$$+ \frac{I(\bar{A}=\bar{a})}{g(0)}\Big[Y - \underbrace{E[Y^a / \bar{L}(1), \bar{A}(1)=\bar{a}]}_{\bar{Q}_1^a(L(1))}\Big]$$

So, summary result,

$$D^*(0) = [E[Y^a / L(0)] - \psi] + \frac{I(A=a)}{g_0(0)}\Big[\bar{Q}_1^a(\bar{L}(1)) - E[\bar{Q}_1^a(\bar{L}(1)) / L(0), A(0)=a(0)]\Big]$$

$$+ \frac{I(\bar{A}=\bar{a})}{g(0)}\Big[Y - E[Y^a / \bar{L}(1), \bar{A}(1)=\bar{a}]\Big]$$

Our parm is $\psi(Q) = E_{P^a}[Y^a] = E\big[E[E(Y^a / \bar{L}^a(1))] / L(0)\big]$

Note: $P^a = Q_0^a Q_1^a Q_2^a$

$$= \cancel{I(0)}\, P(L(0)) \cdot P(\bar{L}(1) / L(0), A(0)) \cdot P(\bar{L}(2) / \bar{L}(1), \bar{A}(1)=\bar{a}(1))$$

$$= E\Big[E\big[E[Y / \bar{L}(1), \bar{A}(1)=\bar{a}(1)] \mid L(0), A(0)=a(0)\big]\Big]$$

Algorithm

① $\bar{Q}_2^a = E[Y / \bar{L}(1), \bar{A}(1)=\bar{a}(1)]$

② $\bar{Q}_1^a = E[\bar{Q}_1^a / L(0), A(0)=a(0)]$

③ $\bar{Q}_0^a = E[\bar{Q}^a] \rightarrow \psi(Q) = \bar{Q}_0^a$

$\boxed{\text{* Read his paper!}}$

11/15/2011 Note: skipped last few classes to prepare for GRE

☆ Taught mainly out of slides today

$$\cancel{\&}_1 = L_1(t+1) = L_1(t)\exp(m(t)\Delta t) \Rightarrow S_1(t) = \exp(m_1(t)\Delta t)$$

$$L_2(t+1) = L_2(t)\exp(m(t)\Delta t) \qquad S_2(t) = \exp(m_2(t)\Delta t)$$

$$\Downarrow$$

$$\frac{\log[S_1(t)]}{\log[S_2(t)]} = \beta$$

Read H Bang, J Robins paper

↳ Doubly robust estimation in missing data & causal inference mode

Back to basics: Consider simple WAY data

IPTW

$$\hat{\psi}_{IPTW} = \frac{\hat{E}\left[\left[\frac{I(A=1)}{g(1|W)} - \frac{I(A=0)}{g(0|W)}\right]Y\right]}{\hat{E}\left[\frac{I(A=1)}{g(1|W)} - \frac{I(A=0)}{g(0|W)}\right]}$$

TMLE

$$\hat{\psi}_{TMLE} = \Psi(\bar{Q}_n^*, Q_{W,n}) = \frac{1}{n}\sum_{i=1}^{n}\left[\bar{Q}_n^*(1,W_i) - \bar{Q}_n^*(0,W_i)\right]$$

To beat an estimator:

consider $\underbrace{\hat{\psi}^0}_{\text{estimator we want to beat}} - \psi^0 = \frac{1}{n}\sum_{i=1}^{n} \underbrace{D^0(P_0)(O_i)}_{\text{IC of } \hat{\psi}^0}$

We construct $Q_n^*, g_n \ni P_n D^*(Q_n^*, g_n) = 0$

$P_n D^0(Q_n^*, g_n) = 0$ ← solving this eqn verifies that we at least match the estimator we want to beat.

In previous paper, updated est. by

$$\hat{P}_\varepsilon(A=1|W) = \frac{1}{1+\exp\{\text{logit } \hat{g}^0(1|W) + \varepsilon_2\left[\frac{\bar{Q}(1,W)}{g(1,W)} + \frac{\bar{Q}(0,W)}{g(0,W)}\right]\}}$$

We solve score eqn: $P_n \underbrace{\left(\frac{\bar{Q}_n^*(1,W)}{g_n^*(1,W)} + \frac{\bar{Q}_n^*(0,W)}{g_n^*(0,W)}\right)(A_i - g_n^*(1|W))}_{D^*_{CAR}(\bar{Q}_n^*, g_n^*)} = 0$

Note: Every IC can be written as $D^* = D^*_{IPTW} + D^*_{CAR}$

$P_n D^*(Q_n^*, g_n^*) = 0$

$P_n D_{CAR}(Q_n^*, g_n) = 0$ $\Rightarrow P_n D_{IPTW}(Q_n^*, g_n^*) = 0 \Rightarrow \Psi(Q_n^*) = P_n\left(\frac{I(A=1)}{g_n^*(1|W)} - \frac{I(A=0)}{g_n^*(0|W)}\right)Y$

So! The TMLE is also an IPTW estimator!
(paper from 2006)

∴ For any estimator, can find IC and apply TMLE to gaurantee that we beat original estimator