11/3/2011

Given $O = (L(0), A(0), L(1), A(1), Y \in L(2)) \sim P_0 \in \mathcal{M}$

our $\mathcal{M} = \{Q_{L(2)}, Q_{L(1)}, Q_{A(0)}, g_{A(0)}, g_{A(1)} : g : G\}$, nonpara.

$$P^{a=(a(0),a(1))}(\ell(0), \ell(1), \ell(2)) = Q_{L(0)}(\ell(0))\, Q_{L(1)}(\ell(1) \mid A(0)=a(0), L(0)=\ell(0))\, Q_{L(2)}(\ell(2) \mid \bar{\ell}(1), \bar{A}(1) = \bar{a}(1))$$

$$L^a = (L(0), L(1), L(2)) \sim P^a$$

Our parm $\Psi(P) = E_{P^a} Y^a$

① We have to find EIC:

$\mathcal{M}(g) = \{Q; g : O\}$, i.e. g is known

the EIC for $\Psi : \mathcal{M}(g) \to \mathbb{R}$ equals EIC for $\Psi : \mathcal{M} \to \mathbb{R}$
i.e. where g is unknown

Need to look at pathwise derivative

$$\underbrace{\Psi(Q(\varepsilon))}_{E_{P^a_{(\varepsilon)}} Y^a} - \underbrace{\Psi(Q)}_{E_{P^a}[Y^a]} = \sum_{\ell} \ell(2) \left[\frac{Q(\varepsilon)(\ell;a) - Q(\ell,a)}{\varepsilon} \right]$$

Note: $Q(\ell, a) = P(L^a = \ell)$
$= P^a(\ell)$

$$= \sum_{\ell, a'} \frac{\ell(2) I(a'=a)}{g(a';\ell)} \left[\frac{Q(\varepsilon)(\ell;a) - Q(\ell,a)}{\varepsilon\, Q(\ell,a')} \right] Q(\ell;a')\, g(a';\ell)$$

as $\varepsilon \to 0$, $\Rightarrow \sum_{a'} \sum_{\ell} \left(\frac{\ell(2) I(a'=a)}{g(a';\ell)} \right) \cdot S(a;\ell)\, Q(\ell,a')\, g(a;\ell)$, & score $S = \dfrac{\frac{\partial}{\partial \varepsilon} Q(\varepsilon) \big|_{\varepsilon=0}}{Q}$

$$= E_P\Big[\underbrace{\Big(Y \frac{I(A=a)}{g(A,L)} - \Psi \Big)}_{\text{is a gradient}} \cdot S(A,L) \Big]$$

i.e. $D(O) = \left[Y \dfrac{I(A=a)}{g(A,L)} - \Psi \right]$ is a gradient of the pathwise derivative

$\hookrightarrow$ cont.

cont. the canonical gradient,

$$D^*_{\mathcal{Q}} = \Pi(D \mid T_Q), \text{ where } T_Q \text{ is the tangent space of } Q = (Q_1, Q_2, Q_3) \text{ in } P = Qg$$

$$= \Pi\left[D \mid T_{Q_0} \oplus T_{Q_1} \oplus T_{Q_2}\right]$$

$\uparrow$ orthogonal sum

T_{Q_j} = tang. space of $Q_{L(j)} = \{ S(L(j) \mid Pa(L(j))) : E(S \mid pa(L(j))) = 0 \}$

$$= \sum_{k=0}^{2} \Pi[D \mid T_{Q_k}]$$

$$= \sum_{k=0}^{2} \left[E[D \mid L(k), Pa(L(k))] - E\left[E[D \mid L(k), Pa(L(k))] \mid Pa(L(k))\right] \right]$$

$$E\left[\frac{YI(A=a)}{g(A,L)} - \psi \,\Big|\, L(0)\right] = E\left[\frac{YI(A=a)}{g(A,L)} \,\Big|\, L(0)\right] - \psi$$

$$= \left(E\left[\frac{YI(A=a)}{g(A,L)} \,\Big|\, L(0)\right] - \psi \right) + E\left[\frac{YI(A=a)}{g} \,\Big|\, L(1) Pa(L(1))\right] - E\left[E\left(Y\frac{I(A=a)}{g} \,\Big|\, L(1), Pa(L(1))\right) \Big|\, Pa(L(1))\right]$$

1 2

$$+ \frac{YI(A=a)}{g(0)} - E\left[Y\frac{I(A=a)}{g(0)} \,\Big|\, Pa(L(2))\right]$$

Lemma: $E\left[Y\frac{I(\bar{A}(k)=\bar{a}(k))}{g(\bar{A}(k),\bar{L})} \,\Big|\, \bar{L}(j), \cancel{\bar{A}(j-1)}\, D^a(1,j)\right]$, $j \le k$

$$= \frac{I(\bar{A}(j-1)=\bar{a}(j-1))}{g_{j-1}(0)} \underbrace{E_{P_a}\left[Y^a \mid \bar{L}(j), \bar{A}(j-1)=\bar{a}(j-1)\right]}_{\bar{Q}^a_j(\bar{L}(j)) \;=\; E[Y^a \mid \bar{L}^a(j)]}$$

Now we can rewrite EIC (w/ Lemma in mind) 2

$$D^*(0) = \left[E[Y^a \mid L(0)] - \psi\right] + \frac{I(A(0)=a(0))}{g_0(0)}\left[E[Y^a \mid \underbrace{\bar{L}(1) Pa(L(1))}_{=\bar{L}(0), A(0)}]\right.$$

$$\left. - E\left[E[Y^a \mid \bar{L}(1), A(0)=a(0)] \mid L(0), A(0)=a(0)\right]\right]$$

$$= \quad + \frac{I(A(0)=a(0))}{g_0(0)}\left[\bar{Q}^a_1(\bar{L}(1)) - E\left[\bar{Q}^a_1(L(1)) \mid L(0), A(0)=a(0)\right]\right]$$

$$+ \frac{I(\bar{A}=\bar{a})}{g(0)} \Big[Y - \underbrace{E[Y^a \mid \bar{L}(1), \bar{A}(1) = \bar{a}]}_{\bar{Q}_1^a(L(1))} \Big]$$

So, summary result,

$$D^*(0) = \big[E[Y^a / L(0)] - \psi \big] + \frac{I(A=a)}{g_0(0)} \Big[\bar{Q}_1^a(\bar{L}(1)) - E[\bar{Q}_1^a(\bar{L}(1)) / L(0), A(0) = a(0)] \Big]$$

$$+ \frac{I(\bar{A}=\bar{a})}{g(0)} \Big[Y - E[Y^a / \bar{L}(1), \bar{A}(1) = \bar{a}] \Big]$$

Our parm is $\psi(Q) = E_{p^a}[Y^a] = E\big[E[E(Y^a / \bar{L}^a(1)] / L(0)]\big]$

Note: $P^a = Q_0^a Q_1^a Q_2^a$

$$= \cancel{I(0)} P(L(0)) \cdot P(\bar{L}(1) \mid L(0), A(0)) \cdot P(\bar{L}(2) \mid \bar{L}(1), \bar{A}(1) = \bar{a}(1))$$

(with "$a(0)$" written above $A(0)$)

$$= E\Big[E\big[E[Y / \bar{L}(1), \bar{A}(1) = \bar{a}(1)] \mid L(0), A(0) = a(0) \big] \Big]$$

Algorithm

① $\bar{Q}_2^a = E[Y / \bar{L}(1), \bar{A}(1) = \bar{a}(1)]$

② $\bar{Q}_1^a = E[\bar{Q}_1^a / L(0), A(0) = a(0)]$

③ $\bar{Q}_0^a = E[\bar{Q}^a] \rightarrow \psi(Q) = \bar{Q}_0^a$

$\boxed{\text{* Read his paper!}}$

11/15/2011 Note: skipped last few classes to prepare for GRE

☆ Taught mainly out of slides today

$$\cancel{B_1} = L_1(t+1) = L_1(t) \exp(m(t) \Delta t) \Rightarrow S_1(t) = \exp(m_1(t) \Delta t)$$

$$L_2(t+1) = L_2(t) \exp(m(t) \Delta t) \quad S_2(t) = \exp(m_2(t) \Delta t)$$

$$\Downarrow$$

$$\frac{\log[S_1(t)]}{\log[S_2(t)]} = \beta$$

Read H Bang, J Robins paper

↳ Doubly robust estimation in missing data & causal inference mode