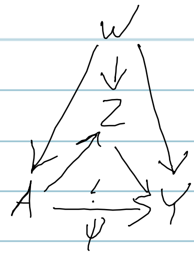
9/27/2011 Note: Was not present for 9/22/11 class

TMLE for natural direct effects

Given



$O=(W,A,Z,Y)$ ← Binary

mediator

For the NPSEM:

$U=(U_W,U_A,U_Z,U_Y)$

$W=f_W(U_W)$

$A=f_A(W,U_A)$

$Z=f_Z(W,A,U_Z)$

$Y=f_Y(W,A,Z,\cancel{Z}\,U_Y)$

Intervening

$Z_a=f_Z(W,A=a,U_Z)$

$Y_{a,z}=f_Y(W,A=a,Z=z,U_Y)$

⇓

1) a controlled direct effect: $E(Y_{1z}-Y_{0z}) = \psi^F(P_{UX})$
2) a natural direct effect: $E(Y_{1Z_0}-Y_{0Z_0}) = \psi^{F'}(P_{UX})$

Suppose $\psi=0$ (i.e. no direct effect)

↳ then under #1, will always see effect b/c controlling z

under #2, would not see effect

↳ So, under this, you have $O\sim(W,A,Z_A,Y_{A,Z_A})$

Identifiability Conditions

1) $(A,Z)\perp Y_{A,Z}\mid W \quad \forall a,z$ } aka randomization assumption
2) $A\perp Z_a\mid W \quad \forall a$
3) $E(Y_{1z}-Y_{0z}\mid Z_0=z,W) = E[Y_{1z}-Y_{0z}\mid W]$ } aka conditional indep.

ie each counterfactual (A&Z) is indep. of the other.

If conditions 1-3 hold, then

$$E(Y_{1Z_0} - Y_{0Z_0}) = \Psi(P_0) := \sum_w P(w) \sum_z p(z|w,0)\left[E(Y|W,A=1,Z) - E(Y|W,A=0,Z)\right]$$

Note: FYI, $\underbrace{E(Y_1 - Y_0)}_{\text{total effect}} = \underbrace{E(Y_{1Z_1} - Y_{1Z_0})}_{NIE} + \underbrace{E(Y_{1Z_0} - Y_{0Z_0})}_{NDE}$

assumptions $\xrightarrow{A1-A2}$

$$\Psi(P_0) = E_W \sum_z p(Z_0 = z|W)\underbrace{E(Y_{1Z} - Y_{0Z}|W)}$$

not the natural direct effect, but still interpretable

A3 gives us $E(Y_{1Z_1} - Y_{0Z}|Z_0 = z, W)$

Target parameter $(\Psi(P_0))$

$$\Psi(P_0) = \sum_w P(w) \sum_z (p(z|w,0))\left[E(Y|W,1,Z) - E(Y|W,0,Z)\right]$$

$$P(O) = P(W)P(A|W)P(Z|W,A)P(Y|W,A,Z)$$

Notation: Let $Q_W(w) = P(W)$

$Q_Z(Z|W,A) = P(Z|W,A)$

$\bar{Q}_Y(W,A,Z) = E(Y|W,A,Z)$

$g(A|W) = P(A|W)$

$$\Rightarrow \Psi(P_0) = \Psi(Q_0) = E_{QW}\left[E_{QZ}(\bar{Q}_Y(W,1,Z) - \bar{Q}_Y(W,0,Z) \mid W, A=0)\right]$$

Z phases out

Finding the E-IC of $\Psi(P_0)$ under $\mathcal{M}_{NP}$

1) Notice $\Psi(P_0) = \underbrace{E_{QW} E_{QZ}[\bar{Q}_Y(W,1,Z)|W,0]}_{\Psi_1(P_0)} - \underbrace{E_{QW}(E(Y|W,A=0))}_{\Psi_0(P_0)}$

(*) We know the EIC for $\Psi_0(P_0) = \frac{I(A=0)}{g(0|W)}(Y - E(Y|W,0)) + E(Y|W,0) - E_W(E(Y|W$

$\Rightarrow$ Suffices to find EIC for $\Psi_1(P_0)$,

$\Rightarrow$ EIC for $\Psi(P_0) = EIC(\Psi_1) - EIC(\Psi_0)$

Want EIC for $\Psi_1(P_0) = \sum_w P(w) \sum_z p(z|w,0) E(Y|W,A=1,Z)$

↳ Use NPMLE approach:

Use the facts: 1) $\mathcal{M}_{NP}$ has only one gradient, namely the canon. grad.

$\Rightarrow$ Need regular & asymp. linear estimator

↳ Its IC will be the EIC

Note: $Pf = E_P f \Rightarrow$ We know $P_0(X) = E_P[I(X = .)]$

$\Psi(P_n)$ is reg. and asymp. linear

↳ We find the IC for $\Psi_1(P_n)$

Can rewrite $\Psi_1(P) = \sum_w P(I(W=w)) \sum_z z \dfrac{P(I(Z=z, A=0, W=w))}{P(I(W=w, A=0))} \sum_y y \dfrac{P(I(Y=y, W=w, A=1, Z=z))}{P(I(W=w, A=1, Z=z))}$

$$\Rightarrow \Psi_1(P) = \Psi_1(Pf : f \in \mathcal{F}), \quad \mathcal{F} = \left\{ \begin{array}{l} I(W=w) \\ I(z, w, 0) \\ I(w, 0) \\ I(y, w, 1, z) \\ I(w, 1, z) \end{array} \right., \quad \begin{array}{l} w \in W \\ z \in Z \\ y \in Y \end{array}$$

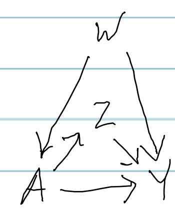
$$\Rightarrow \Psi_1(P_1) - \Psi_1(P_0) \approx \overset{\text{gradient}}{\nabla_{P_0} \Psi} \cdot (P_n - P_0)$$

$$= \left(\frac{\partial \Psi_1}{\partial P_0 f}, f\right) \cdot (P_n f - P_0 f ; f)$$

$$= \sum_f \frac{\partial \Psi_1}{\partial P_0 f} (P_n f - P_0 f)$$

$$= \sum_f \frac{\partial \Psi_1}{\partial P_0 f} \cdot \left(\frac{1}{n} \sum_{i=1}^{n} f(O_i) - P_0 f\right)$$

$$= \frac{1}{n} \sum_{i=1}^{n} \underbrace{\left(\sum_f \frac{\partial \Psi_1}{\partial P_0 f} (f(O_i) - P_0 f)\right)}_{IC(O_i)}$$



9/29/2011 TMLE for Natural Direct Effect (NDE)

$\bar{Q}_Y(A, W, Z) = E(Y \mid W, A, Z)$

$Q_Z(Z \mid W, A) = P(Z \mid W, A)$

$Q_W(W) = P(W)$

$g(A \mid W) = P(A \mid W)$

$$\Psi_Z : (\bar{Q}_Y, Q_Z) \rightarrow \Psi_Z(\bar{Q}_Y, Q_Z) = E_{Q_Z}(\bar{Q}_Y(W, 1, Z) - \bar{Q}_Y(W, 0, Z) \mid W, 0)$$

<u>NDE causal parm</u>

$E(Y_{1, Z_0} - Y_{0, Z_0})$

statistical parm: $\Psi(P_0) = E_{Q_{W_0}}\left[E_{Q_{Z_0}}(\bar{Q}_Y(W, 1, Z) - \bar{Q}_Y(W, 0, Z) \mid W, 0)\right]$

$= E_{Q_{W,0}}[\Psi_Z(Q_0)]$