_{DSAT}$  is the smallest, then it's in V-sat region

$$\begin{aligned} \text{PMOS: } |V_{DS}| &= |1 - 0.926| = 0.074 \text{ V} \\ |V_{GT}| &= |0 - 1| - 0.2 = 0.8 \text{ V} \end{aligned}$$

$$\therefore V_{DS,p} < V_{GT,p} \text{ & } V_{DS,p} < |V_{DSATP}|$$

$\therefore$  it's in Linear region

(b) Calculate  $V_Y$  for  $V_{IN} = V_{DD}$ .

$$V_Y = 0.2V$$

$M_B$  off, but  $M_A$  can't be turned off with 0V at  $V_{GS}$  and  $V_{DS}$ . Thus, the minimum voltage is required for  $M_A$  to work. So  $V_Y$  is at least need to be 0.2V

(c) Ignoring the results from (a) and (b), calculate switching threshold  $V_M$  for the inverter. Is  $V_M$  expected to be greater or smaller than 0.5V? (circle the correct answer)

$V_M > 0.5V$

$V_M > 0.5V$

$$V = IR$$

The  $\frac{W}{L}$  of PMOS is greater than  $\frac{W}{L}$  of NMOS. It means PMOS has smaller resistance than NMOS.

Meanwhile, two transistors are in series, same current.

Since, by Ohm's Law, smaller resistance results in smaller voltage drop at PMOS, it is possible to have  $V_M > 0.5V$ .

What are the modes of operation for M<sub>A</sub> and M<sub>B</sub>?

M<sub>A</sub>: V-Sat

M<sub>B</sub>: SAT

Calculate the value of V<sub>M</sub>.

$$V_M = 0.59V$$

M<sub>B</sub> PMOS:  $V_{DS} : 1 - V_m = 0.41V$

| V<sub>G1</sub> |:  $V_{GS} \cdot V_{th} : |1-0.2-V_m| = 0.8-V_m \rightarrow 0.2 |$  smallest. (v)

→ check

$$V_{DSAT} = 0.3V$$

M<sub>A</sub> NMOS:  $V_{DS} : V_m = 0.59V$

V<sub>G1</sub>:  $V_m - 0.2 = 0.39V$

→ check  $V_{DSAT} = 0.3V \rightarrow$  smallest (v)

Set  $I_p = I_n$ .  $I_0 = k' \frac{W}{2} (V_{G1} \cdot V_m - \frac{V_{min}^2}{2})$

SAT  $100\mu \cdot 4 \cdot \left( \frac{(0.8-V_m)^2}{2} \right) = 130\mu \cdot 1 \cdot \left( (V_m-2) \cdot 0.3 - \frac{0.3^2}{2} \right)$

$$V_m = 0.59V$$