Stat 201A, Fall 2012
HOMEWORK 7 (due Tuesday 9/25)

1. For $i = 1, 2$ let X_i have the Poisson distribution with parameter λ_i, and let the X_i's be independent.

a) Find the m.g.f. of X_1.

b) Use m.g.f.'s to show that the distribution of $X_1 + X_2$ is Poisson, and find its parameter.

2. The bilateral exponential distribution. Let X have density

$$f_X(x) \;=\; \frac{1}{2}e^{-|x|}, \qquad -\infty < x < \infty$$

Find the m.g.f. of X (careful about where it is defined), and use it to find the even moments of X. The odd moments are of course all 0.

3. Let X have m.g.f. ψ_X.

a) Show that for all x,

$$P(X \geq x) \;\leq\; e^{-tx}\psi_X(t) \quad \text{for all } t \geq 0$$

b) Use **a** to show that if X has gamma (r, λ) distribution then

$$P(X \geq 2r/\lambda) \;\leq\; (2/e)^r$$

Compare this to the bound provided by Markov's inequality applied directly to X.

4. A coin that lands heads with chance 1/4 is tossed repeatedly.

a) Write formulas for the following, and then use R to compare the numerical values; you may leave formulas for normal approximations in terms of the normal c.d.f. Φ.
(i) the chance that it takes more than 410 tosses to get the 100th head
(ii) the normal approximations (with and without continuity correction) to the chance in (i)

b) As in **a**, but now for
(i) the chance that the 100th head appears on the 410th toss
(ii) the normal approximation to the chance in (i)

5. Let $X_1, X_2, \ldots, X_n$ be i.i.d. with mean μ and SD σ. Suppose n is large.

a) Find c_n so that $P(\bar{X}_{(n)} - c_n \leq \mu \leq \bar{X}_{(n)} + c_n)$ is approximately 95%. The observed interval $\bar{X}_{(n)} \pm c_n$ is called "an approximate 95%-confidence interval for μ."

b) Specialize to the case of tossing of a coin that lands heads with probability p. Suppose you don't know p and are trying to estimate it by the interval constructed above. You want a confidence level of at least 95%, and are willing to tolerate an interval that has a total width of 0.01 but not more. About how many times do you have to toss?

6. The Weak Law of Large Numbers. Let $X_1, X_2, \ldots$ be i.i.d. with mean μ. Use moment generating functions to show that as $n \to \infty$, the distribution of $\bar{X}_{(n)}$ converges to point mass at μ.

[Follow the method we used in class for the CLT. It's worth noting that a few weeks ago we had used Chebychev's inequality to prove this result in the case where the random variables had both a mean and a variance. You are now proving a stronger result, because you have a bit more technique than you did earlier.]