

Foraging Games: Ideal and Not

Robin Clark Steven O. Kimbrough

October 4, 2022

Contents

1	Introduction	1
2	Preferences	2
3	Ideal Free Ducks	3
4	IFD Modeled in NetLogo	5
4.1	The Foraging Procedure	6
4.2	Behavior of the Pure IFD Configuration	7
5	Beyond Purity	8
6	Discussion of the Models	13
7	Future Work	14
8	Bibliographic Note	14

1 Introduction

Distributions of populations over a landscape are of keen interest in biology and many of the social and behavioral sciences, including linguistics, economics, anthropology, and archeology. We are interested in exploring how individual preferences affect these distributions, in particular how distributions are affected by dynamic preferences built from the situation at hand. Particularly, in dynamic situations the information processing abilities and practices of the relevant agents are crucial. Small changes in their information processing abilities and practices can have large impacts on behavior and the resulting distributions of the agents.

We begin by raising the fundamental question of whether preferences are retrieved or formulated. Following that, we examine in some detail a foraging model built upon and extending the classic ideal free distribution (IFD) model. We find, through agent-based modeling, that very slight departures from the

stringent IFD assumptions lead to significant changes in the resulting distribution. The upshot of the paper weighs in favor of preference construction, both in fact and for modeling.

2 Preferences

Under a widely received view, decisions (alias choices) are made on the basis of preferences.¹ Given a menu of alternatives, decision makers choose based on what they prefer, presumably acting so as to choose what they prefer the most. Preferences, moreover, are in this tradition generally thought to be stable, produced from memory when needed (either directly by retrieval or indirectly by functional transformation of retrieved information), and global in the sense that for all possible choices, a_i versus a_j , the decision maker prefers a_i to a_j (we write $a_i \succ a_j$) or prefers a_j to a_i ($a_j \succ a_i$) or is indifferent between the two ($a_i \sim a_j \sim a_i$).² Further, it is a fundamental tenant in models in economics, game theory, and decision theory that agents are rational in the sense that their preferences conform to the axioms one version of utility theory or another (e.g., (Savage, 1954; Luce and Raiffa, 1957; Krantz et al., 1971; Jeffrey, 1983)). Together, the bundle of these assumption propositions constitute the core of Rational Choice Theory (RCT).

The obvious alternative to retrieval of preferences is to view them as constructed by the agent when needed (and perhaps stored in memory after creation). That is our focus in this paper. There is by now an extensive behavioral literature establishing that indeed preferences are, at least often, so constructed. See (Warren et al., 2011) for a review; see also (Krantz and Kunreuther, 2007). Preferences when empirically studied are often far from stable. They are subject to priming and environmental conditions. They are (often) constructed for the occasion rather than retrieved from memory. A fortiori, they are not global; there simply are not pre-existing preferences for everything. The behavioral and psychology literature in these regards is by now compelling. That literature, however, has not developed much more than a rudimentary account of the mechanisms by which preferences are created or constructed (alias formulated).

Thus, we seek to develop and explore models of preference formation by agents. Where to begin? Benjamin Franklin’s justly celebrated method, expressed in a letter to Joseph Priestly, is the kind of account we seek <https://www.1000minds.com/decision-making/benjamin-franklin>.

In light of empirical findings about preference construction (noted above), a natural interpretation of Franklin’s method is that it is an account, both descriptive and prescriptive, of how preferences may sensibly be constructed (by listing pro and con factors and striking balanced pairs). Franklin proffers the approach as a way to circumvent cognitive limitations. When there are very many pros and cons we cannot keep proper track of them. Franklin thus

¹Terminology is not fully standardized. We do our best to use mainstream senses, but the reader should understand that there is other terminology floating about.

²This is an axiom of utility theory. See Warren et al. (2011) for an authoritative review.

proposes a physical procedure for making the comparisons without decision makers having to use more than all of their limited cognitive resources.

In his letter, Franklin offered a procedure—a “moral or prudential algebra”—by which to employ data available at the time of decision and arrive at a preferred choice. What is preferred is discovered or calculated by this procedure, not dredged up from a stable pool of memories.

Generalizing Franklin’s letter, the principle involved is one of *rule-based preference formation*. Beginning in the next section, we explore this principle for explanatory purposes in the analysis of foraging behavior and preference formation. First, however, we wish to acknowledge and address an objection to our framing and approach. In doing so we introduce some nuance, a refinement of the narrative given so far. The objection is that the distinction between retrieving preferences from memory and constructing them by recalling rules for generating them is a distinction without a real difference. “Of course,” the objection goes, “no one thinks that a forager would have a fixed preference for patch A over patch B. Instead, the forager has a fixed preference for alternatives that maximize food intake. What is fixed and stable and recalled from memory is the rule to prefer more food. If you want to call this preference construction you can, but you are not introducing anything new or different. What’s stable and retrieved is the rule/preference for more food.”

Our response is first that maximizing food intake is a value or goal, not a preference for how to achieve it (by choosing a patch). Second, if stable and simple rules can be identified that account for the behavior in question, then the retrieval-or-construction debate would presumably be resolved in favor of retrieval. But if simple rules are not sufficient for explaining observed behavior, while rather more complex rules are sufficient, then at some point of increased complexity, a retrieval account becomes misleading in a way that a construction account is not. In the end, it matters little where exactly we draw the line. What matters is delineating correctly the mechanisms involved.

In what follows we use agent-based modeling to simulate and investigate foraging. The ideal free distribution (IFD) hypothesis, explained in the sequel, is the immediate target of our modeling. Foragers will be distributed as in the IFD under standard assumptions and Rational Choice Theory. We replicate this finding and go on to demonstrate that small departures from the IFD assumptions lead to large departures from IFD behavior. This creates opportunities for foragers to learn and use revised rules for choosing foraging patches. Whether they do so or not is an empirical question. But to the extent they do, to that extent at least a construction account of preference formation is closer to the mark.

3 Ideal Free Ducks

Food and other resources are often patchy, appearing more in one place than in another, and absent in most places. When this is the case, individuals in a community are typically faced with a strategic decision problem of where to forage

for resources. Without other individuals involved, foraging at the most productive patch, net of transit and other costs, would be optimal. The individual, however, is affected by decisions made by other individuals in its community. Perhaps foraging at a less productive patch will yield an individual more in return than foraging at a more productive patch because the less productive patch is comparatively less crowded.

Thus we have a foraging game. How will it turn out in equilibrium? Fretwell was the first to investigate this framing of foraging theory. In a series of papers Fretwell and Lucas (1969); Fretwell and Calver (1969); Fretwell (1969) and a book, Fretwell (1972), Stephen Fretwell investigated this question both theoretically and empirically. The literature since has blossomed.

Fretwell developed a model he called ideal free distribution (IFD) for predicting the distribution of individuals in a community across a diversity of foraging patches.³ Assuming ideal rationality and identity of capabilities among the individuals, Fretwell posited that at equilibrium individuals would be arrayed among patches such that each individual would obtain an equal amount of resource per unit of time. This *ideal free distribution* (IFD) can be expressed as follows.

Let N_i be the number of foragers at site i and R_i be the level of resources available at site i , which the foragers at the site share equally, then the IFD is

$$\frac{N_i}{\sum_i N_i} = \frac{R_i}{\sum_i R_i} \quad (1)$$

With just two foraging patches this reduces to

$$\frac{N_1}{N_2} = \frac{R_1}{R_2} \quad \text{or equivalently} \quad \frac{N_1}{R_1} = \frac{N_2}{R_2} \quad \text{and in general} \quad \frac{N_i}{R_i} = \frac{N_j}{R_j} \quad \forall i, j \quad (2)$$

As an aside, note that the relationship between the N_i s and the R_i s is exactly that posited by the matching law of Herrnstein (1997), which is widely used in the description of animal learning. Let the B_i s be the behavior alternatives, and the R_i s be the associated reinforcements; then the matching law says that at convergence of learning, we have:

$$\frac{B_i}{B_1 + B_2 + \dots + B_n} = \frac{R_i}{R_1 + R_2 + \dots + R_n} \quad (3)$$

Although it was not designed for modeling strategic decision making, the associated hypothesis of *melioration* (Herrnstein, 1997) addresses the dynamics of learning under the matching law. According to this hypothesis (Herrnstein, 1997, page 77), the adapting/learning agent adjusts its B_i s so that:

$$\frac{R_1}{B_1} = \frac{R_2}{B_2} = \dots = \frac{R_n}{B_n} \quad (4)$$

Returning to Fretwell's ideal free distribution, it is assumed throughout that the sum of the R_i s is fixed, at least comparatively so. In the case of two alternatives, doing R_1 at a time entails not doing R_2 , for example.

³We focus as does the literature on conspecifics in a single habitat.

Moreover, Fretwell’s basic model assumes the following properties of Ideal Free foraging:

- Animals need to forage from patches of varying qualities distributed over a region.
- Animals will tend to distribute themselves over these patches in order to maximize their individual returns.
- They will therefore tend to distribute themselves so that the resources will be distributed evenly, taking into account the fecundity of the resource patch and the number of conspecifics exploiting the patch. This is the insight behind ideal free distributions (Fretwell and Lucas, 1969).
- Elaborating on the above point, suppose that some animal is getting more resources than would be expected if the resources were distributed evenly. This means that some other animal is getting less than expected; the latter animal should, then, move to the patch where the other animal has more than its share. As the foragers reshuffle, they should eventually hit an equilibrium point where all animals are getting the same amount.
- In particular, if F_i is the fecundity of patch i , and n_i is the number of conspecifics at patch i , then:

$$\frac{F_i}{n_i}$$

is the return that an individual forager can expect when occupying i .

- Finally, the basic IFD model assumes that the animals at a patch are dividing resources evenly, so that they all get the same amount. This need not be the case. For example, some animals may be more efficient at foraging, or can bully the other animals, in which case the distribution of resources will not be equal. This should effect the distribution of animals per patch.

4 IFD Modeled in NetLogo

We constructed a model in NetLogo (IFD-Foraging01.nlogo) of foraging based on the ideal free distribution with two food sources (patches). Our model is somewhat more general than the pure IFD model because it allows for the animals to transit non-instantaneously from one patch to the other. Because we based the model on experiments by Harper (1982) involving ducks wintering at a pond, we will henceforth refer to the agents (or “animals”) as *ducks*. The IFD mathematical model supposes that the ducks can move instantaneously from one food source to another. Our NetLogo model permits a number of variations, which turn out to be significant, as we shall see.

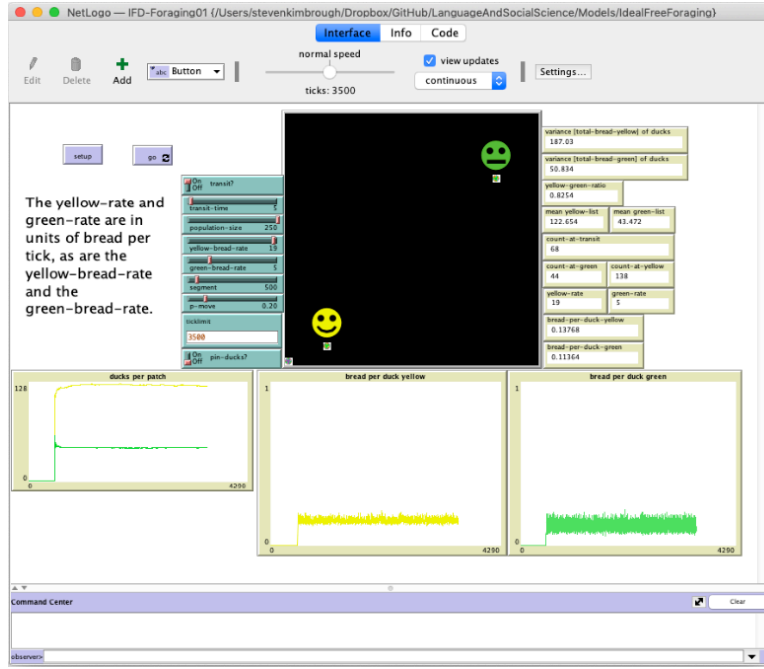


Figure 1: Interface of the IFD-Foraging01.nlogo Model.

4.1 The Foraging Procedure

The IFD-Foraging01.nlogo model is freely available online and gives full access to its code.⁴ Here we sketch in pseudocode the main foraging procedure, occurring at each time step in the execution of the model.

1. Collect statistics on system state.
2. Ask food sources to distribute food according to their schedules
3. Ask animals to collect and record food and determine if they want to move to a different patch.
4. if `transit?` is true
 - (a) Decrement the transit-counter for all animals in transit;
 - (b) For animals that have completed transit, place them at their new destination food source;
 - (c) Put into transit all animals that have decided to move and set their transit counters (indicating the amount of time to be in transit).

⁴<https://github.com/stevenkimbrough/sokpapers/commit/3b3d5ccdaf0bce9d71ca86a34c2e05d11b391ad7>

5. Ask the animals not currently in transit:
 - (a) Increment the time counter for presence of the food-source they are at;
 - (b) Determine whether to move to a new source immediately; this is implemented as the procedure `move-immediate` in the code (see below). The decision depends on the probability of moving `p-move`, the number of animals at the other site and the fecundity of the other site.
 - (c) If `to-move` is `true` and `transit?` is `false`, move immediately to the other site;
 - (d) If `to-move` is `true` and `transit?` is `true`, move to a special transit patch and wait.
 - (e) Otherwise, the animals stay where they are and wait for more food.

Much of the central intuition of the IFD model is captured in `move-immediate` which, in essence, computes the “expected utilities” (values) of the bread⁵ sources by looking at the rate at which bread is tossed out versus the number of animals at the food source. Note that `move-immediate` returns a Boolean value contingent on the expected amount of bread at each patch.

1. If the bread per animal (“duck” in the NetLogo model) is larger at the patch occupied by the duck, return `False`;
2. Otherwise, if the bread per duck is less at the present patch, return `True` with probability `p-move`; otherwise return `False`.

4.2 Behavior of the Pure IFD Configuration

We constructed a “pure” model of the Ideal Free Distribution. This configuration is set in the NetLogo model by switching `pin-ducks?` to off on the Interface tab (see Figure 1). The agents (the foraging animals, which we refer to as ducks) are “ideally rational” They are “free,” meaning they can move instantaneously and without cost to any food patch; and they are “ideal” in the sense that they have perfect information about the fecundity of each food patch, which they use to maximize intake of resources. Furthermore, they know the number of competitors at each food patch, although they cannot predict the future of how many ducks will be at each patch. Thus, in this model the ducks should distribute themselves in such a way as to guarantee that each duck acquires the same amount of food as any other duck, in equilibrium. When we run the model as a pure IFD system, this is exactly what we see. The ducks divide themselves over the food patches in such a way that each duck gets the same amount of food as any other duck, even though the food sources are differentially productive.

⁵We are grateful to Annie Vo for reminding us that ducks should not be fed bread as it messes with them. Here and hereafter, when we say “bread” let us mean “units of healthy food for ducks.”

In a typical case, we ran the model for 3500 time steps with an initial sample of 500 time steps, allowing the ducks to observe the rate of bread distribution at the food patches. The run had (the IFD-Foraging01.nlogo model has) two food patches. In the run there were 250 ducks, with the rate of bread distribution being 15 units of bread per unit time versus 5 units of bread per unit time. The ducks arranged themselves in such a way as to yield 0.080 units of bread per unit of time at the more fecund food patch and 0.079 units of bread per unit time at the less fecund patch, basically identical yields of food per patch. Since the rate of bread distribution at the more fecund patch was three times the rate of the less fecund patch, we expect, then, that three times as many ducks will congregate at the first patch as the number that congregate at the less fecund patch. Again, that is exactly what we see: 187 ducks at the first patch to 63 ducks at the second patch. If we look at the mean of the total bread collected by ducks at the more fecund patch and compare it to the mean of the total bread collected at the less fecund patch, we expect the former to be three times the latter, and it is: 180 units collected at the more fecund patch to 60 at the less fecund patch. In addition, the variance of the total bread collected per food source should be nearly identical and it is: 30.06 for the more fecund patch versus 29.74 for the less fecund patch.

Within a narrow margin of statistical error (noise), the behavior of the Pure IFD NetLogo model (with `transit?` set to off) conforms exactly to the predictions of the ideal free distribution.

5 Beyond Purity

The Pure IFD model exists in a clockwork universe that is at quite a remove from the world of actual foragers, who cannot move instantaneously between food sources and who may not have perfect information about the fecundity of the patches or the number of competitors at each patch. In the next experiment, we will increase the cost of moving from one food source to the other; it will no longer be instantaneous but will rather take some amount of time. We implement this by creating a “transit patch” where ducks are confined for some number of ticks when they decide to switch patches. While confined at the transit patch, the ducks cannot acquire more bread; that is, the ducks temporarily cease foraging while traveling from patch to patch. Now, `transit?` is switched to on.

In the first configuration, when they decide to move from patch to patch, the ducks must spend 5 time ticks in the transit patch, where they receive no resources. Once again, we ran the model for 3500 time steps with an initial sample of 500 time steps for warm up, allowing the ducks to observe the rate of bread distribution at the food patches. As above, the run had two food patches, 250 ducks and the rate of bread distribution being 15 units of bread per unit time versus 5 units of bread per unit time. If we allow the probability of moving to be 1 when a duck decides its yield would be greater at the other patch, we have a striking result. After an initial apparently chaotic period, the number of

ducks at the more productive patch is 2, while the number of ducks at the less productive patch is 1. The remaining 247 ducks are in transit and, therefore, not receiving bread. Thus, the ducks at the more fecund patch get 7.5 units of bread per unit time (there are 15 units of bread distributed per unit of time); the single duck at the less fecund patch receives 5 units of bread per unit time. That is, of course, the maximum any duck could receive per unit of time at that patch. In this sense, a true ideal free distribution would be unattainable, due to the hard limit on the less fecund patch. Of course, an ideal free distribution is out of the question since the vast majority of ducks receive nothing, because they are incarcerated at the transit patch.

This is a largely unsurprising result, given the basic mechanics of the decision making by the foragers and the relative bread rates. A more interesting scenario is suggested by giving the ducks some commitment to their current patch. We can change the probability that they will switch patches once they perceive an asymmetry in the relative distribution of food. Suppose that the probability that a duck will switch patches is 0.2, that is they will transit only twenty percent of the time once they perceive the asymmetry. On this experiment, then, the ducks take time to move from food source to food source but are reluctant to move.⁶ That is, the conditions—number of agents, food rates from the sources, and so on—are the same as the previous experiment but the probability of a duck moving when it perceives an asymmetry between the patches is just 0.2. In a typical run the results are that the ducks at the more fecund patch are getting 0.131 units of bread per time at the patch, while ducks at the less fecund patch are getting 0.102 units of bread per time. After stopping at the default of 3,500 time steps, there are 114 ducks at the more fecund patch compared to 49 ducks at the less fecund patch and 87 ducks in transit.

The results are quite different from those of the pure IFD model, due to the “friction” created by the temporal constraints on movement. The underlying decision rule the ducks are using is the same as in the pure IFD model, but the physical constraints of the world prevent them from achieving a pure ideal free distribution. Notice that the count of the number of ducks at the more profitable patch is depressed compared to the pure IFD model.

The fact that there are fewer ducks at the more profitable patch raises the bread per duck per unit time at that patch. How is this to be explained? In the mathematical model of the IFD the assumption is made that time is continuous, with upshot that each duck makes its decision at an infinitesimal interval that is unique to the duck. In our agent-based model, time is discrete and ducks at sources make their decisions independently whether to move during a finite interval and then move en masse, if they move at all. In consequence, if there’s a more attractive source, while it may have more ducks in residence, more ducks will move if the less attractive source is comparatively under-populated. Conversely, at the less productive source, if it becomes less attractive compared to the other source, fewer ducks will move at any given time step. The outcome of these considerations is that the more attractive source will be relatively under-

⁶We tried this experiment with the pure IFD model and it made no difference.

populated, yielding more bread per duck at that source. This is exactly what we have seen, above.

In order to investigate this, we developed BehaviorSpace experiments to examine the relationship between bread rates at the sources and the number of ducks at the sources. If the distribution were a pure Ideal Free one, the ratio of the number of ducks at the higher yield patch to the number of ducks at the lower yield patch divided by the ratio of the rate of bread distribution at the higher patch to the rate of bread distribution at the lower yield patch should be equal to one. This would be what happens in the pure IFD case where the ducks distribute themselves in such a way that they all get the same amount of bread.

Let:

- (a.) A is the count of ducks at the more fecund food source;
- (b.) B is the count of ducks at the less fecund food source;
- (c.) C is the bread rate at the more fecund food source;
- (d.) D is the bread rate at the less fecund food source.

The graphs in Figure 2 compare C/D (the x-axis) with $(A/B)/(C/D)$ (the y-axis). Under IFD the latter quantity should always be equal to 1, as happens in Figure 2a, the Pure IFD model where transit costs are 0; there is some slight statistical noise but the ducks always arrange themselves as expected with the ducks at the two food sources getting the same amount of bread. When transit from one food source to another involves a real cost, as in Figure 2b, we see that the distribution of bread is not equal across the food sources; in fact, since the ratio is less than 1, there are fewer ducks at the more fecund food source than we would expect under the IFD. In consequence, because there is a smaller population at the more fecund source, the bread per duck should be higher than what would be found at the less fecund source; this is what we observe. In fact, as a result of there being fewer ducks at the more fecund source, the bread per duck at that source is higher than we would expect under an ideal free distribution.

One way to think about the above is that the agent-based model occurs in discrete time so that, in effect, the agents make their decisions about moving in parallel; the agents have no information about what other ducks decide at that time step. Furthermore, once a duck is in transit, it gets no information about the ducks who were already in transit, the ducks who went in transit with the duck, and the ducks who go into transit while the duck itself is transiting; in other words, the cost of transit is to deprive the transitee of information which should condition its choice of destination. Even if a duck in transit got information about the real distribution of ducks, the model provides no way for the duck to use that information while in transit. The pure IFD, on the other hand, takes place in continuous time (and with instantaneous movement) so the ducks always have perfect information about the other ducks and the expected

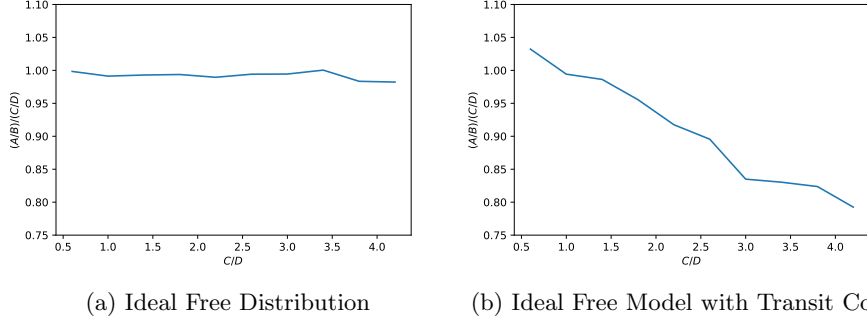


Figure 2: Behavioral comparison with and without transit costs. A is the count of ducks at the more fecund food source; B is the count of ducks at the less fecund food source; C is the bread rate at the more fecund food source; D is the bread rate at the less fecund food source.

returns of the food sources. In other words, discrete time imposes a form of bounded rationality by depriving the ducks of information.

The above results used a certain amount of “stickiness” on the part of the ducks; their probability of moving at any time step is 0.2. If they move immediately when they are dissatisfied with their current food source, that is the probability of moving is 1, then the results are much more chaotic. In essence, the vast majority of ducks are always in transit, landing at a food source only long enough to become dissatisfied and go back into transit. The need for reticence in moving is another illustration of the real costs of transit and reflects something that should be taken into account when analyzing real foraging data.

Rate A	Population at A	Population at B	BPD A	BPD B
3	93.533	156.467	0.032	0.032
5	124.467	125.533	0.040	0.040
10	166.200	83.800	0.060	0.060
15	187.100	62.900	0.080	0.079
20	199.267	50.733	0.100	0.099

Table 1: Bread rate at site B is a constant 5 units per time step. Bread rate at site A varies from 3 to 20 units per time step. Under IFD, Population at A divided by Population at B should be equal to Rate A divided by Rate B, which it is to a close approximation. Also under IFD Bread Per Duck (BPD A) should be equal to BPD B, which it is.

In order to make comparisons more systematically we conducted two BehaviorSpace experiments, each with 10 replications. In the first experiment, we used a Pure IFD configuration (`transit?` switched to off), varying the bread rate at food source A and keeping all else constant. The results are shown in

Rate A	Population at A	Population at B	BPD A	BPD B
3	61.633	96.867	0.049	0.052
5	78.800	79.133	0.064	0.063
10	105.367	58.067	0.095	0.087
15	115.033	47.000	0.131	0.111
20	124.567	38.633	0.162	0.138

Table 2: Bread rate at site B is a constant 5 units per time step. Bread rate at site A varies from 3 to 20 units per time step. Under IFD, Population at A divided by Population at B should be equal to Rate A divided by Rate B, but in our departure from IFD due to transit time this equivalence fails. Also under IFD Bread Per Duck (BPD A) should be equal to BPD B, but transit time costs destroy this equivalence.

Table 1. The second experiment duplicated the first, except that **transit?** was switched to on, violating the Pure IFD assumptions of no travel cost. See Table 2. Comparing the two experiments confirms what we reported for prototypical experiments of the two cases. In the Pure IFD computational model, the predicted Pure IFD obtains. As comparative bread rates change, bread per duck is equal across the two food sources, and the fraction of ducks at each food source is also as predicted by the IFD theory. Contrariwise, in the **transit?** on configuration (with transit cost > 0) and when the fecundity level of food source A is higher than B: (i) the bread per duck systematically varies between the two food sources with ducks at source A receiving more food per unit time than ducks at source B; and (ii) the number of ducks present at site A is lower than it should be under Pure IFD. The effects are robust and systematic.

When IFD conditions obtain, no duck can unilaterally do better by changing its policy for moving between food sources. This is to say that Nash equilibrium conditions rule. What happens when movement has costs? It would seem that the duck’s movement policy may no longer be a Nash equilibrium policy. To investigate this conjecture we conducted another experiment, in which two ducks, one at source A and one at Source B, were “pinned.” That is, two ducks were randomly picked at initialization and made to stay at one or the other of the food sources. Everything else is as set for the experiment of Table 2.

Table 3 shows the difference between the bread collected by a duck pinned at site A, whose rate of bread distribution varies from 3 units to 20 units of bread per time step, a duck pinned at site B, whose rate of bread distribution is held constant at 5 units of bread per time step, and ducks that can freely travel between A and B. The rates are compared for when temporal penalties for travel are disabled (“False”) and when temporal penalties, held at 5 units of time, are enabled (“True”). We can see that the pinned ducks and the free ducks get roughly the same amount of bread when travel penalties are disabled, the Ideal Free Distribution. When temporal penalties are levied, we see that the pinned ducks, who cannot travel, do systematically better than the ducks that are

(Variable Rate at A, Transit)	Duck pinned at A	Duck pinned at B	Average Un- pinned
(3, False)	96.121	95.934	96.000
(3, True)	148.402	153.614	95.556
(5, False)	120.483	119.525	120.000
(5, True)	188.659	187.476	119.451
(10, False)	180.390	179.258	180.001
(10, True)	289.287	269.447	179.199
(15, False)	240.396	238.960	240.003
(15, True)	385.391	329.386	239.053
(20, False)	301.332	295.116	300.014
(20, True)	477.940	381.244	298.955

Table 3: B source is constant at 5; A source is variable. Comparing the average bread collected by a duck pinned at A, a duck pinned at B, and unpinned ducks. Results are the means of 30 replications for each setting.

allowed to transit. This suggests that the best strategy (the Nash equilibrium) is actually not to travel, if travel takes time. More precisely, pinning is shown to be a profitable response when all but one duck, at the other site, are transiting in the usual way. We also see that the pinned duck at site A, the site with generally higher rates of bread distribution, does better than the duck pinned at site B. Table 2 also shows this effect, we think for the same reason. In general, the more prosperous site is proportionately under-represented, although it has more ducks at it; this is because the ducks make their decisions in parallel so that more populated sites will tend to look less attractive. The underlying data shows that the number of ducks at site A is systematically less than the number at site B, when the rate of bread distribution at A is greater than that at B. The odds of this happening by chance is less than one in a billion. The Ideal Free Distribution assumes that ducks make their decisions sequentially and that transit time is zero.

6 Discussion of the Models

With any very idealized model there always lurks the danger of over abstraction and failure of robustness in the face of plausible and realistic changes to the model. We have seen this phenomenon at play with respect to travel time and ideal free distributions. It is hardly surprising that idealized models will fail in the absence of truth in their assumptions. The most interesting subsequent questions are in regard to the nature and magnitude of the changes in model behavior in response to changes in model assumptions. To take one small example, with pinning as in Table 3 and focusing on the last row, we see that the rewards for the pinned ducks greatly exceed those of the unpinned ducks following what would be a Nash equilibrium strategy under IFD conditions. The pinned duck

at source B, the less fecund of the two sources, gets 27% more food than the average unpinned duck (281/299), while the pinned duck at A gets a whopping 60% more (478/299). Exploring these kinds of effects and confronting them with data lies at the heart of strategic modeling and analysis, and the subject of our book, decision games.

The study of foraging is a rich and flourishing subfield of ecology, affording ample vistas of opportunity for modeling and data collection. Our treatment here should be taken as a point of departure into a great realm of modeling possibilities. We have demonstrated that agent-based modeling can both duplicate idealized models and serve as a basis for investigations that relax idealizations in the direction of verisimilitude.

7 Future Work

There is a great deal of work that remains to be done here. Due to space limitations, we have not been able to adequately cover recent work that is quite relevant to this topic, for example Olszewski (2021) (we are grateful to an anonymous referee for pointing this paper out to us). There are, of course, many other papers that would have to be systematically reviewed.

Among the most interesting items for future research, identified by anonymous referees and other discussants, are the following:

- How is the order of events determined? Does order matter?
- Can the ABM model be reduced to a stochastic differential equations model? What advantages does an ABM provide versus other approaches, particularly in the pure model?
- What happens when multiple food sources of similar fecundity are introduced? What if they have different distributions of fecundity values? Would your results remain robust then?

8 Bibliographic Note

In the interest of conserving space, we note the following works related to ideal free distributions related to this paper. Stephens and Krebs (1986) is an early landmark development of optimal foraging theory, which focuses on individual, asocial behavior. Strategic or game theoretic modeling of foraging, which came to be called social foraging, may be said to begin with Fretwell and Lucas (1969); Fretwell and Calver (1969); Fretwell (1969) and Fretwell (1972). Harper (1982), Kennedy and Gray (1993), Tregenza (1995), and Kraft et al. (2002) address important issues as the field developed. Giraldeau and Caraco (2000) synthesizes and develops social foraging models. Stephens et al. (2007) surveys the field of foraging (social and asocial) 20 years after (Stephens and Krebs, 1986). Bettinger (2009) is an accessible treatment of foraging models applied to human hunter-gatherers.

References

- Bettinger, R. L. (2009). *Hunter-Gatherer Foraging: Five Simple Models*. Eliot Werner Publications, Inc., Clinton Corners, NY.
- Fretwell, S. and Lucas, H. (1969). On territorial behavior and other factors influencing habitat distribution in birds: I. theoretical development. *Acta biotheoretica*, 19(1):1–36.
- Fretwell, S. D. (1969). On territorial behavior and other factors influencing habitat distribution in birds. III. Breeding success in a local population of field sparrows. *Acta Biotheoretica*, 19:45–52.
- Fretwell, S. D. (1972). *Populations in a seasonal environment*. Number 5 in Monographs in population biology. Princeton University Press.
- Fretwell, S. D. and Calver, J. S. (1969). On territorial behavior and other factors influencing habitat distribution in birds: II. Sex ratio variation in the Dickcissel. *Acta Biotheoretica*, 19:37–44.
- Giraldeau, L.-A. and Caraco, T. (2000). *Social Foraging Theory*. Princeton University Press, Princeton.
- Harper, D. (1982). Competitive Foraging in Mallards: 'Ideal Free' Ducks. *Animal Behavior*, 30:575–584.
- Herrnstein, R. J. (1997). *The Matching Law: Papers in Psychology and Economics*. Harvard University Press, Cambridge, MA.
- Jeffrey, R. C. (1983). *The Logic of Decision*. University of Chicago Press, Chicago, IL, second edition.
- Kennedy, M. and Gray, R. D. (1993). Can Ecological Theory Predict the Distribution of Foraging Animals? A Critical Analysis of Experiments on the Ideal Free Distribution. *Oikos*, 68(1):158–166. Publisher: [Nordic Society Oikos, Wiley].
- Kraft, J. R., Baum, W. M., and Burge, M. J. (2002). Group choice and individual choices: Modeling human social behavior with the Ideal Free Distribution. *Behavioural Processes*, 57:227–240.
- Krantz, D. H. and Kunreuther, H. (2007). Goals and Plans in Decision Making. *Judgment and Decision Making*, 2(3):34.
- Krantz, D. H., Luce, R. D., Suppes, P., and Tversky, A. (1971). *Foundations of measurement, Vol. I, Additive and polynomial representations*. Academic Press, New York, NY.
- Luce, R. D. and Raiffa, H. (1957). *Games and Decisions: Introduction and Critical Survey*. John Wiley and Sons, Inc., New York, NY.

- Olszewski, W. (2021). Preferences and information processing under vague information. *Journal of Mathematical Economics*, 94:1024–61.
- Savage, L. J. (1954). *The Foundations of Statistics*. John Wiley And Sons, Inc., New York, NY.
- Stephens, D. W., Brown, J. S., and Ydenberg, R. C., editors (2007). *Foraging: Behavior and Ecology*. The University of Chicago Press, Chicago.
- Stephens, D. W. and Krebs, H. R. (1986). *Foraging Theory*. Princeton University Press, Princeton.
- Tregenza, T. (1995). Building on the Ideal Free Distribution. volume 26 of *Advances in Ecological Research*, pages 253 – 307. Academic Press. ISSN: 0065-2504.
- Warren, C., McGraw, A. P., and Van Boven, L. (2011). Values and preferences: defining preference construction. *WIREs Cognitive Science*, 2(2):193–205. .eprint: <https://wires.onlinelibrary.wiley.com/doi/pdf/10.1002/wcs.98>.