An efficient Q-RTM algorithm based on local differentiation operators

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Summary

Under visco-acoustic overburdens such as gas cloud or gas chimney, the quality of images produced by conventional reverse time migration (RTM) can be severely degraded due to the amplitude reduction, phase distortion, traveltime error, and frequency content loss caused by attenuation. In this work, we developed a novel Q-RTM algorithm to efficiently compensate the amplitude loss and correct the phase distortion when finite Q is in presence. Unlike the currently available Q-RTM algorithms based on global operators (pseudo-spectral methods), our Q-RTM approach employs a local differentiation operator (finite-difference) to significantly accelerate the Q compensation computation during wave propagation. Although finite-difference is the most commonly used algorithm for conventional RTM methods, there is no existing finite-difference-based Q-RTM algorithm due to two main challenges: stability issue and dispersion relation compensation using local differentiation operators. We overcome these difficulties by introducing two unique techniques: 1) the negative-τ method for the system stabilization; 2) a multi-stage dispersion optimization algorithm for the phase correction. The numerical examples demonstrate that our Q-RTM algorithm accurately compensates both the phase and the amplitude, while substantially reducing the computational cost, especially for large scale 3D projects.

Introduction

It is well known that seismic images produced by conventional migration algorithms lose structural information below lossy overburdens, especially for highly attenuating media (Traynin et al., 2008, Hu et al., 2011; Zhu et al., 2014). This image information loss due to attenuation includes amplitude decay, phase distortion, depth shift, and lower image resolution. These seismic attenuation effects have severe negative impacts on the interpretation of subsurface structures and they need to be taken into account for rock property characterization and proper AVO analysis. Therefore, it is critical to compensate the attenuation effects in migration images.

Inverse Q filtering technology was widely used in early days (Hargreaves and Calvert, 1991). However, because inverse Q filtering is essentially a space-variant or time-variant filter applied on data prior to migration, this approach is inappropriate for long offsets due to event separation difficulty and it is inaccurate for strong Q heterogeneity scenarios. A prestack Kirchhoff Q-migration method compensates attenuation effects in a more accurate way by honoring the Q absorption effect along the 3D raypath for each trace to be migrated, thus improving the AVO attributes greatly (Traynin et al., 2008). Because one-way wave equation is implemented in frequency domain, the Q compensation effect can easily be incorporated during migration (Dai and West, 1994; Yu et al., 2002).

Reverse time migration (RTM) is believed to have advantages over Kirchhoff migration and one-way wave equation migration for geologically complex structures. Unfortunately, although RTM has become a standard seismic processing technology, there is not much research effort put into Q-compensated RTM. The main reason is that there is no practical time-domain wave propagation modeling algorithm available for Q compensation (Zhu et al., 2014) although the wave propagation algorithm for Q attenuation was successfully developed two decades ago (Blanch et al., 1995). Recently, several Q-RTM algorithms were reported to handle complex geology scenarios (Zhang et al., 2010; Suh et al., 2012; Zhu et al., 2014). These algorithms compensate the Q absorption effects properly and produce excellent migration images, but they are all based on a pseudo-spectral method. As we know, pseudo-spectral methods use a global operator, which is computationally expensive and is not suitable for parallelization. As a result, these algorithms are not widely employed for production in the oil/gas industry.

In this work, we developed a novel Q-RTM algorithm whose propagation engine for Q compensation is based on a local differentiation operator. Therefore, this method is very efficient and this Q-RTM can be easily developed by modifying the existing conventional finite-difference-based RTM codes by adding an amplitude compensation term and several phase correction terms. The main challenge of developing a finite-difference-based Q-compensated wave propagation engine resides in the fact that it is extremely difficult to compensate amplitude and phase simultaneously while maintaining the system stability. To mitigate this difficulty, we proposed two unique techniques. One is the so-called negative-τ method to compensate amplitude without sacrificing stability. The other technique is the multi-stage dispersion optimization scheme for phase distortion recovery. Numerical examples show that the combination of these two techniques can compensate both the amplitude and the phase distortion accurately and efficiently.

Imaging Condition of RTM and Q-RTM

The standard imaging condition of an RTM algorithm without considering attenuation is the zero-lag cross-correlation:

$$I(\mathbf{r}) = \int_0^T S(\mathbf{r},t)R(\mathbf{r},t)dt = \int_0^T \left[W(\mathbf{r},t) * L^S_{PHA}(v)\right]\left[\overline{D}(\mathbf{r},t) * L^R_{PHA}(v)\right]dt, \quad (1)$$

where $\mathbf{r}$ is a subsurface imaging point, W is the source wavelet, $\overline{D}$ is the time-reversed data, $S(\mathbf{r},t)$ is the source-side forward-propagated wavefield and $R(\mathbf{r},t)$ is the receiver-side back-propagated wavefield. L^S_{PHA} and L^R_{PHA} are the phase accumulation effect due to the source-side propagation and the receiver-side back-propagation. T is the length of the recorded data. If the subsurface medium is visco-acoustic, the recorded data are attenuated and the phase of the data is distorted due to the frequency-dependent phase velocity brought by Q. The attenuated data is related to the regular data by

$$D^A(\mathbf{r},t) = D(\mathbf{r},t) * L_{PHD}(v,Q) * L_A(v,Q), \quad (2)$$

where D and D^A are the recorded data in acoustic and visco-acoustic medium respectively, L_{PHD} is the phase distortion introduced by Q, and L_A represents the amplitude reduction due to the attenuation. The time-reversed data is

$$\overline{D}^A(\mathbf{r},t) = D^A(\mathbf{r},T-t) = \overline{D}(\mathbf{r},t) * L_{PHD}^{-1}(v,Q) * L_A(v,Q). \quad (3)$$

Because our goal is to design an imaging condition for attenuated data to obtain the same image point value as (1), apparently, we have the compensated imaging condition as

$$\begin{aligned} I^C(\mathbf{r}) &= \int_0^T \left(W * L^S_{PHA}\right)\left(\overline{D}^A * L_{PHD} * L_A^{-1} * L^R_{PHA}\right)dt \\ &= \int_0^T \left(W * L^S_{PHA} * L^S_{PHD} * L_A^{S^{-1}}\right)\left(\overline{D}^A * L^R_{PHA} * L^R_{PHD} * L_A^{R^{-1}}\right)dt \\ &= \int_0^T \left(W * L^S_{PH} * L_A^{S^{-1}}\right)\left(\overline{D}^A * L^R_{PH} * L_A^{R^{-1}}\right)dt \end{aligned} \quad (4)$$

According to (4), in order to properly compensate the attenuation effect during Q-RTM, we need to boost the amplitude (i.e., reverse the attenuation process L_A) during the propagation while keeping the dispersion relation (i.e., phase change) same as the attenuation process (i.e., L_{PH}).

It is straightforward to simultaneously apply L_A^{-1} and L_{PH} on the wavefield propagation using pseudo-spectral methods. Zhu et al. (2014) gave a convenient equation

$$\frac{1}{{v_0}^2}\frac{\partial^2 p}{\partial t^2} = \nabla^2 p + \beta_1\left(\eta\mathbf{L} - \nabla^2\right)p + \beta_2 \tau \frac{\partial}{\partial t}\mathbf{H}p, \quad (5)$$

where p is the pressure, v_0 is the P-wave velocity at the reference frequency ω_0, $\mathbf{L} = \left(-\nabla^2\right)^{\gamma+1}$, $\mathbf{H} = \left(-\nabla^2\right)^{\gamma+1/2}$, $\gamma = 1/\pi \tan^{-1}\left(1/Q\right)$, $\tau = -{v_0}^{2\gamma-1}{\omega_0}^{-2\gamma}\sin(\pi\gamma)$, $\eta = -{v_0}^{2\gamma}{\omega_0}^{-2\gamma}\cos(\pi\gamma)$. With this equation, if β_1=1 and β_2=1, equation (5) describes an attenuation process. On the other hand, if β_1=1 and β_2=-1, then the equation describes a compensation process because the amplitude loss is reversed while the dispersion relation is same as the Q attenuation process. However, pseudo-spectral methods are not computationally efficient for large scale problems. Furthermore, pseudo-spectral methods are not suitable for domain decomposition and parallelization. Therefore, a method based on finite-difference or other local differentiation operators is more preferable for Q-RTM. Unfortunately, local differentiation operator based methods cannot be applied directly to equation (5) because of the fractional spatial derivative operator in $\mathbf{H}$ and $\mathbf{L}$, which are natively global operators.

Negative-τ Method

In order to develop a finite-difference-based Q compensation wave propagation engine, first we investigate the finite-difference wave propagation algorithm for Q attenuation – the τ method (Blanch et al., 1995):

$$\frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho}\nabla p \quad (6)$$

$$\frac{\partial p}{\partial t} = -\lambda\left(1 + n\tau\right)\nabla\cdot\mathbf{v} - \sum_{i=1}^{n} r_i + Q \quad (7)$$

$$\frac{\partial r_i}{\partial t} = -\frac{1}{\tau_{\sigma i}} r_i - \frac{\lambda\tau}{\tau_{\sigma i}}\nabla\cdot\mathbf{v}, \quad (8)$$

where $\mathbf{v}$ is the particle velocity, p is the pressure, λ is the bulk modulus, ρ is the mass density, r is the memory variable, Q is the source term, τ and τ_σ are optimized parameters to approximate a constant Q over a specific frequency range. In equation (6), there is no fractional derivative involved because the fractional derivative terms are approximated by the summation of the memory variables. Consequently, it is straightforward to apply finite-difference method on equation (6) for wave propagation with Q attenuation. To investigate the dispersion relation and the dissipation property of equation (6), we derived the complex velocity from (6) as follows:

$$v_c = \sqrt{\frac{\lambda}{\rho}}\sqrt{\left(1 + n\tau\right) - \sum_{i=1}^{n}\frac{\tau}{\omega^2{\tau_{\sigma i}}^2 + 1} + j\sum_{i=1}^{n}\frac{\omega\tau\tau_{\sigma i}}{\omega^2{\tau_{\sigma i}}^2 + 1}} \quad (9)$$

The real part of v_c represents the phase velocity of the attenuation system (i.e., the dispersion relation) and the imaginary part represents the amplitude reduction effect. The ratio of the real part of v_c^2 to the imaginary part is defined as Q, which is positive for attenuation. In Q-RTM, we need to design a negative Q for compensation purpose.

Observing formulation (9), we notice that if we replace τ_σ by - τ_σ, (8) becomes

$$\frac{\partial r_i}{\partial t} = \frac{1}{\tau_{\sigma i}} r_i + \frac{\lambda\tau}{\tau_{\sigma i}}\nabla\cdot\mathbf{v}. \quad (10)$$

Then the phase velocity remains the same but the sign of the imaginary part of v_c is flipped, which means, by doing this, the phase velocity or the dispersion relation does not change but the positive Q (attenuation effect) is replaced by a negative Q (compensation effect). Furthermore, the absolute value of Q does not change. This procedure seems to be a perfect strategy for Q compensation. Unfortunately, according to the control system theory, the system consisting of (6), (7), and (10) is unconditionally unstable due to the poles on the right half *S*-plane.

Instead of modifying τ_σ, we propose an alternative approach to design an engine for Q compensation, the so-called negative-τ method. In the negative-τ method, the parameter τ is replaced by –τ and (8) becomes

$$\frac{\partial r_i}{\partial t} = -\frac{1}{\tau_{\sigma i}} r_i - \frac{\lambda \tau}{\tau_{\sigma i}} \nabla \cdot \mathbf{v} \,. \tag{11}$$

The negative-τ method has no stability issue because the right *S*-plane poles are eliminated. The resulting absolute Q value of the negative-τ method is slightly larger than the specified Q values, as shown in Figure 1(a). Actually, this inaccuracy in Q values is not the main disadvantage of the negative-τ method. The main limitation of this method is that the phase velocity becomes

$$v_{ph} = \sqrt{\frac{\lambda}{\rho}} \sqrt{(1+n\tau) + \sum_{i=1}^{n} \frac{\tau}{\omega^2 \tau_{\sigma i}{}^2 + 1}} \,, \tag{12}$$

plotted in Figure 1(b) (red curves) in comparison with the ideal velocity for Q compensation (blue curves).

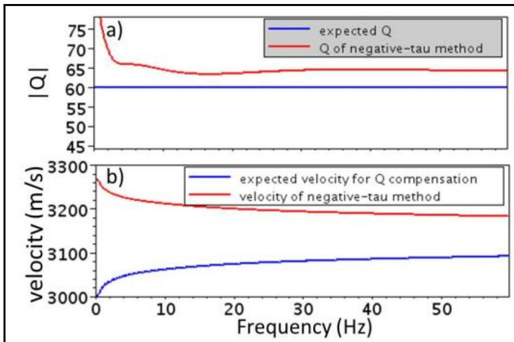

Figure 1: Dispersion relation of negative-τ method: a) absolute Q value; b) phase velocity.

Based on the analysis shown in Figure 1, this deviation of the phase velocity from the ideal phase velocity is unacceptable. With the negative-τ method, the already distorted phase of the data will be distorted further instead of being corrected during the Q compensation process, leading to totally wrong phases in the Q-RTM images.

Optimized Negative-τ Method

To overcome this drawback of the negative-τ method, we propose a multi-stage dispersion optimization algorithm to improve the dispersion relation curve of the negative-τ method. The idea of the multi-stage dispersion optimization algorithm is to modify the real part of the complex velocity of the Q compensation equation system through polynomial approximation, intending to design a frequency-dependent phase velocity profile to match the ideal phase velocity for Q compensation (the blue curve in Figure 1(b)).

First, we convert the original negative-τ method, a first-order partial differential equation system (PDE), to a second order PDE system as follows:

$$\frac{\partial^2 p}{\partial t^2} = \frac{\lambda}{\rho}(1+n\tau)\nabla^2 p - \sum_{i=1}^{n} s_i + \frac{\partial Q}{\partial t} \tag{13}$$

$$\frac{\partial s_i}{\partial t} = -\frac{1}{\tau_{\sigma i}} s_i - \frac{\lambda \tau}{\rho \tau_{\sigma i}} \nabla^2 p \,, \tag{14}$$

which is equivalent to the equation system (7),(8), and (11). Then, we introduce several dispersion optimization terms in (13) to improve the dispersion relation curve of the system:

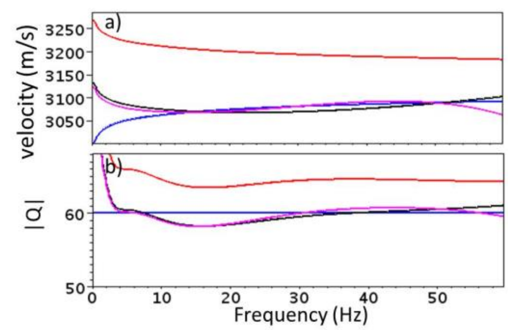

Figure 2: Dispersion relation of optimized negative-τ method. blue: ideal case; red: negative-τ method; black: 2-stage optimized; magenda: 3-stage optimized. a) phase velocity; b) absolute Q.

$$\frac{\partial^2 p}{\partial t^2} = \frac{\lambda}{\rho}(1+n\tau)\nabla^2 p - \left(\alpha - \beta \frac{\partial^2}{\partial t^2} - \gamma \frac{\partial^4}{\partial t^4}\right)\nabla^2 p \\ - \sum_{i=1}^{n} s_i + \frac{\partial Q}{\partial t} \,. \tag{15}$$

For ease of implementation, (15) can be further modified to

$$\frac{\partial^2 p}{\partial t^2} = \frac{\lambda}{\rho}(1+n\tau)\nabla^2 (p + p^c) - \sum_{i=1}^{n} s_i + \frac{\partial Q}{\partial t} \tag{16}$$

$$p^c \approx -\alpha p + \beta \frac{\lambda}{\rho}\nabla^2 p + \gamma \frac{\lambda}{\rho}\nabla^2\left(\frac{\lambda}{\rho}\nabla^2 p\right). \tag{17}$$

The right-hand-side of (17) consists of 3 terms, thus we call this scheme as 3-stage dispersion optimization. Dispersion optimization parameters, α, β, γ, are obtained through a polynomial fitting algorithm and they are only dependent on the Q values. In practice, we can also neglect the third term, which results in a 2-stage dispersion optimization scheme. More stages of optimization give better dispersion relation performance but also imply higher computational cost and may cause a stability problem. The phase velocity and the corresponding Q for compensation are plotted in Figure 2. Apparently, both phase velocity and Q values are more accurate than the original negative-τ method.

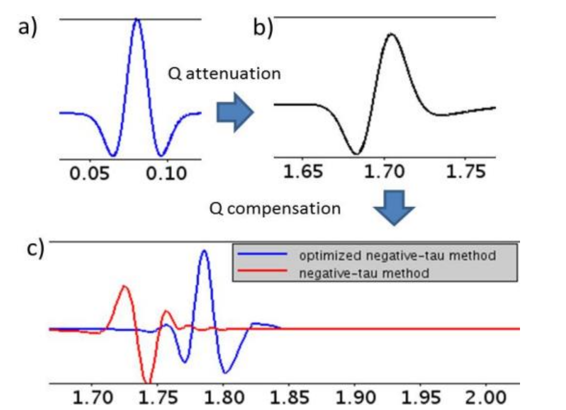

Figure 3: Numerical simulation of Q attenuation and Q compensation. a) Ricker source wavelet; b) recorded attenuated data; c) Q compensation result: blue – optimized negative-τ method; red – negative-τ method.

A simple numerical experiment is conducted to verify the Q compensation effect. As shown in Figure 3, first, we propagate a Ricker wavelet (Figure 3(a)) with attenuation to obtain the attenuated data (Figure 3(b)). Then, we reverse the data in time domain and back-propagate the time-reversed data to the source location with Q compensation to obtain the compensated data (Figure 3(c)). The results in Figure 3 are consistent with the theoretical analysis presented in Figure 2. As shown in Figure 3, the Q-compensated data using the optimized negative-τ method is very close to the expected ideal wavelet – a Ricker wavelet. However, the original negative-τ method produces a severely phase-distorted wavelet.

Numerical Results

To demonstrate the performance of this new Q-RTM algorithm based on the optimized negative-τ method, we apply it on the Sigsbee model. The 1/Q model is shown in Figure 4(a), where the Q value in the two Q anomalies is approximately 60. The conventional RTM using unattenuated data (Figure 4(b)) is regarded as the reference solution. As expected, the conventional RTM with attenuated data (Figure 4(c)) is unable to provide clear structrual information and it cannot resolve those scattering points under the Q bodies. Both the optimized negative-τ Q-RTM (Figure 4(e)) and the pseudo-spectral Q-RTM (Figure 4(d)) using attenuated data produce high quality images almost identical to the reference solution shown in Figure 4(b), which is our ultimate goal of Q compensation. This numerical experiment validated that this new Q-RTM algorithm based on finite-difference operators has the capability of compensating both the amplitude and the phase accurately with substantially reduced computational cost in comparison with the pseudo-spectral-based Q-RTM.

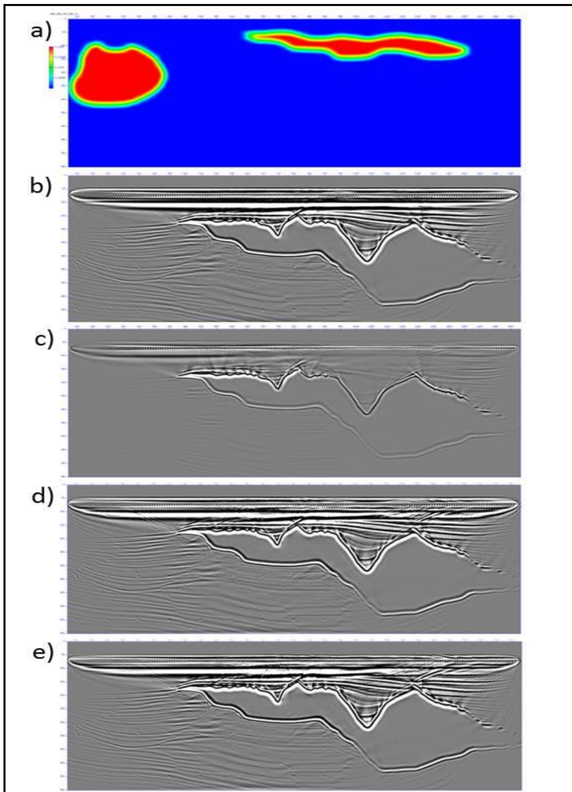

Figure 4: a) 1/Q model; b) conventional RTM with unattenuated data; c) conventional RTM with attenuated data; d) pseudo-spectral Q-RTM; e) optimized negative-τ Q-RTM.

Conclusions

For the first time, a Q-RTM algorithm based on local differentiation operators was developed to improve the imaging quality when there are visco-acoustic overburdens in presence. Because this Q-RTM algorithm is designed for finite-difference schemes, it is more efficient and more suitable for parallelization and GPU implementation than the currently existing pseudo-spectral-based Q-RTM algorithm, especially for large scale 3D production. The main challenge in developing a finite-different-based Q compensation engine is to compensate the wave amplitude while maintaining the dispersion relation same as the attenuation process. We overcame this difficulty by introducing two novel approaches – the negative-τ method and the multi-stage dispersion optimization algorithm. The former realizes amplitude compensation without triggering any instability issue and the latter corrects the phase distortion. The numerical examples validated that this Q-RTM algorithm compensates both the amplitude and the phase distortion accurately and produces high quality images almost identical to the pseudo-spectral Q-RTM while saving the computational cost significantly.

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