First, note that any $n \in \mathbb{N}$, we have $n = m(m+1)/2 + j$ for some $m \in \mathbb{N}$ and $0 \leq j \leq m$. Thus, for any $n \in \mathbb{N}$, define the interval $A_n = \left[\frac{j}{m+1}, \frac{j+1}{m+1}\right]$ and the sequence of random variables, $\{X_n\}$ on $[0,1]$ by

$$X_n(x) = \begin{cases} m+1 & \text{if } x \in A_n \\ 0 & \text{otherwise} \end{cases}$$

Hence, $X_1(x) = 2 \cdot I_{[0,1/2]}(x)$, $X_2(x) = 2 \cdot I_{[1/2,1]}(x)$, $X_6(x) = 4 \cdot I_{[0,1/4]}(x) \ldots$

Now, we first establish that X_n cannot converge in quadratic mean to 0:

$$\begin{aligned} E(X_n^2) &= \sum_{x \in \text{range}(X_n)} x^2 P(X_n = x) \\ &= (m_n+1)^2 P(X_n = m_n + 1) \\ &= (m_n+1)^2 \cdot \frac{1}{m_n+1} \\ &= m_n + 1 \end{aligned}$$

And clearly $m_n \to \infty$ as $n \to \infty$.

Next, we establish that $X_n \xrightarrow{P} 0$. Let $\epsilon > 0$[1], and find $m \in \mathbb{N}$ such that $\epsilon > 1/(m+1)$. Then, we have that

$$P(X_{m(m+1)/2} \geq \epsilon) = P(X_{m(m+1)/2} = m+1) = \frac{1}{m+1} \xrightarrow{m \to \infty} 0$$

Hence, our "moving mountain" demonstrates a function which converges in probability to 0, but does not converge in quadratic mean to 0. Moreover, it's clear that X_n fails to converge point-wise to the zero function: for any $x \in [0,1]$ the sequence $X_n(x)$ diverges. So, we've actually found a measurable function on the space $[0,1]$ which has converges in measure to the zero function, but fails to converge in any other fashion (point-wise, quadratic mean, or uniform).

[1] and for the sake of an interesting result, suppose $\epsilon \ll 1$