Q1

$\frac{I_o}{I_i}$

**1. Pushing it to the Limit! [15 pts]**
Prove that the short-circuit unity current gain (transit frequency) of an NMOS transistor is $g_m/(C_{gs}+C_{gd})$. Neglect all extrinsic capacitors. (Hint: You will need to make an approximation; assume that the $sC_{gd}$ term is small compared to $g_m$.)

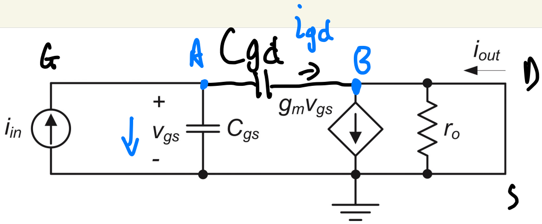



**Figure 3-9:** Small-signal circuit model for finding the MOSFET's transit frequency.

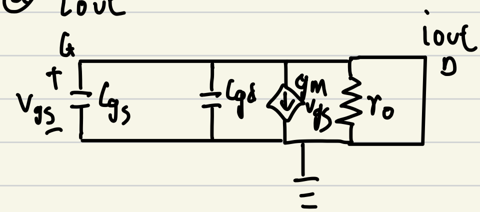
$\frac{1}{j\omega C} = Z_c$

$\frac{1}{sC} = Z_c$

Cgd is parallel with Cgs

$C_{tot} = C_{gd} + C_{gs}$

@ iout




$$i_{out} = g_m V_{gs} = g_m\, i_{in}\, C_{tot}$$

$$= g_m\, i_{in} \frac{1}{s(C_{gd}+C_{gs})}$$

$$\left|\frac{i_{out}}{i_{in}}\right| = \frac{g_m}{s\cdot(C_{gd}+C_{gs})} = \frac{g_m}{j\omega(C_{gd}+C_{gs})} = 1$$

$$\boxed{\omega_T = \frac{g_m}{C_{gs}+C_{gd}}}$$

# Q2

**2. Common-Source with Source Degeneration [25 pts]**

Consider the circuit shown below.

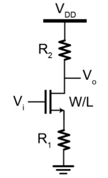
a) Draw the small-signal model of this circuit. Show that the small-signal dc-gain is $\frac{v_o}{v_i} = -\frac{g_m R_2}{1+g_m R_1}$ if the transistor is in saturation and the body is tied to the source.

b) Redo part (a), but now tie the body to ground. (Note: The body is tied to the source for all other parts of this problem.)

c) Calculate the maximum value of $R_2$ for each of the following bias currents: 50, 150, and 225 μA. Use $R_1$ = 8 kΩ, $k_n'$ = 125 μA/V², $W/L$ = 8, and $V_{DD}$ = 5 V. Calculate the resulting small-signal gain for each current, assuming the transistor is in saturation.

d) This circuit has a form of negative feedback. Show that the gain becomes independent of $g_m$ under "strong feedback" where $g_m R_1 \gg 1$.

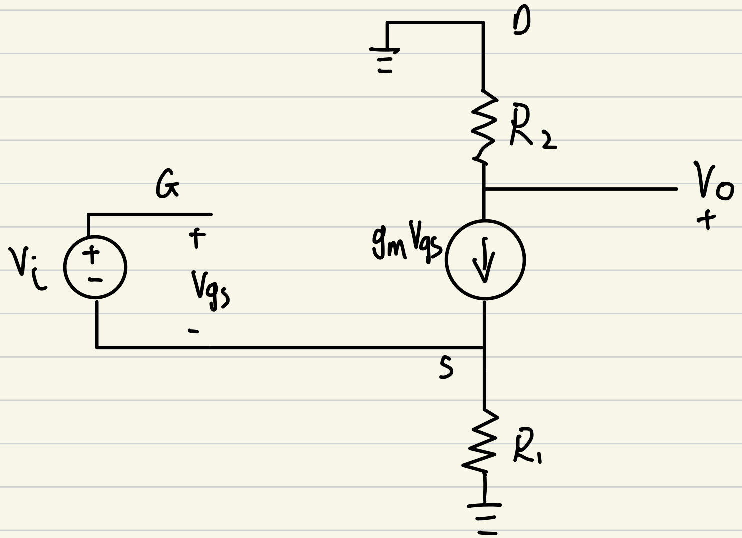
e) Derive an expression for the circuit's output impedance (*i.e.*, the impedance seen looking into $v_o$). Take into account finite output impedance.




3. Stacked Amplifier [20 pts]

$$V_o = -g_m V_{gs} \cdot R_2$$

a)




$$V_i = V_{gs} + g_m V_{gs} \cdot R_1 \qquad V_o = -g_m V_{gs} \cdot R_2$$

$$= V_{gs}(1+g_m R_1)$$

$$\frac{V_o}{V_i} = \frac{-g_m V_{gs} R_2}{V_{gs}(1+g_m R_1)} = \boxed{\frac{-g_m R_2}{1+g_m R_1}}$$

b)

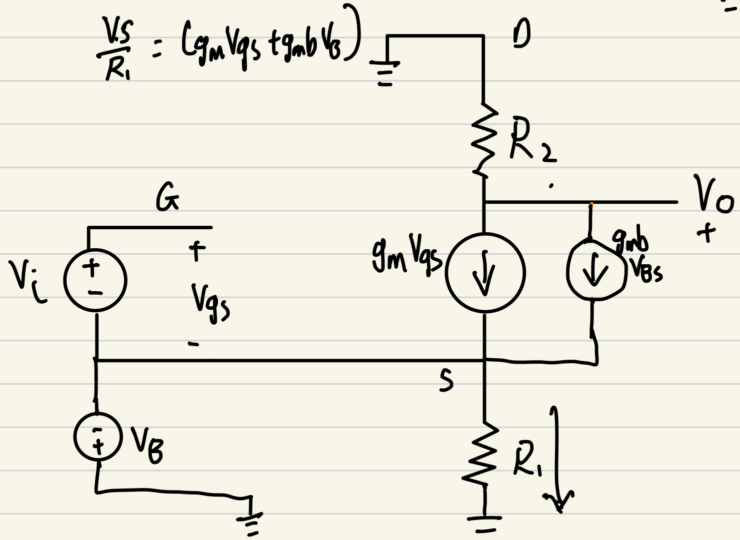
$$\frac{V_o}{R_2} = -(g_m V_{gs} + g_{mb} V_B)$$

$$\frac{V_S}{R_1} = (g_m V_{gs} + g_{mb} V_B)$$

$V_{BS} = V_B - V_S$

$= -V_S$

$V_{gs} = V_i - V_S$




$$V_i = (g_m V_{gs} + g_{mb}(-V_S)) \cdot R_1 + V_{gs} \qquad V_o = -(g_m V_{gs} + g_{mb}(-V_S)) \cdot R_2$$

$V_S = V_i - V_{gs}$

$$V_i = (g_m V_{gs} + g_{mb}(V_{gs} - V_i)) \cdot R_1 + V_{gs}$$

$$V_i = R_1 g_m V_{gs} + R_1 g_{mb} V_{gs} - R_1 g_{mb} V_i + V_{gs}$$

$V_{gs} = V_i + \frac{R_1 V_o}{R_2}$

$$V_i (1 + R_1 g_{mb}) = V_{gs}(R_1 g_m + R_1 g_{mb} + 1)$$

$V_{gs} - \frac{R_1 V_o}{R_2}$

$$V_i = \frac{V_{gs}(R_1 g_m + R_1 g_{mb} + 1)}{1 + R_1 g_{mb}}$$

$$V_o = -(g_m V_{gs} + g_{mb}(V_{gs} - V_i)) \cdot R_2$$

$$= -g_m V_{gs} \cdot R_2 - g_{mb} V_{gs} R_2 + g_{mb} V_i \cdot R_2$$

$$= -g_m V_{gs} \cdot R_2 - g_{mb} V_{gs} R_2 + g_{mb}\left(V_{gs} - \frac{R_1 V_o}{R_2}\right) \cdot R_2$$

$$V_o = -g_m V_{gs} \cdot R_2 - g_{mb} V_{gs} R_2 + g_{mb} V_{gs} R_2 - R_1 V_o \cdot g_{mb}$$

$$V_o(1+R_1 g_{mb}) = V_{gs}(-g_m R_2 - g_{mb} R_2 + g_{mb} R_2)$$

$$V_o = \frac{V_{gs}(-g_m R_2)}{1+R_1 g_{mb}}$$

$$\frac{V_o}{V_i} = \frac{V_{gs}(-g_m R_2)}{1+R_1 g_{mb}} \cdot \frac{1+R_1 g_{mb}}{V_{gs}(R_1 g_m + R_1 g_{mb} + 1)}$$

$$= \boxed{\frac{-g_m R_2}{g_m R_1 + g_{mb} R_1 + 1}}$$

c) $I_D = 50uA \quad R_1 = 8k$

$$I_D = \frac{1}{2} k_n' \cdot \frac{W}{L} \cdot V_{ov}^2$$

$$V_{ov} = V_{GS} - V_t$$

$$V_S = V_{R_1} = I_D \cdot R_1 = 0.4V$$

$$50u = \frac{1}{2} \cdot 125u \cdot 8 \cdot V_{ov}^2$$

$$V_{ov} = \frac{\sqrt{10}}{10} V \approx 0.316V$$

$$g_m = \frac{2I_D}{V_{ov}} = \frac{2 \cdot 50u}{\frac{\sqrt{10}}{10}} \approx 3.16 \cdot 10^{-4}$$

Since in saturation $V_{DS} = V_{ov}$

$$V_{DS} = V_D - V_S$$

$$\frac{\sqrt{10}}{10} V = V_D - 0.4V$$

$$V_D = \frac{4+\sqrt{10}}{10} V$$

$$R_2 = \frac{V_{DD} - V_D}{I_D} = \frac{5 - \frac{4+\sqrt{10}}{10}}{50uA} \approx \boxed{85.68\ k\Omega}$$

$$|A_v| = \left| -\frac{g_m R_2}{1+g_m R_1} \right| = \frac{3.16 \cdot 10^{-4} \cdot 85.68k}{1+3.16 \cdot 10^{-4} \cdot 8k} \approx \boxed{7.68}$$

---

$I_D = 150uA \qquad V_S = I_D \cdot R_1 = 1.2V$

$$150u = \frac{1}{2} \cdot 125u \cdot 8 \cdot V_{ov}^2$$

$$V_{ov} = \frac{\sqrt{30}}{10} V$$

$$V_{DS} = V_D - V_S$$

$$\frac{\sqrt{30}}{10} = V_D - 1.2V$$

$$V_D = \frac{12+\sqrt{30}}{10}$$

$$R_2 = \frac{5 - \frac{12+\sqrt{30}}{10}}{150u} \approx \boxed{21.68\ k\Omega}$$

$$|A_v| = \left| \frac{g_m R_2}{1+g_m R_1} \right|$$

$$g_m = \frac{2 \cdot 150u}{\frac{\sqrt{30}}{10}} = 5.477 \cdot 10^{-4}$$

$$= \frac{5.477 \cdot 10^{-4} \cdot 21.68k}{1+5.477 \cdot 10^{-4} \cdot 8k} \approx \boxed{2.21}$$

$I_D = 225\ uA$ $\quad V_S = I_D \cdot R_1 = 1.8\ V$

$$225u = \frac{1}{2} \cdot 125u \cdot 8 \cdot V_{ov}^2$$

$$V_{ov} = \frac{3\sqrt{5}}{10}\ V$$

$$V_{DS} = V_D - V_S$$

$$\frac{3\sqrt{5}}{10} = V_D - 1.8$$

$$V_D = \frac{18 + 3\sqrt{5}}{10}$$

$$R_2 = \frac{5 - \frac{18+3\sqrt{5}}{10}}{225\ u} \approx \boxed{11.24\ k\Omega}$$

$$|A_v| = \frac{g_m R_2}{1 + g_m R_1}$$

$$g_m = \frac{2 \cdot 225 \cdot u}{\frac{3\sqrt{5}}{10}}$$

$$\approx 6.71 \cdot 10^{-4}$$

$$= \frac{6.71 \cdot 10^{-4} \cdot 11.24k}{1 + 6.71 \cdot 10^{-4} \cdot 8k}$$

$$\approx \boxed{1.18}$$

d) Based on the gain from a

$$\frac{V_o}{V_i} = -\frac{g_m R_2}{1+g_m R_1}$$

when $g_m R_1 >> 1$

$$\frac{V_o}{V_i} = -\frac{\cancel{g_m} R_2}{\cancel{g_m} R_1} \Rightarrow g_m \text{ is canceled } = \frac{-R_2}{R_1}$$

The gain becomes independent of $g_m$

e) Looking into $V_o$

$V_i$ is grounded

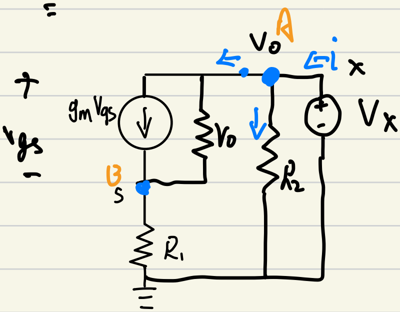
$V_{gs} = V_g - V_s$

$V_{gs} = -V_s$

$=$

$V_{gs}$ , $V_s = g_m V_{gs} \cdot R_1$

$$R_{eq} = \frac{V_x}{i_x} \qquad i_x = -g_m V_{gs}$$




@ A

$$i_o = \frac{V_o}{R_2} + \frac{V_{gs}}{r_o} + g_m V_{gs}$$

@ B

$$g_m V_{gs} + \frac{V_{gs}}{r_o} = \frac{V_o - V_{gs}}{R_1}$$

$$R_1 g_m V_{gs} + \frac{R_1 V_{gs}}{r_o} = V_o - V_{gs}$$

$$V_{gs}\left(1 + R_1 g_m + \frac{R_1}{r_o}\right) = V_o$$

$$i_o = \frac{V_o}{R_2} + \frac{V_o}{\left(1+R_1 g_m + \frac{R_1}{r_o}\right) r_o} + g_m \frac{V_o}{\left(1+R_1 g_m + \frac{R_1}{r_o}\right)}$$

$$= V_o\left(\frac{1}{R_2} + \frac{1}{\left(1+R_1 g_m + \frac{R_1}{r_o}\right)\cdot r_o} + \frac{g_m}{\left(1+R_1 g_m + \frac{R_1}{r_o}\right)}\right)$$

$$i_o = V_o\left(\frac{\left(1+R_1 g_m + \frac{R_1}{r_o}\right)\cdot r_o + R_2 + R_2 r_o g_m}{R_2 \cdot \left(1+R_1 g_m + \frac{R_1}{r_o}\right)\cdot r_o}\right)$$

$$\frac{V_o}{i_o} = \boxed{\frac{R_2 \cdot \left(1+R_1 g_m + \frac{R_1}{r_o}\right)\cdot r_o}{\left(1+R_1 g_m + \frac{R_1}{r_o}\right)\cdot r_o + R_2 + R_2 r_o g_m}}$$

$M_2 \quad V_{DS}$

# Q3

**3. Stacked Amplifier [20 pts]**

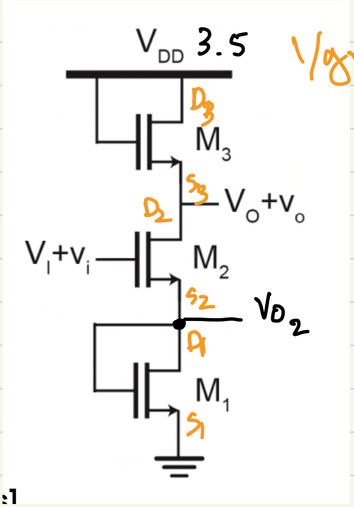
Consider the circuit shown below. Initially, assume all NMOS devices are identical, $\lambda = 0$, and $V_{DD} = 3.5$ V.

a) For what range of dc input voltages ($V_I$) are all the transistors in saturation? Determine the corresponding output range ($V_O$).

b) Assuming a dc input voltage within this range, what is the small-signal gain?

c) Assuming $V_{t0}$ = 300 mV, $k_n'$ = 40 μA/V², $W/L$ = 5, and $V_{OV,min}$ = 150 mV, what is the required dc input voltage and $V_{DD}$ needed to ensure that all devices are in saturation?

d) If you wanted to gain 5 V/V, how would you size $M_1$ and $M_3$ while leaving $M_2$ at $W/L$ =10? Does this change the requirement in part (c)? The limit on $W/L$ is 35.




## a)

Assume $M_2$ also in Saturation

$M_3$: Will be in saturation since $V_{DS} = V_{GS}$, then $V_{DS} > V_{GS} - V_t$

$$V_{GS} - V_t = 3.5 - V_O - V_t > 0$$

$$3.5 - V_t > V_O$$

$$V_O < 3.5 - V_t$$

$$V_O + V_t < 3.5$$

$M_2$: $V_{DS} = V_O - V_{O2}$

$V_{GS} = V_I - V_{O2}$

$$V_{DS} > V_{GS} - V_t \Rightarrow V_O - V_{O_2} > V_I - V_{O_2} - V_t$$

$$\boxed{V_O + V_t > V_I}$$

$M_1$: will be in Saturation as well, since $V_{DS} = V_{GS}$, then $V_{DS} > V_{GS} - V_t$, However $V_{GS} - V_t > 0$

$$V_{GS,1} = V_{O_2} = V_{DS,1} > V_t$$

In $M_2$ $\quad V_{GS,2} - V_t > 0 \quad = V_I - V_{O_2} - V_t > 0$

$$= V_I - V_{DS,1} - V_t > 0$$

so

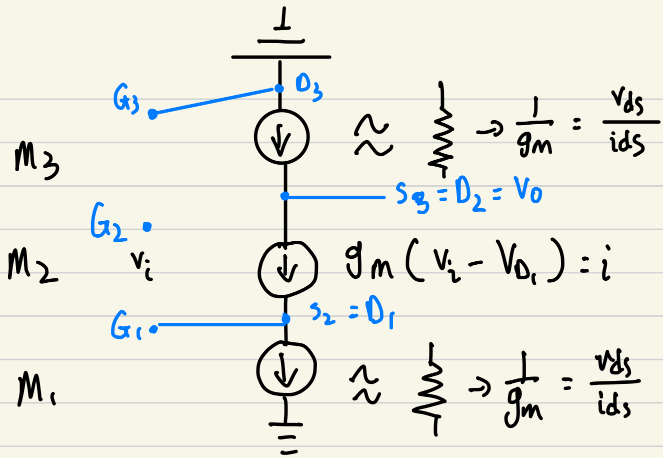
$$V_I > V_{DS,1} + V_t > V_t + V_t$$

$$\boxed{2V_t < V_{O_2} + V_t < V_I < V_O + V_t < 3.5}$$

Based on the range of $V_I$ above

$$\boxed{V_t < V_O < 3.5 - V_t}$$

b)




$$V_i = (V_i - V_{o_1}) + g_m (V_i - V_{o_1}) \cdot \frac{1}{g_m}$$

$$V_o = -g_m (V_i - V_{o_1}) \cdot \frac{1}{g_m}$$

$$\left|\frac{V_o}{V_i}\right| = \left|\frac{-(V_i - V_{o_1})}{2(V_i - V_{o_1})}\right| = \boxed{0.5}$$

c)

$$V_{OV} = V_{GS} - V_t \Rightarrow V_{GS} = 450mV$$

$V_{DS} > V_{OV}$

$$M_3: \quad I_{D_3} = \underbrace{\frac{1}{2} \cdot k_n' \cdot \frac{W}{L}}_{A} \cdot (V_{GS} - V_t)^2$$

$V_{GS} = V_{DD} - V_O$

$$= A(V_{DD} - V_O - V_t)^2$$

$$M_2: \quad I_{D_2} = A \cdot (V_I - V_{O_2} - V_t)^2$$

$V_{GS} = V_I - V_{O_2}$

$$M_3: \quad I_{D_1} = A \cdot (V_{O_2} - V_t)^2$$

$$I_{D_3} = I_{D_2} = I_{D_1}$$

$$V_{DD} - V_O - V_t = V_I - V_{O_2} - V_t = V_{O_2} - V_t = 150m\,V$$

1 $V_{DD} - V_O - 300m = 150m$

$V_{DD} - V_O = 450m$

2 $V_I - V_{O_2} = 450mV$

3 $V_{O_2} = 450mV$

Use 3 solve 2 $\rightarrow$ $\boxed{V_I = 900mV}$

$V_{DS,2} \geq V_{OV}$

$V_O - V_{O_2} = 150mV$

$V_O = 600mV$

Use 1

$V_{DD} - V_O = 450m \Rightarrow \boxed{V_{DD} = 1.05V}$

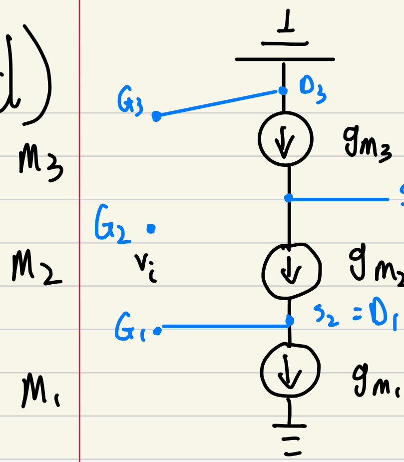
Check:

✓ $M_3$ $V_{DS} = 1.05 - 0.6 = 450mV \geq 150mV$

✓ $M_2$ $V_{DS} = 600m - 450m \geq 150mV$

✓ $M_1$ $V_{DS} = 450m \geq 450m - 300mV$

d)




$g_{m_3}(-V_O) = i$

$g_{m_2}(v_i - V_{D_1}) = i$

$g_{m_1}(V_{D_1}) = i$

$\frac{1}{2} k_n' \cdot \frac{W}{L} \cdot V_{OV}^2$

$g_{m_3} - V_O = g_{m_1} V_{D_1}$

$-\frac{g_{m_3}}{g_{m_1}} V_O = V_{D_1}$

$g_m = \sqrt{2 i_D k_n \frac{W}{L}}$

$g_m = \sqrt{\frac{W}{L}} \cdot A$

$g_m^2 = \frac{W}{L} \cdot A^2$

$$g_{m_1} V_{D_1} = g_{m_2} V_i - g_{m_2} V_{D_1}$$

$$V_{D_1} = \left(\frac{g_{m_2}}{g_{m_1} + g_{m_2}}\right) V_i$$

$$-g_{m_3} V_O = g_{m_2}\left(V_i - V_i \frac{g_{m_2}}{g_{m_1} + g_{m_2}}\right)$$

$$-g_{m_3} V_O = V_i \left(g_{m_2} - \frac{g_{m_2}^2}{g_{m_1} + g_{m_2}}\right)$$

$$\left|\frac{V_O}{V_i}\right| = \left|\frac{-1}{g_{m_3}}\left(g_{m_2} - \frac{g_{m_2}^2}{g_{m_1} + g_{m_2}}\right)\right| = 5$$

$$g_{m_2} - \frac{g_{m_2}^2}{g_{m_1} + g_{m_2}} = 5 g_{m_3}$$

$$\frac{g_{m_2}(g_{m_1} + g_{m_2}) - g_{m_2}^2}{g_{m_1} + g_{m_2}} = 5 g_{m_3}$$

$$\frac{g_{m_1} g_{m_2}}{g_{m_1} + g_{m_2}} = 5 g_{m_3}$$

$$\frac{\sqrt{10}\, g_{m_1}}{\sqrt{10} + g_{m_1}} = 5 g_{m_3}$$

$$\frac{\sqrt{10}\, g_{m_1}}{\sqrt{10} + g_{m_1}} = 5 g_{m_3}$$

Let's assume $g_{m_1} = \sqrt{35} \cdot A \Rightarrow \boxed{\left(\frac{W}{L}\right)_1 = 35}$

then solving above equation, we get

$$g_{m_3} = \frac{-2\sqrt{35} + 7\sqrt{10}}{25} \cdot A$$

$$\boxed{\left(\frac{W}{L}\right)_3 \approx 0.17}$$

This wouldn't change the requirement in part c), since we only change the $\left(\frac{W}{L}\right)$ ratio to maintain same current across each transistor

# Q4

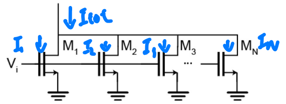
**4. Parallel/Series Transistors [20 pts]**

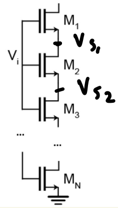
Consider the circuits shown below. Each transistor has the same *W/L*.

a) Derive an expression for *W* and *L* for a single transistor that replaces, and is equivalent to, the multiple transistors. (Note: Neglect the body effect.)

b) For the parallel transistors, re-derive (a) without requiring all the transistors to have the same *W*.

c) For the series transistors, re-derive (a) without requiring all the transistors to have the same *L*. (Note: Neglect the body effect.)




a)

Parallel: For $M_1$ to $M_N$

$$I_d = \frac{1}{2} k_n' \cdot \frac{W}{L} (V_i - V_t)^2 = \underbrace{\frac{1}{2} k_n' (V_i - V_t)^2}_{A} \cdot \frac{W}{L} \quad k$$

$$A \cdot \frac{W}{L}$$

A is the same for all transistors

$$I_{tot} = I_1 + I_2 + I_3 + \dots I_N$$
$$= A \cdot \frac{W_1}{L_1} + A \cdot \frac{W_2}{L_2} + A \cdot \frac{W_3}{L_3} + \dots + A \cdot \frac{W_N}{L_N}$$

Since $\frac{W_1}{L_1} = \frac{W_2}{L_2} = \dots = \frac{W_N}{L_N}$

$$I_{tot} = A \frac{W}{L} \cdot N$$

Replace all transistors with single one

$$I_{tot} = A \cdot \left(\frac{W}{L}\right)_{eq} = A \cdot \frac{W}{L} \cdot N$$

So $\boxed{\left(\frac{W}{L}\right)_{eq} = N \cdot \frac{W}{L}}$

$V_i - V_{s_1} - V_t > 0$

$V_i - V_t > V_{s_1}$

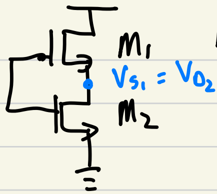
$V_{s_1} = V_{DS_2}$

$V_{GS} - V_t = V_{ov}$

$V_i - V_t = V_{ov}$

Series

$V_i$




$M_1$: $I_D = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2$

$= \frac{1}{2} k_n' \frac{W}{L} (V_i - V_{s_1} - V_t)^2$

$M_2$: $k_n \frac{W}{L} \left( (V_i - V_t) \cdot V_{s_1} - \frac{1}{2} V_{s_1}^2 \right)$

If $M_2$ in Sat, then $M_1$ is in cutoff

If $M_1$ in Sat, then $M_2$: $V_{DS_2} = V_{s_1}$ in Triode Region

$\Downarrow$

$V_i - V_{s_1} - V_t > 0$

$V_i - V_t > V_{s_1}$

$\Downarrow$

$V_{DS_2} < (V_{GS2} = V_i - V_t)$

$V_{s_1} < V_i - V_t$

$I_D = \frac{1}{2} k_n' \frac{W}{L} (V_i - V_{s_1} - V_t)^2 = k_n \frac{W}{L} \left( (V_i - V_t) \cdot V_{s_1} - \frac{1}{2} V_{s_1}^2 \right)$

$= \frac{1}{2} \frac{W}{L} (V_i^2 - 2V_i V_{s_1} - 2 V_i V_t + V_{s_1}^2 + 2 V_{s_1} V_t + V_t^2)$

$= \frac{W}{L} ( V_i V_{s_1} - V_t V_{s_1} - \frac{1}{2} V_{s_1}^2 )$

$= \frac{1}{2} V_i^2 - V_i V_{s_1} - V_i V_t + \frac{1}{2} V_{s_1}^2 + V_{s_1} V_t + \frac{1}{2} V_t^2$

$= V_i V_{s_1} - V_t V_{s_1} - \frac{1}{2} V_{s_1}^2$

$= \frac{1}{2} V_i^2 - 2 V_i V_{s_1} - V_i V_t + V_{s_1}^2 + 2 V_t V_{s_1} + \frac{1}{2} V_t^2$

$= \frac{1}{2} (V_i - V_t)^2 - 2 V_i V_{s_1} + V_{s_1}^2 + 2 V_t V_{s_1}$

Series

when n transistor in series

the equivalent transistor has the same width but $n \cdot L$

$$\left(\frac{W}{L}\right)_{eq} = \frac{W}{N \cdot L}$$

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b) $I_{tot} = I_1 + I_2 + I_3 + \dots I_N$

$$= A \cdot \frac{W_1}{L_1} + A \cdot \frac{W_2}{L_2} + A \cdot \frac{W_3}{L_3} + \dots + A \cdot \frac{W_N}{L_N}$$

Since W is different, but same Length

$$I_{tot} = A \cdot \left(\frac{\sum_{n=1}^{N} W_n}{L}\right) = A \cdot \frac{W_1}{L} + A \cdot \frac{W_2}{L} + \dots = A\left(\frac{W_1 + W_2 + \dots W_N}{L}\right)$$

Replace all transistors with single one

$$I_{tot} = A \cdot \left(\frac{W}{L}\right)_{eq} = A \cdot \frac{\sum_{n=1}^{N} W_n}{L}$$

So $\boxed{\left(\frac{W}{L}\right)_{eq} = \frac{\sum_{n=1}^{N} W_n}{L}}$

c) Different $L$, but same $W$

$$\left(\frac{W}{L}\right)_{eq} = \frac{W}{\sum_{n=0}^{N} L_n}$$

Q5

**5. Switches [20 pts]**
We often need to use transistors as switches in circuits. In particular, a whole class of discrete-time circuits relies on making good switches. Your supervisor at *Triton Industries* has asked you to make and characterize some switches for their upcoming design.

a) Derive an expression as a function of $W/L$ for the "on-resistance" of an NMOS switch with the gate tied to $V_{DD}$. (Note: Neglect the body effect.) Use a $V_{DD}$ = 1.8V and $V_t$ = 500 mV.
b) Using Cadence, sweep $V_{DS}$ from $-V_{DD}/2$ to $V_{DD}/2$ and $W/L$ with $V_G = V_{DD}$ and $V_S = V_{DD}/2$. The body should be tied to $V_{SS}$ and $L$ =500 nm. How should you size the switch if you want the on-resistance to be less than 1 kΩ across the entire range? Submit your schematic and simulation results. (Hint: You need to do two sweeps! Re-read the tutorial if you are unsure how to do this.)
c) Repeat (a) - (b) using a PMOS. Use $V_G$ = 0 V instead of $V_{DD}$ and $V_B = V_{DD}$. Assume $|V_{tp}| = V_t$.
d) Repeat (a) - (b) using an NMOS and PMOS in parallel. Size the PMOS $k$ times larger where $k$ is the ratio between parts (b) and (c). Explain where this ratio arises from. Use the correct $V_G$ for each transistor.

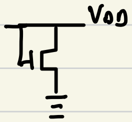
when use transistors as switches. transistor are usually in triode region

a)

$$R_{(ON)} = \frac{V_{DS}}{i_D}$$

$V_{GS} = V_{DS}$

$V_{GS} = V_{DD}$
$V_{DS} = V_{DD}$



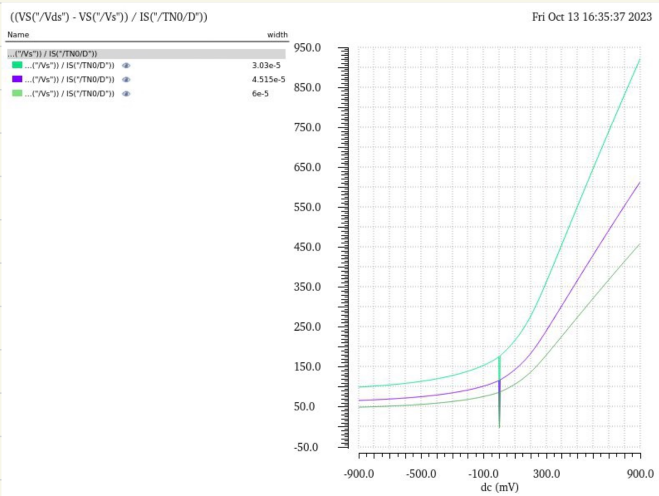

$$I_D = \frac{1}{2} k_n' \frac{W}{L} \left( V_{OV} V_{DS} - \frac{1}{2} V_{DS}^2 \right)$$

Assuming $V_{DS} << 1$, so $2^{nd}$ term can be neglected

$i_D = k'_n \frac{W}{L} V_{OV} \cdot V_{DS}$ $\quad V_{OV} = V_{GS} - V_{t,n} = 1.8 - 0.9 - 0.5$

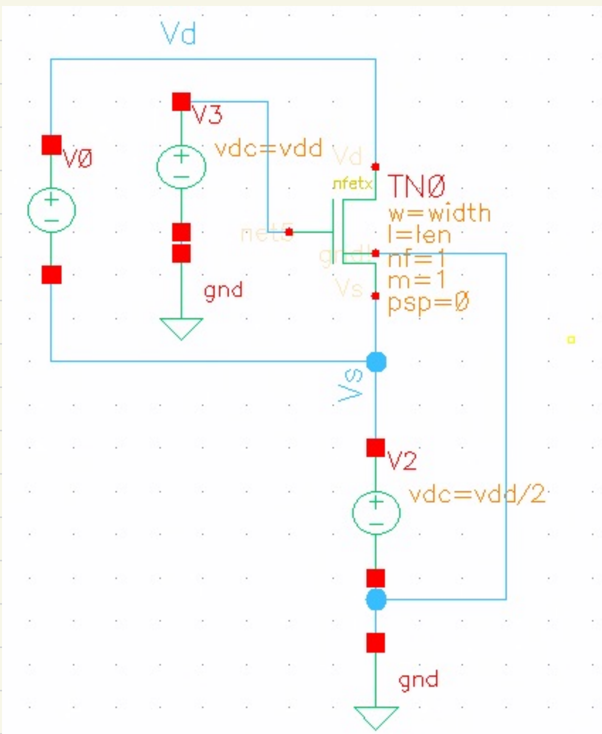
$$\boxed{R_{on} = \frac{V_{DS}}{i_D} = \frac{1}{k'_n \frac{W}{L} \cdot V_{OV}} = \frac{1}{k'_n \frac{W}{L} \cdot V_{OV}}}$$

b)

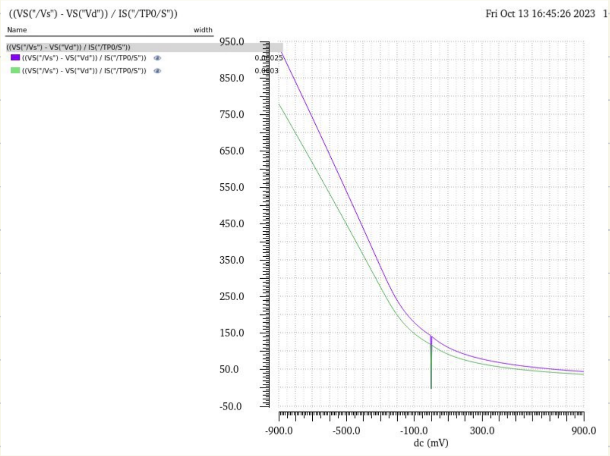



$W = 3.03e^{-5}$ satisfy the requirement

$= 0.0303m$

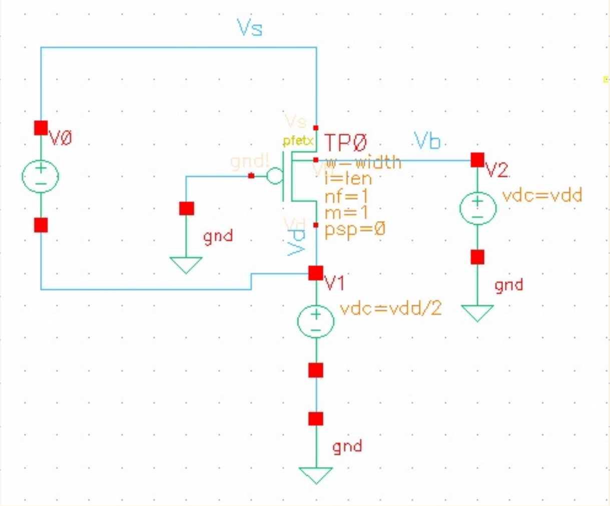


c)




W = 0.00025 satisfy the requirement

= 0.25m




$$i_D = k'_p \frac{W}{L}\left(V_{OV} v_{SD} - \frac{1}{2} v_{SD}^2\right)$$

Assuming $v_{SD} << 1$

the 2nd term can be neglect

$$i_D = k'_p \frac{W}{L} (V_{ov} v_{SD})$$

$$R_{on} = \frac{v_{SD}}{i_D} = \boxed{\frac{1}{k'_p \frac{W}{L} \cdot V_{ov}}}$$

d) $k = \frac{0.25m}{0.0303m} \approx 8.25$

Assuming $V_{SD} << 1$, $V_{DS} << 1$ and both are in triode region, $2^{nd}$ term can be neglect

| $i_{D,n}$ | $i_{D,p}$ |
|---|---|
| $k'_n \left(\frac{W}{L}\right)_n \cdot V_{ov} v_{DS}$ | $k'_p \left(\frac{W}{L}\right)_p \cdot V_{ov} v_{SD}$ |

$$8.25 \left(\frac{W}{L}\right)_n = \left(\frac{W}{L}\right)_p$$

$$R_{on} = \frac{v_{DS}}{i_D} = \frac{v_{SD}}{i_D}$$

$$R_{on,n} = \frac{1}{k'_n \left(\frac{W}{L}\right)_n \cdot 0.4} = \frac{5}{2} \frac{1}{k'_n \left(\frac{W}{L}\right)_n}$$

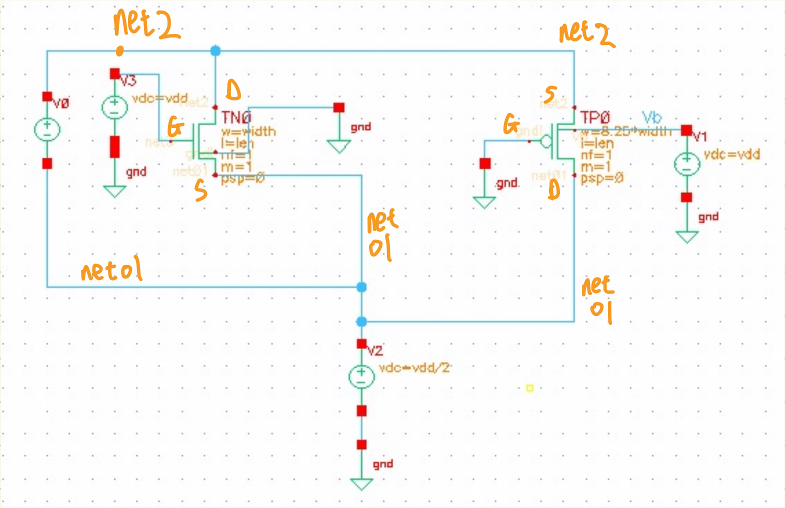
$$R_{on,p} = \frac{1}{k'_p \left(\frac{W}{L}\right)_p \cdot V_{ov}} = \frac{1}{k'_p \left(\frac{W}{L}\right)_n \cdot 8.25 \cdot 1.3} = \frac{40}{429} \frac{1}{k'_p \left(\frac{W}{L}\right)_n}$$

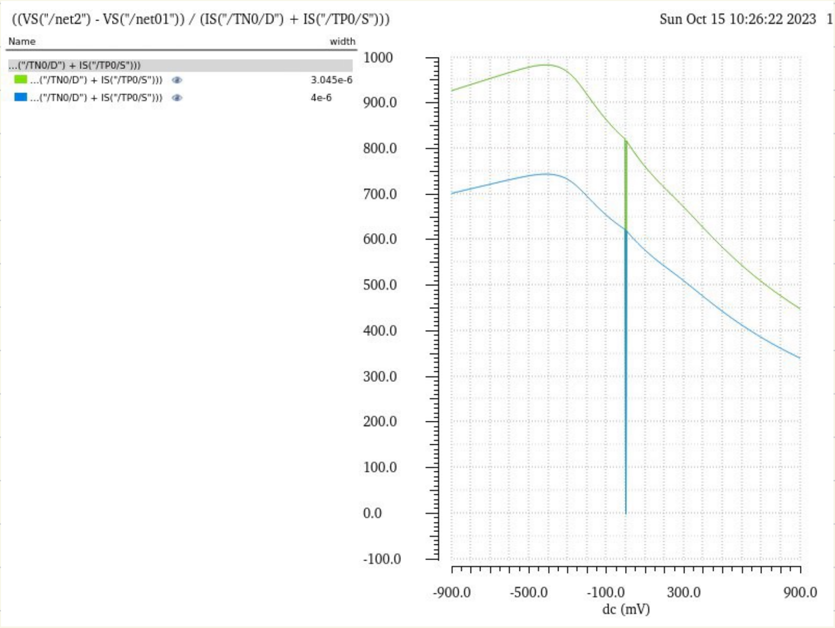
$$R_{eq} = R_{on,n} \parallel R_{on,p}$$

$$= \frac{1}{\frac{1}{\frac{5}{2} \cdot \frac{1}{k'_n \left(\frac{W}{L}\right)_n}} + \frac{1}{\frac{40}{429} \cdot \frac{1}{k'_p \left(\frac{W}{L}\right)_n}}} = \frac{1}{\frac{2}{5} k'_n \left(\frac{W}{L}\right)_n + \frac{429}{40} k'_p \left(\frac{W}{L}\right)_n}$$

where this ratio arises from?

In the PMOS, the mobility of holes is less than the mobility of electrons, In order to maintain the same current across transistors, PMOS need to have wider width to maintain the same current as NMOS




(a) Width = 3.045 um while Length = 500n
the Ron < 1k across the entire range