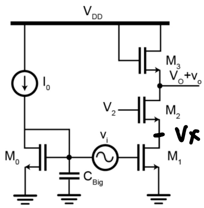
$V_{DS} \geq V_{OV}$

1. **Cascoded Amplifier with Replica Bias [20 pts]**
Consider the circuit shown below. For all parts of the problem, use $(W/L)_{0,1,2} = 6$, $(W/L)_3 = 1/n\times(W/L)_1$, (*i.e.*, the L for M_3 is increased by a factor of "n" relative to M_1), $V_{DD} = 4$ V, $k_n' = 120\ \mu A/V^2$, and $V_{t0} = 300$ mV. Neglect channel length modulation and all other capacitances. Assume that C_{BIG} is an ac short.
   a) Calculate the minimum and maximum values for V_O and V_2 to keep all transistors in saturation if $\gamma = 0$, $n = 2$, and $I_0 = 30\ \mu A$.
   b) Repeat part (a) with $\gamma = 0.5$ and $\varphi_f = 400$ mV. (Hint: Think about the voltage range for the source of M_2.)
   c) If the devices are all in saturation, derive an expression for the small-signal voltage gain. Use your numerical results from part (b) to compute the gain.



$\left(\frac{W}{L}\right)_{0,1,2} = 6$

$\left(\frac{W}{L}\right)_3 = \frac{1}{n}\left(\frac{W}{L}\right)_1$

$V_{DD} = 4V$

$V_{OV_1} = V_{OV_2}$

0

a) M_0, M_3 is diode-connected, so it's always in saturation

M_0, M_1, M_2, M_3 has the same current

$$V_{OV_0} = V_{OV_1} = V_{OV_2} = \sqrt{\frac{2I_D}{k_n' \cdot \frac{W}{L}}} = \sqrt{\frac{2 \cdot 30u}{120u \cdot 6}} = \frac{\sqrt{3}}{6}$$

$$V_{OV_3} = \sqrt{\frac{2I_D}{k_n'\left(\frac{W}{L}\right)_3}} = \sqrt{\frac{2I_D}{k_n' \cdot \left(\frac{W}{L}\right)_1 \cdot \frac{1}{2}}} = \frac{\sqrt{6}}{6} = V_{DD} - V_O - V_{t_3}$$

$\boxed{V_O = 3.29}$ V_O is fixed

For M_2

$V_2 = V_X + V_{OV} + V_t$ $\qquad V_X \geq V_{OV}$

$V_2 \geq 2V_{OV} + V_t$

$V_2 - V_X \geq V_t$ $\qquad V_D - V_X \geq V_2 - V_X - V_t$

$V_2 \leq V_O + V_t$

$$2V_{OV} + V_t \leq V_2 \leq V_O + V_t$$

$$\boxed{\begin{array}{c} 0.8774\text{ V} \leq V_2 \leq 3.59\text{ V} \\ V_O = 3.29 \end{array}}$$

$V_t = V_{t0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{|2\phi_F|}\right)$ 0.9518

3.553

b) $V_{SB0}, V_{SB1} = 0 \qquad V_{SB2}, V_{SB3} \neq 0$

$V_X = V_0 - V_{ov2} \quad \text{or} \quad V_X = V_{ov,1}$

$V_{ov1} \leq V_X \leq V_0 - V_{ov2}$

$V_0 - 0$

With Body effect $V_{t,2}, V_{t,3}$ is affected

$$V_0 = V_{DD} - V_{ov3} - \underbrace{\left(V_{t0} + \gamma\left(\sqrt{2\phi_F + V_0} - \sqrt{2\phi_F}\right)\right)}_{V_{t3}}$$

$V_0 = 2.79\,V$

$V_0 = 3.29$

$\frac{\sqrt{3}}{6} \leq V_X \leq 2.79 - \frac{\sqrt{3}}{6}$

$V_{ov} = \frac{\sqrt{3}}{6}$

$V_{ov3} = \frac{\sqrt{6}}{6}$

0.37448

$V_2 = 2V_{ov} + \left(V_{t0} + \gamma\left(\sqrt{2\phi_F + V_{ov}} - \sqrt{|2\phi_F|}\right)\right)$

$= 0.9518\,V$

$V_0 - V_{ov2} = 2.5$ 0.76

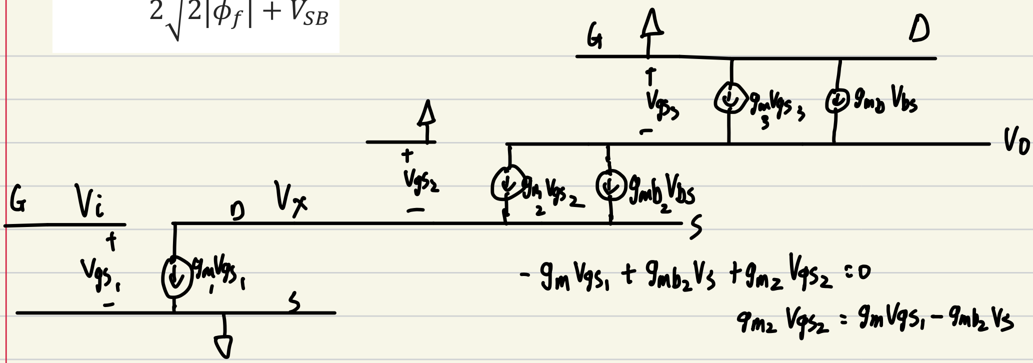
$V_2 = V_0 + \left(V_{t0} + \gamma\left(\sqrt{2\phi_F + V_0 - V_{ov2}} - \sqrt{|2\phi_F|}\right)\right)$

$= 3.55108\,V$

$$\boxed{\begin{gathered} 0.9518 < V_2 < 3.55108\,V \\ V_0 = 2.79\,V \end{gathered}}$$

c)

$$g_{mb} \approx \frac{\gamma g_m}{2\sqrt{2|\phi_f| + V_{SB}}}$$



$$-g_m V_{gs_1} + g_{mb_2} V_s + g_{m_2} V_{gs_2} = 0$$

$$g_{m_2} V_{gs_2} = g_m V_{gs_1} - g_{mb_2} V_s$$

$$g_{m_1} V_{gs_1} = g_{m_2} V_{gs_2} + g_{mb_2} V_{BS} = g_{m_3} V_{gs_3} + g_{mb,3} V_{BS}$$

$$g_{m_1}(V_i) = g_{m_3}(0 - V_o) + g_{mb,3}(0 - V_o)$$

$$\boxed{\frac{V_o}{V_i} = \frac{-g_{m_1}}{g_{m_3} + g_{mb,3}}}$$

$$g_{m2} = g_{m1} = \frac{2I_D}{V_{OV1}} = \frac{60u}{\frac{\sqrt{3}}{6}} = 207.85u$$

$$g_{m3} = \frac{2I_D}{V_{OV3}} = \frac{60u}{\frac{\sqrt{6}}{6}} = 147u$$

$$g_{mb,3} \approx \frac{0.5 \cdot 147u}{2\sqrt{2 \cdot 0.4 + V_{S,B}}}$$

$$V_{SB3} = V_O - 0$$
$$= 2.79$$

$$g_{mb,3} = 1.939 \cdot 10^{-5}$$

$$g_{mb,2} \approx \frac{0.5 \cdot 207.85u}{2 \cdot \sqrt{2 \cdot 0.4 + V_{S_2B}}}$$

$$V_{S_2B} = V_X$$
$$\frac{\sqrt{3}}{6} \leq V_X \leq 3.3$$

$$25.66u \leq g_{mb,2} \leq 49.8u$$

$$\boxed{\frac{v_o}{v_i} = \frac{-g_{m1}}{g_{m3} + g_{mb,3}} = -1.249}$$

Q2

2. Multistage Amplifier [20 pts]

Consider the multistage amplifier circuit shown below. For all parts, use L = 1 µm, k_n' = 80 µA/V², k_p' = 50 µA/V², $|V_{t0}|$ = 400 mV, V_{DD} = 3 V, and $-V_{SS}$ = -3 V. There is a ratiometric constraint imposed for process variation reasons: $n = I_2/I_1 = R_1/R_2 = (W/L)_2/(W/L)_1$. Neglect channel length modulation and the body effect unless specifically stated otherwise.

a) *Briefly* (less than 4 sentences) explain how this circuit works and what function each transistor performs.

b) Determine the operating point and values for R_1/R_2. Assume that I_{BIAS} = 150 µA, I_{M2}/I_{M1} = 2, $W_{A,B}$ = 14 µm, W_1 = 8 µm, and V_O = 0 V (by design).

c) *Briefly* comment on how the constraint equations make the result in part (b) "obvious" (in retrospect, of course).

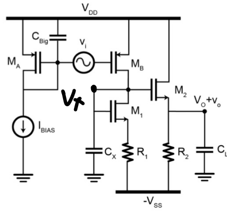
d) How do the results in part (b) change if the body effect is considered?

e) How do the results in part (b) change if the threshold voltage is 750 mV instead?

f) Derive an expression for the small-signal voltage gain. Calculate the gain using the values from (b).

g) Derive an expression for the -3dB bandwidth using ZVTCs in terms of C_X, C_L, I_{BIAS}, μ_n, V_{OV1}, n, V_{SS}, and L. Assume that g_mR >> 1. (Hint: You need to calculate C_{gs}, think about how you can do that.)

h) Use the expression from (g) to find an expression for V_{OV} (in terms of C_L, n, I_{BIAS}, μ_n, and L) that maximizes the bandwidth.



$L = 1um$

$k'_n = 80u$

$k'_p = 50u$

$|V_t| = 400m$

$V_{DD} = 3$ V

$-V_{SS} = -3$ V

$n = \frac{I_2}{I_1} = \frac{R_1}{R_2} = \frac{\frac{W}{L_2}}{\frac{W}{L}_1}$

a) M_A copies I_{Bias} to M_B with $k=1$

M_B works as a current source

M_1 copies current I_1 to M_2 with $k = \frac{(\frac{W}{L})_2}{(\frac{W}{L})_1}$

M_2 is the CD Amplifier

$150u = \frac{1}{2} k_n' \overset{8}{(\frac{W}{L})_1} V_{ov_1}^2$

$V_{ov_1} = \frac{\sqrt{30}}{8}$:

$300u = \frac{1}{2} k_n' \overset{16}{(\frac{W}{L})_2} V_{ov_2}^2$

$V_{ov_2} = \frac{\sqrt{30}}{8}$

b) $I_{Bias} = 150u$ $\quad \frac{I_{M_2}}{I_{M_1}} = 2$

$I_1 = I_{Bias} = 150uA \quad I_2 = 300uA$

$R_2 = \frac{0-(-V_{ss})}{I_2} = \frac{3}{300u} = 10\,000\ \Omega$

$\frac{W_1}{L_1} = \frac{8}{1} \cdot n = \frac{W_2}{L_2}$

$R_1 = R_2 \cdot n = 20\,000\ \Omega$

$W_2 = 16u$

$I_{M_2} = 300u = \frac{1}{2} k_n' \cdot (\frac{W}{L})_2 (V_G - V_O - V_t)^2$

$\frac{\sqrt{30}}{8} + V_{t,n} = V_{GS,2}$

$V_{GS_2} = 1.085$ V

$V_{G,2} = 1.085$ V

$\frac{V_S - (-V_{ss})}{R_1} = I_{M_1}$

$V_S - (-3) = 3$

$V_S = 3 - 3$

$= 0$

$$V_{OVA} = V_{OVB} = \sqrt{\frac{2I_D}{k_p' \frac{W}{L}}} = \frac{\sqrt{21}}{7} \approx 0.655\ V$$

$$\frac{I_{M1}}{I_{M2}} = \frac{\frac{1}{2}k_n'\left(\frac{W}{L}\right)_1 (V_{G1}-V_{S1}-V_{tn})^2}{\frac{1}{2}k_n'\left(\frac{W}{L}\right)(V_{G2}-V_{S2}-V_{tn})^2}$$

$V_S = V_O = 0$

$V_{G1} = V_{G2}$

$$\frac{I_{M1}}{I_{M2}} = \frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2} = \frac{1}{2}$$

$$V_{OV1} = \frac{\sqrt{30}}{8} \qquad V_{OV2} = \frac{\sqrt{30}}{8}$$

c) $$R_1 = \frac{V_S - (-V_{SS})}{I_{M1}} \qquad R_2 = \frac{0 - (-V_{SS})}{I_{M2}}$$

since M_2 and M_1 work as a current mirror circuit They must have the same V_{OV} so the equation becomes:

$\hookrightarrow V_S = 0$

$$R_1 \cdot I_{M1} = R_2 \cdot I_{M2}$$

$$\frac{I_{M1}}{I_{M2}} = \frac{R_2}{R_1}$$

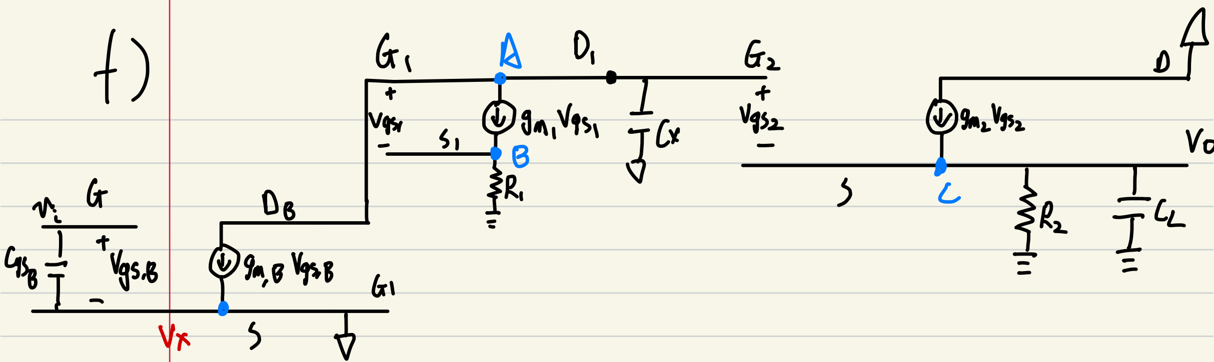
d) If the body effect is considered. Body tie to $-V_{SS}$

then $V_{SB} = 0 - (-V_{SS}) = 3V$.

so the result $V_{t1,2} \uparrow \rightarrow I_{M1}, I_{M2} \downarrow \Rightarrow V_{OV1,2} \downarrow$

e) Similar to part d) $V_{t1,2} \uparrow \rightarrow$ circuit is harder to turn on. $I_{M1}, I_{M2} \downarrow$ and $V_{OV1,2} \downarrow$

f)



$$@A \quad -g_{m,B}(V_i) = g_{m_1}(V_A - V_{s_1}) \quad (1)$$

$$@B \quad g_{m_1}(V_A - V_{s_1}) = \frac{V_{s_1}}{R_1}$$

$$g_{m_1}R_1 V_A - g_{m_1}R_1 V_{s_1} = V_{s_1}$$

$$V_{s_1}(1+g_{m_1}R_1) = g_{m_1}R_1 V_A$$

$$V_{s_1} = \frac{g_m R_1}{1+g_m R_1} V_A$$

$$\frac{1+g_m R_1 - g_m R_1}{1+g_m R_1} = \frac{1}{1+g_m R_1}$$

$$(1) \quad -g_{m,B} V_i = g_{m_1} V_A - g_{m_1} \frac{g_m R_1}{1+g_{m_1}R_1} V_A$$

$$-g_{m,B} V_i = V_A g_{m_1} \left(1 - \frac{g_m R_1}{1+g_{m_1}R_1}\right)$$

$$V_A = \frac{-g_{m,B}}{\frac{g_{m_1}}{1+g_m R_1}} V_i$$

$$@C \quad g_{m_2}(V_A - V_o) = \frac{V_o}{R_2}$$

$$g_{m_2}R_2\left(\frac{-g_{m,B}(1+g_m R_1)}{g_{m_1}} \cdot V_i - V_o\right) = V_o$$

$$\frac{-g_{m_2} g_{m,B} R_2 (1+g_m R_1)}{g_{m_1}} V_i = V_o(1+g_{m_2}R_2)$$

$$\boxed{\frac{V_o}{V_i} = \frac{-g_{m_2} g_{m,B} R_2 (1+g_m R_1)}{g_{m_1}(1+g_{m_2}R_2)}}$$

$$\frac{V_o}{V_i} = \frac{-g_{m1} g_{m2} g_{m,B} R_2 R_1 - g_{m2} g_{mB} R_2}{g_{m1} + g_{m} \cdot g_{m2} R_2}$$

$$g_{m,b} = \frac{2I_D}{V_{ov,b}} = \frac{2 \cdot 150u}{\frac{\sqrt{21}}{7}} = 458u$$

$$V_{ov,b} = \sqrt{\frac{2I_D}{k_p \frac{W}{L}}} = \frac{\sqrt{21}}{7}$$

$R_1 = 20000\Omega$

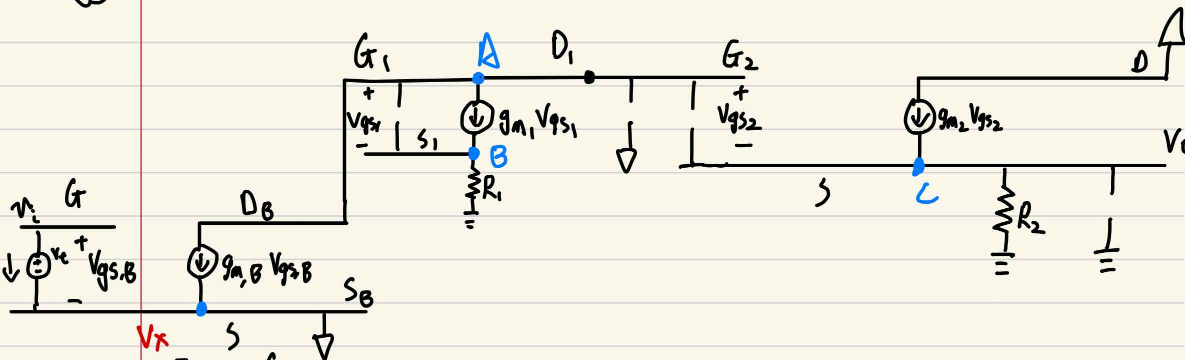
$R_2 = 10000\Omega$

$$g_{m1} = \frac{2I_D}{V_{ov1}} = \frac{2 \cdot 150u}{\frac{\sqrt{30}}{8}} = 438u$$

$$g_{m2} = \frac{300u \cdot 2}{\frac{\sqrt{30}}{8}} = 876u$$

$$\boxed{\frac{V_o}{V_i} = -9.16}$$

g) $C_{gs} = \frac{2}{3} WL\, C_{ox}$ $\qquad C_{ox} = \frac{k_n'}{u_n}$



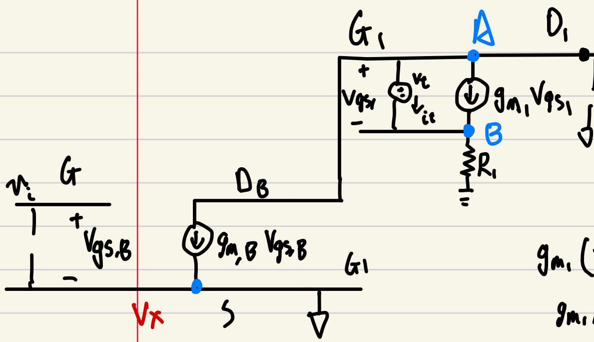
For C_{gs_B}

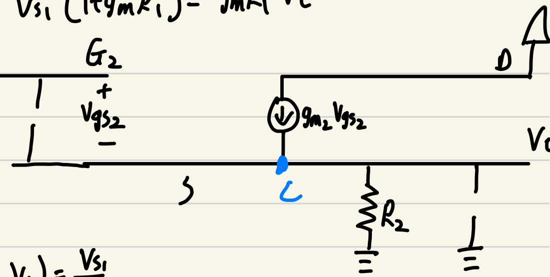
$R = 0$

$\tau_{C_{gs_B}} = 0$

$$V_{s_1} = g_m (V_t - V_{s_1}) \cdot R_1$$

$$V_{s_1}(1 + g_m R_1) = g_m R_1 \cdot V_t$$



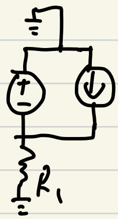


$$g_{m_1}(V_t - V_{s_1}) = \frac{V_{s_1}}{R_1}$$

$$g_{m_1} R_1 V_t - g_{m_1} R_1 V_{s_1} = V_{s_1} \Rightarrow V_{s_1}(1 + g_{m_1} R_1) = V_t \, g_{m_1} R_1$$

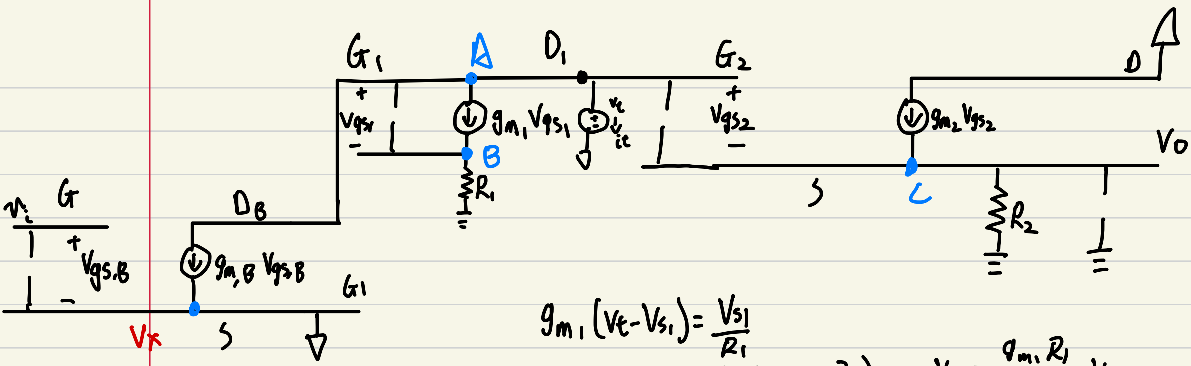
$$V_{s_1} = \frac{g_m R_1}{1 + g_{m_1} R_1} V_t$$

For C_{gs_1}
@A



$$i_t = g_{m_1}(0 - V_t)$$

$$\frac{v_t}{i_t} = \frac{1}{g_{m_1}}$$



$$g_{m_1}(V_t - V_{s_1}) = \frac{V_{s_1}}{R_1}$$

$$g_{m_1} R_1 V_t = V_{s_1}(1+g_{m_1}R_1) \qquad V_{s_1} = \frac{g_{m_1}R_1}{1+g_{m_1}R_1}\cdot V_t$$

For ✓ C_x

@ A

$$-i_t - g_{m_1}(V_t - V_{s_1}) = 0$$

$$-i_t = g_{m_1}\left(V_t - \frac{g_{m_1}R_1}{1+g_{m_1}R_1}\cdot V_t\right)$$

$$\frac{1+g_{m_1}R_1 - g_{m_1}R_1}{1+g_{m_1}R_1}$$

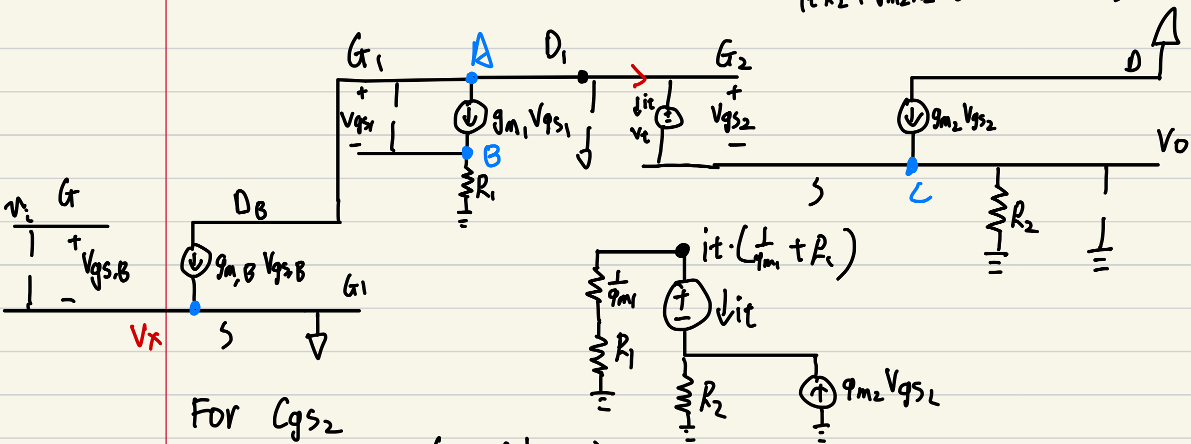
$$-i_t = g_{m_1}V_t\left(1 - \frac{g_{m_1}R_1}{1+g_{m_1}R_1}\right)$$

$$-i_t = V_t\left(\frac{g_{m_1}}{1+g_{m_1}R_1}\right)$$

$$\frac{V_t}{i_t} = \frac{-(1+g_{m}R_1)}{g_{m_1}}$$

$$\left(i_t + g_{m2}(V_t - V_c)\right)\cdot R_2 = V_c$$

$$i_t R_2 + g_{m2} R_2 V_t = V_c(1 + g_{m2} R_2)$$



For C_{gs2}

$$V_{gs2} = \left(i_t \cdot \left(\frac{1}{g_{m1}} + R_1\right) - i_t \cdot \left(\frac{1}{g_{m1}} + R_1\right) + V_t \right)$$

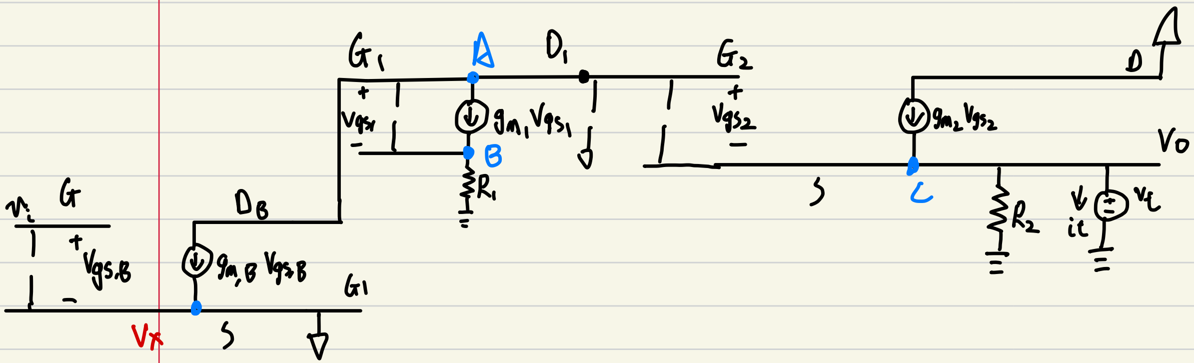
$$i_t + g_{m2} V_t = \frac{i_t \cdot \left(\frac{1}{g_{m1}} + R_1\right) - V_t}{R_2}$$

$$i_t R_2 + g_{m2} R_2 V_t = \frac{i_t}{g_{m1}} + i_t \cdot R_1 - V_t$$

$$i_t g_{m1} R_2 + g_{m2} R_2 g_{m1} V_t = i_t + i_t g_{m1} R_1 - V_t g_{m1}$$

$$i_t (g_{m1} R_2 - 1 - g_{m1} R_1) = V_t (g_{m1} + g_{m2} g_{m1} R_2)$$

$$\frac{V_t}{i_t} = \frac{g_{m1} R_2 - 1 - g_{m1} R_1}{g_{m1} + g_{m2} g_{m1} R_2}$$



For C_L:

$$g_{m2}(0 - V_t) - \frac{V_t}{R_2} - i_t = 0$$

$$-V_t(g_{m2} \cdot R_2 + 1) = i_t \cdot R_2$$

$$\frac{V_t}{i_t} = \frac{R_2}{1 + g_{m2} \cdot R_2}$$

$$\tau_{C_L} = C_L \cdot \frac{R_2}{1 + g_{m2} R_2}$$

$$|t_{-3}| = \frac{1}{b_1} = \frac{1}{\left(C_L \cdot \frac{R_2}{1+g_{m1}R_2} + C_{gs2}\left(\frac{g_{m1}R_2 - 1 - g_{m1}R_1}{g_{m1} + g_{m2}g_{m1}R_2}\right) + C_x \frac{1+g_m R_1}{g_{m1}} + C_{gs1}\frac{1}{g_{m1}}\right)\cdot 2\pi}$$

$$g_{m2} = \frac{2 \cdot I_2}{V_{ov2}} = \frac{2 \cdot n \cdot I_{Bias}}{V_{ov1}}$$

$$g_{m1} = \frac{2 \cdot I_{Bias}}{V_{ov1}}$$

$$C_{gs2} = \frac{2}{3} W_2 L_2 C_{ox}$$

$$n \cdot I_B = \frac{1}{2} u_n C_{ox} \left(\frac{W}{L}\right)_2$$

$$C_{ox} = \frac{n I_B}{\frac{1}{2} u_n \frac{W_2}{L_2}}$$

$$C_{gs2} = \frac{2}{3} W_2 L_2 \frac{n I_B \cdot 2 L_2}{u_n W_2}$$

$$= \frac{\frac{2}{3} 2n I_B \cdot L_2^2}{u_n}$$

$$R_1 = \frac{0 - (-V_{ss})}{I_{Bias}}$$

$$R_2 = \frac{0 - (-V_{ss})}{n I_{Bias}}$$

If $g_m R \gg 1$

$$C_L \cdot \frac{R_2}{1+g_{m2}R_2} = \frac{1}{g_{m2}}$$

$$C_x \frac{1+g_m R_1}{g_{m1}} = R_1$$

$$C_{gs2}\left(\frac{g_{m1}(R_2 - R_1)}{g_{m1}(1+g_{m2}R_2)}\right) = C_{gs2} \frac{R_2 - R_1}{(1+g_{m2}R_2)}$$

$$= \frac{R_2}{g_{m2}R_2} - \frac{R_1}{g_{m2}R_2}$$

$$= \frac{1}{g_{m2}} - \frac{R_1}{g_{m2}R_2}$$

$$g_{m1} = \frac{2 \cdot I_{Bias}}{V_{ov}}$$

$$g_{m2} = \frac{2n I_{Bias}}{V_{ov}}$$

$$R_1 = \frac{V_{ss}}{I_{Bias}}$$

$$R_2 = \frac{V_{ss}}{n I_{Bias}}$$

$$C_{gs2} = \frac{\frac{4}{3} n I_B \cdot L^2}{u_n}$$

$$C_{gs1} = \frac{\frac{4}{3} I_B \cdot L^2}{u_n}$$

$$\boxed{|f_{-3}| = \frac{1}{\left(\frac{V_{ov}}{2nI_B}\cdot C_L + \frac{\frac{4}{3}nI_B\cdot L^2}{u_n}\cdot\left(\frac{V_{ov}}{2nI_{Bias}} - \frac{V_{ov}}{2I_{Bias}}\cdot n\right) + C_x\cdot\frac{V_{SS}}{I_{Bias}} + \frac{\frac{4}{3}nI_B\cdot L^2}{u_n}\cdot\frac{V_{ov}}{2I_{Bias}}\right)\cdot 2\pi}}$$

h) Take the derivative of $|f_{-3}|$

Then find expression for V_{ov}

Q3

3. Cascoded Differential Amplifier Biasing [25 pts]

Consider the circuit shown below. A "magic battery" is used to establish the bias point of the cascode relative to other nodes in the circuit. For all parts, use (W/L) = 15 μm/1μm except for M_L which has L = 1 μm, V_{DD} = 4 V, $-V_{SS}$ = -1.5 V, k_n' = 120 μA/V², V_{t0} = 400 mV, λ = 0.1 μm/V.

a) *Briefly* explain why the "magic battery" is needed to bias M_2. (Hint: Think about the effect of changing V_{CM}.)

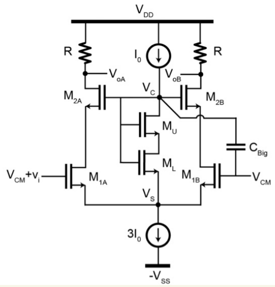
b) Derive an expression for the dc voltage V_{CS} as a function of $n=(W_U/W_L)$ and I_O.

c) What is V_{CS} if n = 5 and I_O = 120 μA?

d) Draw the differential-mode small-signal model for the circuit. Assume that C_{BIG} creates an ac short.

e) What is the maximum R value that keeps M_2 in saturation using the bias voltage computed in part (c)? Show that M_1 also remains in saturation. Assume that V_S = 0 V.

f) Calculate the maximum small-signal gain that can be achieved between v_i and v_{oA}. (Use your result from part (e).)



$I_A + I_B + I_o = 3I_o$

a) In order to provide enough overdrive voltage to drive both M_2 and M_1 as $2V_{ov} + V_t$

$V_{cm} \uparrow \Rightarrow I_{A,B} \uparrow \Rightarrow V_{o,A} \downarrow \Rightarrow M_2$ will be in triode

$V_{cs} - V_t = a \qquad -V_x + V_s := -b$

$V_c - V_s - V_t = a \qquad V_x - V_s = b \qquad V_c - V_s - V_t - V_x + V_s$

$a + V_s - V_x \quad a-b \qquad (a - b + V_t)$

b)

$$M_U: \frac{1}{2} k_n' \left(\frac{W}{L}\right)_U \cdot (V_c - V_x - V_t)^2 = I_0 .$$

$$M_L: k'n \left(\frac{W}{L}\right)_L \cdot \left((V_{cs} - V_t) \cdot (V_x - V_s) - \frac{1}{2} (V_x - V_s)^2 \right) = I_0 .$$

$$\frac{1}{2} W_u \cdot (a-b)^2 = W_L \left(a \cdot b - \frac{1}{2} b^2 \right)$$

$$\frac{1}{2} n (a^2 - 2ab + b^2) = a \cdot b - \frac{1}{2} b^2$$

$$\frac{1}{2} n a^2 - nab + \frac{1}{2} n b^2 - ab + \frac{1}{2} b^2 = 0$$

$$b^2 \left(\frac{1}{2} n + \frac{1}{2} \right) - b (a + an) + \frac{1}{2} n a^2$$

$$b = \frac{a + an \pm \sqrt{(a + an)^2 - 4 \left(\frac{1}{2} n + \frac{1}{2}\right) \cdot \frac{1}{2} n a^2}}{n+1}$$

$$a^2 + 2a^2 n + a^2 n^2 - 2na^2 \left(\frac{1}{2} n + \frac{1}{2} \right)$$

$$a^2 + 2a^2 n \quad a^2 n^2 - n^2 a^2 - a^2 n$$

$$a^2 + a^2 n = a^2 (n+1)$$

$$b = \frac{a + an \pm a\sqrt{n+1}}{n+1} = \frac{a(1 + n \pm \sqrt{n+1})}{n+1}$$

$$\frac{I_0}{k'n \left(\frac{W}{L}\right)_L} = \left(a \cdot b - \frac{1}{2} b^2 \right)$$

$$= \frac{a^2 (1 + n + \sqrt{n+1})}{n+1} - \frac{1}{2} \frac{a^2 (1 + n + \sqrt{n+1})^2}{(n+1)^2}$$

$$= \frac{a^2 \left((n+1)(1 + n + \sqrt{n+1}) - \frac{1}{2} (1 + n + \sqrt{n+1})^2 \right)}{(n+1)^2}$$

$$= \frac{a^2 \, n(n+1)}{(n+1)^2 \quad 2} = \frac{a^2 n}{(n+1) \cdot 2}$$

$$\frac{a^2 n}{(n+1)\cdot 2} = \frac{I_o}{k_n' W_L}$$

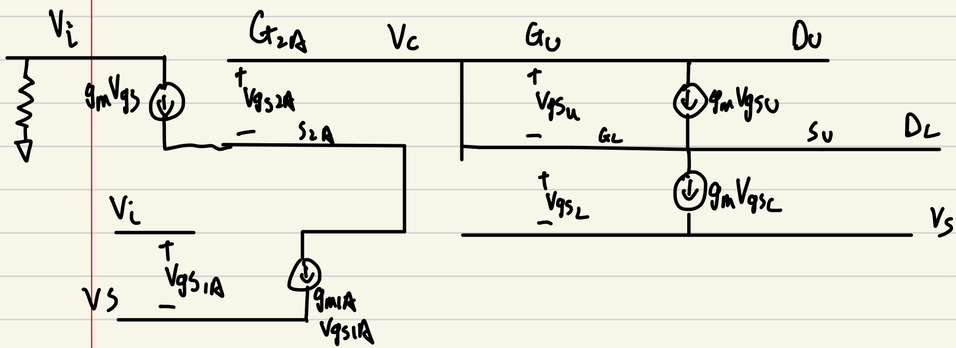
$$a^2 = \frac{I_o \cdot (n+1)\cdot 2}{k_n' \; W_L \; n}$$

$$\boxed{V_{GS} = \sqrt{\frac{I_o \cdot (n+1)\cdot 2}{k_n' \; W_L}} + V_t}$$

c) $n = 5 \quad I_o = 120u$

$$V_{GS} = \sqrt{\frac{120u\,(5+1)\cdot 2}{120u \cdot 15}} + 0.4 \approx 1.294\ V$$

d)



Vi
G2A
Vc
GU
DU
gmVgs
+
Vgs2A
−
S2A
+
Vgsu
−
GL
gmVgsU
SU
DL
Vi
+
Vgs1A
−
VS
gm1A
Vgs1A
+
VgsL
−
gmVgsL
Vs

e)

$V_{CS} = V_C = 1.294\,V$

$I_A + I_B + I_0 = 3I_0$ $\qquad$ $I_A = I_B = I_0 = 120u$

$2V_{ov} + V_t = V_C$

$V_{ov} = 0.447\,V$

$V_{0A} - V_{ov} \geq V_{ov}$

$V_{0A} \geq 2V_{ov}$

$V_{DD} - I_0 \cdot R \geq 2V_{ov}$

$\frac{V_{DD} - 2V_{ov}}{I_D} \geq R$ $\qquad$ $\boxed{R \leq 25.883\,k}$

$V_{0A} = V_{DD} - I_0 R = 0.894$

$V_G - V_{GS} = V_S = V_G - (V_{ov} + V_t)$

$= V_G - 0.847$

$= V_C - 0.847 = 0.447\,V$

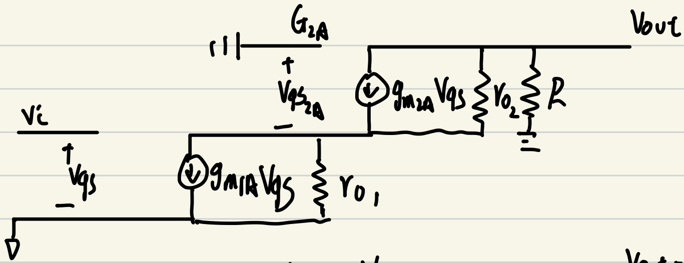
$\therefore V_{S,2A} - 0 \geq V_{ov}$

$0.447 = 0.447\,V$ $\qquad$ $\therefore M_1$ also in saturation

$r_{o_1} = r_{o_2}$

$r_o = \frac{1}{g_{ds}} \simeq \frac{1}{\lambda I_D}$

f)



$$g_{m1A}(V_i) + \frac{V_x}{r_{o_1}} = g_{m2A}(-V_x) + \frac{V_{out}-V_x}{r_{o_2}}$$

$$r_o g_{m_1} V_i + V_x = -g_{m_2} r_o V_x + V_{out} - V_x$$

$$V_x(1+1+g_{m_2} r_o) = V_{out} - r_o g_{m_1} V_i$$

$$V_x = \frac{V_{out} - r_o g_{m_1} V_i}{2 + g_{m_2} r_o}$$

$$-g_{m2A}(-V_x) = \frac{V_{out}-V_x}{r_o} + \frac{V_{out}}{R}$$

$$R r_o g_{m2A} = V_{out}\cdot R - V_x \cdot R + r_o V_{out}$$

$$r_o R g_{m_2} + \frac{V_{out}\cdot R - r_o R g_m V_i}{(2+g_{m_2} r_o)} = V_{out}(R + r_o)$$

solved by symbolab

$$V_{out} = -V_i\left(\frac{R g_{m_1} g_{m_2} r_o^2 + R g_{m_1} r_o}{g_{m_2} r_o^2 + 2r_o + R}\right)$$

$$\boxed{\frac{V_{out}}{V_i} = -\frac{R g_{m_1}(g_{m_2} r_o^2 + r_o)}{g_{m_2} r_o^2 + 2r_o + R}}$$

$r_o = \frac{1}{\lambda I_D}$

$g_{m_1} =$

$$g_m = \frac{2I_D}{V_{OV}} = \sqrt{2I_D \mu C_{ox} \frac{W}{L}(1+\lambda V_{DS})} \cong \sqrt{2I_D \mu C_{ox} \frac{W}{L}}$$

$$r_0 = \frac{1}{\lambda I_D} = \frac{1}{0.1 \cdot 120u} = \frac{250000}{3}$$

$$g_{m_2} = g_{m_1} = \sqrt{2 \cdot 120u \cdot 120u \cdot 15(1+0.1V_{DS})} = 671.8u$$

$$V_{DS_1} = V_{DD} - I_D \cdot R - V_{S_2} = 0.447$$

$$V_{DS_2} = V_{S_2} = 0.447$$

$$\boxed{\frac{V_{out}}{V_i} = -16.9973}$$

Q4

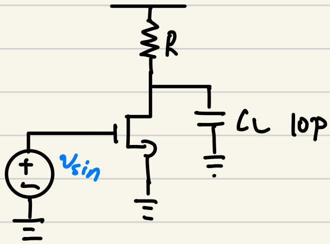
-V_{ss}

4. Common Source Amplifier Design [35 pts]
Design an amplifier with a gain of 25 dB and a -3dB bandwidth of 5 MHz. The amplifier must drive a load capacitance C_L of 10 pF and have an output swing of 600 mV_{pp}. Minimize the power dissipation, but do not forget the company policy of $V_{OV} \geq 150$ mV. The maximum allowed *W*/*L* size ratio for any device is 500. If your *W* or *L* sizes exceed the area constraints in Cadence, consider using "multiplicity" or "number of fingers" to avoid this. Submit all your hand calculations, schematic, and simulation results showing dc operating point, ac response, and transient response. Briefly describe your design methodology. **Note:** Zero points will be given for designs not supported by hand calculations. There may be slight discrepancies between your hand calculations, but you should be able to explain why and how you "tweaked" the circuit to get it to meet the above specifications.

$$g_m = \frac{2I_D}{V_{OV}}$$

BW ~ 5.17M

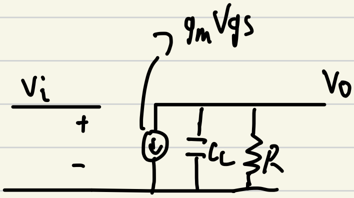
gain ~ 24dB



① Gain

$$25 = 20\log_{10} x$$
$$\frac{5}{4} = \log_{10} x$$
$$10^{\frac{5}{4}} = x$$
$$x = 17.78$$

$$V_{out} = 600m$$
$$V_i = \frac{600m}{17.78} = 33.7m$$



$$A_v = k_n' \frac{W}{L} V_{ov} \cdot R$$

$$g_m V_i = \frac{V_o}{R} + sC_L \cdot V_o$$
$$g_m R\, V_i = V_o + sC_L \cdot R \cdot V_o$$
$$g_m R = V_o(1 + sC_L R)$$
$$\frac{V_o}{V_i} = \frac{g_m R}{1 + \frac{s}{RL}}$$

① $g_m V_i = \frac{V_o}{R}$

$$A_v = \frac{V_o}{V_i} = g_m R$$

② $\tau = C_L \cdot R$

$$5M = f = \frac{1}{2\pi R \cdot C_L} \Rightarrow R \approx 3183\,\Omega$$

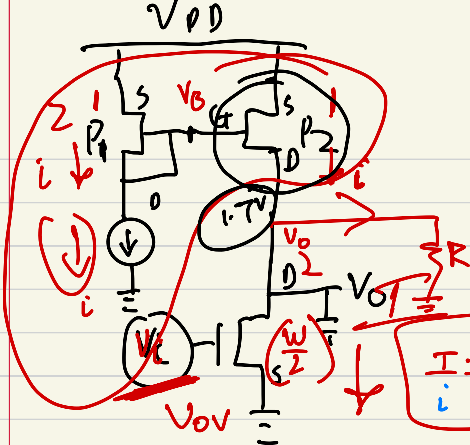
③ $R = 3183\,\Omega$ $\quad g_m = 5.586m = \frac{A_v}{R}$

④ $V_{ov} = 155mV$, $I_D = g_m \cdot V_{ov} \cdot \frac{1}{2} = 432.9u$

⑤ Assuming 155mV as V_{ov} ($V_G = 462m$), $k_n' = 267.2\,uA/V^2$

$$g_m = k_n' \frac{W}{L} V_{ov} \Rightarrow \frac{W}{L} = 134.89 \Rightarrow \frac{W}{L} = \frac{135}{1}$$

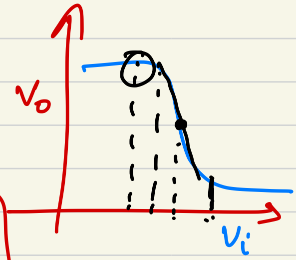
Tweak V_{ov}

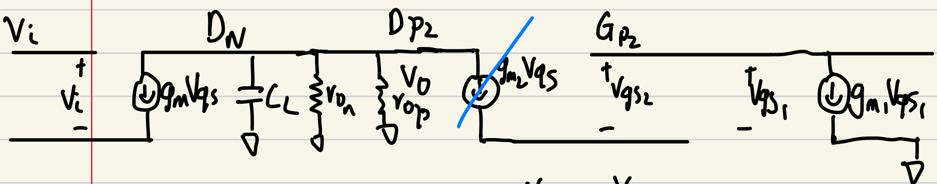


$$g_m = \frac{2I_D}{V_{ov}}$$

$$g_{ds} = \lambda I_D$$

$$\boxed{\underset{i}{I} = K_n'\left(\frac{W}{2}\right) V_{ov}^2}$$





$$g_m V_i = \quad + \frac{V_o}{r_{on}} + \frac{V_o}{r_{op}}$$

$$g_m r_{on} r_{op} V_i = \quad + r_{op} V_o + r_{on} V_o$$

$$g_m r_{on} r_{op} V_i = V_o(r_{op} + r_{on})$$

$$\frac{V_o}{V_i} = \frac{g_m r_{on} r_{op}}{r_{op} + r_{on}} = A_v$$

$$\frac{1}{g_{ds}} = r$$

$$g_{ds} = \lambda I_D$$

$$\lambda = \frac{1}{V_A}$$

$$R_{C_L}: \quad i_t = \frac{V_t}{r_n} + \frac{V_t}{r_p}$$

$$i_t(r_n r_p) = (r_p + r_n) V_t$$

$$R_{C_L} = \frac{r_n r_p}{r_p + r_n}$$

$$r = \frac{1}{\lambda I_D}$$

Assume $r_n = r_p$ $\quad \lambda$

$$f = \frac{1}{2\pi R_{C_L} \cdot C_L} \Rightarrow R_{C_L} = 3183\,\Omega \;,\; = \frac{r^2}{2r}$$

$$r = 6366\,\Omega$$

$$\frac{\frac{1}{\lambda_n I_D} \cdot \frac{1}{\lambda_p I_D}}{\frac{1}{\lambda_n I_D} + \frac{1}{\lambda_p I_D}}$$

$$\frac{\lambda_p I_D + \lambda_n I_D}{\lambda_n I_D \lambda_p I_D}$$

$$A_v = \frac{r^2 \cdot g_m}{2r} = 17.78$$

$$\frac{r}{2} g_m = 17.78$$

$$g_m = 5.586\,m$$

If $r = \frac{1}{\lambda I_D}$ $\lambda \propto L$, $L = 2um$ $\lambda = 0.2$

$I_D = \frac{1}{\lambda r} = 785u.$ $g_m = 5.586m$

$g_m = k_n' \cdot \frac{W}{L} \cdot V_{OV}$ $V_t = 307.2m$

$V_{OV} = 177.8$

① $\frac{W}{L} = 117 \Rightarrow W = 234$

↳ $VIN = 556m$

↳ $BW = 3.579M$

Gain = 29

② Decrease L to have more current

$L = 1um$

$VIN = 582m$

$BW = 5.189M$

Gain = 30

$I_{dc} = 1.4m$

$\frac{W}{L} = \frac{227}{1u}$

③ I_D is the knob to have more BW

$$\text{gain} = \left(\overset{\uparrow g_m}{-\frac{2I_D}{V_{OV}}}\right) \overset{\uparrow r_n \| r_p}{R_{out}} \downarrow$$

$$BW = \frac{1}{2\pi R_{out} C_L} \downarrow$$

① Calculate Rout

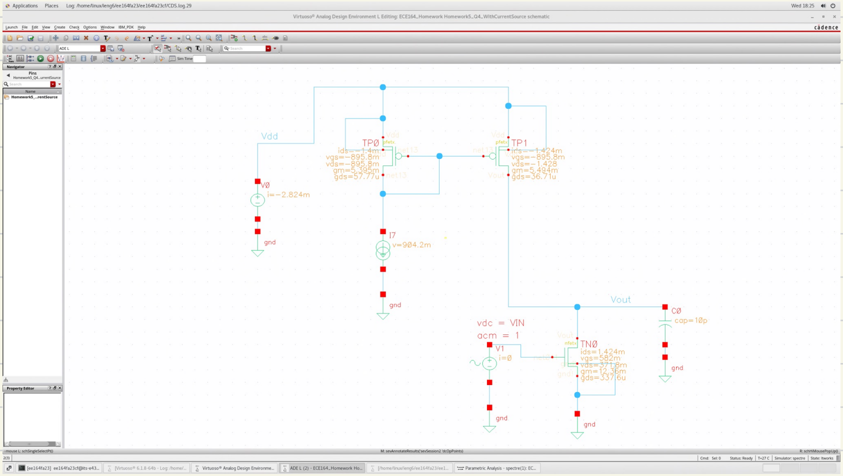
② I_D, g_m

③ VIN

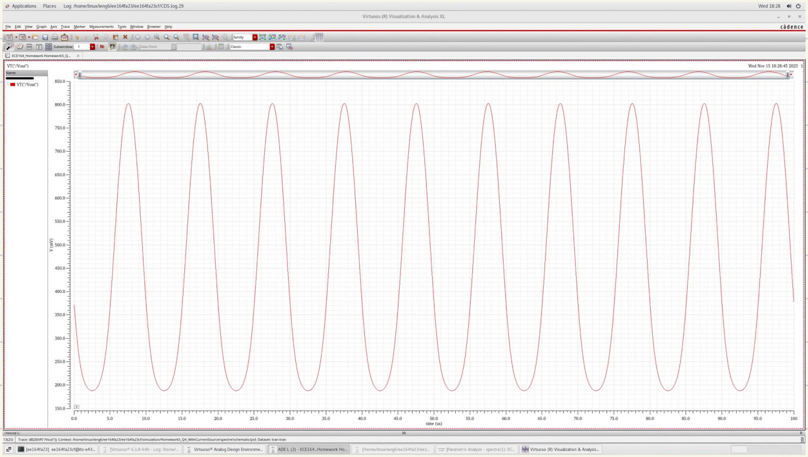
① Fix I_D, $\frac{W}{L}$

Find VIN

DC Operating Point



AC



BW 5.189M

Gain 30.2 dB