Note: $Pf = E_P f \Rightarrow$ We know $P_S(X) = E_P[I(X = .)]$

$\Psi(P_n)$ is reg. and asymp. linear

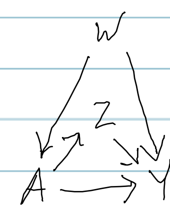
↳ We find the IC for $\Psi_1(P_n)$

Can rewrite $\Psi_1(P) = \sum_w P(I(W=w)) \sum_z z \dfrac{P(I(Z=z, A=0, W=w))}{P(I(W=w, A=0))} \sum_y y \dfrac{P(I(Y=y, W=w, A=1, Z=z))}{P(I(W=w, A=1, Z=z))}$

$$\Rightarrow \Psi_1(P) = \Psi_1(Pf : f \in \mathcal{F}), \quad \mathcal{F} = \left\{ \begin{array}{l} I(W=w) \\ I(z, w, 0) \\ I(w, 0) \\ I(y, w, 1, z) \\ I(w, 1, z) \end{array} \right., \quad \begin{array}{l} w \in W \\ z \in Z \\ y \in Y \end{array}$$

$$\Rightarrow \Psi_1(P_1) - \Psi_1(P_0) \approx \overset{\text{gradient}}{\nabla_{P_0}} \Psi \cdot (P_n - P_0)$$
$$= \left(\frac{\partial \Psi_1}{\partial P_0 f}, f\right) \cdot (P_n f - P_0 f ; f)$$
$$= \sum_f \frac{\partial \Psi_1}{\partial P_0 f} (P_n f - P_0 f)$$
$$= \sum_f \frac{\partial \Psi_1}{\partial P_0 f} \cdot \left(\frac{1}{n} \sum_{i=1}^{n} f(O_i) - P_0 f\right)$$
$$= \frac{1}{n} \sum_{i=1}^{n} \underbrace{\left(\sum_f \frac{\partial \Psi_1}{\partial P_0 f} (f(O_i) - P_0 f) \right)}_{IC(O_i)}$$

9/29/2011 TMLE for Natural Direct Effect (NDE)



$\bar{Q}_Y(A, W, Z) = E(Y \mid W, A, Z)$

$Q_Z(Z \mid W, A) = P(Z \mid W, A)$

$Q_W(W) = P(W)$

$g(A \mid W) = P(A \mid W)$

$$\Psi_Z : (\bar{Q}_Y, Q_Z) \rightarrow \Psi_Z(\bar{Q}_Y, Q_Z) = E_{Q_Z}(\bar{Q}_Y(W, 1, Z) - \bar{Q}_Y(W, 0, Z) \mid W, 0)$$

<u>NDE causal parm</u> ?

$E(Y_{1, Z_0} - Y_{0, Z_0})$

statistical parm: $\Psi(P_0) = E_{Q_{W_0}}[E_{Q_{Z_0}}(\bar{Q}_Y(W, 1, Z) - \bar{Q}_Y(W, 0, Z) \mid W, 0)]$

$= E_{Q_{W,0}}[\Psi_Z(Q_0)]$

EIC for $\Psi(P_0)$:

$$D^*(P) = \left[\frac{I(A=a)}{g(a|W)}\frac{Q_Z(z|W,0)}{Q_Z(z,W,a)} - \frac{I(A=0)}{g(0|W)}\right]\left(Y - \bar{Q}(W,A,Z)\right) \Rightarrow D^*_{Y|A,W,Z}$$

$$+ \frac{I(A=0)}{g(0|W)}\left(\bar{Q}_Y(W,1,Z) - \bar{Q}_Y(W,0,Z) - \Psi_Z(\bar{Q}_Y, Q_Z)\right) \Rightarrow D^*_{Z|A,W}$$

$$+ \Psi_Z(\bar{Q}_Y, Q_Z) - E_{QW}\left(\Psi_Z(\bar{Q}_Y, Q_Z)\right) \Rightarrow D^*_W$$

TMLE for $\Psi(Q)$, $Q = (Q_j, j)$

1) $L_j(Q_j)$

2) $Q_j(\varepsilon, Q, g) \ni \left\langle \frac{d \log Q_j(\varepsilon, Q, g)}{d\varepsilon}\Big|_{\varepsilon=0} \right\rangle \supset \left\langle D^*_j(Q_j, Q, \varepsilon) \right\rangle$

- for $\bar{Q}_Y$:
  1. $L_Y(\bar{Q}_Y) = (Y - \bar{Q}(W,A,Z))^2$
  2. $\bar{Q}^0_Y, g^0$: $\bar{Q}^0_Y(\varepsilon) = \bar{Q}^0_Y + \varepsilon\left(\frac{I(A=a)}{g(a|W)}\frac{Q_Z(z|W,0)}{Q(z,W,a)} - \frac{I(A=0)}{g(0|W)}\right)$
- Given $\bar{Q}_Y, \vec{\Psi}_Z$:
  1. $L_Z(\vec{\Psi}_Z) = I(A=0)\left(\bar{Q}_Y(W,1,Z) - \bar{Q}_Y(W,0,Z) - \vec{\Psi}_Z(\bar{Q}_Y)\right)^2$
  2. $\bar{Q}^0_Y, g^0, \vec{\Psi}^0_Z$: $\vec{\Psi}^0_Z(\bar{Q}^0_Y)(\varepsilon) = \vec{\Psi}^0_Z(\bar{Q}^0_Y) + \varepsilon\frac{1}{g(0|W)}$

Implementation

step 0. $\bar{Q}^0_Y, g^0,$ ~~Q^0_Z or $P(A|W,Z)$~~ $\vec{\Psi}^0_Z : Q \mapsto \Psi^0_Z(Q)$ estimates $\Psi_Z(Q_{Y_0}, Q_{Z_0})$, $Q_Z{}^0$ or $P^0(A|W,Z)$

1. $\varepsilon^0_Y = \arg\min P_n L_Y(\bar{Q}^0_Y(\varepsilon, g^0, Q_Z))$
   $\bar{Q}^*_Y = \bar{Q}^0_Y(\varepsilon^0_Y) \Rightarrow P_n\, Q^*_{Y|A,W,Z}(\bar{Q}^*_Y, Q^*_Z, g^0)$
2. $\varepsilon^0_Z = \arg\min P_n L_Z(\vec{\Psi}^0_Z(\bar{Q}^*_Y)(\varepsilon))$
   func. of W $\vec{\Psi}^*_Z = \vec{\Psi}^0_Z(\bar{Q}^*_Y)(\varepsilon^0_Z) \Rightarrow P_n D^*_Z(Q^*_Y, \vec{\Psi}^*_Z, g^0)$
3. TMLE Ψ^* for $\Psi(P_0)$ is
   $\Psi^* = \frac{1}{n}\sum_{i=1}^{n} \vec{\Psi}^*_Z(W_i) \Rightarrow P_n D^*_W(\Psi^*_Z, \Psi^*) = 0$ solves

* Robustness of EIC $\Rightarrow$ inherits good qualities

$P_0 D^*(Q_Y, Q_Z, \Psi_Z, g; \Psi_0) = 0$ if

1. $\bar{Q}_Y = \bar{Q}_{Y,0}$, $\Psi_Z(Q_{Y,0}) = \Psi_{Z,0}$
2. $g = g_0$, $\bar{Q}_Y = \bar{Q}_{Y,0}$
3. $g = g_0$, $\frac{Q_Z(z|W,0)}{Q_Z(z|W,a)}$ = true
4. $g = g_0$, $\frac{P(0|W,Z)}{P(a|W,Z)}$ = true.