Extending the Analysis: Supplement to ECF Chapter

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September 21, 2017

We now investigate the generalization to many random games and all five types or roles of players. For each of the five types of players, we randomly generate 1,000 game setups, instead of just one as in the previous section. We then examine how the individual strategies for each type of player would fare in the 1,000 setups and we report summary statistics. We proceed by type of player in the subsections that follow.

To solve the best response problem with formulate as a Knapsack problem as in the previous section and use the Bang-for-Buck heuristic, implemented in Python.

Player	Type	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	Sum
0	1	0	0	0	0	0	0	30	20	20	10	80
1	1	0	0	0	0	0	0	0	30	25	25	80
2	1	0	0	0	0	0	25	20	20	10	5	80
3	1	0	0	0	0	0	0	0	0	30	50	80

Table 1: Strategies of all players type 1 for ECF Game 1.

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Player 1	5	15	25	35	45	55	65	75	85	95
Player 2	95	5	15	25	35	45	55	65	75	85
Player 3	85	95	5	15	25	35	45	55	65	75
Player 4	75	85	95	5	15	25	35	45	55	65
Player 5	65	75	85	95	5	15	25	35	45	55
Sums	325	275	225	175	125	175	225	275	325	375

Table 2: Data set 1 for ECF Game 1.

0.1 Focal Player: Role 1

Table 3 reproduces Table 1 with the addition of the project values for players in role 1 (presented in Table 2, page 1). We see a general pattern of allocating thalers across the higher value projects, from player 1's perspective. Notice that player 3 is the most concentrated, dividing its thalers between projects P9 and P10. Player 2 is the most dispersed in its allocations.

	Prefs:	5	15	25	35	45	55	65	75	85	95	
Player	Type	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	Sum
0	1	0	0	0	0	0	0	30	20	20	10	80
1	1	0	0	0	0	0	0	0	30	25	25	80
2	1	0	0	0	0	0	25	20	20	10	5	80
3	1	0	0	0	0	0	0	0	0	30	50	80

Table 3: Strategies of all players type 1 for ECF Game 1 (after Table 1).

Looking now at Table 4 we can see how the strategies of the four role 1 players fared during 1,000 randomly generated ECF games. As in our simple example from the previous section, Bang-for-Buck, the optimal strategy (although estimated by a heuristic) does by far the best overall. Also as in the previous case, the strategy of player 3 does far better than any of the other player 1 strategies. In fact, the four strategies can be ranked (intuitively) by the degree to which they focus their thalers on projects P9 and P10. The ranking is $3 > 1 > 0 > 2$, and this is exactly the performance order of the mean.

player	count	mean	std	min	25%	50%	75%	max
0	1000.0	22.015	35.1	0.0	0.0	0.0	15.0	100.0
1	1000.0	49.355	51.5	0.0	0.0	15.0	95.0	185.0
2	1000.0	10.85	24.6	0.0	0.0	0.0	15.0	100.0
3	1000.0	111.69	53.0	0.0	95.0	100.0	180.0	195.0
BfB	1000.0	163.825	36.0	90.0	115.0	185.0	185.0	255.0

Table 4: Summary statistics for player role 1 strategies in ECF game 1.

0.2 Focal Player: Role 2

Table 5 presents the strategies chosen by the 15 type 2 (role 2) players in the game, along with the preference scores (project values) for each of the projects (from Table 2). The pattern in evidence is broadly that of the role 1 players, in Table 3. Most of the thalers are allocated to high-value projects, but here there is considerably more spread in the disbursements.

| | Prefs: | 95 | 5 | 15 | 25 | 35 | 45 | 55 | 65 | 75 | 85 | |
Player	Type	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	Sum
4	2	20	0	0	0	0	0	0	20	20	20	80
5	2	20	20	0	0	0	0	0	0	20	20	80
6	2	15	20	0	0	0	0	0	0	30	15	80
7	2	19	10	5	0	0	0	5	8	15	18	80
8	2	34	0	0	0	0	0	0	0	12	34	80
9	2	30	0	0	0	0	0	0	0	20	30	80
10	2	30	0	0	0	0	0	0	0	25	25	80
11	2	10	0	0	0	0	0	20	20	20	10	80
12	2	30	1	0	0	0	0	0	4	20	25	80
13	2	35	0	0	0	0	0	0	0	25	20	80
14	2	30	0	0	0	0	0	0	10	20	20	80
15	2	0	0	50	0	0	0	0	20	10	0	80
16	2	30	0	0	0	0	0	0	0	20	30	80
17	2	30	0	0	0	0	0	0	0	25	25	80
18	2	16	0	0	0	8	8	8	8	16	16	80

Table 5: Strategies of all players type 2 for ECF Game 1.

Table 6 tells the story of how the various strategies fared. Bang-for-Buck of course does comparatively well. As expected, it does about as well as it did in the role of player 1. None of the player 2 strategies, however come close to the play 3 strategy in role 1, even though the payoffs for role 2 are a one-step rotation of the payoffs for role 1. Notice that player 4 in Table 6 has the highest of the mean scores (71.45) among the players. This player corresponds to the 5th strategy in Table 5, there labeled player 8. This strategy is arguably the most concentrated on the higher-value strategies and may be compared to strategy 1 in Table 3, whose mean is 49.36 in Table 4. This suggests a structural difference in the game between the two roles.

player	count	mean	std	min	25%	50%	75%	max
0	1000.0	40.395	54.5	0.0	0.0	5.0	85.0	255.0
1	1000.0	41.465	54.2	0.0	0.0	5.0	85.0	255.0
2	1000.0	47.985	56.0	0.0	0.0	5.0	80.0	255.0
3	1000.0	36.99	52.4	0.0	0.0	5.0	85.0	255.0
4	1000.0	71.45	67.3	0.0	0.0	85.0	95.0	255.0
5	1000.0	62.04	65.8	0.0	0.0	75.0	95.0	255.0
6	1000.0	63.64	65.9	0.0	0.0	75.0	95.0	255.0
7	1000.0	28.26	45.2	0.0	0.0	0.0	75.0	180.0
8	1000.0	56.915	63.4	0.0	0.0	5.0	90.0	255.0
9	1000.0	65.285	64.5	0.0	0.0	75.0	95.0	255.0
10	1000.0	51.415	60.7	0.0	0.0	5.0	90.0	255.0
11	1000.0	10.735	27.1	0.0	0.0	0.0	5.0	160.0
12	1000.0	62.04	65.8	0.0	0.0	75.0	95.0	255.0
13	1000.0	63.64	65.9	0.0	0.0	75.0	95.0	255.0
14	1000.0	38.405	53.2	0.0	0.0	5.0	85.0	255.0
BfB	1000.0	173.765	52.4	80.0	160.0	170.0	185.0	260.0

Table 6: Summary statistics for player role 2 strategies in ECF game 1.

0.3 Focal Player: Role 3

Table 7 presents the strategies chosen by the 12 type 3 (role 3) players in the game, along with the preference scores (project values) for each of the projects (from Table 2). The pattern in evidence is broadly that of the role 1 players, in Table 3. Most of the thalers are allocated to high-value projects, but here there is considerably more spread in the disbursements.

	Prefs:	85	95	5	15	25	35	45	55	65	75	
Player	Type	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	Sum
19	3	20	20	0	0	0	0	0	0	20	20	80
20	3	32	0	0	0	0	0	0	0	24	24	80
21	3	30	30	0	0	0	0	0	0	10	10	80
22	3	25	40	0	0	0	0	0	0	0	15	80
23	3	20	0	0	0	0	0	0	20	20	20	80
24	3	30	0	0	0	0	0	0	0	20	30	80
25	3	20	25	0	0	0	0	5	5	10	15	80
26	3	30	50	0	0	0	0	0	0	0	0	80
27	3	24	25	1	1	1	1	1	1	1	24	80
28	3	20	20	0	0	0	0	0	0	20	20	80
29	3	35	0	0	0	0	0	0	0	25	20	80
30	3	0	0	0	0	0	0	0	20	30	30	80

Table 7: Strategies of all players type 3 for ECF Game 1.

Table 8 tells the story of how the various strategies fared. Bang-for-Buck, of course, does comparatively well. As expected, it does about as well as it did in the roles of players 1 and 2. As in the case of player 2, none of the player 3 strategies come close to the player 3 strategy in role 1, even though the payoffs for role 3 are a two-step rotation of the payoffs for role 1. Notice that player 7 in Table 8 has the highest of the mean scores (74.45) among the players. This player corresponds to the 8th strategy in Table 7, there labeled player ID 26. This strategy is arguably the most concentrated on the higher-value strategies and may be compared to strategy 1 in Table 3, whose mean is 49.36 in Table 4. This suggests a structural difference in the game between the two roles.

player	count	mean	std	min	25%	50%	75%	max
0	1000.00	59.40	58.84	0.00	0.00	65.00	95.00	235.00
1	1000.00	72.33	64.43	0.00	0.00	65.00	140.00	235.00
2	1000.00	59.90	56.63	0.00	0.00	75.00	95.00	255.00
3	1000.00	68.04	57.44	0.00	0.00	85.00	95.00	255.00
4	1000.00	45.81	53.72	0.00	0.00	0.00	75.00	235.00
5	1000.00	64.50	62.40	0.00	0.00	65.00	95.00	235.00
6	1000.00	52.86	55.12	0.00	0.00	65.00	95.00	235.00
7	1000.00	74.45	54.12	0.00	0.00	95.00	95.00	255.00
8	1000.00	62.50	59.45	0.00	0.00	75.00	95.00	235.00
9	1000.00	59.40	58.84	0.00	0.00	65.00	95.00	235.00
10	1000.00	68.47	62.66	0.00	0.00	65.00	95.00	235.00
11	1000.00	61.83	52.77	0.00	0.00	65.00	75.00	235.00
BfB	1000.00	188.93	47.78	65.00	160.00	180.00	235.00	320.00

Table 8: Summary statistics: player role 3 strategies in ECF game 1.

0.4 Focal Player: Role 4

Table 9 presents the strategies chosen by the 16 type 4 (role 4) players in the game, along with the preference scores (project values) for each of the projects (from Table 2). The pattern in evidence is broadly that of the role 1 players, in Table 3. Most of the thalers are allocated to high-value projects, but here there is considerably more spread in the disbursements.

| | Prefs: | 75 | 85 | 95 | 5 | 15 | 25 | 35 | 45 | 55 | 65 | |
Player	Type	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	Sum
31	4	10	50	0	0	0	0	0	0	10	10	80
32	4	15	50	0	0	0	0	0	0	0	15	80
33	4	45	0	0	0	0	0	0	0	0	35	80
34	4	35	1	0	0	1	1	1	1	10	30	80
35	4	20	8	5	0	0	0	5	7	15	20	80
36	4	25	0	0	0	0	0	0	15	20	20	80
37	4	20	0	0	0	0	0	0	20	20	20	80
38	4	20	20	20	0	0	0	0	0	0	20	80
39	4	30	0	0	0	0	0	0	20	10	20	80
40	4	15	40	0	0	0	0	0	0	10	15	80
41	4	35	0	0	0	0	0	0	0	35	10	80
42	4	0	0	40	0	0	0	0	0	20	20	80
43	4	20	30	0	0	0	0	0	10	5	15	80
44	4	40	0	0	0	0	0	0	0	0	40	80
45	4	20	20	20	0	0	0	0	0	0	20	80
46	4	15	15	30	0	0	0	0	0	10	10	80

Table 9: Strategies of all players type 4 for ECF Game 1.

Table 10 tells the story of how the various strategies fared. Bang-for-Buck, of course, does comparatively well. As expected, it does about as well as it did in the roles of players 1 and 2. As in the case of player 2, none of the player 3 strategies come close to the player 3 strategy in role 1, even though the payoffs for role 3 are a two-step rotation of the payoffs for role 1. Notice that player 7 in Table 8 has the highest of the mean scores (74.45) among the players. This player corresponds to the 8th strategy in Table 7, there labeled player ID 26. This strategy is arguably the most concentrated on the higher-value strategies and may be compared to strategy 1 in Table 3, whose mean is 49.36 in Table 4. This suggests a structural difference in the game between the two roles.

player	count	mean	std	min	25%	50%	75%	max
0	1000.00	72.05	48.20	0.00	55.00	85.00	85.00	205.00
1	1000.00	68.47	48.00	0.00	0.00	85.00	85.00	205.00
2	1000.00	75.08	54.88	0.00	0.00	75.00	140.00	225.00
3	1000.00	60.91	55.48	0.00	0.00	65.00	85.00	225.00
4	1000.00	40.06	46.67	0.00	0.00	0.00	65.00	205.00
5	1000.00	42.53	47.43	0.00	0.00	55.00	65.00	225.00
6	1000.00	38.78	44.55	0.00	0.00	0.00	65.00	150.00
7	1000.00	36.91	45.87	0.00	0.00	0.00	65.00	205.00
8	1000.00	38.91	48.77	0.00	0.00	0.00	65.00	225.00
9	1000.00	65.16	51.38	0.00	0.00	85.00	85.00	205.00
10	1000.00	68.16	47.55	0.00	55.00	55.00	75.00	195.00
11	1000.00	35.71	42.52	0.00	0.00	0.00	65.00	150.00
12	1000.00	52.85	49.88	0.00	0.00	65.00	85.00	225.00
13	1000.00	69.36	54.64	0.00	0.00	65.00	140.00	225.00
14	1000.00	36.91	45.87	0.00	0.00	0.00	65.00	205.00
15	1000.00	33.59	43.58	0.00	0.00	0.00	65.00	150.00
BfB	1000.00	173.85	38.40	75.00	140.00	160.00	205.00	280.00

Table 10: Summary statistics: player role 4 strategies in ECF game 1.

0.5 Focal Player: Role 5

Table 11 presents the strategies chosen by the 3 type 5 (role 5) players in the game, along with the preference scores (project values) for each of the projects (from Table 2). The pattern in evidence is only very broadly that of the role 1 players, in Table 3. The highest value projects for role 5 are P2, P3, P4, but there is no investment at all in the two highest value projects P3 and P4. Instead the players focus on the two ends of the project list, P1–P2 and P9–P10.

	Prefs:	65	75	85	95	5	15	25	35	45	55	
Player	Type	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	Sum
47	5	10	30	0	0	0	0	0	10	20	10	80
48	5	20	50	0	0	0	0	0	0	0	10	80
49	5	0	50	0	0	0	0	0	0	30	0	80

Table 11: Strategies of all players type 5 for ECF Game 1.

Table 12 tells the story of how the various strategies fared. Bang-for-Buck, of course, does well. Because no player really focused on its high-value projects all players do comparatively poorly.

player	count	mean	std	min	25%	50%	75%	max
0	1000.00	38.06	43.31	0.00	0.00	22.50	65.00	165.00
1	1000.00	49.88	45.81	0.00	0.00	55.00	75.00	195.00
2	1000.00	45.59	41.53	0.00	0.00	45.00	75.00	195.00
BfB	1000.00	148.32	30.54	55.00	120.00	165.00	165.00	240.00

Table 12: Summary statistics: player role 5 strategies in ECF game 1.

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