# Calculations on Rolling Hedges

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Let:

- $S(t)$: the spot price per unit (of natural gas) at time $t$. (Our units will be 1 million Btu, which is how natural gas is priced in the U.S., in \$/mmBtu.)
- $F(t)$: the one period (month) futures contract at time $t-1$ for one unit of gas, the price is denoted $\$F(t)$.
- $p$: the premium for the futures contract, so $\$F(t+1) = S(t) + p$. Our analysis will treat $p$ as negligible.
- $X(t)$: random change in period (month) in spot price, so $S(t+1) = S(t) + X(t)$.
- $b$: the brokerage per period (monthly) interest rate
- $c$: the firm's weighted cost of capital per period (monthly) interest rate
- $n$: the number of periods (months) to be hedged with a rolling hedge. Assuming for now that $n = 240$, so 20 years.

Here's how my somewhat stylized model of a rolling hedge for natural gas works. There are $n$ lines of hedges, one for each month of delivery. Consider for now how a single line works, line $i \in \{1, 2, \ldots n\}$.

At period $t = 0$ the hedging firm contracts to buy $F(1)$. The firm immediately deposits $\$F(1)$ in its brokerage account. (We are assuming no margins, i.e., 100% in cash. Standardly, margins are 10–20% cash. This can save the hedging firm money, but it also introduces margin risk.)

At periods $t < i$ the hedging firm "rolls over" the hedge. In the model, the firm instantaneously and simultaneously does several things. First, it pays for its $F(t)$ contract, drawing from its brokerage account, thereby purchasing the unit of gas. The firm then immediately sells the unit for $S(t)$ (the current spot price) and contracts for a $F(t+1)$ at $S(t) + p$ (and we neglect $p$ in the accounting). Having just withdrawn $\$F(t)$ from its brokerage account, the firm now deposits $S(t)$ into it, which will be used in the next period to pay for the futures contract, $F(t+1)$.

This process repeats itself until the last period in the line. At period $t = i$, the hedger buys the unit of gas for $\$F(i)$, withdrawing from its brokerage account, takes delivery of the unit of gas, and uses the gas during the upcoming period to generate electricity. The firm then closes its brokerage account associated with line $i$ and takes home the money. How much is in the account? During each period $t$ the account has contained $\$F(t)$ plus accrued interest. Let us assume for present purposes that the $\$F(t)$ amount each period is a constant $\$F(1)$. (It should be approximately so on average.) Then the accrued amount in the account at the closing is (approximately)

$$closingAmount(i) = \$F(1)e^{b \cdot i} - \$F(1) \tag{1}$$

At this point, the firm has invested $\$F(1)$ and after a spell it has come into possession of a unit of gas, a closing amount in its brokerage account, and has incurred an opportunity cost:

$$oppCost(i) = \$F(1)e^{c \cdot i} - \$F(1) \tag{2}$$

This nets out at $i$ to a cost of:

$$realPrice = oppCost(i) - closingAmount(i)$$

The present value of this amount, this cost of purchasing the gas, is

$$PV(i) = \frac{realPrice}{e^{c \cdot i}} \tag{3}$$

The net present value, the total cost of purchasing the unit of gas at period $i$ is

$$NPV(i) = \$F(1) + PV(i) \tag{4}$$

And this is what the hedging firm is paying in full for a unit of gas in period $i$. There is no free lunch.

What the firm pays in total for $n$ periods of hedging is

$$NPV = \sum_{i=1}^{n} NPV(i) \tag{5}$$

Finally, the levelized unit cost of the gas, which may be compared to the LCOE of the PPA, is the capital recovery factor times the NPV, where the capital recovery factor is

$$CRF(c, n) = \frac{c(1+c)^n}{(1+c)^{n-1}} \tag{6}$$

Let's look at some example calculations. First, a single line. Let: $i = 120$ (ten years). $b = 0.02/12 = 0.0016666$. $c = 0.06/12 = 0.005$. $\$F(1) = 2.67$. Then closingAmount = 0.5911453. oppCost = 2.195057. realPrice_i = 1.6039. PV_i = 0.8802. NPV_i = 3.5502.

I have to complete the calculations for all periods together, but it is clear that NPV_i = 3.5502 is a high number compared to 2.67.

We can put this together in a single model. See `reserve-fund-hedging-calculations.ipynb`, a Python Jupyter notebook for implementation of the model. At $\$F(1)$ = \$2.419, the monthly unit cost (dollars per million Btu of gas) is \$5.64. This is the Henry Hub price. The delivered price to the natural gas power plant will be higher, about 10% above $\$F(1)$. Even so, at \$5.64 this works out to $7.6 \times 5.64$/mmBtu = \$42.864/MWh or \$0.0429/kWh. PPAs for VRE are widely available at prices lower than \$0.0429/kWh.

Points arising:

1. This scheme can be interpreted as another form of the reserve fund measure of volatility cost.

2. The rolling hedge will indeed reduce price volatility, but the power plant retains or gains other forms of risk as well, while at the same time having to charge to cover the cost of the hedge (mainly the interest rate in this analysis). The plant, in particular, runs the risk of buying too much or too little on the futures market. Even so, the interest costs will likely be determinative in favoring a power purchase agreement with a renewable generator.

3. To the objection: But renewables are variable too, even more so! Reply: That's irrelevant. Given an existing natural gas power plant the question is: Net of variability costs is it or is it not cheaper to acquire a power purchase agreement with a renewable generator (to substitute for some of the natural gas)? Answer: it's going to be cheaper under a broad range of likely circumstances.

EIA natural gas prices: `https://www.eia.gov/dnav/ng/ng_pri_sum_dcu_nus_m.htm`.

EIA gas prices for utilities: `https://www.eia.gov/dnav/ng/hist/n3045us3m.htm`

EIA Henry Hub prices `https://www.eia.gov/dnav/ng/hist/n3045us3m.htm`

Henry Hub futures: `https://www.cmegroup.com/trading/energy/natural-gas/natural-gas.html`

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