1. NMOS Bias Circuit Amplifier [20 pts]

Consider the circuit shown below. For all calculations, assume $k_n' = 125\ \mu A/V^2$, $V_t = 0.3$ V, $W/L = 8$, $V_{DD} = 2$ V, $C_X = 250$ pF, $C_Y = 50$ pF. Neglect channel length modulation, body effect, and all transistor capacitances.

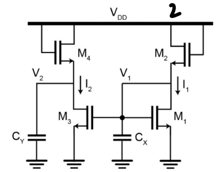
a) Calculate V_1, I_1, V_2, and I_2.

b) Re-calculate V_2 and I_2 with $(W/L)_{3,4} = 20$.

c) Calculate the zero-value time constants for C_X and C_Y. Use $(W/L)_{3,4} = 20$.

d) Which time constant limits the bandwidth of the circuit?

e) *Briefly* qualitatively explain how your answers for (a)-(d) would change (decrease, no change, or increase) if you accounted for the body effect. (Hint: Consider where the bulk must be tied for all transistors.)



$k_n' = 125 \frac{uA}{V^2}$

$V_t = 0.3V$

$\frac{W}{L} = 8$

$V_{DD} = 2$

$C_X = 250pF \quad C_Y = 50pF$

a) M_1, M_2, M_4 are diode connected transistors (Sat)

C_Y and C_X are open Circuit

M_4: $I_2 = \frac{1}{2} \cdot k_n' \frac{W}{L} \cdot (V_{DD} - V_2 - V_t)^2$ ① $\qquad V_{DD} - V_2 - V_t = V_1 - V_t$

Assuming M_3 in Sat region $\qquad V_{DS} = V_2 > V_1 - V_t$

$I_2 = \frac{1}{2} k_n \frac{W}{L} (V_1 - V_t)^2$ ②

M_2: $I_1 = \frac{1}{2} k_n' \frac{W}{L} \cdot (V_{DD} - V_1 - V_t)^2$ ③ $\qquad 1.7 - V_1 = V_1 - 0.3$

M_1: $I_1 = \frac{1}{2} k_n' \frac{W}{L} \cdot (V_1 - V_t)^2$ ④ $\qquad 2 = 2V_1$, $V_1 = 1$

③ = ④

$V_{DD} - V_1 - V_t = V_1 - V_t$

$V_{DD} = 2V_1$

$\boxed{V_1 = 1}V$

$I_1 = \frac{1}{2} \cdot 125u \cdot 8 \cdot 0.7^2$

$= \boxed{245uA}$

① = ②

$V_{DD} - V_2 - V_t = 1 - 0.3$

$2 - V_2 = 1$

$\boxed{V_2 = 1\ V}$

For M_3

$V_{DS} = V_2 - 0 = 1\ V$

$V_{GS} = 1\ V$

$V_{DS} > V_{GS} - V_t = 0.7$

so M_3 in Sat

then ② = ④

$I_2 = I_1 = 245uA$

$\boxed{V_2 = 1\ V}$

b) Assume M_3 in Sat

$$M_3: \ I_2 = \frac{1}{2} k_n' \cdot 20 \cdot (V_1 - V_t)^2$$

$$= \frac{1}{2} \cdot 125u \cdot 20 \cdot (1 - 0.7)^2$$

$$= 612.5 \ uA$$

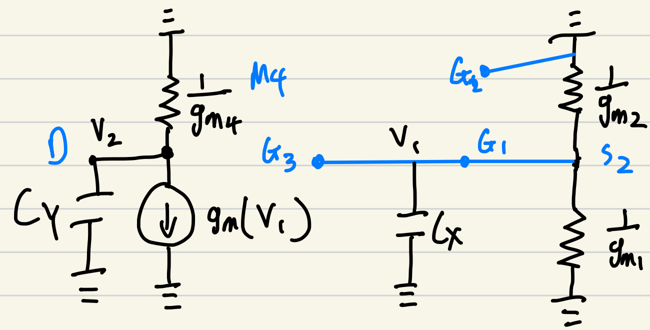
$$M_4: \ I_2 = \frac{1}{2} k_n' \cdot 20 \cdot (V_{DD} - V_2 - V_t)^2$$

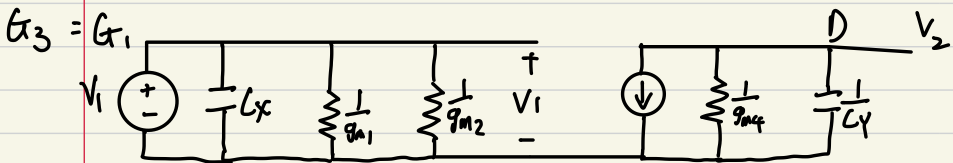
$$612.5u = \frac{1}{2} \cdot 125u \cdot 20 \cdot (1.7 - V_2)^2$$

$$\sqrt{0.49} = 1.7 - V_2$$

$$V_2 = 1 \ V$$

c)





$$I_X = \left(\frac{1}{g_{m1} + g_{m2}}\right) \cdot C_X$$

$$I_Y = \frac{1}{g_{m4}} \cdot C_Y =$$

$$g_{m2} = g_{m1} = k'_n \left(\frac{W}{L}\right)_{(1,2)} V_{OV} = 125u \cdot 8 \cdot 0.7 = 700u \frac{A}{V}$$

$$g_{m3} = g_{m4} = k'_n \left(\frac{W}{L}\right)_{(3,4)} V_{OV} = 125u \cdot 20 \cdot 0.7 = 1750u \frac{A}{V}$$

$$\tau_x = \frac{1}{g_{m1} + g_{m2}} \cdot C_x = \frac{g_{m1} \cdot g_{m2}}{g_{m2} + g_{m1}} \cdot C_x = 714 \cdot 250p$$

$$= \boxed{1.786 \cdot 10^{-7} s}$$

$$\tau_y = \frac{1}{g_{m4}} \cdot C_y = \frac{1}{1750u} \cdot 50p = \boxed{28.6 \, ns}$$

d) $\frac{1}{\tau_y} = 35 MHz$ $\quad \frac{1}{\tau_x} = 5.6 M \, Hz$

So $\boxed{\tau_x \text{ limits the bandwidth of the circuit}}$

e) a) b) Body effect $\uparrow V_t$, $\rightarrow$ $\boxed{I_1 \text{ and } I_2 \downarrow}$

↳ increase Threshold.

$\boxed{V_1, V_2 \downarrow}$

c) Body effect $\uparrow V_t \Rightarrow g_m \downarrow$

$\downarrow$

$\tau_x, \tau_y \uparrow$

d) One of the pole is dominant,

so no change

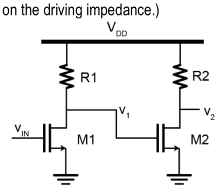
Q2

$R_2(V_{in})$

2. NMOS Cascade Amplifier [20 pts]

Consider (again) the circuit shown below. Assume that (magically) this circuit is biased with V_{IN} and V_1 such that node V_2 is at $V_{DD}/2$. Further, assume that M_1 and M_2 have the same W/L, bias current, negligible channel length modulation, and are in saturation.

a) Symbolically derive an expression for R_2 with respect to V_{IN} satisfying the above conditions.
b) Draw the small-signal model of this circuit. Include only C_{gs} and C_{gd} capacitors.
c) Using only KCL/KVL, derive an expression for the transfer function $v_2(s)/v_{in}(s)$. Simplify your expression as much as possible and pull out the gain term, which has no frequency dependency.
d) *Briefly* explain why node V_{IN} will likely not matter in ZVTC calculations.
e) Using ZVTCs, derive an expression to estimate the dominant pole.
f) Instead of using ZVTCs, estimate the bandwidth using Miller's Theorem. (Hint: Consider where the dominant pole is in this circuit based on the driving impedance.)



$V_2 = V_{DD}/2$

$V_2 = V_{DD} - I \cdot R_2$

$I R_2 = \frac{V_{DD}}{2}$

$R_2 = \frac{V_{DD}}{2I}$

$M_1 : I_D = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2$
$= \frac{1}{2} k_n' \frac{W}{L} (V_{in} - V_t)^2$

$M_2 : I_D = \frac{1}{2} k_n' \frac{W}{L} (V_1 - V_t)^2$

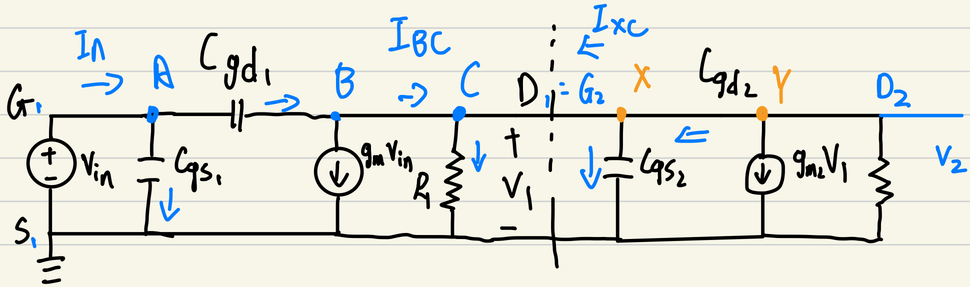
$V_2 = V_{DD} - I \cdot R_2 \Rightarrow I R_2 = \frac{V_{DD}}{2}$

$I = \frac{V_{DD}}{2 R_2}$

$\frac{V_{DD}}{R_2} = \frac{1}{2} k_n' \frac{W}{L} (V_{in} - V_t)^2$

$$\boxed{R_2 = \frac{V_{DD}}{2 \cdot \frac{1}{2} k_n' \frac{W}{L} (V_{in} - V_t)^2}}$$

b)



c) $$\frac{V_2(s)}{V_{in}(s)} = \frac{V_1(s)}{V_{in}(s)} \cdot \frac{V_2(s)}{V_1(s)}$$

@A

1 $$I_n - sC_{gs_1} V_{in} - sC_{gd_1}(V_{in} - V_1) = 0$$

@B

$$sC_{gd_1}(V_{in} - V_1) - g_{m_1} V_{in} = I_{BC}$$

2 @C

$$I_{BC} - \frac{V_1}{R_1} + I_{xc}$$

@D

$$-sC_{gs_2} \cdot V_{in} + sC_{gd_2}(V_2 - V_1) = I_{xc}$$

3 @ Y $$-sC_{gd_2}(V_2 - V_1) - g_{m_2} V_1 = \frac{V_2}{R_2}$$

−

Solve ③

$$-R_2 s C_{gd_2}(V_2 - V_1) - g_{m_2} R_2 V_1 = V_2$$

$$-R_2 s C_{gd_2} V_2 + R_2 s C_{gd_2} V_1 - g_{m_2} R_2 V_1 = V_2$$

$$V_1 (R_2 s C_{gd_2} - g_{m_2} R_2) = V_2 (1 + R_2 s C_{gd_2})$$

$$V_2 = \frac{V_1 (R_2 s C_{gd_2} - g_{m_2} R_2)}{(1 + R_2 s C_{gd_2})}$$

Solve ②

$$\left(s C_{gd_1} (V_{in} - V_1) - g_{m_1} V_{in}\right) - \frac{V_1}{R_1} + \left(-s C_{gs_2} \cdot V_{in} + s C_{gd_2} V_2 - s C_{gd_2} V_1\right)$$

$$s C_{gd_1} V_{in} - s C_{gd_1} V_1 - g_{m_1} V_{in} - \frac{V_1}{R_1} - s C_{gs_2} V_{in} + s C_{gd_2} V_2 - s C_{gd_2} V_1$$

V_{in} : $V_{in}(s C_{gd_1} - g_m - s C_{gs_2}) +$

V_1 : $V_1 \left(-s C_{gd_1} - \frac{1}{R_1} + s C_{gd_2} \cdot \frac{(R_2 s C_{gd_2} - g_{m_2} R_2)}{(1 + R_2 s C_{gd_2})} - s C_{gd_2}\right) = 0$

$$V_{in}(s C_{gd_1} - g_{m_1} - s C_{gs_2}) = V_1 \left(s C_{gd_1} + \frac{1}{R_1} - s C_{gd_2} \frac{R_2 s C_{gd_2} - g_{m_2} R_2}{1 + R_2 s C_{gd_2}} + s C_{gd_2}\right)$$

$$\frac{V_1}{V_{in}} = \frac{s C_{gd_1} - g_{m_1} - s C_{gs_2}}{s C_{gd_1} + \frac{1}{R_1} - s C_{gd_2} \cdot \frac{R_2 s C_{gd_2} - g_{m_2} R_2}{1 + R_2 s C_{gd_2}} + s C_{gd_2}}$$

$$\frac{V_2(s)}{V_{in}(s)} = \frac{V_2}{V_1} \cdot \frac{V_1}{V_{in}}$$

$$\frac{R_2 s C_{gd_2} - g_{m_2} R_2}{1 + R_2 s C_{gd_2}} \cdot \frac{s C_{gd_1} - g_{m_1} - s C_{gs_2}}{s C_{gd_1} + \frac{1}{R_1} - s C_{gd_2} \cdot \frac{R_2 s C_{gd_2} - g_{m_1} R_2}{1 + R_2 s C_{gd_2}} + s C_{gd_2}}$$

Denominator $(1 + R_2 s C_{gd_2})\left(s C_{gd_1} + \frac{1}{R_1} + s C_{gd_2}\right) - s C_{gd_2} \cdot \left(R_2 s C_{gd_2} - g_{m_2} R_2\right)$

$$s C_{gd_1} + \frac{1}{R_1} + s C_{gd_2} + s^2 R_2 C_{gd_2} C_{gd_1} + \frac{s R_2 C_{gd_2}}{R_1} + \cancel{s^2 R_2 C_{gd_2}^2}$$

$$- \cancel{s^2 R_2 C_{gd_2}^2} + s g_{m_2} R_2 C_{gd_2}$$

$$= s\left(C_{gd_1} + C_{gd_2} + \frac{R_2 C_{gd_2}}{R_1} + g_{m_2} R_2 C_{gd_2}\right) + \frac{1}{R_1} + s^2 R_2 C_{gd_2} C_{gd_1}$$

Numerator $\left(R_2 s C_{gd_2} - g_{m_2} R_2\right) \cdot \left(s C_{gd_1} - g_{m_1} - s C_{gs_2}\right)$

$$= s^2 R_2 C_{gd_1} C_{gd_2} - s g_{m_2} R_2 C_{gd_1} - s g_{m_1} R_2 C_{gd_2} + g_{m_1} g_{m_2} R_2 + s g_{m_2} R_2 C_{gs_2}$$

$$= s^2 \left(R_2 C_{gd_1} C_{gd_2}\right) + s\left(g_{m_2} R_2 C_{gs_2} - g_{m_2} R_2 C_{gd_1} - g_{m_1} R_2 C_{gd_2}\right)$$

$$+ g_{m_1 m_2} R_2$$

$$\frac{V_2}{V_{in}} = \frac{g_{m_1} g_{m_2} R_2 + s\left(g_{m_2} R_2 C_{gs_2} - g_{m_2} R_2 C_{gd_1} - g_{m_1} R_2 C_{gd_2}\right) + s^2 \left(R_2 C_{gd_1} C_{gd_2}\right)}{s\left(C_{gd_1} + C_{gd_2} + \frac{R_2 C_{gd_2}}{R_1} + g_{m_2} R_2 C_{gd_2}\right) + \frac{1}{R_1} + s^2 R_2 C_{gd_2} C_{gd_1}}$$

Mulipy $\frac{R_1}{R_1}$

$$\frac{V_2}{V_{in}} = \frac{g_{m1} g_{m2} R_1 R_2 + s(g_{m2} R_1 R_2 C_{gs2} - g_{m2} R_1 R_2 C_{gd1} - g_{m1} R_1 R_2 C_{gd2}) + s^2 (R_1 R_2 C_{gd1} C_{gd2})}{1 + s(R_1 C_{gd1} + R_1 C_{gd2} + R_2 C_{gd2} + g_{m2} R_1 R_2 C_{gd2}) + s^2 R_1 R_2 C_{gd2} C_{gd1}}$$

If $g_{m1} = g_{m2}$ $C_{ds1} = C_{ds2}$ $C_{gs1} = C_{gs2}$

Numerator $= g_m^2 R_1 R_2 + s(g_m R_1 R_2 C_{gs} - g_m R_1 R_2 C_{gd} - g_m R_1 R_2 C_{gd}) + s^2 (R_1 R_2 C_{gd}^2)$

$$= g_m^2 R_1 R_2 \left(1 + s\left(\frac{C_{gs}}{g_m} - 2\frac{C_{gd}}{g_m}\right) + s^2 \frac{C_{gd}^2}{g_m^2}\right)$$

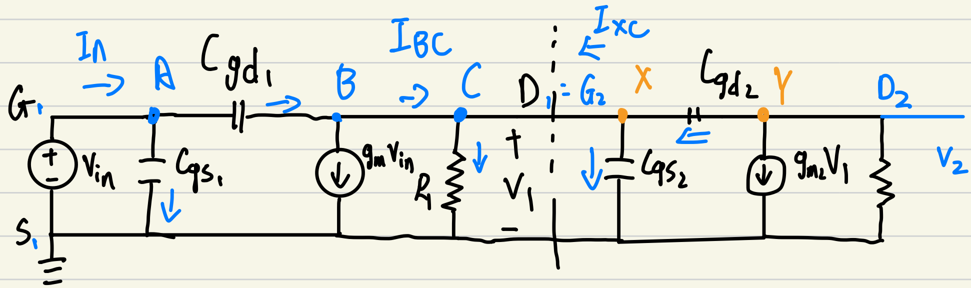
Denominator $1 + s(2R_1 C_{gd} + R_2 C_{gd} + g_m R_1 R_2 C_{gd}) + s^2 R_1 R_2 C_{gd}^2$

$$\boxed{\frac{V_2(s)}{V_1(s)} = \frac{g_m^2 R_1 R_2 \left(1 + s\left(\frac{C_{gs}}{g_m} - 2\frac{C_{gd}}{g_m}\right) + s^2 \frac{C_{gd}^2}{g_m^2}\right)}{1 + s(2R_1 C_{gd} + R_2 C_{gd} + g_m R_1 R_2 C_{gd}) + s^2 R_1 R_2 C_{gd}^2}}$$

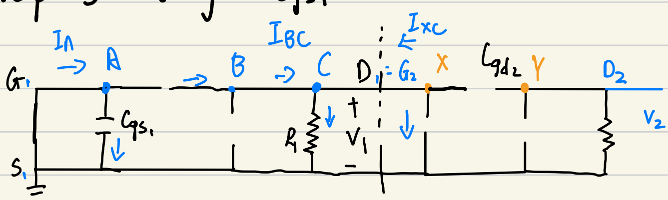
$g_m^2 R_1 R_2$ is the gain

d) In the ZVTC, Vin is grounded, which means it won't affect the time constant $b_1 = \sum \tau_j$

e)

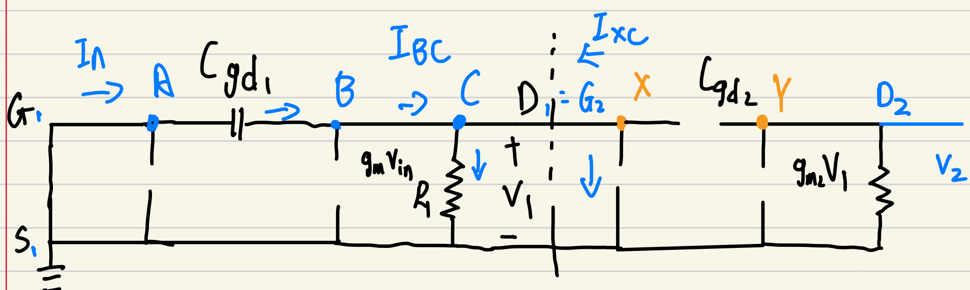


step 1) Fing τ_{gs_1}



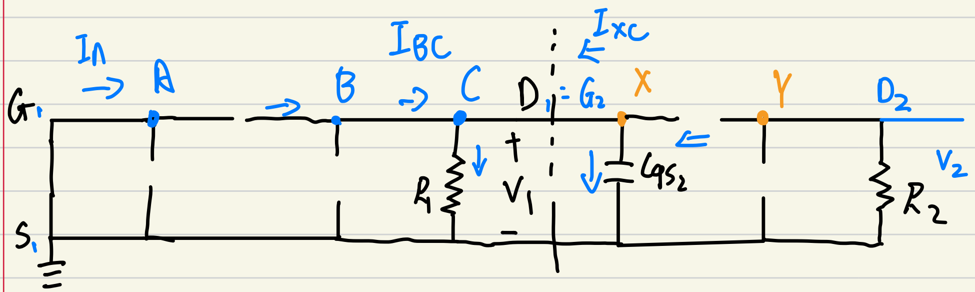
$$\tau_{gs_1} = 0$$

step2) Find τ_{gd}



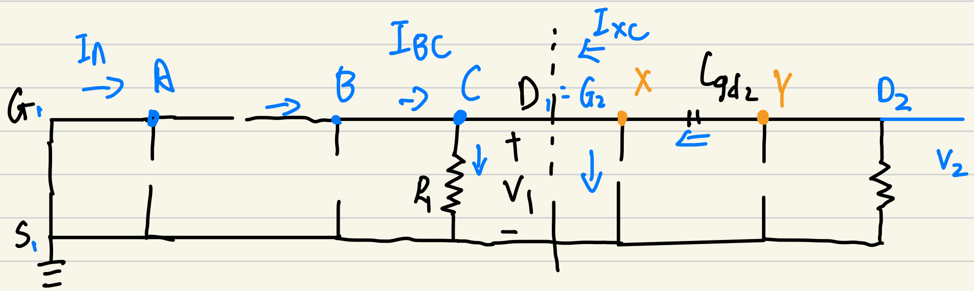
$$\tau_{gd_1} = C_{gd_1} \cdot R_1$$

Step 3) Find C_{gs_2}



$$\tau_{C_{gs_2}} = R_1 \ C_{gs_2}$$

Step 4) Find τC_{gd_2}



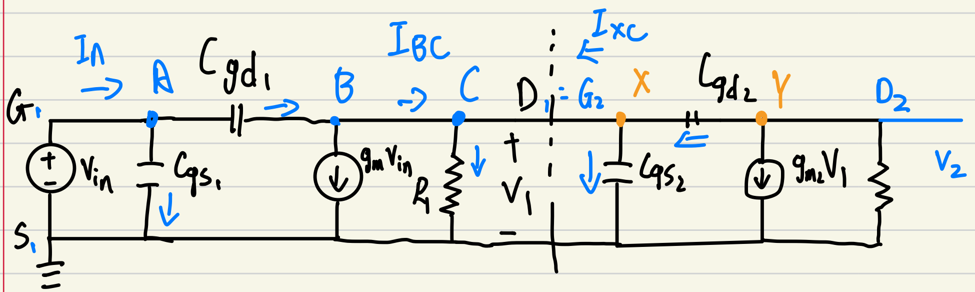
$$\tau_{C_{gd_2}} = (R_1 + R_2) \ C_{gd_2}$$

$$b_1 = \tau_{gs_1} + \tau_{gd_1} + \tau_{Cgs_2} + \tau_{Cgd_2}$$

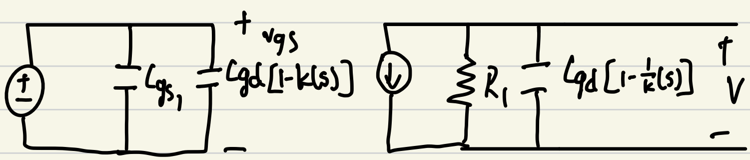
$$= 0 + C_{gd_1} \cdot R_1 + C_{gs_2} \cdot R_1 + (R_1 + R_2) C_{gd_2}.$$

$$= R_1 C_{gd_1} + R_1 C_{gs_2} + (R_1 + R_2) C_{gd_2}$$

t)



$k_1(s)$



$$k_1(s) = \frac{V_1(s)}{V_{gs}(s)}$$

KCL @ B

$$s C_{gd_1}(V_{gs} - V_1) - g_m V_{gs} = I_{BC}$$

KCL @ C

$$I_{BC} - \frac{V_1}{R_1} = 0$$

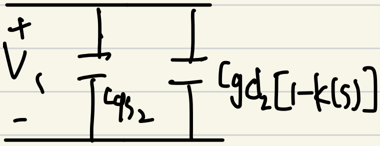
$$s C_{gd_1} V_{gs} - s C_{gd_1} V_1 - g_m V_{gs} = \frac{V_1}{R_1}$$

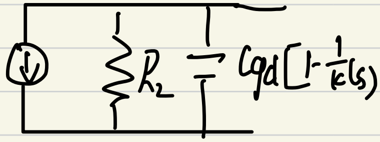
$$v_{gs}\,(s C_{gd_1} - g_m) \cdot R_1 = V_1 (1 + s R_1 C_{gd_1})$$

$$\frac{V_1(s)}{V_{gs}(s)} = \frac{-R_1 g_m + s R_1 C_{gd_1}}{1 + s R_1 C_{gd_1}}$$

$$k_1(s) = -R_1 g_m \left(\frac{1 - s\frac{C_{gd_1}}{g_m}}{1 + s R_1 C_{gd_1}} \right) \approx -g_m R_1$$

$$K_2(s) = \frac{V_2(s)}{V_1(s)}$$





KCL @ Y

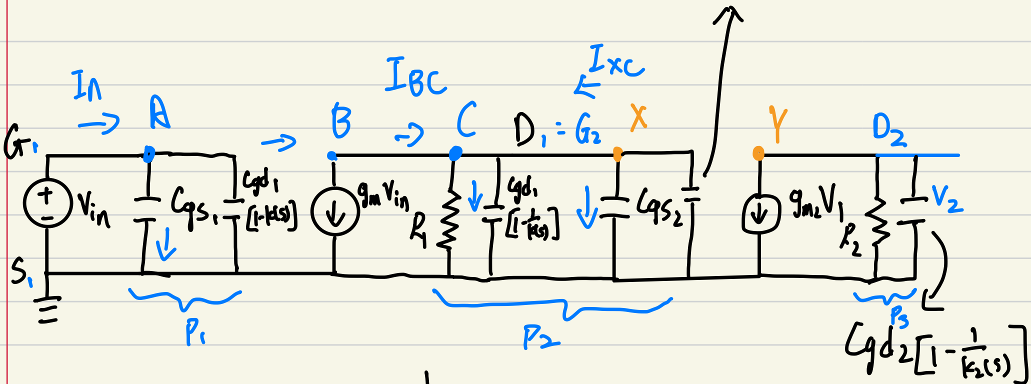
$$sC_{gd_2}(V_1 - V_2) - g_m V_1 = \frac{V_2}{R_2}$$

$$sC_{gd_2}V_1 - sC_{gd_2}V_2 - g_m V_1 = \frac{V_2}{R_2}$$

$$V_1 R_2 (sC_{gd_2} - g_m) = V_2 (1 + sR_2 C_{gd_2})$$

$$\frac{V_2}{V_1} = \frac{-g_m R_2 \left(1 - \frac{sC_{gd_2}}{g_m}\right)}{1 + sR_2 C_{gd_2}} \approx -g_m R_2$$

$$K_1(s) = -g_m R_1 \qquad K_2(s) = -g_m R_2 \qquad C_{gd_2}[1-k(s)]$$



$$P_1 = 0 = \frac{-1}{0 \cdot \left(C_{gs_1} + C_{gd_1}[1-k_1(s)]\right)}$$

$$P_2 = \frac{-1}{R_1\left(C_{gd_1}[1-\frac{1}{k_1(s)}] + C_{gs_2} + C_{gd_2}[1-k_2(s)]\right)}$$

$$P_3 = \frac{-1}{R_2\left(C_{gd_2}[1-\frac{1}{k_2(s)}]\right)} = \frac{-1}{R_2\left(C_{gd_2}(1+\frac{1}{g_m R_2})\right)}$$

P_2 will be the dominant Pole

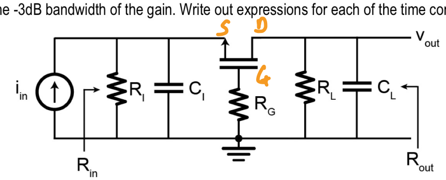
$$\boxed{P_2 = \frac{-1}{R_1\left(C_{gd}(1+\frac{1}{g_m R_1}) + C_{gs_2} + C_{gd_2}(1+g_m R_2)\right)}}$$

Q3

3. Transimpedance Amplifier [20 points]

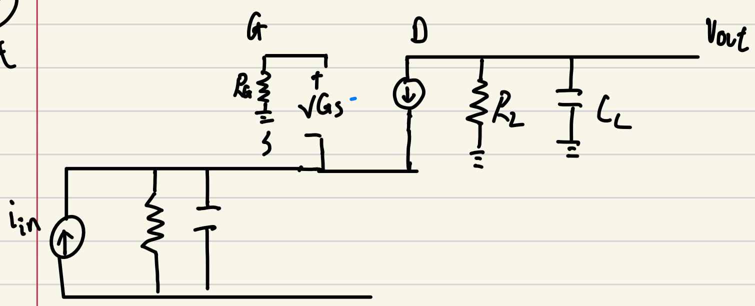
Consider the circuit shown below. For all calculations, use g_m = 4 mS, C_{gs} = 120 fF, R_i = 10 kΩ, C_i = 20 fF, R_G = 100 Ω, C_L = 150 fF, and R_L = 5 kΩ. Neglect the body effect, channel length modulation, and all other capacitances. Assume that the transistor is in *saturation.*

a) Draw the *simplified* small-signal equivalent circuit, including the small-signal model of the transistor.
b) Calculate the low-frequency transimpedance gain (v_{out}/i_{in}). (Hint: Neglect capacitive effects.)
c) Calculate the low-frequency input impedance, R_{in}.
d) Calculate the low-frequency output impedance, R_o.
e) Estimate the -3dB bandwidth of the gain. Write out expressions for each of the time constants.

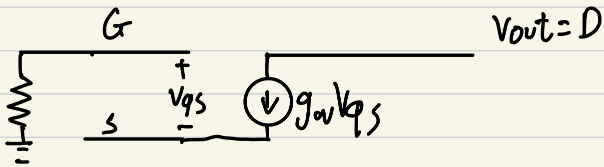


a)

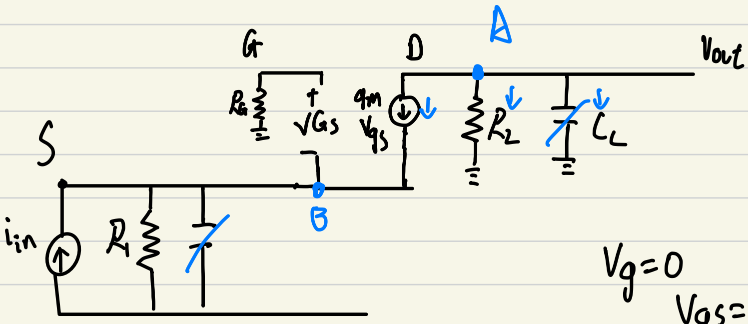
Circuit



Transistor



b)



$V_g = 0$

$V_{gs} = -V_s$

KCL @ A

$$-g_m V_{gs} - \frac{V_{out}}{R_L} = 0 \Rightarrow V_{out} = -g_m V_{gs} R_L$$

KCL @ B

$$i_{in} + g_m V_{gs} = \frac{V_s}{R_I}$$

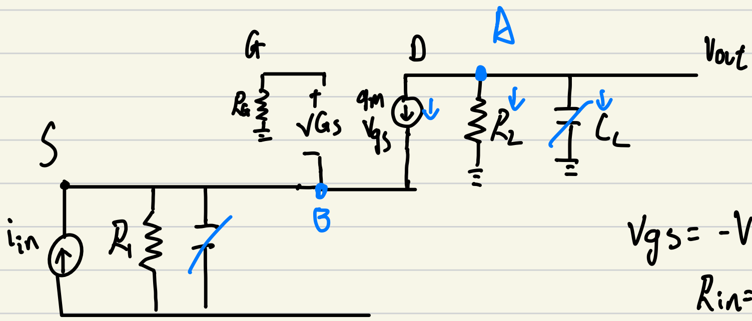
$\frac{i}{v} \cdot \frac{v}{i}$

$$i_{in} + g_m(-V_s) = V_s$$

$$i_{in} R_I + g_m V_s \cdot R_I = V_s \Rightarrow i_{in} = \frac{V_s(1+g_m R_I)}{R_I}$$

$$\frac{V_{out}}{i_{in}} = \frac{-g_m -V_s \cdot R_L \cdot R_I}{V_s(1+g_m R_I)} = \frac{R_L R_I g_m}{1+g_m R_I} = \boxed{4878 \quad \frac{V}{A}}$$

c)



$V_{gs} = -V_s$

$R_{in} = \frac{V_{in}}{i_{in}}$

$$V_{in} = (i_{in} + g_m V_{gs}) \cdot R_I = V_s$$

$$= R_I i_{in} + g_m - V_s \cdot R_I = V_s$$

$$R_I \cdot i_{in} = V_s (1 + g_m R_I)$$

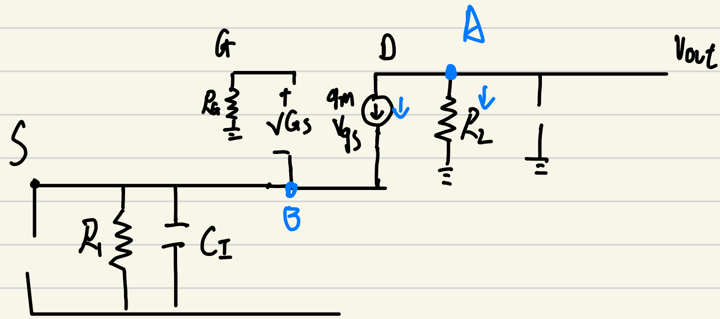
$$\boxed{R_{in} = \frac{V_{in}}{i_{in}} = \frac{R_I}{1 + g_m R_I} = 243.9\ \Omega}$$

d) $R_o = \frac{V_{out}}{i_{out}}$ $\quad i_{out} = -g_m V_{gs}$

$$V_{out} = -g_m V_{gs} R_L$$

$$R_o = \frac{-g_m V_{gs} R_L}{-g_m V_{gs}} = \boxed{5K\ \Omega}$$

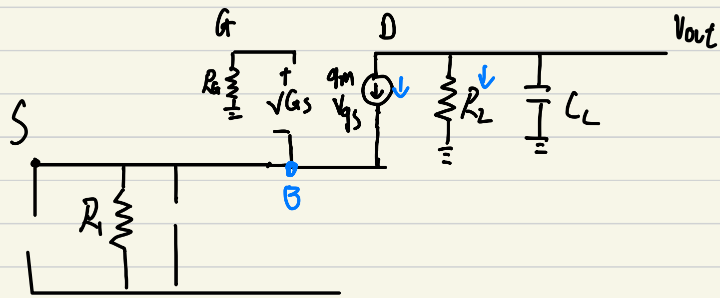
e)



step 1) Find τ_{CI}

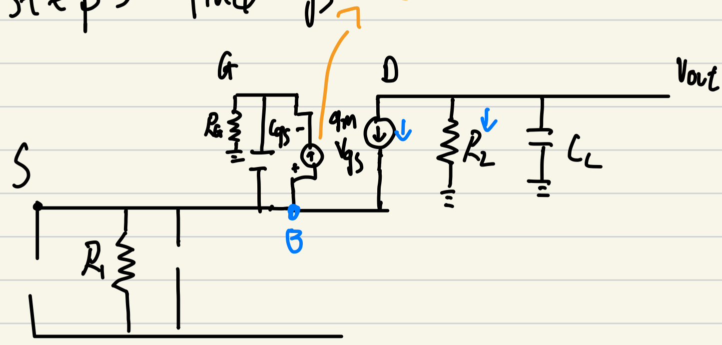
$$\boxed{\tau_{CI} = R_{in} \cdot C_I = 2 \cdot 10^{-10}\,s}$$

step 2) Find τ_{CL}



$$\boxed{\tau_{CL} = R_L \, C_L = 7.5 \cdot 10^{-10}\,s}$$

Step 3 find C_{gs}



KCL @ B

$$-i_t - \frac{V_S}{R_i} + g_m V_t = 0$$

$$V_{gs} = V_t = i_t \cdot R_G - V_S$$

$$V_S = i_t \cdot R_G - V_t$$

$$g_m V_t = \frac{i_t R_G - V_t}{R_i} + i_t$$

$$g_m R_i V_t = i_t R_G + i_t R_i - V_t$$

$$V_t (1 + g_m R_i) = i_t (R_i + R_G)$$

$$\frac{V_t}{i_t} = \frac{R_i + R_G}{1 + g_m R_i} = R_t$$

$$\tau_{C_{gs}} = \frac{R_i + R_G}{1 + g_m R_i} \cdot C_{gs} = \boxed{2.956 \cdot 10^{-11} s}$$

step 4) find f_{W-3dB}

$$\boxed{\begin{aligned} f_{-3dB} &= \left|\frac{1}{b_1}\right| = \frac{1}{\tau_{Cgs} + \tau_{cc} + \tau_{ci}} = 1.02 \text{ GHz} \\ W_{-3} &= \frac{1}{2\pi f} = 0.16 \text{ GHz} \end{aligned}}$$

Q4

4. All NMOS Filter [10 pts]

Consider the circuit shown below. Use $W_1 = 15\ \mu m$, $L_1 = 5\ \mu m$, $W_2 = 12\ \mu m$, $L_2 = 15\ \mu m$, $V_B = 2.2$ V, $V_{t0} = 350$ mV, $V_{DD} = 4.5$ V, $\lambda = 0.1$ 1/V, $k_n' = 120\ \mu A/V^2$, and $C_{ox} = 4\ fF/\mu m^2$.

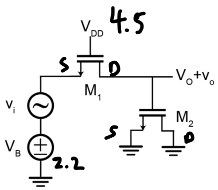
a) Analyze the DC operating point of this circuit (all node voltages, branch currents, and region of operation for each transistor.)

b) Sketch the small-signal model, including capacitors. Neglect all extrinsic capacitors.

c) Calculate the transfer function $v_o(s)/v_i(s)$.

d) Sketch the magnitude and phase of the transfer function.

e) *Briefly*, qualitatively explain what happens if V_B is increased.



$V_{t0} = 350 mV$ $\lambda = 0.1$

$V_i = 0$ $k_n' = 120 u$

$V_o = 0$ $C_{ox} = 4\ fF$

M_1 $\frac{W}{L} = 3$

M_2 $\frac{W}{L} = \frac{12}{15} = \frac{4}{5}$

a)

M_1 $V_{GS} = V_{DD} - V_B = 4.5 - 2.2 = 2.3\ V$ $V_{OV} = 2.3 - 350mV = 1.95V$

$V_{DS} = V_{out} - V_B$

Assuming M_1 in Saturation

$$I_{D_1} = \frac{1}{2} k_n' \cdot \left(\frac{W}{L}\right)_1 \cdot V_{OV}^2 \cdot (1 + \lambda V_{DS})$$

M_2: $V_{DS} = 0$

$V_{GS} = V_{out}$ $V_{OV} = V_{out} - V_t$

$V_{DS} \leq V_{OV}$

Assuming M_2 in Triode region

$$I_{D_2} = k_n' \left(\frac{W}{L}\right)_2 \left(V_{DS} V_{OV} - \frac{1}{2} V_{DS}^2\right) = 0$$

And $\boxed{I_{D_1} = I_{D_2} = 0A}$

So M_1 is in cutoff or Triode region

$$I_{D_1} = k_n' \left(\frac{W}{L}\right)_1 \left(V_{DS} V_{OV} - \frac{1}{2} V_{DS}^2\right) = 0$$

$$V_{DS} = 0 = V_{out} - V_B$$

$$\boxed{V_{out} = V_B = 2.2V}$$

M_2: $V_{OV} = 2.2 - 350mV = 1.85V$

$V_{DS} < V_{OV}$ & $V_{OV} > 0$

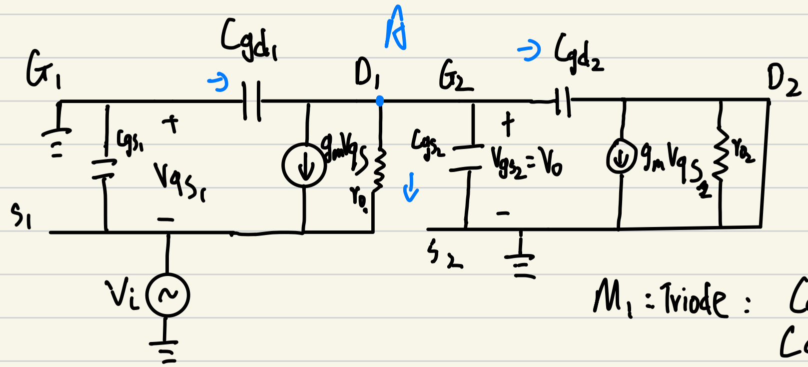
$\boxed{\text{So } M_2 \text{ is in Triode Region}}$

M_1: $(V_{DS} = 0) < (V_{OV} = 1.95V)$

$V_{DS} < V_{OV}$ & $V_{OV} > 0$

$\boxed{\text{So } M_1 \text{ is in Triode Region}}$

b)



M_1 : Triode : $C_{gd_1} = \frac{1}{2} WLC_{ox}$

$C_{gs_1} = \frac{1}{2} WLC_{ox}$

$V_{gs_1} = 0 - V_i = -V_i$

M_2 : Triode : $C_{gd_2} = \frac{1}{2} WLC_{ox}$

c) @ A

$$sC_{gd_1}(0 - V_o) + g_m V_i - \frac{V_o}{r_o} - sC_{gs_2} V_o - sC_{gd_2}(V_o - 0) = 0$$

$$-sC_{gd_1} V_o + g_m V_i - \frac{V_o}{r_{o_1}} - sC_{gs_2} V_o - sC_{gd_2} V_o = 0$$

$$-sr_{o_1} C_{gd_1} V_o + g_m V_i - V_o - sr_{o_1} C_{gs_2} V_o - sr_{o_1} C_{gd_2} V_o = 0$$

$$g_m V_i = V_o (1 + sr_{o_1} C_{gd_1} + sr_o C_{gs_2} + sr_{o_1} C_{gd_2})$$

$$\frac{V_o}{V_i} = \frac{g_m}{1 + sr_{o_1} C_{gd_1} + sr_{o_1} C_{gs_2} + sr_{o_1} C_{gd_2}}$$

$$= \frac{g_m}{1 + s \underbrace{r_{o_1} (C_{gd_1} + C_{gs_2} + C_{gd_2})}_{\beta_1}}$$

M_1 : Triode : $C_{gd1} = \frac{1}{2} W_1 L_1 C_{ox} = 150 f\ F$

$C_{gs1} = \frac{1}{2} W_1 L_1 C_{ox} = 150 f\ F$

M_2 : Triode : $C_{gd2} = \frac{1}{2} W_2 L_2 C_{ox} = 360 f\ F$

$C_{gs2} = \frac{1}{2} W_2 L_2 C_{ox} = 360 f\ F$

$r_o = \frac{1}{g_{ds}}$

$$g_{ds} = \frac{dI_d}{dV_{ds}} \left(k_n' \frac{W}{L} \left(V_{ov} V_{ds} - \frac{1}{2} V_{ds}^2 \right) \right) \Big|_{V_{ds}}$$

$$= \left(k_n' \frac{W}{L} \left(V_{ov} - V_{ds} \right) \right)$$

$$g_{ds1} = k_n' \left(\frac{W}{L} \right)_1 \left(V_{ov_1} - V_{ds} \right)$$

$$= 120u \cdot 3 \cdot 1.95 = 702u$$

$r_{o1} = 1424.5\ \Omega$

$$g_{ds2} = k_n' \left(\frac{W}{L} \right)_2 \cdot \left(V_{ov} - V_{ds} \right)$$

$$= 120u \cdot \frac{12}{15} \cdot 1.85 = 177.6u$$

$r_{o2} = 5630.6\ \Omega$

$$g_m = \frac{dI_d}{dV_{GS}} \left(k_n' \frac{W}{L} \left((V_{GS} - V_t) V_{ds} - \frac{1}{2} V_{ds}^2 \right) \right)$$

$$= k_n' \frac{W}{L} \left(V_{ds} - \frac{1}{2} V_{ds}^2 \right) = 0$$

due to $V_{ds} = 0$ in M_1, M_2

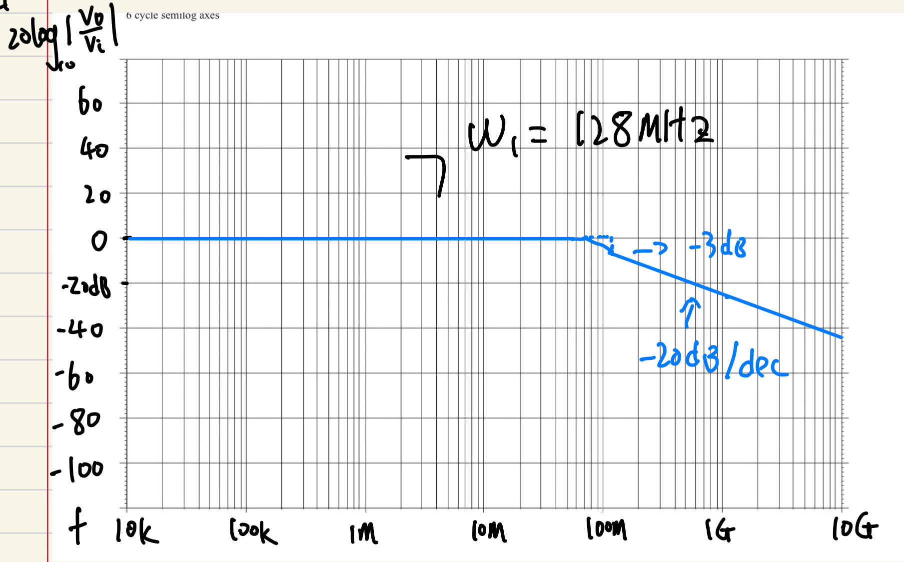
$$H(s) = \frac{g_m}{1 + s \underbrace{r_{o1} (C_{gd1} + C_{gs2} + C_{gd2})}_{p_1}}$$

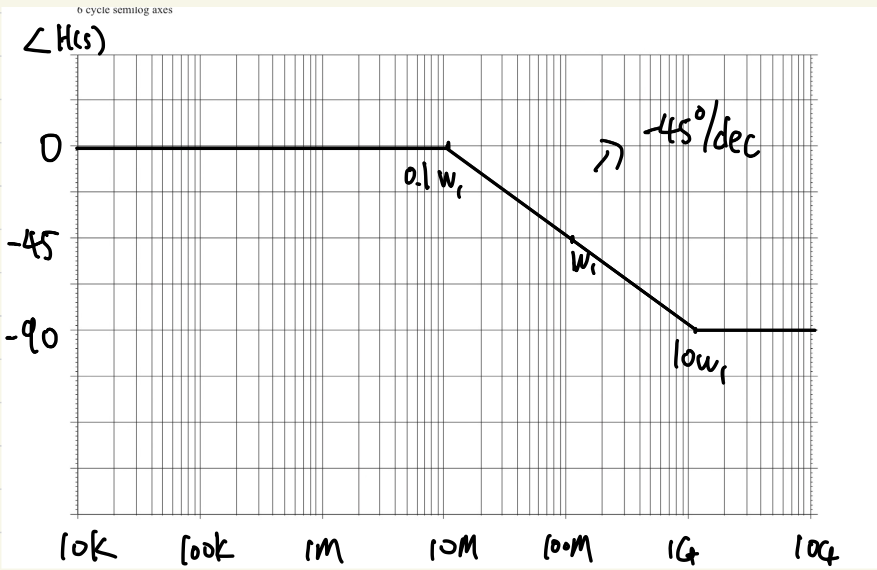
$$P_1 = \frac{1}{2\pi\ r_{o1} (C_{gd1} + C_{gs2} + C_{gd2})}$$

$$= 0.128\ GHz$$

$$= \boxed{128\ MHz}$$

d


$20\log\left|\frac{V_o}{V_i}\right|$
6 cycle semilog axes
60
40
20
0
-20dB
-40
-60
-80
-100
f
10k
100k
1M
10M
100M
1G
10G
$w_1 = 128MHz$
-3dB
-20dB/dec


6 cycle semilog axes
∠H(s)
0
-45
-90
$0.1w_1$
w_1
$10w_1$
-45°/dec
10k
100k
1M
10M
100M
1G
10G

e)

From part a

$V_{out} = V_B$

① so $V_B \uparrow \Rightarrow V_{out} \uparrow$

② $V_{GS_1} = V_{DD} - V_B$ $\quad V_{OV} = (V_{DD} - V_B) - V_t$

$\therefore V_B \uparrow \rightarrow V_{GS} \downarrow$

If $V_B \uparrow\uparrow$ until $V_B = V_{DD} \Rightarrow V_{OV} < 0$

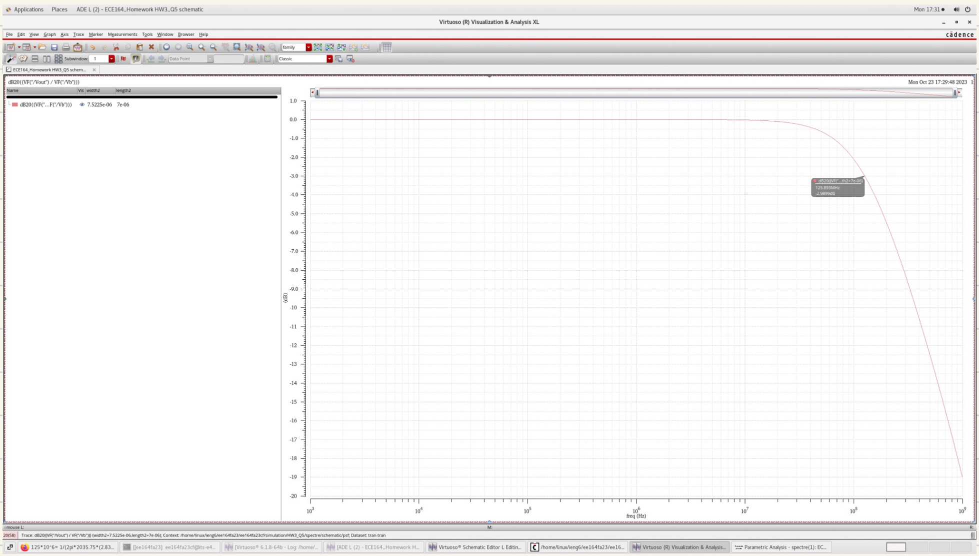
M_1 will be in cut-off region

Q5

5. Filter Design [30 pts]

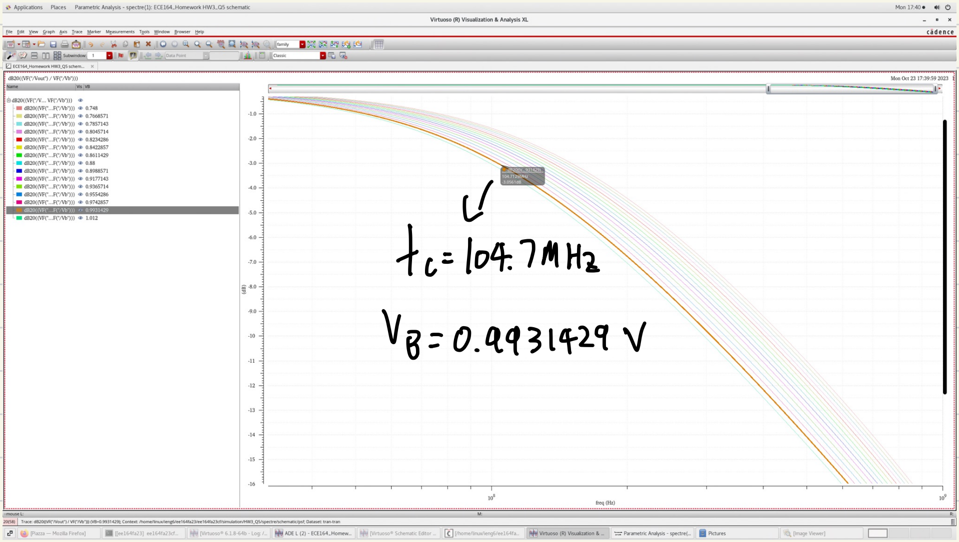
You are off on your own! Redesign the circuit shown in question 4 for our technology. The specifications for this circuit are: f_c = 125 MHz and be tunable ±15% by changing V_B. (Remember, V_{DD} = 1.8 V). Submit a schematic annotated with the DC operating point and a plot of the AC response showing that the circuit can meet the tuning specification. Lastly, *__briefly__* describe your design methodology. (Hint: If you are unsure where to start, I would recommend connecting an NMOS device like the device connected in M_2 and sweeping the L to extract C_{ox}. It is probably a good idea to make the area large enough not to be determined by parasitic/extrinsic capacitance. From there, you can appropriately size M_1.)

f_c



$f_c = 125.893\,MHz \qquad |H(s)| \approx -3\,dB$

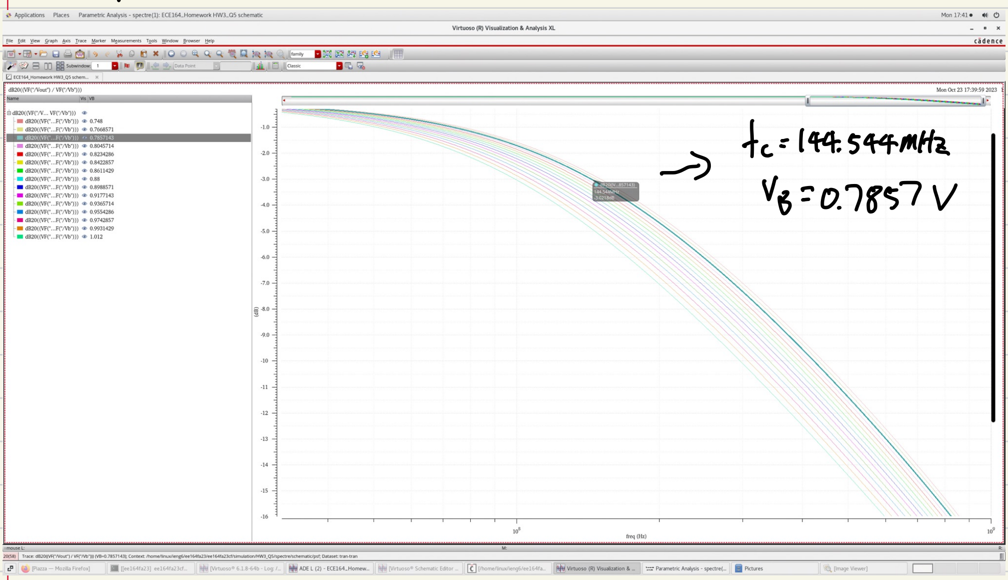
Lower Bound



$f_c = 104.7\,\mathrm{MHz}$

$V_B = 0.9931429\ \mathrm{V}$

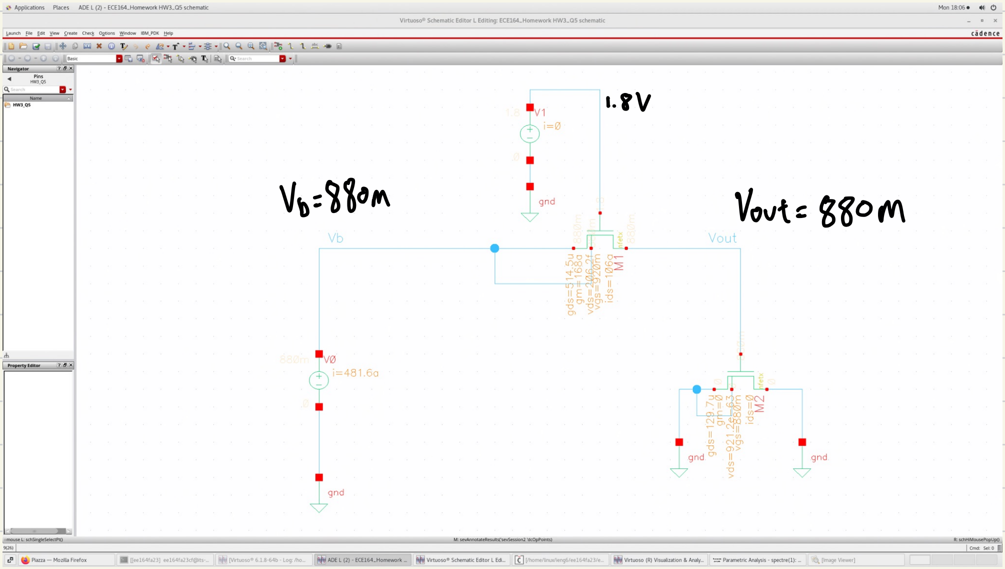
Higher Bound



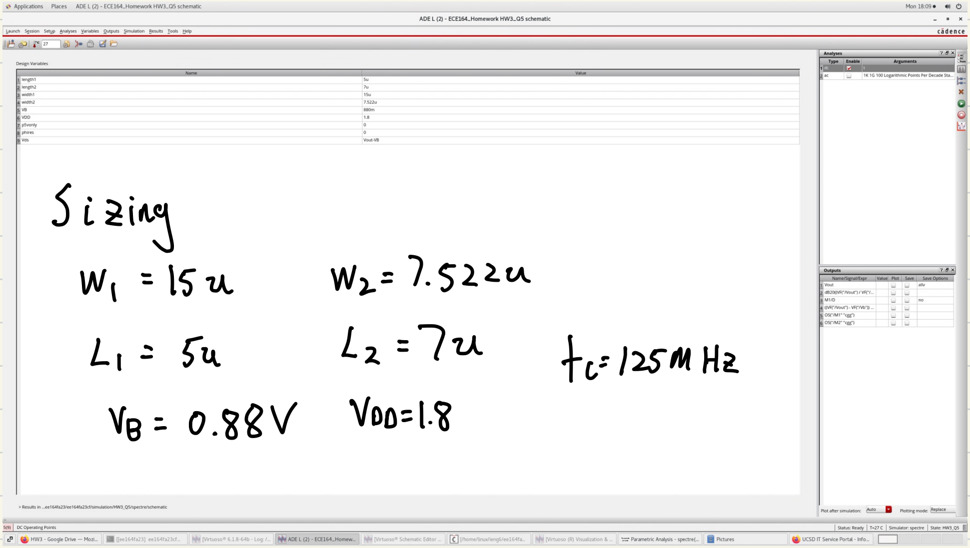
$f_c = 144.544\,\mathrm{MHz}$

$V_B = 0.7857\ \mathrm{V}$

DC Operating Point



Setup



Sizing

$W_1 = 15u$ $\quad$ $W_2 = 7.522u$

$L_1 = 5u$ $\quad$ $L_2 = 7u$ $\quad$ $f_c = 125\,MHz$

$V_B = 0.88V$ $\quad$ $V_{DD} = 1.8$

Design Methology

① Using $\underset{15u}{W_1}$ and $\underset{5u}{L_1}$ from Q4. keep them constant

② Calcuate $r_o = \dfrac{1}{k_n' \left(\frac{W}{L}\right)_1 (V_{DD} - V_B - V_t)}$ set

$\underset{262.7\frac{uA}{V^2}}{k_n'}$ and $\underset{0.307V}{V_t}$ derived from HW1, $V_B = 0.88V$

③ $f_c = \dfrac{1}{2\pi\, r_{o1} (C_{gd1} + C_{gs2} + C_{gd2})}$

$C_{gd1} = \frac{1}{2} W_1 L_1 C_{ox}$

$C_{gs2} + C_{gd2} = W_2 L_2 C_{ox}$

④ Find Cox by plotting C_{gg1} vs. L_1

$C_{gg1,2} = W_1 \cdot \underbrace{L_1}_{x} \cdot C_{ox}$

$C_{gg} = \underbrace{W_1 \cdot C_{ox}}_{k} \cdot L_1 = kx$

slope

After finding k, we know W_1, then Cox can be solved

$C_{ox} = 7.56 \cdot 10^{-3}$

⑤ Since we know r_{o1} . $C_{gd1} = \frac{1}{2} W_1 L_1 C_{ox}$

I assume $W_2 = L_2 = x$

so

$$125MHz = \frac{1}{2\pi \cdot r_o \cdot (C_{gd1} + x^2 \cdot C_{ox})}$$

$x = 9.09u$

⑥ Then try to stimulate this

$f_c = 125MHz$ is not found

⑦ keep adjusting W_2 and L_2 simultaneously

Until find $f_c = 125MHz$ $|H(s)| = -3dB$

⑧ Varying V_B to find upper and lower bound.

to get $\pm 15\%$ f_c