GEC Workshop | Summer 2022

**Discussion 3**

# Digital Integrated Circuits

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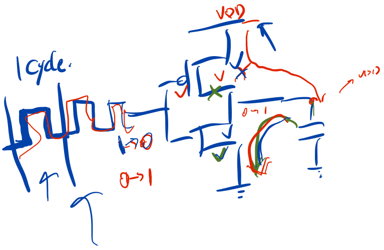


① charging
- Energy from power supply $C \cdot V_{DD}^2$ ①
  - ||
- Heat dissipation $\frac{1}{2} C V_{DD}^2$ ①
  - +
- Energy stored on cap $\frac{1}{2} C V_{DD}^2$ ②

+

② discharging
- Energy from power supply $= 0$
- heat dissipation $= \frac{1}{2} C V_{DD}^2$ ②

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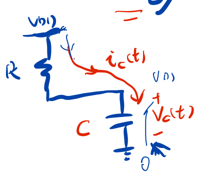
Energy in charging

① Energy from power supply




$Q(t) = C \cdot V_c(t)$ — ①

$\frac{dQ}{dt} = i_c(t) = C \cdot \frac{dV_c(t)}{dt}$

$P_{VDD} = V_{DD} \cdot i_c(t)$

$E_{VDD} = \int P_{VDD} \cdot dt = \int V_{DD} \cdot i_c(t) \cdot dt$

$= \int V_{DD} \cdot C \cdot \frac{dV_c(t)}{\cancel{dt}} \cdot \cancel{dt}$

$= \int_0^{V_{DD}} V_{DD} \cdot C \cdot dV_c(t)$

$= \int_0^{V_{DD}} V_{DD} \cdot C \cdot dV$

$= C \cdot V_{DD}^2$

$\int_{V_i}^{V_o} V_{DD} \cdot dV$

$= V_{DD} \cdot C (V_o - V_i)$

② Energy stored on cap


$\Delta V = \frac{\Delta Q}{C}$

$\Delta Q = \Delta V \cdot C$

$\Delta E = \Delta Q \cdot V$

$\Delta E = \Delta V \cdot V \cdot C$

$\int \Delta E = \int C \cdot V \cdot dV$

$= \int_0^{V_{DD}} C \cdot V \cdot dV$

$= \frac{1}{2} C \cdot V^2 \Big|_0^{V_{DD}}$

$= \frac{1}{2} C V_{DD}^2$

$\int_{V_i}^{V_o} C \cdot V dV$

$= \frac{1}{2} C (V_o^2 - V_i^2)$

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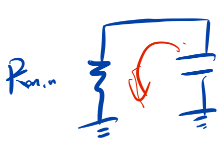
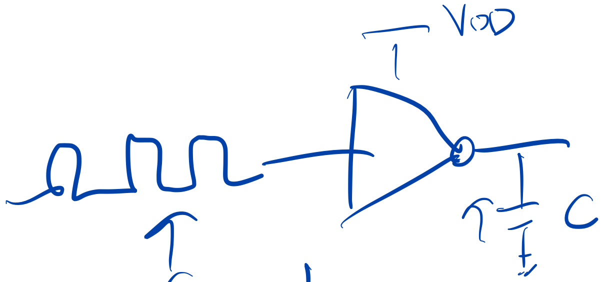
discharging phase




$E_{supply} = 0$

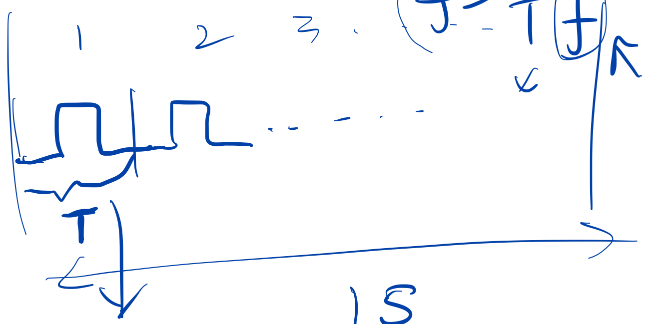
$E_{heat} = E_{store} = \frac{1}{2} C V_{DD}^2$




$P = ?$

$$f = \frac{1}{T}$$

$$f = \frac{1}{T} \quad f$$




$C \cdot V_{DD}^2$

$$P = \frac{E}{T}$$

$$P = f \cdot C \cdot V_{DD}^2$$

# Digital Electronic Circuits
## Extra Problem *(Discussion)*

### Finding propagation delay

**a)** A CMOS inverter drives an identical copy of itself. Find the total load capacitance $C_L$ by the driver. Use the parameters provided in the table. Ignore wire capacitance. Given $\left(\frac{W}{L}\right)_p = \frac{3\mu m}{1\mu m}$, and $\left(\frac{W}{L}\right)_n = \frac{1\mu m}{1\mu m}$.

**b)** Find $t_{pLH}$ for the inverter driving a total capacitance of $C_L$ from last problem. Assume an ideal step voltage on $V_{in}$ and approximate the charging current by averaging the initial and final drain currents.

**c)** Suppose the inverter drives a total capacitance of $45fF$ at $4.1GHz$. How much power does it consume assuming $V_{DD} = 5V$?

$P = C \cdot V^2 \cdot f$

| Parameter | PMOS | NMOS |
|---|---|---|
| $V_{T0}$ $(V)$ | -0.8 | 0.8 |
| $\mu C_{ox}$ $(\mu A/V^2)$ | 100 | 300 |
| $\gamma$ $(V^{1/2})$ | 0 | 0 |
| $W_{min}$ $(\mu m)$ | 1.0 | 1.0 |
| $L_{min}$ $(\mu m)$ | 1.0 | 1.0 |
| $\lambda$ $(V^{-1})$ | 0.0 | 0.0 |
| VDD $(V)$ | 5 | |

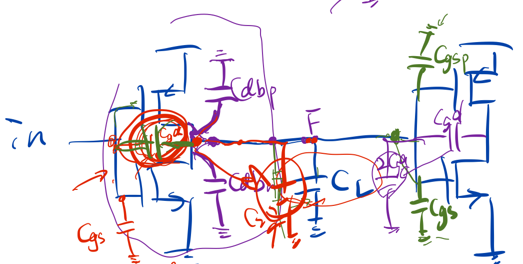
| Capacitance/Width $(fF/\mu m)$ | PMOS | NMOS |
|---|---|---|
| $C_{gs}$ | 1.5 | 1.4 |
| $C_{gd}$ | 1.5 | 1.4 |
| $C_{db}$ | 1.0 | 0.9 |
| $C_{sb}$ | 1.0 | 0.9 |


1st stage　　2nd stage

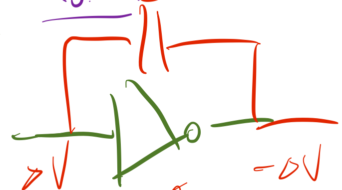
$C_L = C_{int} + C_{ext}$

$(C_{dbp} + C_{dbn}) + 2\cdot(C_{gdp} + C_{gdn})$　　$C_{gsp} + C_{gsn} + 2(C_{gdp} + C_{gdn})$

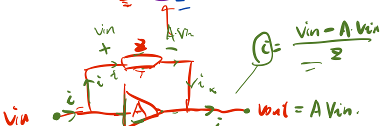
miller effect




$C_{gd} \Rightarrow Z_C = \frac{1}{j\omega C_{gd}}$



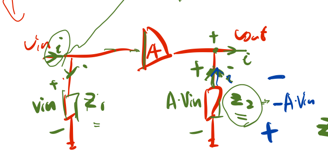

$A \approx -1$




$i = \frac{V_{in} - A V_{in}}{Z} = \frac{V_{in}(1-A)}{Z}$

miller effect

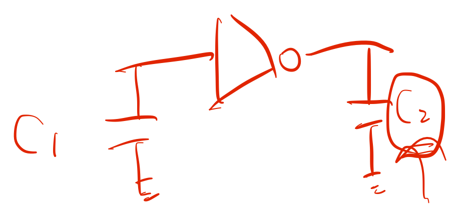
equivalent circuit




$Z_1 = \frac{V_{in}}{i} = \frac{V_{in}}{\frac{V_{in}(1-A)}{Z}} = \frac{Z}{1-A}$

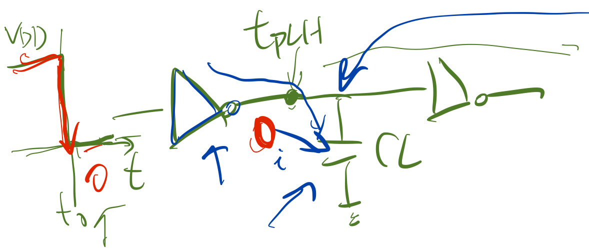
$Z_2 = \frac{-A \cdot V_{in}}{i} = \frac{-A V_{in}}{\frac{V_{in}(1-A)}{Z}} = -\frac{AZ}{1-A} = \frac{Z}{1-\frac{1}{A}}$

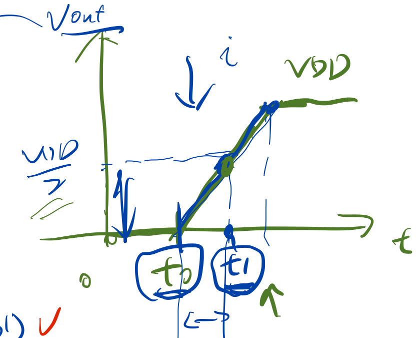
$Z_2 = \frac{Z}{1-A} = \frac{\frac{1}{j\omega C_{gd}}}{1+1}$




$= \frac{1}{2 j\omega C_{gd}}$

$= \frac{1}{j\omega C_2}$




$$t_{pLH} = \frac{Q}{I_{avg}} = \frac{C_L \cdot \Delta V}{I_{avg}} = \frac{C_L \cdot \frac{V_{DD}}{2}}{I_{avg}}$$

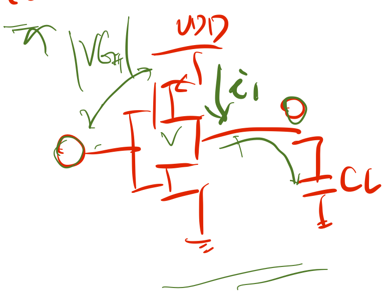
$t_{pLH} = t_1 - t_0$

@ $t_0$ = $i_1$ — initial

@ $t_1$ = $i_2$ — final

$$i_{avg} = \frac{i_1 + i_2}{2}$$

@ $t_0^+$




$|V_{GT}| = |V_{DD}| - |V_t| = 5 - 0.8 = 4.2V$

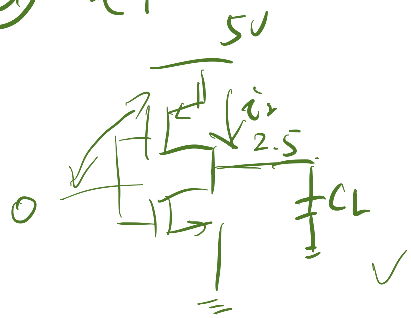
$|V_{DS}| = |V_{DD}|$

$|V_{thp}| = 0.8V$

$V_{DD} = 5V$

$$i_1 = \frac{\mu C_{ox}}{2} \left(\frac{W}{L}\right)_p \cdot |V_{GTp}|^2$$

@ $t_1$

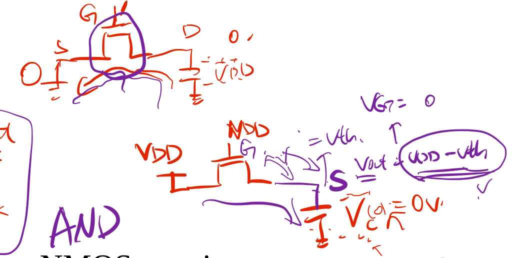



$|V_{GTP}| = 4.2V$

$|V_{DS}| = 2.5V$

$\Rightarrow$ linear $(|V_{DS}| < |V_{GT}|)$

$$i_2 = \frac{\mu C_{ox}}{2} \left(\frac{W}{L}\right)_p \left(2|V_{GT}||V_{DS}| - |V_{DS}|^2\right)$$

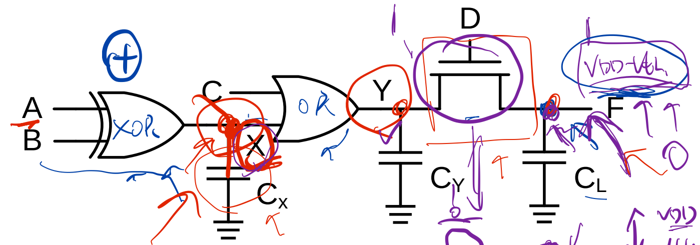


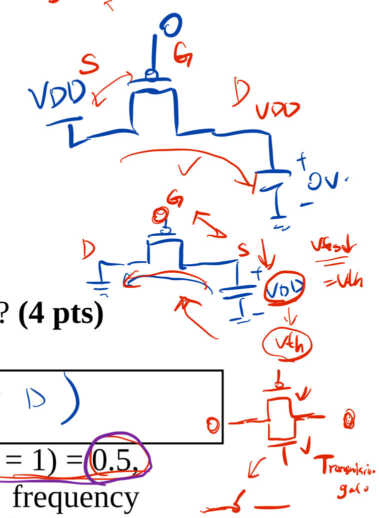
| Vin | VG | Vout |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

AND

**Problem 2: Power and Energy (20 pts)**

Consider the following circuit, with XOR, OR gates and one NMOS transistor.



(a) What logic function is implemented by this circuit (inputs: A, B, C, and D)? **(4 pts)**

or and xor

| F = f(A B C D) |
|---|

(b) Assume the probably of logic 1 for inputs: p(A = 1) = 0.5, p(B = 1) = 0.5, p(C = 1) = 0.2, p(D = 1) = 0.3, capacitance: $C_X = C_Y = 10$ fF, $C_L = 20$ fF, frequency f = 100 MHz, $V_{DD} = 1$V, threshold voltage $V_{TN} = 0.2$ V, $V_{TP} = -0.3$ V. Calculate the average switching power $P_{sw}$ of the circuit (logic gates and input D are powered from $V_{DD}$). **(12 pts)**

XOR

| A | B | X |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

$$P_{sw} = P_{sw,X} + P_{sw,Y} + P_{sw,F}$$

$$P = \boxed{\alpha \cdot C \cdot V^2 \cdot f}$$

$$P(X = 0 \to 1) = \alpha_{X\,0\to1}$$

$$= \boxed{P(X=0)} \cdot P(X=1)$$

$$\{1 - P(X=0)\}$$

| p(X = 1) = |
|---|
| $\alpha_{X:0\to1}$ = |

| X | C | Y |
|---|---|---|
| 0 | 0 | |
| 0 | 1 | |
| 1 | 0 | |
| 1 | 1 | |

$$P(X=0) = P(A=0)\,P(B=0) + P(A=1) \cdot P(B=1)$$

$$(1 - P(A=1))$$

| p(Y = 1) = |
|---|
| $\alpha_{Y:0\to1}$ = |

$$P = \alpha \cdot C \cdot V_{DD}^2 f = (\alpha_X \cdot C_X + \alpha_Y \cdot C_Y) \cdot V_{DD}^2 f$$

| $P_{sw,X} + P_{sw,Y}$ = |
|---|

| D | Y | F |
|---|---|---|
| 0 | 0 | |
| 0 | 1 | |
| 1 | 0 | |
| 1 | 1 | |

| |
|---|
| **p(F = 1) =** |
| **$\alpha_{F:0\to1}$ =** |

$= \alpha_F \cdot C_F \cdot V_{DD} \cdot (V_{DD} - V_{th}) \cdot f$

| |
|---|
| **$P_{sw,F}$ =** |

(c) Calculate the heat energy dissipation for charging and discharging $C_X$, $C_Y$, and $C_L$. **(8 pts)**

charging

$E_{supply} = C \cdot V_{DD} \cdot (V_o - V_i)$

$E_{cap} = \frac{1}{2} C (V_o^2 - V_i^2)$

$V_i = 0$

$E_{heat} = E_{supply} - E_{cap}$

discharging

$E_{heat} = E_{cap}$

| Heat Energy | Charging | Discharging |
|---|---|---|
| Node X ($C_X$) | | |
| Node Y ($C_Y$) | | |
| Node F ($C_L$) | $V_o = V_{DD} - V_{th}$ | |
| **Total $E_{heat}$** | | |

$F = (A(B+C))'$