Stat 201A, Fall 2012
HOMEWORK 9 (due Tuesday 10/9)

1. There are two boxes labeled I and II. There are 5 balls in each box. Of these 10 balls, 4 are white and 6 are blue. At time n, one ball is picked at random from each box, and these two balls are switched (that is, each is moved to the other box). Let X_n be the number of white balls in Box I at time n. Does the process $\{X_n\}$ have a stationary distribution, and if so, do you recognize it? About what fraction of the time do you expect Box I to have no white balls?

2. Let $\{X_n\}$ be a finite state-space, irreducible, aperiodic Markov chain. As usual, let p_{ij} denote the one-step transition probability from i to j, and let π be the stationary distribution of the chain. Let m_{ij} denote the expected time to reach state j given that the chain starts in state i.

a) Show that

$$m_{ij} \;=\; 1 + \sum_{k \neq j} p_{ik} m_{kj}$$

b) Multiply both sides of the equation above by π_i, and then sum (carefully) to show that $m_{jj} = 1/\pi_j$.

3. In what follows, please refer to the notation used in lecture on Wednesday 10/3 and Friday 10/5.

Let $\hat{Y}$ be the best linear predictor of Y based on X_1 and X_2, so that

$$\hat{Y} \;=\; \mu_Y + c_{*1}(X_1 - E(X_1)) + c_{*2}(X_2 - E(X_2))$$

Let $\hat{X}$ be the best linear predictor of X_2 based on X_1. Show that the best linear predictor of Y based on X_1 is

$$\mu_Y + c_{*1}(X_1 - E(X_1)) + c_{*2}(\hat{X} - E(X_2))$$

[One way is to repeatedly use the fact that "best linear predictor" is the same as "error is uncorrelated with each predictor variable." We proved that in class.]