Stat 201A, Fall 2012
HOMEWORK 3 (due Tuesday 9/4)

1. For $1 \le i \le n$, let U_i be i.i.d. uniform on the unit interval. Let $M = \min\{U_1, U_2, \ldots, U_n\}$. Find the distribution, expectation, and SD of M. On the same horizontal axis, sketch (roughly, by hand) the density of M for $n = 10$ and $n = 100$. Give a brief intuitive explanation for how the two densities differ.

[To find the distribution, recall what we said in lecture about finding probabilities associated with extrema. To connect with the text, notice that this problem is 2.14.7 with many bells and whistles attached. The hint for that problem should seem very familiar.]

2. Let X have the Poisson (μ) distribution. Find $SD(X)$ by finishing the calculation that we started in class.

3. There are many close links between the binomial and the Poisson. You've seen one in class. Here's another.

A coin lands heads with probability p. It is tossed N times, where N is Poisson with parameter λ. Let X be the number of heads. Find the distribution of X. Identify this as a well known distribution and find its parameter or parameters.

[Find the possible values, carefully. For each possible value x, partition the event $\{X = x\}$ according to the value of N; and then use the two great rules: multiplication and addition.]

4. A coin lands heads with probability p. If it is tossed n times, where n is a fixed number, then clearly the number of heads and the number of tails are dependent random variables. But as you've seen before, randomizing a parameter can really affect dependence and independence. With that as preamble, do 2.14.11(b).

[Write the event $\{X = x, Y = y\}$ in terms of X and N. The result of Problem 2 will be very useful.]

5. 3.8.7. This is the "tail integral" formula for expectation, analogous to tail sums in the discrete case. The hint is useful because the basic formula for expectation is the integral of a product. Use the fact that f is the derivative of F, and the derivative of x is a really easy function ...

6. For $1 \le i \le n$, let T_i be exponential with rate λ_i, and assume the T_i's are independent.

a) Use the result of the previous problem to find $E(T_1)$.

b) Let $M = \min\{T_i : 1 \le i \le n\}$. Find the distribution (do you recognize it?) and expectation of M.

7. Here you will derive two ways of computing covariance.

a) Prove the first statement in Theorem 3.19, that says covariance is the "mean of the products minus the product of the means." This formula should be treated like the basic formula for expectation, and like "variance is the mean of the square minus the square of the mean." Use it only when the distributions of X, Y, and XY are very simple. Occasionally it is useful in proving some general fact about covariance. But the key to covariance is part **b**.

b) 3.8.14, but just do the easy case $m = n = 2$. The general case is proved by routine algebra that you don't have to do. The result is the most useful computational tool for covariance, and is called "bilinearity of covariance."

8. Covariance has awful units. For example, if X is weight in pounds and Y is height in inches, the units of covariance are "inch pounds." Nobody understands that. So it's a great idea to get rid of the units if possible. Read the definitions 3.18. The first is covariance. The second is correlation, which divides covariance by the two SDs to get rid of those awful units and create a pure number.

By the way, the notation for correlation is not the letter p. It's *rho*, the Greek r.

The goal of this problem is to prove the second statement in Theorem 3.19, that correlation is between -1 and 1. There are many ways of doing this but I'd like you to use the following steps.

a) Let $X^* = (X - \mu_X)/\sigma_X$ be "X in standard units." Show that $E(X^*) = 0$, $SD(X^*) = 1$, and $E(X^{*2}) = 1$.

b) Show that $\rho(X, Y) = E(X^*Y^*)$. There's no real calculation here; just rewrite the definition of correlation in the right way.

c) Both $(X^* + Y^*)^2$ and $(X^* - Y^*)^2$ are non-negative random variables, and hence so are their expectations. Compute the two expectations and apply the results of the previous two parts to show that $-1 \le \rho(X, Y) \le 1$.