check out:

1. sample mean, variance, moment (sample variance is $c\chi^2$)
2. simple random sampling
3. WLLN
4. Chebyshev, Markov, Cauchy-Schwartz

Change of variables: given $Y = g(X)$, and $f_X(x)$ the density of X,

$$f_Y(y) = \sum_{x:g(x)=y} \frac{f_X(g^{-1}(y))}{\left| \left. \frac{dg}{dx} \right|_{x=g^{-1}(y)} \right|}$$

Example: $X \sim N(0,1)$, and $Y = X^2$:

1. $\text{range}(Y) = [0, \infty)$,
2. $y = g(x) = x^2 \Rightarrow g'(x) = 2x$, and beware: $x = \pm\sqrt{y}$.
3. For $x = \sqrt{y}$:
$$\frac{(2\pi)^{-1/2} e^{-(\sqrt{y})^2/2}}{|2\sqrt{y}|} = \frac{e^{-y/2}}{\sqrt{2\pi}(2\sqrt{y})}$$
For $x = -\sqrt{y}$:
$$\frac{(2\pi)^{-1/2} e^{-(-\sqrt{y})^2/2}}{|-2\sqrt{y}|} = \frac{e^{-y/2}}{\sqrt{2\pi}(2\sqrt{y})}$$
Hence,
$$f_Y(y) = \frac{e^{-y/2}}{\sqrt{2\pi}(2\sqrt{y})} + \frac{e^{-y/2}}{\sqrt{2\pi}(2\sqrt{y})} = \frac{y^{-1/2} e^{-y/2}}{\sqrt{2\pi}}$$

Convolution: for $X \perp\!\!\!\perp Y$, if $W = X + Y$, then

$$\begin{aligned} f_W(w) &= \int_{supp(Y)} f_{X,Y}(w-y, y)\, dy \\ &= \int_{supp(X)} f_{X,Y}(x, w-x)\, dx \\ &= \int_{supp(X)} f_X(x) f_Y(w-x)\, dx \\ &= f_X * f_Y(w) \end{aligned}$$

Poisson Process: if $N_{(0,1)}$ = # of arrivals in $(0,1)$, and $N_{(0,1)} \sim$ Poisson(λ), then $N_{(0,t)} \sim$ Poisson(λt). If T_1 is the time until the first arrival, then $T_1 \sim$ Exp(λ). Hence, if T_r is time until r^{th} arrival, $T_r = W_1 + W_2 + \cdots + W_r$ where $W_i \overset{iid}{\sim}$ Exp(λ), hence $T_r \sim \Gamma(r, \lambda)$.

"Thinning" the Poisson Process: Given a Poisson process with rate λ, and supposing each arrival is killed with probability p (independent of the rest of process), if X is the Poisson process for the particles who live and Y is for the particles who die, $X \perp\!\!\!\perp Y$, $X \sim$ Poisson(λq), $Y \sim$ Poisson(λp). Think about this like a random generator spits out particles of type A or B, with the chance of A being p. Then, provided the generic observational random variable is Poisson(λ), the type A observational random variable will be Poisson(λp).

Γ **tricks:** Given $Z \sim N(0,1)$, $Z^2 \sim \chi^2_{(1)} = \Gamma(1/2, 1/2)$. But, $X_1 \perp\!\!\!\perp X_2$, $X_i \sim \Gamma(r_i, \lambda)$ has that $X_1 + X_2 \sim \Gamma(r_1 + r_2, \lambda)$. Hence, for $Z_i \overset{iid}{\sim} N(0,1)$, $\sum_{i=1}^n Z_i^2 \sim \chi^2_{(n)} = \Gamma(n/2, 1/2)$.

Moments:

1. $\mu_k = E(X^k)$ (doesn't always exist)
2. if $j < k$ and μ_k exists, then μ_j exists.

MGF:

1. $\psi_X = E(e^{tX})$ (doesn't always exist)
2. $\psi(0) = 1$
3. $\psi_{aX+b}(t) = e^{tb}\psi_X(at)$
4. If $X \perp\!\!\!\perp Y$, $\psi_{X+Y}(t) = \psi_X(t)\psi_Y(t)$
5. If $\psi_X(t)$ exists in a nhd of 0, $\mu_k = \psi_X^{(k)}(0) < \infty$, for all $k \in \mathbb{N}$. (Inspiration for $E(e^{tx}) = \sum_{k=0}^\infty E(X^k)t^k/k!$.)
6. ψ_X, ψ_Y existing in nhd of 0 and $\psi_X \equiv \psi_Y$ implies $X \sim Y$.
7. If $\{X_n\}$ is a sequence of RV's and $\psi_{X_n} \to \psi_X$ a.e. in a nhd of 0, then $X_n \xrightarrow{\mathcal{L}} X$. That is, $F_{X_n} \to F_X$ at all points of continuity of F_X.

Common MGF's:

X	$\psi_X(t)$
c, constant	e^{ct}
$I_A,\ P(A) = p$	$pe^t + q$
Binom(n, p)	$(pe^t + q)^n$
$N(\mu, \sigma^2)$	$\exp\left\{\mu t + \sigma^2 t/2\right\}$
Exp(λ)	$\lambda(\lambda - t)^{-1} I(t < \lambda)$
$\Gamma(r, \lambda)$	$\lambda^r(\lambda - t)^{-r} I(t < \lambda)$