

**Stat 201A, Fall 2012**  
**HOMEWORK 6 (due Thursday 9/20)**

All problems below refer to a homogeneous Poisson process with rate  $\lambda$ . As in class,  $N_{(a,b)}$  is the number of arrivals in the time interval  $(a,b)$ , and  $T_r$  is the time till the  $r$ th arrival.

**1.** Fix  $0 < a < b < t$ . Given  $N_{(0,t)} = n$ , find the conditional joint distribution of  $(N_{(0,a)}, N_{(a,b)}, N_{(b,t)})$ . Identify this as a well known distribution and find its parameters.

**2.** Fix integers  $r \leq n$ . Find the conditional density of  $T_r$  given  $N_{(0,1)} = n$ . Identify this as a well known distribution and find its parameters.

[Find  $P(T_r \in dt | N_{(0,1)} = n)$ .]

**3.** Continuing Problem 2: Assume  $n > 1$ . Find the conditional joint density of  $T_1$  and  $T_n$  given  $N_{(0,1)} = n$ . In what way is this joint density related to i.i.d. uniform random variables?

**4.** Fix  $r$  and let  $s \geq 0$  be an integer. Find the correlation between  $T_r$  and  $T_{r+s}$ . Describe the behavior of this correlation as  $s$  gets large.