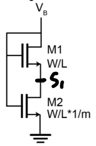
Q1

1. **Magic Battery Biasing [15 pts]**
Consider the circuit shown below. This is the circuit that was shown in several of the biasing schemes shown in the lecture. Remember that the "*m*" factor in this problem is always an integer number (and less than 20 in ECE 164), indicating the fractional *W/L* for the "lower" device (M_2) as compared to the "upper" device (M_1) *W/L*. Assume both devices have the same V_t and k_n'. Neglect channel length modulation.
   a) What are all the possible permutations for the region of operation for both transistors in this circuit? Support how you arrived at your answer through calculations (harder) or device physics (easier).
   b) Derive an expression for V_{DS} for M_2 as a function of V_B.
   c) Derive an expression to determine V_B for a given I_D.
   d) Determine the appropriate "*m*" value for $I_D = 200\ \mu A$ and $W/L = 10$ to get $V_B = 2$ V. Assume $k_n' = 80\ \mu A/V^2$ and $V_t = 500$ mV. Neglect the body effect and channel length modulation.
   e) *Briefly* (in two sentences or less) suggest and justify how to get better "manufacturability" in terms of the dimensions from part (d). (Hint: You should have gotten a non-integer *m* value in (d), how would you fix this?)



2. **Source Follower [20 pts]**

$V_{DS_1} = V_B - V_{S_1} = V_{ov_1} + V_t$

$V_{GS_1} = V_B - V_{S_1}$

$V_{ov_1} = V_B - V_{S_1} - V_t$

$V_{GS_2} = V_B$

$V_{ov_2} = V_B - V_t$

a) From HW2, we can knon M_2 is always in triode region

① Because If M_2 and M_1 are both in saturation

$$V_{ov}^2 = \frac{2I_D}{k_n \frac{W}{L}} \qquad V_{ov_2}^2 = \frac{2I_D}{k_n W} \cdot Lm \qquad V_{ov_1}^2 = \frac{2I_D}{k_n W} \cdot L \qquad \Rightarrow V_{ov_2} > V_{ov_1}$$

② $V_{DS_1} > V_{ov_1}$ $\quad V_{DS_2} > V_{ov_2}$ for sat $\quad V_{DS_2} = V_{GS_1} - V_{GS_2}$

$= V_B - V_{S_1} - V_B$

$V_{GS_1} = V_{ov_1} + V_t \qquad V_{GS_2} = V_{ov_2} + V_t \qquad$ ③ $V_{DS_2} = V_{ov_1} - V_{ov_2}$

② and ① $\Rightarrow$ $V_{ov_1} > 2V_{ov_2}$ $\Leftarrow$ contradicts ①
So they can't both be in saturation

And since $V_{ov_2} > V_{ov_1}$, $I_{D_2} > I_{D_1}$ (can't be true in series connection $\Rightarrow$ $\therefore$ $\boxed{M_2 \text{ is in triode region.}}$

M_1 is in saturation due to diode connection b)

$$V_B - V_t = a$$

$$a^2 + \frac{a^2}{m}\cdot 2$$

$$2a^2\left(\frac{1}{2}+\frac{1}{2m}\right) = a^2 + \frac{a^2}{m}$$

(1) $\frac{1}{2}\cancel{kn'}\cancel{\frac{W}{L}}(V_B - V_{DS} - V_t)^2 = \cancel{kn'}\cancel{\frac{W}{L}}\frac{1}{m}\left[(V_B - V_t)\cdot V_{DS} - \frac{1}{2}V_{DS}^2\right]$

$$\frac{1}{2}(a - V_{DS})^2 = \frac{1}{m}\cdot a\cdot V_{DS} - \frac{1}{2m}V_{DS}^2$$

$$\frac{1}{2}(a^2 - 2aV_{DS} + V_{DS}^2) = \frac{a}{m}\cdot V_{DS} - \frac{1}{2m}\cdot V_{DS}^2$$

$$\frac{1}{2}a^2 - aV_{DS} + \frac{1}{2}V_{DS}^2 - \frac{a}{m}V_{DS} + \frac{1}{2m}\cdot V_{DS}^2 = 0$$

$$V_{DS}^2\left(\frac{1}{2}+\frac{1}{2m}\right) - V_{DS}\left(a+\frac{a}{m}\right) + \frac{1}{2}a^2$$

$$V_{DS} = \frac{a + \frac{a}{m} \pm \sqrt{a^2 + a^2\frac{2}{m} + a^2\frac{1}{m^2} - 4\cdot\frac{1}{2}a^2\left(\frac{1}{2}+\frac{1}{2m}\right)}}{1+\frac{1}{m}}$$

$$V_{DS} = \frac{am + a \pm m\sqrt{\cancel{a^2} + a^2\cdot\frac{2}{m} + a^2\frac{1}{m^2} - \cancel{a^2} - a^2\frac{1}{m}}}{m+1}$$

$$V_{DS} = \frac{am + a \pm am\sqrt{\frac{2}{m} + \frac{1}{m^2} - \frac{1}{m}}}{m+1}$$

$$V_{DS} = \frac{am + a \pm a\sqrt{2m + 1 - m}}{m+1}$$

$$\boxed{V_{DS} = \frac{am + a \pm a\sqrt{m+1}}{m+1}}$$

$$\boxed{V_{DS} = \frac{(V_B - V_t)m + (V_B - V_t) \pm (V_B - V_t)\sqrt{m+1}}{m+1}}$$

c)

$$V_{DS} = \frac{am + a \pm a\sqrt{m+1}}{m+1} = \frac{a(m+1 \pm \sqrt{m+1})}{m+1}$$

M_2:

$$I_D = k_n' \cdot \frac{W}{L} \cdot \frac{1}{m}\left[(V_B - V_t) \cdot V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

$$I_D = k_n' \cdot \frac{W}{L} \cdot \frac{1}{m}\left[a \cdot V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

$$\frac{I_D}{k_n'\frac{W}{L} \cdot \frac{1}{m}} = \frac{a^2(m+1+\sqrt{m+1})}{m+1} - \frac{1}{2}\frac{a^2(m+1+\sqrt{m+1})^2}{(m+1)^2}$$

$$= \frac{a^2(m+1)(m+1+\sqrt{m+1})}{(m+1)^2} - \frac{1}{2}\frac{a^2(m^2+2m+2m\sqrt{m+1}+1+2\sqrt{m+1}+m+1)}{(m+1)^2}$$

$$= \frac{a^2}{(m+1)^2} \cdot \left(\cancel{m^2} + \cancel{m} + \cancel{m} + \cancel{1} + \cancel{m\sqrt{m+1}} + \cancel{\sqrt{m+1}} - \cancel{\tfrac{1}{2}}m^2 - \cancel{m} - \cancel{m\sqrt{m+1}} - \cancel{\tfrac{1}{2}} - \cancel{\sqrt{m+1}} - \cancel{\tfrac{1}{2}}m - \cancel{\tfrac{1}{2}}\right)$$

$$= \frac{a^2}{(m+1)^2}\left(\frac{1}{2}m^2 + \frac{1}{2}m\right)$$

$$= \frac{a^2}{(m+1)^2}\left(\frac{1}{2}m(m+1)\right)$$

$$\frac{I_D}{k_n'\frac{W}{L} \cdot \frac{1}{m}} = \frac{a^2 m}{2(m+1)}$$

$$\frac{I_D \cdot 2(m+1)}{k_n'\frac{W}{L} \cdot \frac{1}{m}} \cdot \frac{1}{m} = a^2$$

$$\sqrt{\frac{I_D \cdot 2(m+1)}{k_n'\frac{W}{L}}} = V_B - V_t$$

$$\boxed{V_B = \sqrt{\frac{I_D \cdot 2(m+1)}{k_n'\frac{W}{L}}} + V_t}$$

d)

$$V_B = \sqrt{\frac{I_D \cdot 2(m+1)}{k_n' \frac{W}{L}}} + V_t$$

$$\frac{(V_B - V_t)^2 \cdot k_n' \frac{W}{L}}{I_D \cdot 2} - 1 = m$$

$$\Rightarrow \underbrace{\frac{(V_B - V_t)^2 \cdot k_n'}{I_D \cdot 2}}_{C} \cdot \frac{W}{L} - 1 = m$$

$$\frac{(2-0.5)^2 \cdot 80u \cdot 10}{400u} - 1 = m$$

$$\boxed{m = 3.5}$$

$c = -0.55$

$$C \frac{W}{L} - 1 = m$$

$$\frac{3.5+1}{4+1} = \frac{10}{x}$$

e)

In order to get better "manufacturability", we need to get a integer of m, we need adjust $\frac{W}{L}$.

For example : $\frac{m_{\text{from c)}} + 1}{m_{\text{integer}} + 1} = \frac{10}{x}$

If $m_{\text{integer}} = 4 \Rightarrow x = \frac{W}{L} = \frac{100}{9}$

$m_{\text{integer}} = 3 \Rightarrow x = \frac{W}{L} = \frac{80}{9}$

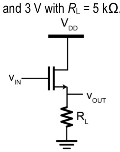
Q2

2. **Source Follower [20 pts]**
Consider the source follower circuit shown below. For all calculations, assume $W/L = 200/1$, $V_{DD} = 4.5$ V, $V_{t0} = 400$ mV, $\gamma = 0.8$, $\varphi_f = 0.5$ V, $\lambda = 0.1/L$ $\mu m V^{-1}$ and $k_n' = 80$ $\mu A/V^2$.

a) Suppose that the output voltage swings from 1.8 to 3 V. Calculate the corresponding voltage swing at the gate node. Perform calculations for $R_L = 5$ kΩ and $R_L = 2$ MΩ as well as for the body connected to the source and ground. Summarize your results in a table and explain how the two different bulk connections affect the required input voltages.

b) Draw the small-signal equivalent circuit (use $V_{BS} = 0$ V). Derive an analytical expression for the low-frequency small-signal gain $a_v = v_o/v_i$ as a function of R_L, V_I, and V_O.

c) Calculate the small-signal gain at $V_O = 1.8$ and 3 V with $R_L = 5$ kΩ. (Still using $V_{BS} = 0$ V).



$$\lambda = \frac{0.1}{1\mu m} \cdot 1\mu m$$
$$= 0.1$$

a) when body connect to source $V_{BS} = 0$. no body effect

Assuming transistor in saturation

$$\frac{V_{out}}{R_L} = \frac{1}{2} k_n' \cdot \frac{W}{L} \cdot (V_{in} - V_{out} - V_t)^2 \cdot (1 + 0.1 \cdot (V_{DD} - V_{out}))$$

@ $R_L = 5\,k\Omega$, solved by symbolab

V_{out} : 1.8V $\boxed{V_{in} = 2.388V}$ V_{out} : 3V $\boxed{V_{in} = 3.655V}$ $\Delta V_{in} = 1.267V$

@ $R_L = 2M\Omega$

$V_{out} = 1.8V$ $\boxed{V_{in} = 2.21V}$ V_{out} : 3V $\boxed{V_{in} = 3.413V}$ $\Delta V_{in} = 1.203V$

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when body connect to ground $V_{SB} = V_{out} - 0$.

$$V_t = V_{t0} + 0.8 \cdot \left(\sqrt{2 \cdot 0.5 + V_{out}} - \sqrt{2 \cdot 0.5}\right)$$

@ $V_{out} = 1.8V$ $V_t = 0.94$ V @ $V_{out} = 3V$ $V_t = 1.2V$

@ $R_L = 5k$, $V_{out} = 1.8V$, $V_t = 0.94V$

$\boxed{V_{in} = 2.93V}$

@ $R_L = 5k$, $V_{out} = 3V$, $V_t = 1.2V$

$\boxed{V_{in} = 4.46V}$

ⓐ $R_L = 2M\Omega$ $V_{out} = 1.8V$ $V_t = 0.94V$

$\boxed{V_{in} = 2.75V}$

ⓐ $R_L = 2M\Omega$ $V_{out} = 3V$ $V_t = 1.2V$

$\boxed{V_{in} = 4.213V}$

Body to Source	$R_L = 5k$	$R_L = 2M$
ΔV_{in}	1.267V	1.203V

Body to Grand

ΔV_{in}	1.53V	1.463V

When $V_{BS} = 0$ $V_t = 400mV$, the required input voltage is less than when $V_{BS} \neq 0$

Because it increases the V_t due to Body effect when $V_{BS} \neq 0$

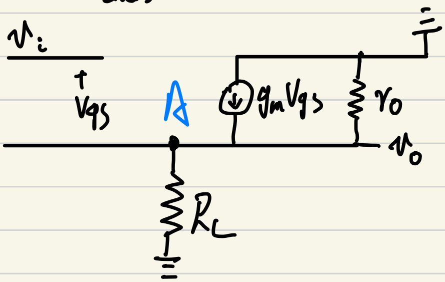
$\therefore$ Larger V_{in} is needed

$$g_m = \frac{dI_d}{dV_{gs}} = \frac{1}{2}k_n'\frac{W}{L}(V_{gs}-V_t)^2(1+\lambda V_{DS}) = k_n'\frac{W}{L}(1+\lambda V_{DS})(V_{gs}-V_t)$$

b)

$$g_{ds} = \frac{dI_d}{dV_{DS}} = \frac{1}{2}k_n'\frac{W}{L}(V_{gs}-V_t)^2(1+\lambda V_{DS}) = \frac{1}{2}k_n'\frac{W}{L}\cdot(V_{gs}-V_t)^2\cdot\lambda$$

$1+\lambda x$



$$V_{gs} = v_i - v_o$$

①
$$\frac{v_o}{R_L} + \frac{v_o}{r_o} = g_m V_{gs}$$

$$g_m = k_n'\frac{W}{L}\cdot(V_{in}-V_{out}-0.4)\cdot(1+\lambda V_{ds}) = 0.016(V_{in}-V_{out}-0.4)(1+\lambda V_{DS})$$

$$g_{ds} = \frac{1}{2}k_n'\frac{W}{L}\cdot(V_{in}-V_{out}-0.4)^2\cdot 0.1 = 800u(V_{in}-V_{out}-0.4)^2$$

$$r_o = \frac{1}{g_{ds}}$$

$$v_o r_o + v_o R_L = g_m(v_i - v_o)\cdot r_o R_L$$

$$v_o r_o + v_o R_L = g_m\cdot r_o R_L\cdot v_i - g_m r_o R_L v_o$$

$$v_o(r_o + R_L + g_m r_o R_L) = g_m\cdot r_o R_L\cdot v_i$$

$$\boxed{\frac{v_o}{v_i} = \frac{g_m r_o R_L}{r_o + R_L + g_m r_o R_L}}$$

$$\frac{v_o}{v_i} = \frac{0.016(V_I-V_o-0.4)(1+\lambda(V_{DD}-V_o))\cdot\frac{1}{800u(V_I-V_o-0.4)^2}\cdot 5k}{\frac{1}{800u(V_I-V_o-0.4)^2} + 5k + 0.016(V_I-V_o-0.4)(1+\lambda(V_{DD}-V_o))\cdot\frac{1}{800u(V_I-V_o-0.4)^2}\cdot 5k}$$

c) from a) $R_L = 5k$

1. $V_{out}: 1.8V$, $V_{in} = 2.388V$

2. $V_{out}: 3V$, $V_{in} = 3.655V$

1.

$$g_m = 0.016(2.388-1.8-0.4)[1+0.1\cdot(4.5-1.8)]$$
$$= 3.82016m$$

$$r_o = \frac{1}{800u(2.388-1.8-0.4)^2} = 3.5367\cdot 10^4$$

$$\frac{v_o}{v_i} = \frac{g_m r_o R_L}{r_o + R_L + g_m r_o R_L} = \boxed{0.9295}$$

2.

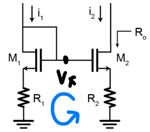
$$g_m = 0.0047$$
$$g_{ds} = 5.2020\cdot 10^{-5}$$
$$r_o = 1.9223\cdot 10^4$$

$$\boxed{\frac{v_o}{v_i} = 0.9490}$$

solved by matlab

Q3

3. **Source Degenerated Current Source [15 pts]**
Consider the current mirror configuration shown below. Assume that M_1 and M_2 are identical transistors. Neglect r_o (except for part c), all devices capacitors, and the back-gate effect.
a) Derive an expression showing the relationship between I_1 and I_2 in terms of the resistor and V_{OV} values.
b) What is the ratio of I_1/I_2 if the overdrive voltages are the same?
c) Derive an expression for the small-signal output impedance. Neglect both C_{gs} and C_{gd}.



4. **Transimpedance Amplifier [20 pts]**

$$V_{gs_1} = V_g - i_1 R_1$$
$$V_{gs_2} = V_g - i_2 R_2$$
$$V_{gs_1} - V_{gs_2} = -i_1 R_1 + i_2 R_2$$

Assume both in saturation

$$V_{ov_1} = V_g - I_1 \cdot R_1 - V_t$$

$$\frac{I_1}{I_2} = \frac{\frac{1}{2} k_n' \frac{W}{L} (V_g - i_1 R_1 - V_t)^2}{\frac{1}{2} k_n' \frac{W}{L} (V_g - i_2 R_2 - V_t)^2}$$
$$I_1 \cdot V_{ov_2}^2 = I_2 \cdot V_{ov_1}^2$$

M_1:
$$V_{DS_1} = V_g - I_1 R_1 = V_{ov_1} + V_t$$
$$V_{GS_1} = V_g - I_1 R_1 = V_{ov_1} + V_t \Rightarrow V_g = V_{ov_1} + V_t + I_1 R_1$$
$$V_{ov_1} = V_g - I_1 R_1 - V_t$$

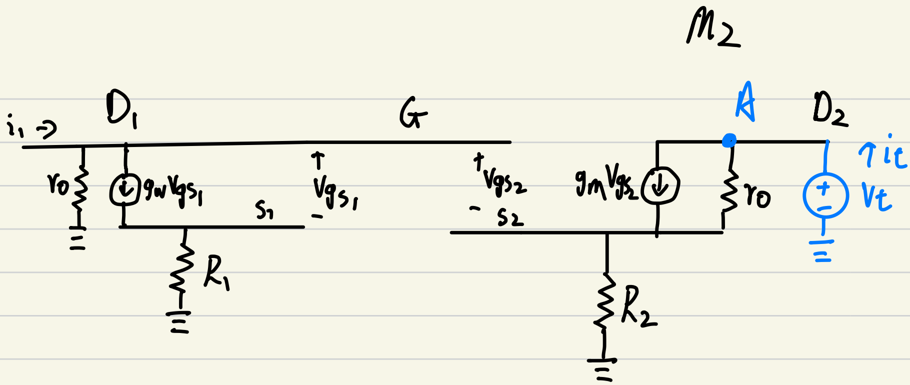
M_2
$$V_{GS_2} = V_g - I_2 R_2 = V_{ov_2} + V_t \Rightarrow V_g = V_{ov_2} + V_t + I_2 R_2$$
$$V_{ov_2} = V_g - I_2 R_2 - V_t$$

$$\boxed{V_{ov_1} + I_1 R_1 = V_{ov_2} + I_2 R_2}$$

b) If $V_{ov_1} = V_{ov_2}$

$$I_1 R_1 = I_2 R_2$$
$$\boxed{\frac{I_1}{I_2} = \frac{R_2}{R_1}}$$

c)



@A

$$i_t = \frac{V_t - i_t \cdot R_2}{r_o} + g_m(-i_t \cdot R_2) \qquad V_{gs} = i_t r_o - V_s = -i_t \cdot R_2$$

$$i_t \cdot r_o = V_t - i_t \cdot R_2 - i_t\, g_m R_2 \cdot r_o \qquad V_s = i_t \cdot R_2$$

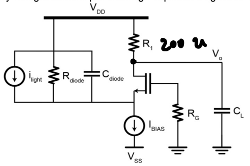
$$V_t = i_t(r_o + R_2 + g_m R_2 r_o)$$

$$\frac{V_t}{i_t} = \boxed{r_o + R_2 + g_m R_2 r_o}$$

Q4

4. Transimpedance Amplifier [20 pts]
Consider the circuit shown below that is to be used as a transimpedance amplifier for small-signal currents coming from a photodiode, modeled using the equivalent circuit as shown. For all calculations, use R_{diode} = 118 kΩ, C_{diode} = 25 fF, C_L = 100 fF, R_1 = 8 kΩ, R_G = 2 kΩ, k_n' = 50 μA/V², V_t =400 mV, V_{DD} = 3 V, V_{SS} = 0 V, C_{ox} = 15 fF/μm², C_{ov}' = 1.5 fF/μm, and L = 2 μm. Neglect r_o, the body effect, and all device capacitors other than C_{gs} and C_{gd}. Determine the necessary I_{BIAS}, V_{OV}, and W/L to set the MOS drain current at 200 μA and g_m at 750 μS. Assume NMOS in saturation.

a) Draw the small-signal model of the circuit.
b) Assuming R_G to be zero, symbolically find the small-signal transimpedance transfer function.
c) Assuming R_G to be zero, estimate this circuit -3dB bandwidth using the ZVTC method.
d) Considering a non-zero R_G, estimate this circuit -3dB bandwidth using the ZVTC method.
e) *Briefly* explain why designers often place a large capacitor to ground in parallel with R_G.



In sat

$C_{gs} = \frac{2}{3} WLC_{ox} + C_{ov}$

$C_{gd} = C_{ov}$

$V_{ov} = V_g - V_s - V_t$
$0.93 = 0 - V_s$
$V_s = -0.93$

DC operating point

$g_m = \frac{2I_D}{V_{ov}} \Rightarrow V_{ov} = \frac{2I_D}{g_m} = \frac{400u}{750u} = \frac{8}{15} \approx \boxed{0.53\ V}$

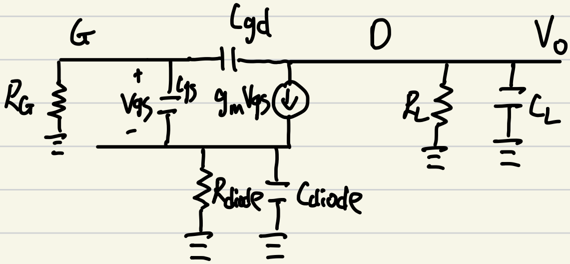
$I_{Bias} = I_D = 200u\ A$

$I_D + \frac{V_{DD} - V_S}{R_D} = I_{Bias}$
$200u + 33.3u = I_{Bias}$
$\boxed{I_{Bias} = 233.3\ uA}$

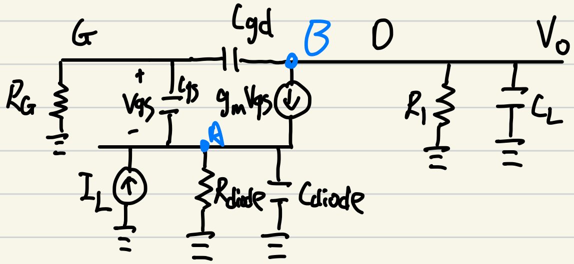
$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2$
$200u = \frac{1}{2} \cdot 50u \cdot \frac{W}{L} \cdot \left(\frac{8}{15}\right)^2$
$\frac{W}{L} = \frac{225}{8} = \boxed{28.125}$

$L = 2u$
$W = 56.25u$

a)



b)



$V_G = 0$

$V_{gs} = -V_s$

@ A

$$I_L + sC_{gs}(-V_s) + g_m(-V_s) = \frac{V_s}{R_{diode}} + sC_{diode} \cdot V_s$$

$$I_L \cdot R_{diode} = V_s\left(1 + sR_d C_{diode} + sR_{diode}C_{gs} + g_m R_{diode}\right)$$

$\frac{V_s}{I_L} \cdot \frac{V_o}{V_s}$

$$V_s = \frac{I_L R_{diode}}{1 + sR_d C_{diode} + sR_d C_{gs} + g_m R_d} \Rightarrow \frac{V_s}{I_L}$$

@ B

$$sC_{gd}(0 - V_o) - g_m(-V_s) - \frac{V_o}{R_1} - sC_L V_o = 0$$

$$R_L g_m V_s = V_o\left(sR_1 C_{gd} + 1 + sR_1 C_L\right)$$

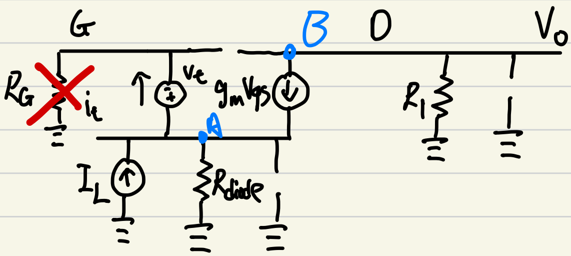
$$\frac{V_o}{V_s} = \frac{g_m R_1}{1 + sR_1 C_{gd} + sR_1 C_L}$$

$$\frac{V_o}{I_L} = \frac{V_s}{I_L} \cdot \frac{V_o}{V_s} = \boxed{\frac{R_{diode}}{1 + sR_d C_{diode} + sR_d C_{gs} + g_m R_d} \cdot \frac{g_m R_1}{1 + sR_1 C_{gd} + sR_1 C_L}}$$

$R_d = R_{diode}$

$C_{gs} = \frac{2}{3} WL C_{ox} + C_{ov}$

c)



① For C_{gs} $\qquad V_{gs} = (0 - V_t)$

@A

$$-i_t + g_m(-V_t) = \frac{V_t}{R_d}$$

$$-i_t R_d - g_m R_d V_t = V_t$$

$$-i_t R_d = V_t(1 + g_m R_d)$$

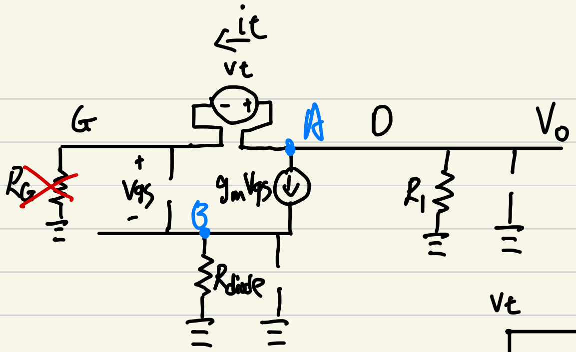
$$\frac{V_t}{i_t} = \frac{R_d}{1 + g_m R_d}$$

$$\tau_{gs} = C_{gs} \cdot \frac{R_d}{1 + g_m R_d}$$

$$= \left(\frac{2}{3} WL C_{ox} + C_{ov}\right) \cdot \frac{R_d}{1 + g_m R_d}$$

$$= \left(\frac{2}{3} \cdot 56.25u \cdot 2u \cdot 15 fF/um^2 + 1.5f \cdot 56.25\right) \cdot \frac{118k}{1 + 118k \cdot 750u}$$

$$= 1.59 ns$$

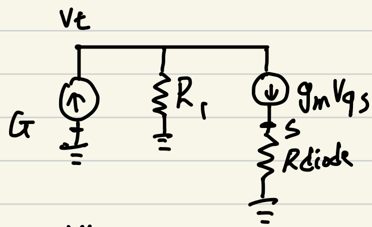


$V_{gs} = -V_s$

$+g_m V_s$

$$V_{gs} = -V_s$$

$$g_m V_{gs} = i_t + \frac{V_t}{R_1}$$



② Find τ_{cgd}

@A $\quad -i_t - g_m V_{gs} + \frac{0 - V_t}{R_1} \quad \Rightarrow \frac{V_t}{R_1} = -i_t - g_m V_{gs} = g_m V_s - i_t$

@B

$$\frac{V_s}{R_d} = g_m V_{gs} = -i_t - \frac{V_t}{R_1}$$

$$V_s = -i_t \cdot R_d - V_t \frac{R_d}{R_1}$$

$$-i_t - g_m(-V_s) = \frac{V_t}{R_1} \Rightarrow -i_t - g_m\left(i_t R_d + V_t \frac{R_d}{R_1}\right) = \frac{V_t}{R_1}$$

$$-i_t \cdot R_1 - g_m R_1 R_d \cdot i_t - g_m R_d V_t = V_t$$

$$-i_t(R_1 + g_m R_1 R_d) = V_t(1 + g_m R_d)$$

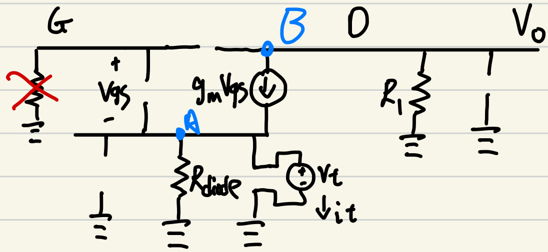
$$\frac{V_t}{i_t} = \frac{R_1 + g_m R_1 R_d}{1 + g_m R_d}$$

$$\tau_{cgd} = C_{gd} \cdot R_1 = C_{ov} \cdot \frac{R_1 + g_m R_1 R_d}{1 + g_m R_d}$$

$$= C_{ov}' \cdot W \cdot R_1$$

$$= 1.5 \frac{fF}{um} \cdot 56.25\, um \cdot R_1$$

$$= 0.675\, ns$$



③ Find τ_{Cdiode} $\qquad V_{gs} = 0 - V_s = -V_t$

@ A

$$-\frac{V_t}{R_d} + g_m(-V_t) - i_t = 0$$

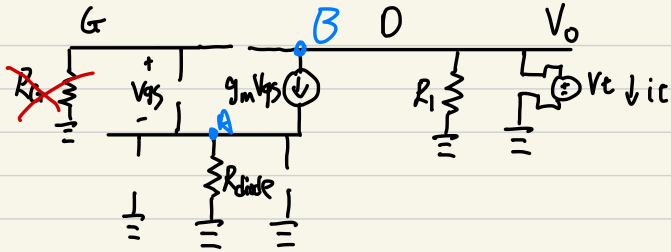
$$-V_t - g_m R_d \cdot V_t = i_t$$

$$-V_t(1 + g_m R_d) = i_t$$

$$\frac{V_t}{i_t} = \frac{1}{1 + g_m R_d}$$

$$\tau_{Cdiode} = C_{diode} \cdot \frac{1}{1 + g_m R_d}$$

$$= 25f \cdot \frac{1}{1 + 750u \cdot 118k} = 0.28fs$$



④ find τ_{CL} $\qquad V_{gs} = -V_s$

@B $\quad -g_m V_{gs} - i_t = \frac{V_t}{R_1} \qquad\qquad g_m V_{gs} = -i_t - \frac{V_t}{R_1}$

@A $\quad g_m V_{gs} = \frac{V_s}{R_D}$

$$V_s = -i_t \cdot R_D - V_t \frac{R_D}{R_1}$$

$$-g_m\left(i_t \cdot R_D + V_t \frac{R_D}{R_1}\right)R_1 - i_t \cdot R_1 = V_t$$

$$-i_t\left(g_m R_D R_1 + R_1\right) = V_t\left(1 + g_m R_D\right)$$

$$\frac{V_t}{i_t} = \frac{g_m R_D R_1 + R_1}{1 + g_m R_D}$$

$$\tau_{CL} = 100f \cdot \frac{g_m R_D R_1 + R_1}{1 + g_m R_D} = 0.8\,ns$$

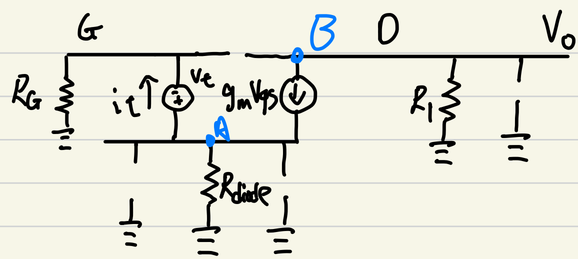
$$= \frac{R_1(1 + g_m R_D)}{1 + g_m R_D}$$

$$f_{-3} = \left|\frac{1}{b_1}\right| = \frac{1}{\tau_{cL} + \tau_{C diode} + \tau_{cgs} + \tau_{cgd}}$$

$$= \frac{1}{0.8n + 0.28f + 1.59n + 0.675n}$$

$$= 0.326 \text{ GHz}$$

d)



① τ_{cgs}

$V_t: V_{gs} = i_t \cdot R_G - V_s \Rightarrow V_s = i_t \cdot R_G - V_t$

ⓐ $-i_t - \frac{V_s}{R_d} + g_m V_t = 0$

$$-i_t - \left(i_t \frac{R_G}{R_d} - \frac{V_t}{R_d}\right) + g_m V_t = 0$$

$$-R_d i_t - i_t R_G + V_t + g_m R_d V_t = 0$$

$$V_t(1+g_m R_d) = i_t(R_d + R_G)$$

$$\frac{V_t}{i_t} = \frac{R_d + R_G}{1+g_m R_d}$$

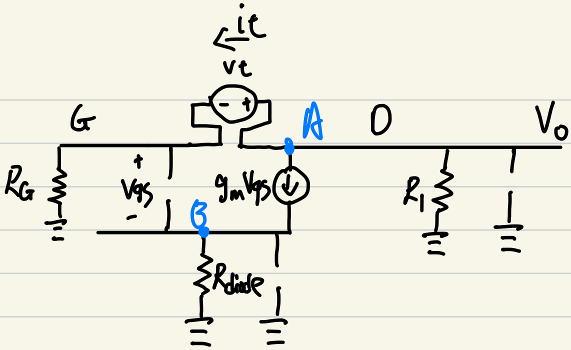
$$\tau_{cgs} = C_{gs} \cdot \frac{R_d + R_G}{1+g_m R_d}$$

$$= \left(\frac{2}{3} W L C_{ox} + C_{ov}\right) \cdot \frac{R_d + R_G}{1+g_m R_d}$$

$$= \left(\frac{2}{3} \cdot 56.25u \cdot 2u \cdot 15fF/um^2 + 1.5f \cdot 56.25\right) \cdot \frac{118k + 2k}{1+118k \cdot 750u}$$

$$= 1.21 \cdot 10^{-12} \cdot \frac{118k + 2k}{1+118k \cdot 750u}$$

$$= 1.622n$$



② τ_{cgd}

$$V_{gs} = it \cdot RG - V_s$$

@B
$$\frac{V_s}{R_d} = g_m(it \cdot R_G - V_s)$$

$$V_s = g_m Rd \cdot R_G \cdot it - V_s \cdot Rd$$

$$V_s(1+Rd) = g_m Rd \cdot R_G \cdot it$$

$$V_s = \frac{g_m R_d R_G}{1+Rd} \cdot it$$

@A
$$-it - g_m\left(itR_G - it\frac{g_m R_d R_G}{1+R_d}\right) + \frac{Vt}{R_1} = 0$$

$$\frac{Vt}{R_1} = it\left(1 + g_m R_G - \frac{g_m R_d R_G}{1+Rd}\right)$$

$$\frac{Vt}{it} = R_1 \cdot \left(1 + \frac{g_m R_G + g_m R_d R_d - g_m R_d R_G}{1+Rd}\right)$$

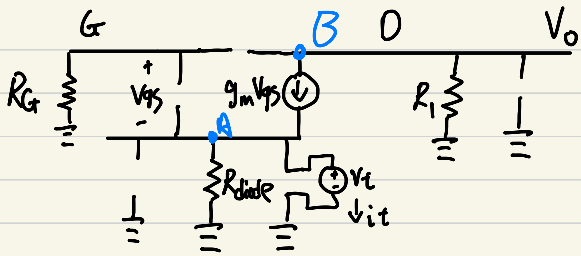
$$\frac{Vt}{it} = R_1\left(1 + \frac{g_m RG}{1+Rd}\right)$$

$$\tau_{cgd} = C_{gd} \cdot R_1 = C_{ov} \cdot R_1 \left(1 + \frac{g_m R_G}{1+Rd}\right.$$

$$= C_{ov}' \cdot W \cdot R_1 \left(1 + \frac{g_m R_G}{1+Rd}\right)$$

$$= 1.5\,\frac{fF}{um} \cdot 56.25\,um \cdot 8000.1$$

$$= 0.675\,ns$$



③ Find $\tau_{C_{diode}}$ same as c), because $V_g = 0$

$$\tau_{C_{diode}} = C_{diode} \cdot \frac{1}{1+g_m R_d}$$

$$= 25f \cdot \frac{1}{1+750u \cdot 118k} = 0.28\,fs$$

④ Find τ_{C_L} same as c) because $V_g = 0$

$$\tau_{C_L} = C_L \cdot R_1$$

$$= 100f \cdot 8k$$
$$= 0.8\,ns$$

$$f_{-3} = \left|\frac{1}{b_1}\right| = \frac{1}{\tau_{C_L} + \tau_{C_{diode}} + \tau_{C_{gs}} + \tau_{C_{gd}}}$$

$$= \frac{1}{0.8n + 0.28f + 1.622n + 0.675n}$$

$$= 0.323\ GHz$$

e)

Having a large capacitor in parallel with R_G will increase the BW of the circuit.

V_{ss}

Q5

5. Bias Network Design [30 pts]

Your boss has asked you to design the bias network for a colleague's car engine sensor in your next assignment at Triton Industries. The bias network must generate a 20 μA current with g_m = 200 μS (you don't need EXACTLY these values but get as close as you can and justify any discrepancies). Based on your nominal value of g_m, it cannot vary by more than ±10% over -40 to 130 °C or supply variations of ±10% with a nominal 1.8 V supply. You decide to use a **constant g_m bias** circuit to bias the amplifier. Design the bias network and determine the dimensions of all transistors. Due to area constraints, you can only use a resistor up to 40 kΩ and a maximum L of 10 μm. Also, don't forget that $K > 20$ is forbidden, and your circuit needs a startup circuit! (Hint: Consider using $m = 4$ to simplify your equations.) Submit a schematic annotated with the operating point, transient simulation showing correct startup (ramp V_{DD} from 0 to 1.8V), and sweeps showing that g_m is stable over the temperature and voltage range.

$$g_m = 200us = \frac{2\left(1-\frac{1}{\sqrt{m}}\right)}{R_2}$$

$g_m \pm 10\%$ over -40° to 130°C
over ±10% 1.8
(1.62 → 1.98)

$L_{max} = 10um$ $\quad I = 20u$

$R_{max} = 40k\Omega$

① Start from $m=4$ → $R_2 = 5k\Omega$

$k'_n = 267.2\,uA/V^2$ $\quad k'_p = 49.1\,uA/V^2$

② $I_{ref} = \frac{V_{ov_1}}{R_2}\left(1-\frac{1}{\sqrt{m}}\right)$, Find $V_{ov_1} = \frac{1}{5}$

NMOS: M_1: $V_{ov_1} = \sqrt{\frac{2I_D}{k'_n \frac{W}{L}}}$ $\quad \frac{W_1}{L_1} = \frac{2I_D}{k'_n V_{ov_1}^2} = \frac{625}{167}$

PMOS: M_1: $V_{ov_{4p}} = \sqrt{\frac{2I_D}{k'_p \frac{W}{L}}}$ $\quad \frac{W}{L} = \frac{2I_D}{k'_n V_{ov_1}^2} = \frac{10000}{491}$

NMOS

If $L_{max} = 10um$, $W_N = 37.425um$

M_2: $V_{ov_{2,n}} = \sqrt{\frac{2I_D}{k'_n \cdot \frac{W}{L}\cdot 4}} = 0.1$ $\quad \frac{W}{L}\cdot 4$

PMOS

If L_{max} = 10um, W_p = 203.666 u

Inverter

W_PMOS : 0.1u

W_NMOS : 100 u

$V_{in} < 0.265$ Turn ON → Output 1

$V_{in} > 0.265$ Turn OFF → Output 0

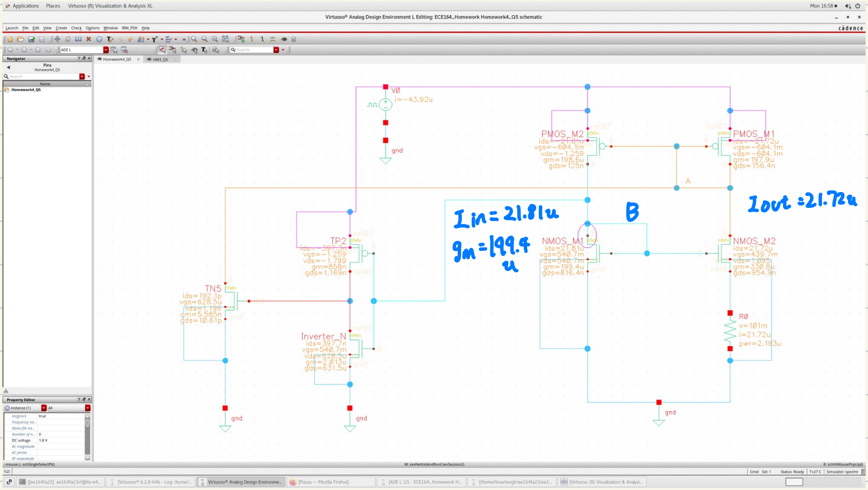
Temperature

	@ −40°C	@ 130°C	Tolerance
	gm = 214 u	184 u	180u ≤ gm << 220 u

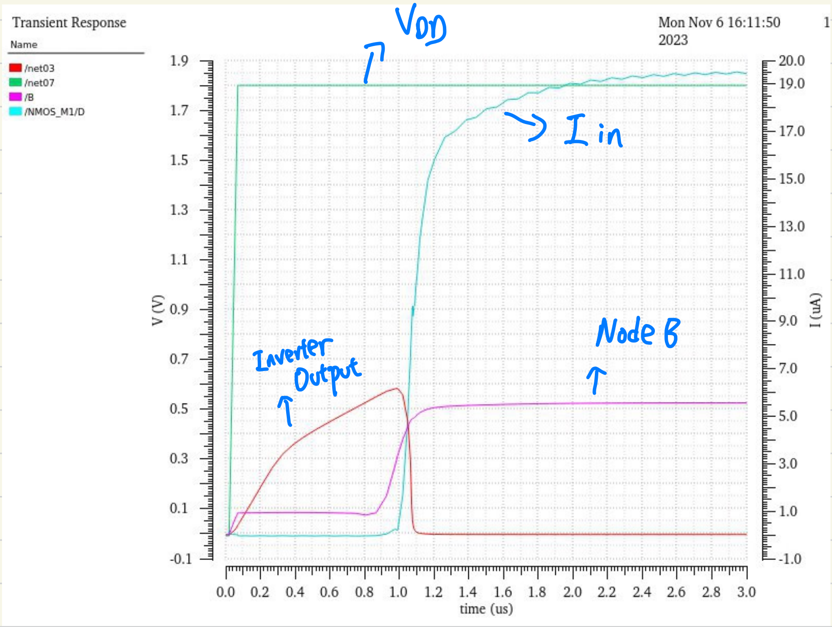
VDD	@ 1.62 V	@ 1.98V	Tolerance
	gm = 198.4	200.4u	180u ≤ gm ≤ 220 u

In my answer, I_{IN} is little off, it's 21.81uA

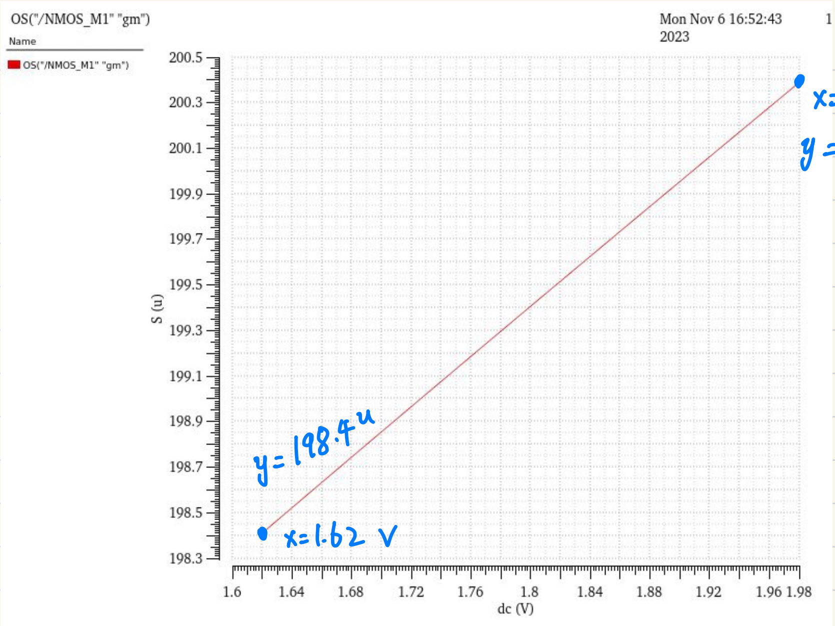
Operating Point



Transient



gm VS. V_{DD}



gm VS. T