Lean Six Sigma Green Belt Certification Course

simplilearn

Statistical Process Control

Learning Objectives

By the end of this lesson, you will be able to:

- Explain the basics of Statistical Process Control
- Choose appropriate traditional control charts
- Differentiate between CUSUM and EWMA Charts

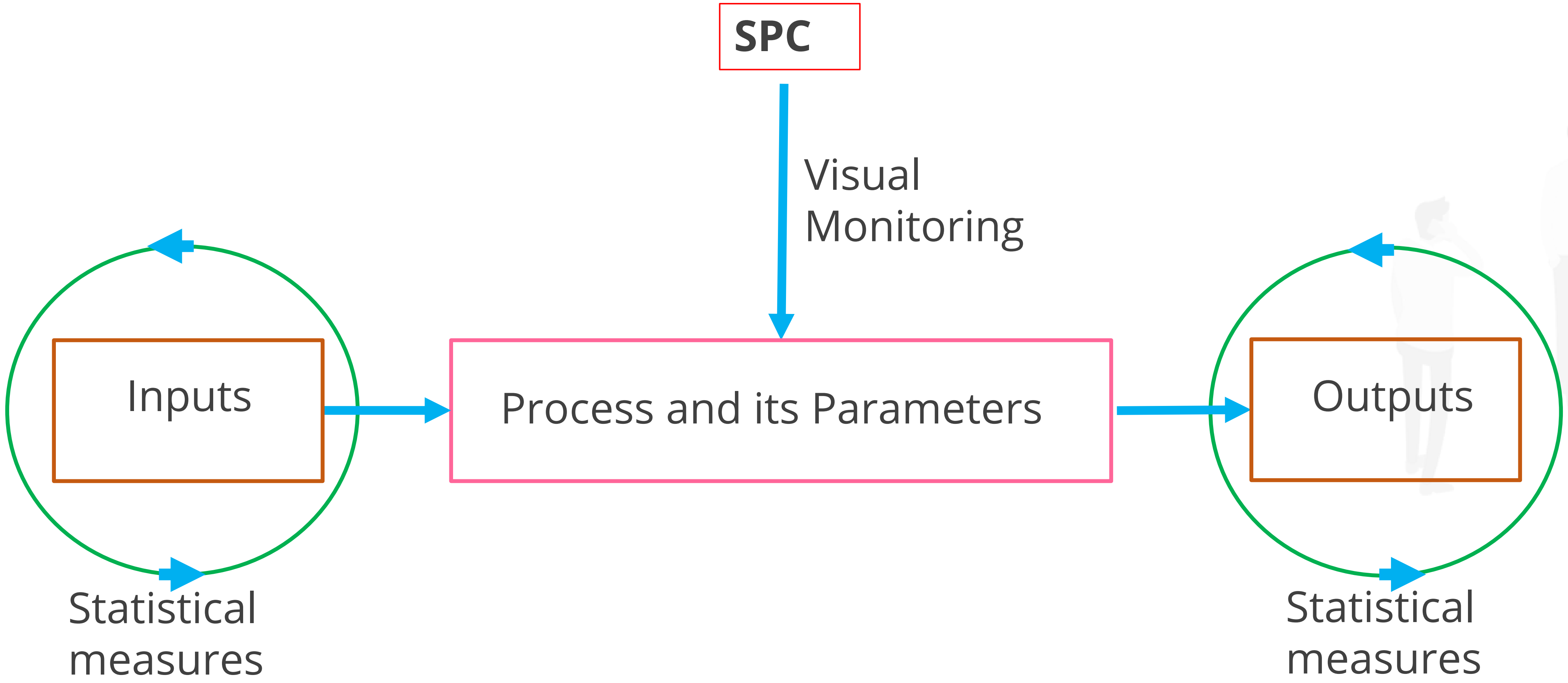
DIGITAL OPERATIONS

Statistical Process Control (SPC) Basics

simplilearn

Statistical Process Control Basics

Walter A Shewhart developed statistical process control in 1924.



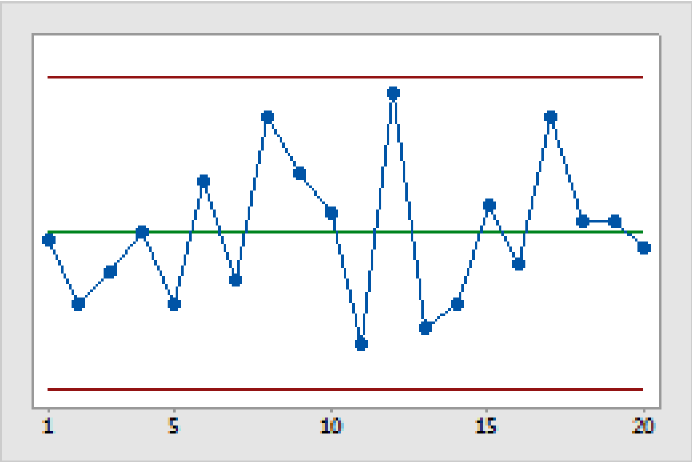
Benefits of SPC Basics

SPC

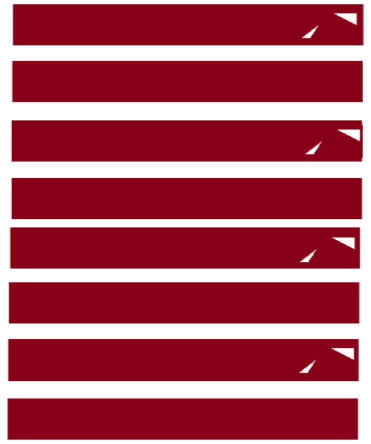
- separates → Special causes of variability
- separates → Common causes of variability
- recognizes → Unexpected changes in the process output
- identifies stable zones → Variables with unknown specification limits
- useful external information → Continuous process improvement and online monitoring

The concept of Process Control can be used in the Measure Phase, when we check for data stability.

Features of Common Cause Variation



- Small variations in raw materials
- Variations due to manual interventions in manual processes
- Responses from machines or systems, etc.



Aware of common cause variation

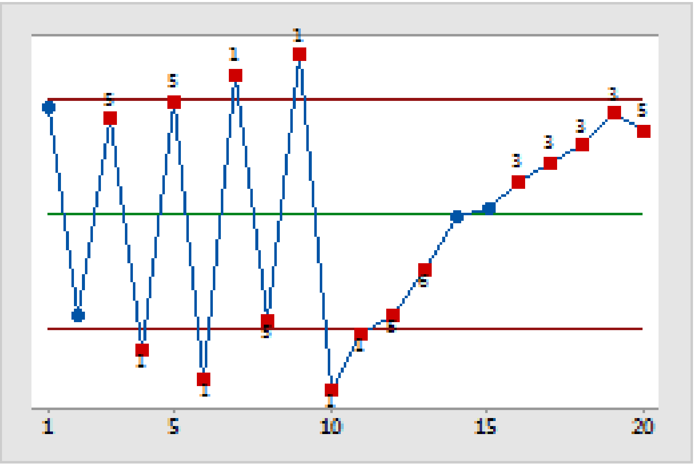
Part of the Process

Frequency → Predictability

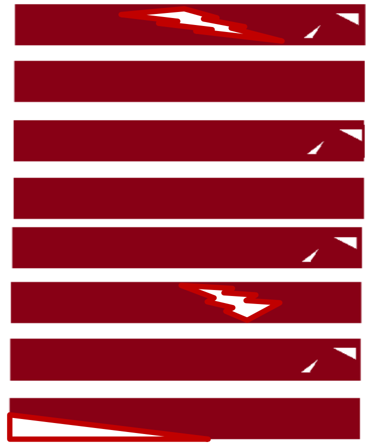
Repetitive → Huge investment and time to identify and remove them

Within tolerance or specification limits

Statistical Process Control Basics



- Machine or system crash
- Delay in supply of raw materials
- Huge variations in the raw materials, etc.



Frequency → Predictability

Part of the Process

Non-repetitive → Less investment and time to identify and remove them

Variation affects the flow of the process due to which the defects appear

Statistical Process Control Basics

Subgroup or Sample → A collection of units that are produced or created under the same set of conditions

Rational Subgroup
Represents the process at a particular point in time
Measurements must be taken within a short span of time but should also be independent of each other

Statistical Process Control Basics

Approach 1

Sample → Every other group of 10 consecutive units

- Used to detect process shifts
- A process shift happens over time and is reflected in the variation of the output variables

Example:

Process → 10 consecutive units

↓ sample

every other group

Approach 2

Sample → Random for all process output over the sampling interval

- Used to make decisions about the acceptance of a product
- Detecting the shifts of the output variable to an out of control state and back into an in-control state between samples

Example:

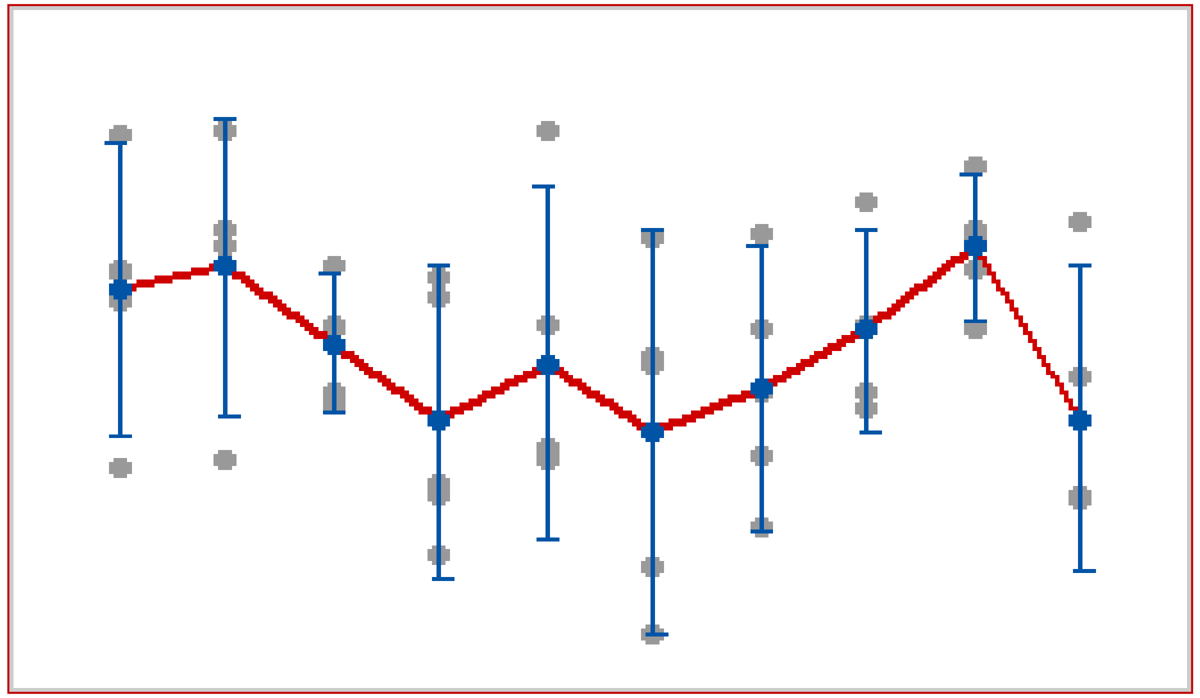
Process → 100 parts per hour

↓ sample

5 randomly selected parts every hour

Statistical Process Control Basics

Rational subgrouping refers to the selection of subgroups or samples in a way that if special causes are present, chance for differences between subgroups will be maximized and chance for differences due to special causes within a subgroup will be minimized.



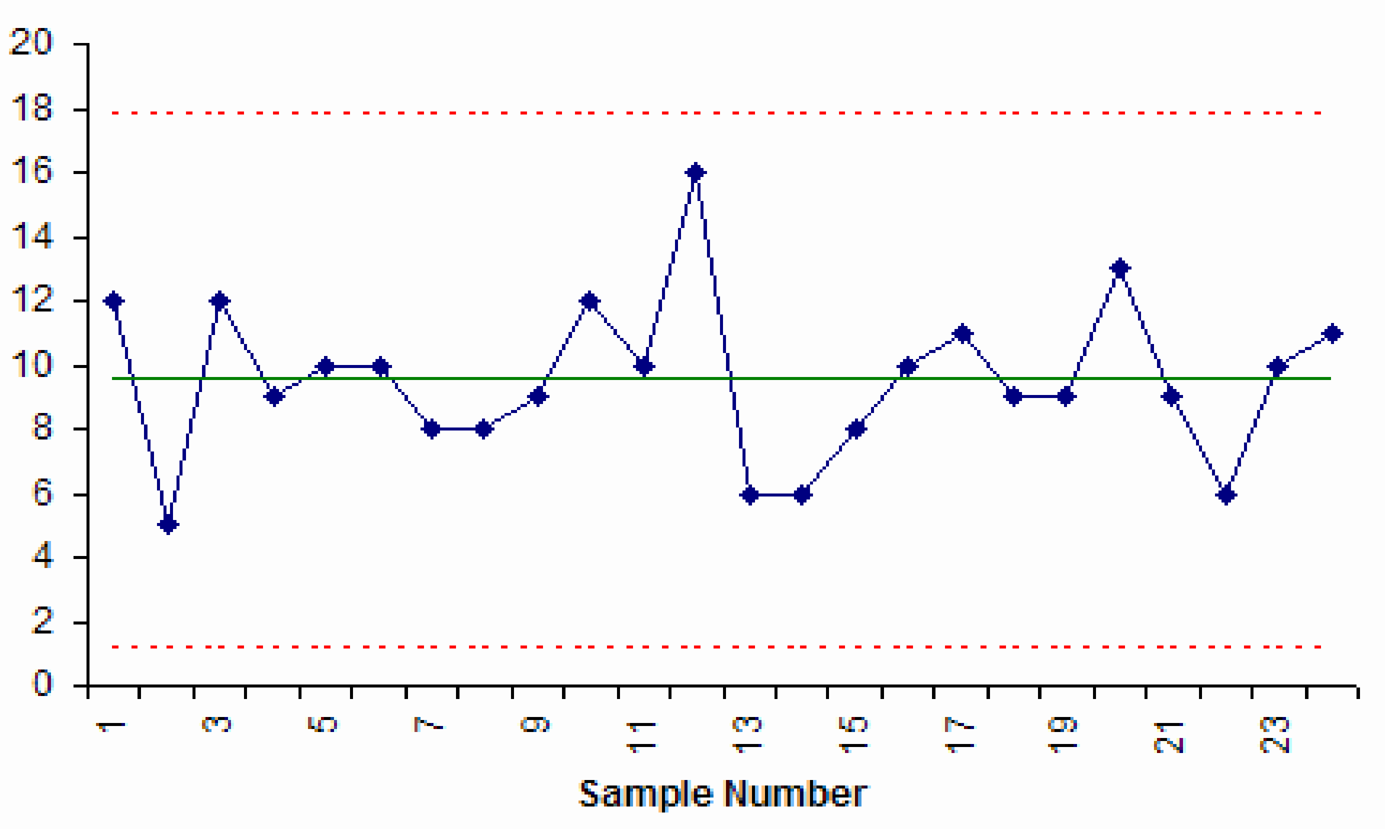
The interval bars represent the within-subgroup variation

The red line that connects the averages of the consecutive subgroups represents the between-subgroup variation

http://support.minitab.com/en-us/minitab/17/topic-library/quality-tools/control-charts/data/within-subgroup-and-between-subgroup-variation/

Statistical Process Control Basics

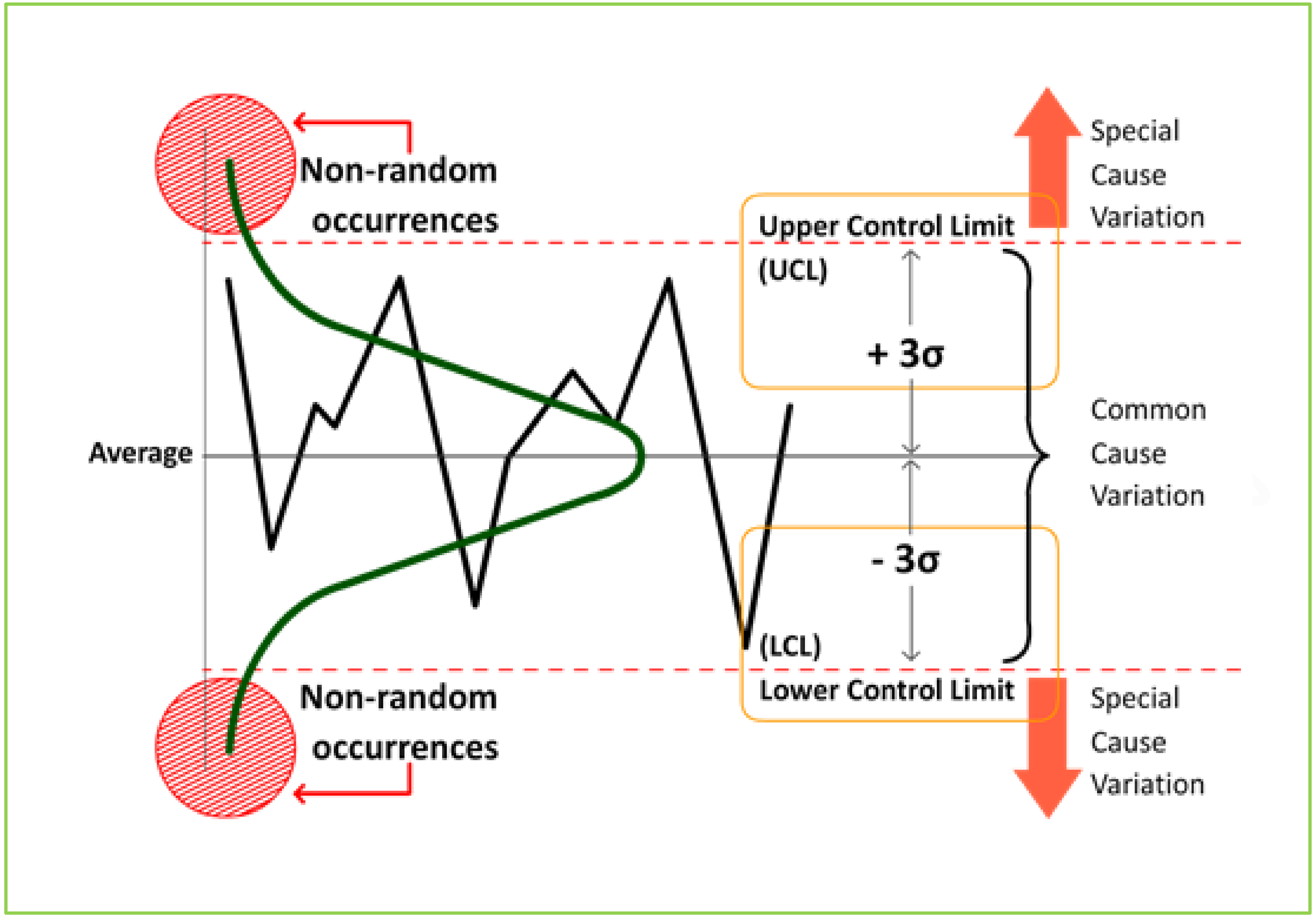
Walter Shewhart, 1920s



Characteristics of control charts are as follows:

- Similar to run or trend charts, with an addition of a control limit line and an average or center line
- Can be used with discrete or continuous data
- Control limits (UCL and LCL) are typically set at approximately three standard deviations from the center line
- Specification limits (USL and LSL) normally do not appear on them

Statistical Process Control Basics



simplilearn

Statistical Process Control Basics

Control Charts → Tracking Process Statistics over time

Control Charts → Detecting the presence of Special Causes

Process is in control → Most of the points fall within the bounds of the control limits

Process is in control → Points do not display any non-random patterns

A process is in control when most of the points fall within the bounds of the control limits and the points do not display any non-random patterns.

Statistical Process Control Basics

Standard Control Chart

Uses control limits

σ_{mean} from the data's grand average ($\overline{\overline{X}}$, average of the sample averages, or μ).

The probability of an out-of-control point when the process has not changed → 0.27%

2σ ↑ the chance of type I or alpha error

4σ ↑ the chance of type II or beta error

Walter Shewhart had set 3σ limits on control charts with the belief that when the process goes beyond these limits, it needs correction.

Statistical Process Control Basics

An Out-Of-Control (OOC) condition is indicated if one of the following is true:

1 point is outside the Control Limits (either above UCL or below LCL)

p(f) = 0.27%

8 consecutive points are above the Center Line (CL) or consecutively below the CL

p(f) = $(0.5)^8$ = 0.39%

2 out of 3 points are within 1 σ_{mean} of either the UCL or the LCL

p(f) = $\frac{3!}{(2!1!)(0.023)^2(0.477)}$ = 0.08% for one side

6 to 8 points are consecutively increasing or decreasing

p(f) = $(0.5)^6$ or $(0.5)^8$ = 1.6% to 0.39%

Tips

Identify the purpose for data collection and try to determine what kind of data may be needed for measurement

Identify measures that are used daily

Some Tips

If you don’t adapt data to filter any noise factors from the process, the control chart will show you wrong results

DIGITAL OPERATIONS

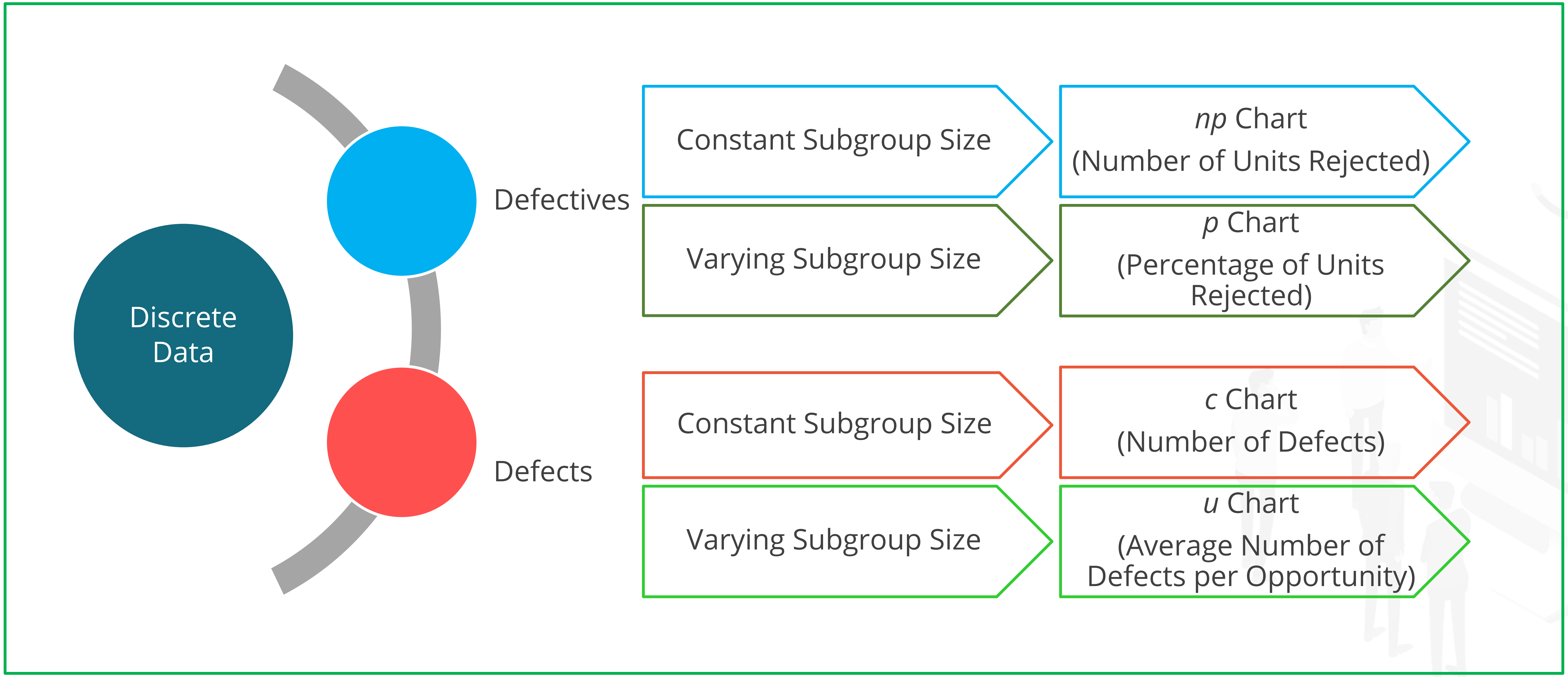
Traditional Control Charts

Choosing an Appropriate Control Chart

Continuous data sampling

- Individual Data Points (Pulling one sample at fixed frequency)
  - I-MR Chart
    (Depicts the variability of individual characteristics over time)
- Subgroups (Taking periodic group data)
  - $\bar{X}$ and *R* Chart
    (If n is between 2 and 9)
  - $\bar{X}$ and *s* Chart
    (When standard deviation is calculated and $n \geq 10$)

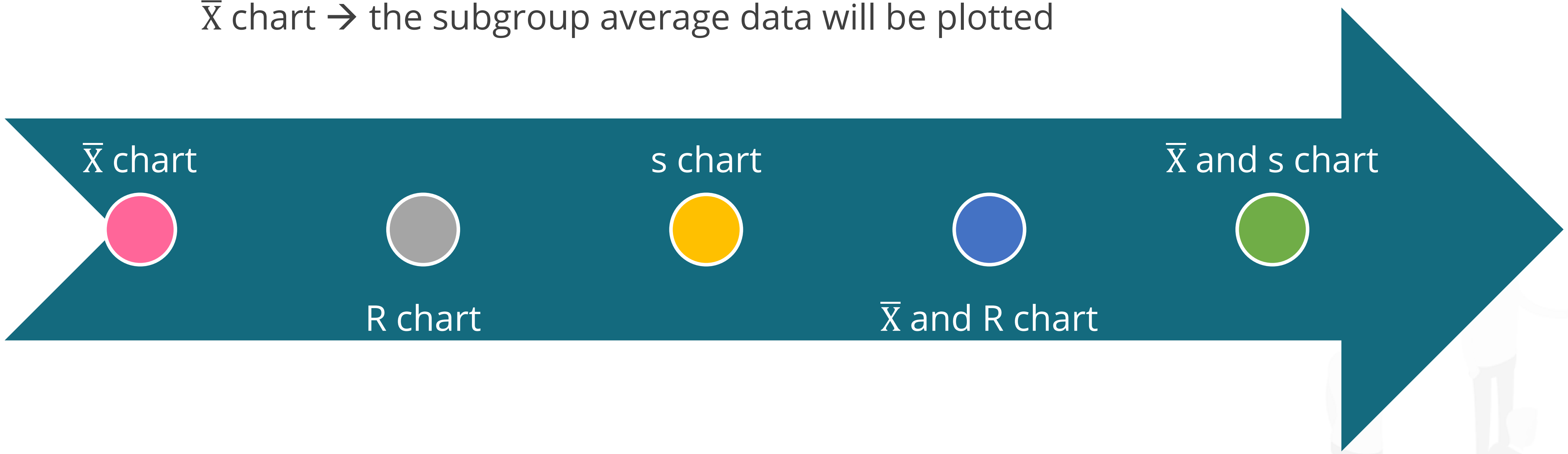
Choosing an Appropriate Control Chart



$\bar{X}$ Chart Principles

$\bar{X}$ → average of each subgroup of data

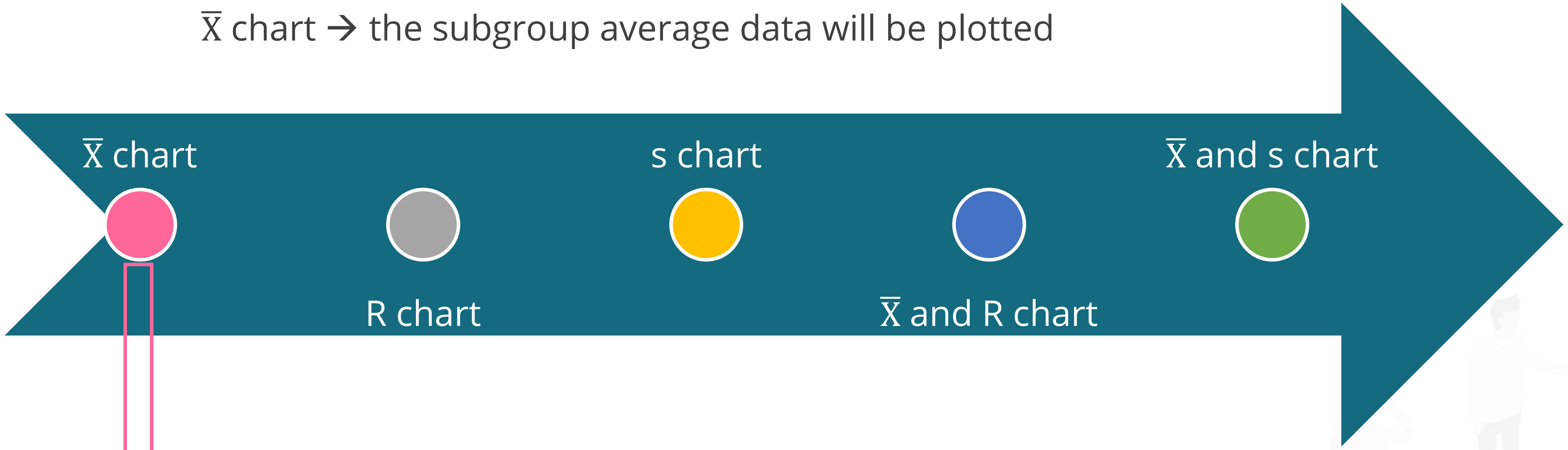
$\bar{X}$ chart → the subgroup average data will be plotted



$\bar{X}$ Chart Principles

$\bar{X}$ → average of each subgroup of data

$\bar{X}$ chart → the subgroup average data will be plotted

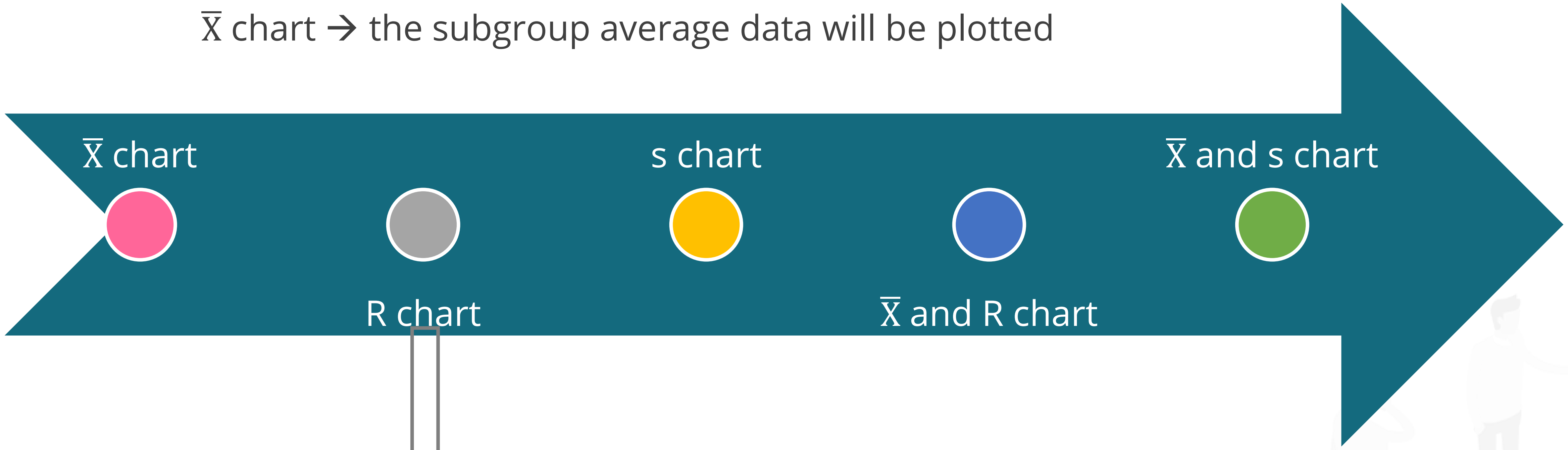


- It is the plot of the means of the subgrouped data
- It shows inter-subgroup or between-subgroup variation
- The control limits are calculated based on mean of means, range or standard deviation, and other factors

$\bar{X}$ Chart Principles

$\bar{X}$ → average of each subgroup of data

$\bar{X}$ chart → the subgroup average data will be plotted

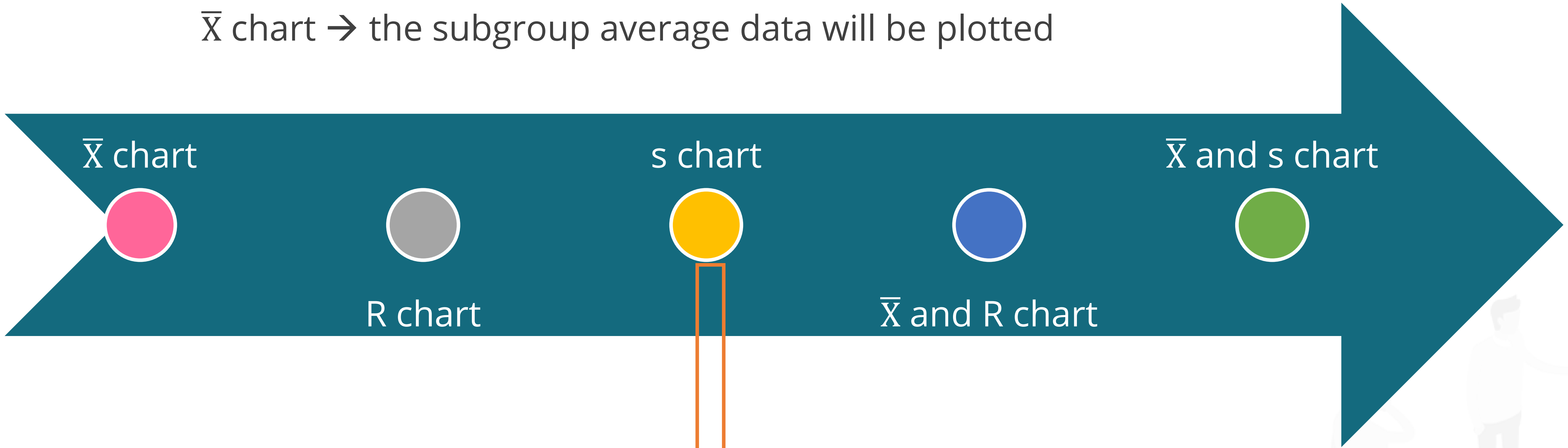


- ❑ It is the plot of the value of subgroup range
- ❑ The R chart shows intra subgroup
- ❑ One of the most sensitive charts to track and identify special causes of variation
- ❑ Can be plotted with any type of data

$\bar{X}$ Chart Principles

$\bar{X}$ → average of each subgroup of data

$\bar{X}$ chart → the subgroup average data will be plotted



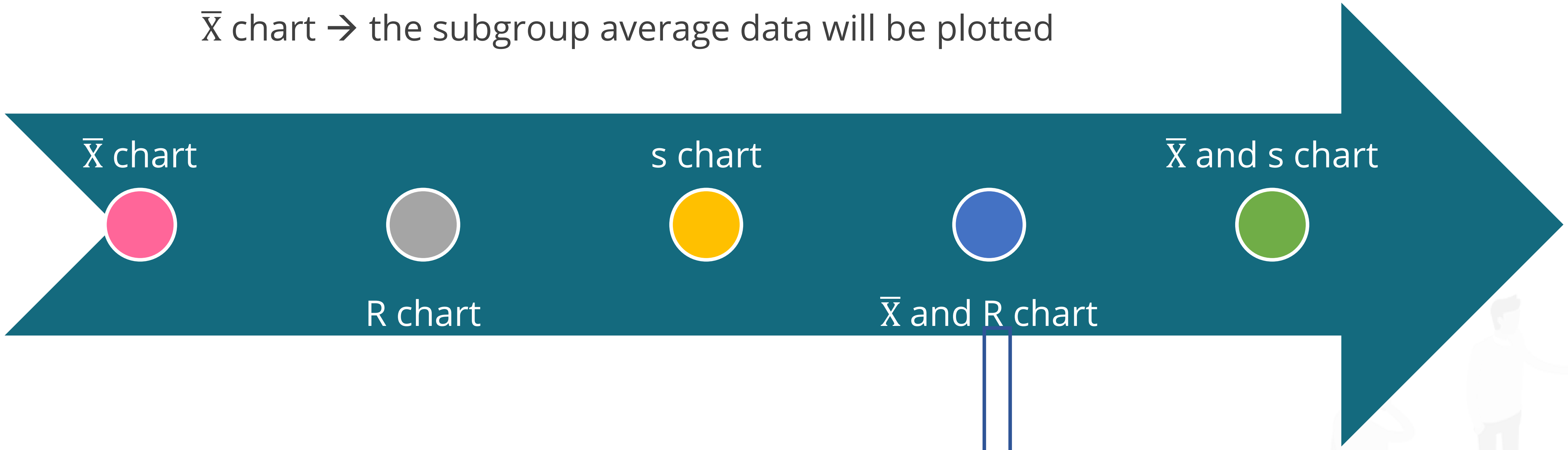
- It is the plot of the standard deviation of the subgroup range
- One of the most sensitive charts to track and identify special causes of variation
- Can be plotted with any type of data

simplilearn

$\bar{X}$ Chart Principles

$\bar{X}$ → average of each subgroup of data

$\bar{X}$ chart → the subgroup average data will be plotted

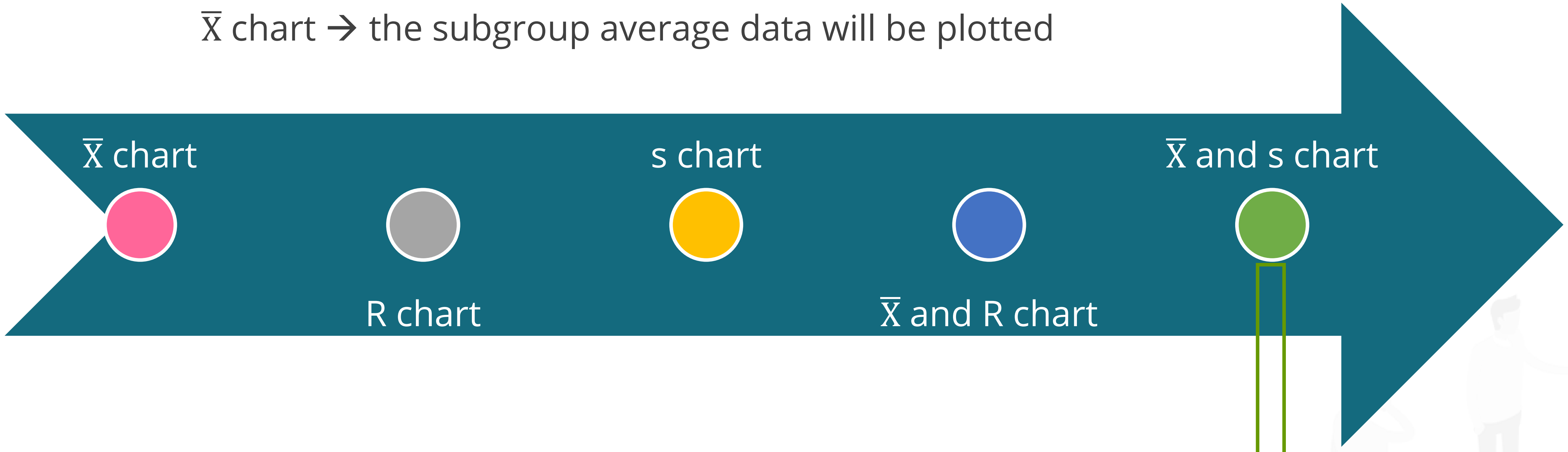


- It is of the same subgrouped data as the $\bar{X}$ and s chart
- One chart is the X bar and the other is the R chart
- Can be plotted with any type of data

$\bar{X}$ Chart Principles

$\bar{X}$ → average of each subgroup of data

$\bar{X}$ chart → the subgroup average data will be plotted



- It is of the same subgrouped data as the $\bar{X}$ and R chart
- One chart is the X bar and the other is the s chart
- Can be plotted with any type of data

Control Limit Formulas

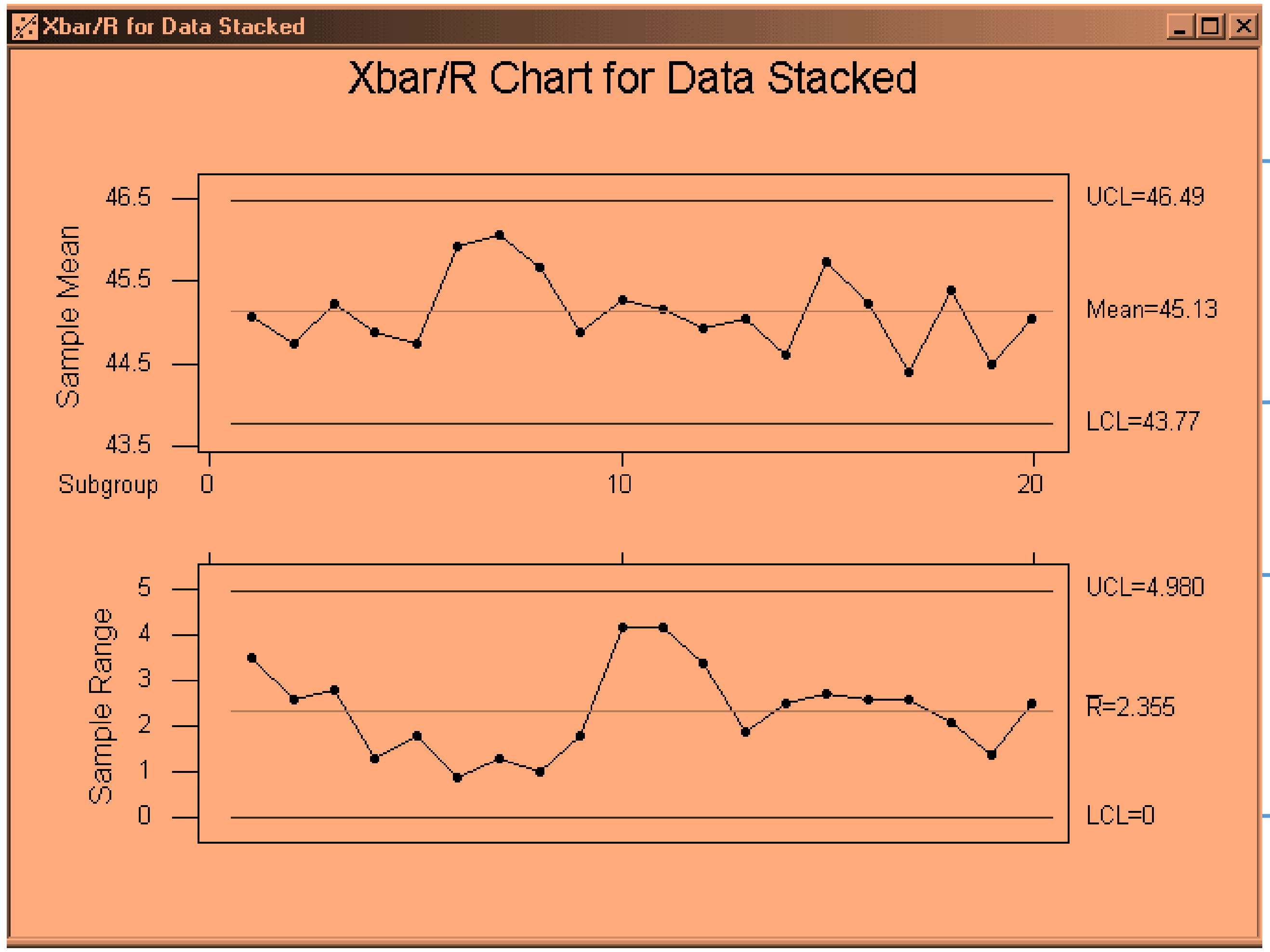
Control Limit Formulas

	Centerline	Control Limits		σ_x
X bar and R Charts	$CL_{\bar{X}} = \bar{\bar{X}}$	$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$	$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$	$\frac{\bar{R}}{d_2}$
	$CL_R = \bar{R}$	$UCL_R = D_4\bar{R}$	$LCL_R = D_3\bar{R}$	
X bar and s Charts	$CL_{\bar{X}} = \bar{\bar{X}}$	$UCL_{\bar{X}} = \bar{\bar{X}} + A_3\bar{S}$	$LCL_{\bar{X}} = \bar{\bar{X}} - A_3\bar{S}$	$\frac{\bar{s}}{c_4}$
	$CL_s = \bar{s}$	$UCL_s = B_4\bar{s}$	$LCL_s = B_3\bar{s}$	

Control Chart Constants

	Chart for Averages	Chart for Ranges (R)			Chart for Averages	Chart for Standard Deviation (s)		
	Control Limits Factor	Divisors to Estimate σ_x	Factors for Control Limits		Control Limits Factor	Divisors to estimate σ_x	Factors for Control Limits	
Subgroup size (n)	A_2	d_2	D_3	D_4	A_3	c_4	B_3	B_4
2	1.880	1.128	-	3.267	2.659	0.7979	-	3.267
3	1.023	1.693	-	2.574	1.954	0.8862	-	2.568
4	0.729	2.059	-	2.282	1.628	0.9213	-	2.266
5	0.577	2.326	-	2.114	1.427	0.9400	-	2.089
6	0.483	2.534	-	2.004	1.287	0.9515	0.030	1.970
7	0.419	2.704	0.076	1.924	1.182	0.9594	0.118	1.882
8	0.373	2.847	0.136	1.864	1.099	0.9650	0.185	1.815
9	0.337	2.970	0.184	1.816	1.032	0.9693	0.239	1.761
10	0.308	3.078	0.223	1.777	0.975	0.9727	0.284	1.716
15	0.223	3.472	0.347	1.653	0.789	0.9823	0.428	1.572
25	0.153	3.931	0.459	1.541	0.606	0.9896	0.565	1.435

UCL and LCL in $\overline{X}$ and *R* Chart



$UCL_{\bar{X}} = \overline{\overline{X}} + A_2 \overline{R}$

$LCL_{\bar{X}} = \overline{\overline{X}} - A_2 \overline{R}$

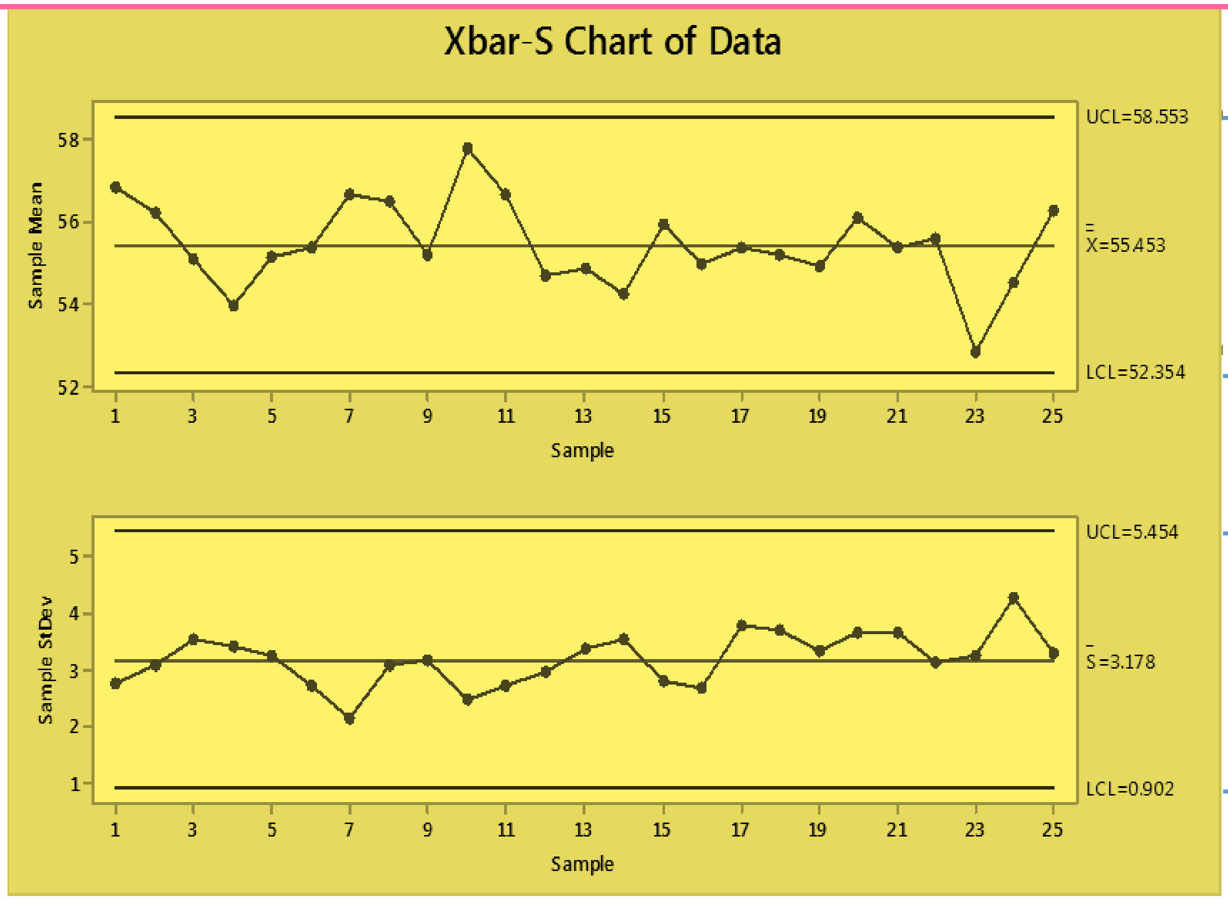
$UCL_R = D_4 \overline{R}$

$LCL_R = D_3 \overline{R}$

$\overline{\overline{X}}$ → grand average and $\overline{R}$ → average of the range.

A2, D3, and D4 are values from the control chart table

UCL and LCL in $\overline{X}$ and *s* Chart



$$UCL = \overline{\overline{X}} + A_3\,\overline{S}$$
$$LCL = \overline{\overline{X}} - A_3\,\overline{S}$$

$$UCL = B_4\,\overline{S}$$
$$LCL = B_3\,\overline{S}$$

- S → Standard deviation of each subgroup data
- The data is divided into subgroups.
- Standard deviation is calculated for each subgroup.

Values for A3, B3, and B4 are constant and are taken from the control chart table. $\overline{X}$ and *s* charts are used to track process variation where the subgroup sample size ≥ 9.

$\bar{X}$ and *R* and Subgroup Data

Q

Establish 1 σ process limits for the data set shown. Use the table of control chart constants for values of A_2, D_3, and D_4.

Table for control chart constants

n	A_2	D_3	D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00

$\bar{X}$ Chart

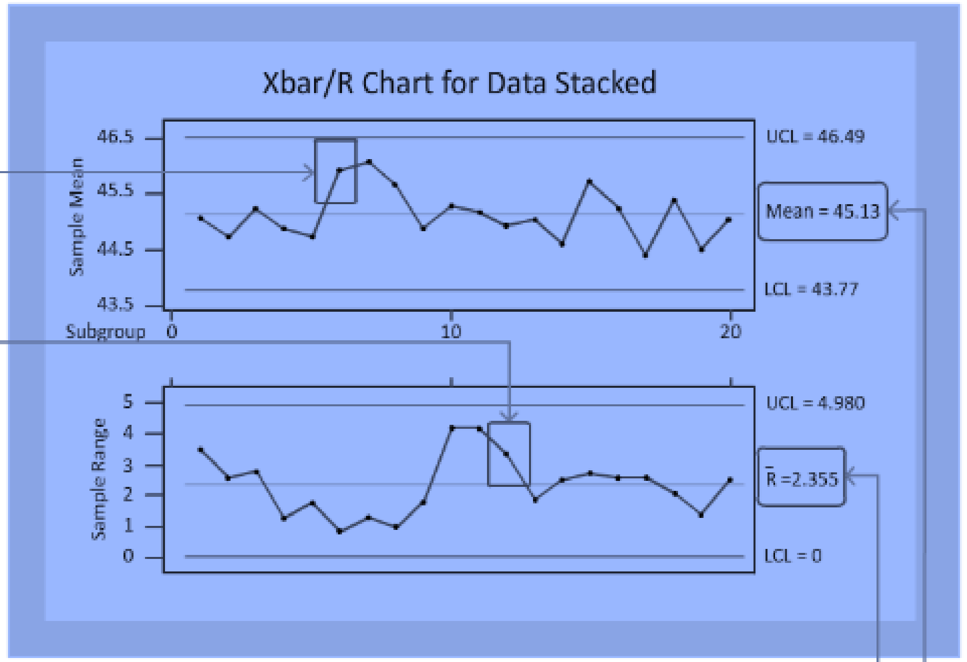
	X1	X2	X3	X4	X5
SG 1	43.8	43.7	47.2	46.3	44.4
SG 2	44.7	43.2	45.7	45.8	44.4
SG 3	45.3	43.8	44.3	46.2	46.6
SG 4	45.4	44.1	44.6	45.3	45.0
SG 5	43.8	45.6	44.6	44.8	45.0
SG 6	45.7	46.0	45.6	45.9	46.5
SG 7	46.5	45.6	45.7	46.9	45.6
SG 8	46.1	45.8	45.5	45.9	45.1
SG 9	44.5	44.0	45.4	45.8	44.7
SG 10	47.8	43.6	44.5	46.0	44.5
SG 11	45.5	45.4	42.8	47.0	45.1
SG 12	46.8	43.5	43.4	46.0	45.0
SG 13	44.2	44.7	46.1	44.5	45.8
SG 14	44.6	44.7	45.2	43.0	45.5
SG 15	46.0	46.0	45.0	44.5	47.2
SG 16	46.3	43.7	44.8	46.0	45.4
SG 17	43.2	43.0	45.6	44.8	45.4
SG 18	45.2	45.1	46.9	45.0	44.8
SG 19	44.6	44.5	44.6	43.7	45.1
SG 20	45.6	44.2	46.0	43.5	45.9

$\overline{X}$ and *R* and Subgroup Data

A

- In $\overline{X}$ and *R* chart, point SG 6 is the point of change in the process from below the center line to above the center.
- No points are outside control limits in the given process; examine points 6 and 7 on $\overline{X}$ chart, and points 10 and 11 on the *R* chart for rule #4 (If 2 out of 3 points are within 1 σ_{mean} of either the UCL or the LCL).

	X1	X2	X3	X4	X5	Xbar	Range
SG 1	43.8	43.7	47.2	46.3	44.4	45.1	3.5
SG 2	44.7	43.2	45.7	45.8	44.4	44.8	2.6
SG 3	45.3	43.8	44.3	46.2	46.6	45.2	2.8
SG 4	45.4	44.1	44.6	45.3	45.0	44.9	1.3
SG 5	43.8	45.6	44.6	44.8	45.0	44.8	1.8
SG 6	45.7	46.0	45.6	45.9	46.5	45.9	0.9
SG 7	46.5	45.6	45.7	46.9	45.6	46.1	1.3
SG 8	46.1	45.8	45.5	45.9	45.1	45.7	1.0
SG 9	44.5	44.0	45.4	45.8	44.7	44.9	1.8
SG 10	47.8	43.6	44.5	46.0	44.5	45.3	4.2
SG 11	45.5	45.4	42.8	47.0	45.1	45.2	4.2
SG 12	46.8	43.5	43.4	46.0	45.0	44.9	3.4
SG 13	44.2	44.7	46.1	44.5	45.8	45.1	1.9
SG 14	44.6	44.7	45.2	43.0	45.5	44.6	2.5
SG 15	46.0	46.0	45.0	44.5	47.2	45.7	2.7
SG 16	46.3	43.7	44.8	46.0	45.4	45.2	2.6
SG 17	43.2	43.0	45.6	44.8	45.4	44.4	2.6
SG 18	45.2	45.1	46.9	45.0	44.8	45.4	2.1
SG 19	44.6	44.5	44.6	43.7	45.1	44.5	1.4
SG 20	45.6	44.2	46.0	43.5	45.9	45.0	2.5
					Average	45.13	2.36



simplilearn

$\overline{X}$ and *s* and Subgroup Data

Q

The data in subgroups with 10 samples in each subgroup is given here along with the $\overline{X}$ chart. Using this data, find out if the process is in control.

Table for control chart constants

n	A_3	B_3	B_4
2	2.659	0	3.267
3	1.954	0	2.568
4	1.628	0	2.266
5	1.427	0	2.089
6	1.287	0.030	1.970
7	1.182	0.118	1.882
8	1.099	0.185	1.815
9	1.032	0.239	1.761
10	0.975	0.284	1.716

$\overline{X}$ Chart

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
SG 1	53.7	60.3	55.5	53.8	58.9	60	60	53.7	55.6	56.9
SG 2	60.4	57.6	58.4	60.3	51.7	55.3	57.1	53.4	52.7	55.2
SG 3	52	52.7	51.7	60.5	54.9	58.4	60.3	51.3	55.5	53.6
SG 4	54.5	50.4	53.6	59.8	50.1	50	57.3	51.8	55.7	56.7
SG 5	51	58.1	59.4	51	54.8	59.1	55	56.7	51.3	55
SG 6	59.3	59.7	55.3	57.7	53.5	53.1	51.8	53.4	55.3	54.4
SG 7	52.5	59.1	55.8	55.5	57.9	56.8	59.9	57.1	54.8	57.5
SG 8	55.1	59	51.8	56.1	60.7	58.6	58.7	58.7	52.6	53.6
SG 9	51.9	57.3	51	55.5	53.9	50.8	59.9	56.8	56.6	58.3
SG 10	59	59.1	59.8	54.4	60.4	60.3	58.7	56.6	54.1	55.3
SG 11	53.7	57.7	52.6	53.3	57	58.4	57.3	56.5	59.2	60.9
SG 12	57.6	55.4	50.8	57.2	50.2	52.6	56.6	59	54.2	53.4
SG 13	58	50	55.4	51.7	59.4	53.8	52.5	56.8	59	51.9
SG 14	53.4	50.2	51.5	57.5	53.3	59.8	50	55.7	52	58.8
SG 15	53.2	59.5	59.9	56.2	57.7	54.2	51.9	52.9	57.1	56.6
SG 16	58.4	52.8	55.1	58.5	54.6	53.6	55.5	58	52.9	50.6
SG 17	51.2	54.1	53.5	59	50.2	60.7	51.3	57.1	58.3	58.3
SG 18	51.9	54.9	60.6	54.6	58.7	55.2	50	60.3	51.3	54.4
SG 19	59.2	51.8	58.6	52.5	52.5	51.9	60.2	56	54.5	52
SG 20	50.7	54.5	55.2	60.2	50.9	60.3	58.7	60.1	55.5	54.7
SG 21	59.1	53.9	51.4	50.6	56.3	53.1	60.6	60.3	52.9	55.3
SG 22	57.8	57.4	54.5	53.7	52.3	53.1	60.5	58.9	51.1	56.9
SG 23	50.8	50.3	50.1	58.3	57.4	52.4	51.3	50.1	51.2	56.3
SG 24	57.9	51.7	56.8	50.6	50	50.1	51.1	60.3	56.9	60.1
SG 25	52.6	60.4	55.7	53.8	60.7	57.7	50.4	56.6	56.6	58.4

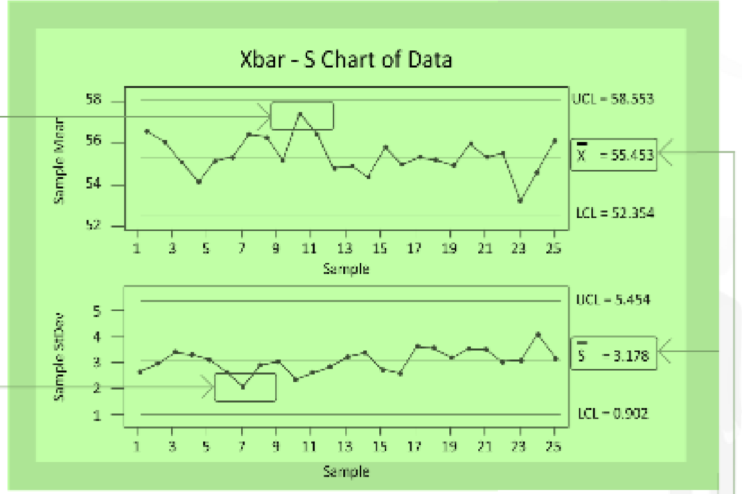
$\bar{X}$ and *s*: Constructing Chart

A

- The $\bar{X}$ chart point SG 10 is the variation of the point from the mean.
- Also, points 4, 10, and 23 have more variation from the center. These points can be analyzed further.
- The points are within the limits, and hence the process is in control.

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	Xbar	Std dv
SG 1	53.7	60.3	55.5	53.8	58.9	60	60	53.7	55.6	56.9	56.8	2.76
SG 2	60.4	57.6	58.4	60.3	51.7	55.3	57.1	53.4	52.7	55.2	56.2	3.06
SG 3	52	52.7	51.7	60.5	54.9	58.4	60.3	51.3	55.5	53.6	55.1	3.51
SG 4	54.5	50.4	53.6	59.8	50.1	50	57.3	51.8	55.7	56.7	54	3.4
SG 5	51	58.1	59.4	51	54.8	59.1	55	56.7	51.3	55	55.1	3.24
SG 6	59.3	59.7	55.3	57.7	53.5	53.1	51.8	53.4	55.3	54.4	55.4	2.7
SG 7	52.5	59.1	55.8	55.5	57.9	56.8	59.9	57.1	54.8	57.5	56.7	2.15
SG 8	55.1	59	51.8	56.1	60.7	58.6	58.7	58.7	52.6	53.6	56.5	3.09
SG 9	51.9	57.3	51	55.5	53.9	50.8	59.9	56.8	56.6	58.3	55.2	3.17
SG 10	59	59.1	59.8	54.4	60.4	60.3	58.7	56.6	54.1	55.3	57.8	2.45
SG 11	53.7	57.7	52.6	53.3	57	58.4	57.3	56.5	59.2	60.9	56.7	2.7
SG 12	57.6	55.4	50.8	57.2	50.2	52.6	56.6	59	54.2	53.4	54.7	2.96
SG 13	58	50	55.4	51.7	59.4	53.8	52.5	56.8	59	51.9	54.9	3.35
SG 14	53.4	50.2	51.5	57.5	53.3	59.8	50	55.7	52	58.8	54.2	3.54
SG 15	53.2	59.5	59.9	56.2	57.7	54.2	51.9	52.9	57.1	56.6	55.9	2.78
SG 16	58.4	52.8	55.1	58.5	54.6	53.6	55.5	58	52.9	50.6	55	2.66
SG 17	51.2	54.1	53.5	59	50.2	60.7	51.3	57.1	58.3	58.3	55.4	3.78
SG 18	51.9	54.9	60.6	54.6	58.7	55.2	50	60.3	51.3	54.4	55.2	3.68
SG 19	59.2	51.8	58.6	52.5	52.5	51.9	60.2	56	54.5	52	54.9	3.34
SG 20	50.7	54.5	55.2	60.2	50.9	60.3	58.7	60.1	55.5	54.7	56.1	3.64
SG 21	59.1	53.9	51.4	50.6	56.3	53.1	60.6	60.3	52.9	55.3	55.4	3.63
SG 22	57.8	57.4	54.5	53.7	52.3	53.1	60.5	58.9	51.1	56.9	55.6	3.11
SG 23	50.8	50.3	50.1	58.3	57.4	52.4	51.3	50.1	51.2	56.3	52.8	3.22
SG 24	57.9	51.7	56.8	50.6	50	50.1	51.1	60.3	56.9	60.1	54.6	4.24
SG 25	52.6	60.4	55.7	53.8	60.7	57.7	50.4	56.6	56.6	58.4	56.3	3.3

Average	55.453	3.17



simplilearn

I-MR Chart Principles

I-MR Charts

I-chart

Plot of the individual data points

MR-chart

Plot of the moving range of the previous individuals

Used

- When subgroup variation = 0 or no subgroups exist; and
- With data points from destructive testing or batch processing, or summary data from a time period

I-MR Chart Principles

I-MR charts → sensitive to trends, cycles, patterns, and normality.

Control limits of the I-MR chart are calculated using a similar method as the $\overline{X}$ and R chart.

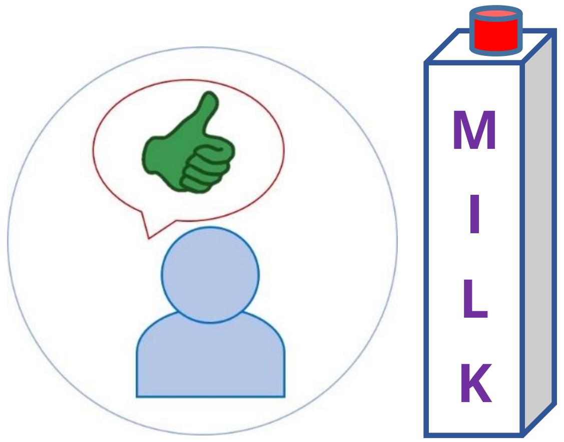
Charts for Individuals	$CL_X = \overline{X}$	$UCL_X = \overline{X} + E_2\overline{R}$	$LCL_X = \overline{X} - E_2\overline{R}$
	$CL_R = \overline{R}$	$UCL_R = D_4\overline{R}$	$LCL_R = D_3\overline{R}$

I-MR and Individual Data: Example

Q

The QC department measures the strength of its milk cartons once in every hour. Is the process in control?

- Since the data is individual data, the I-MR chart will be used here.
- This is an example of a destructive test.



Strength of the milk carton

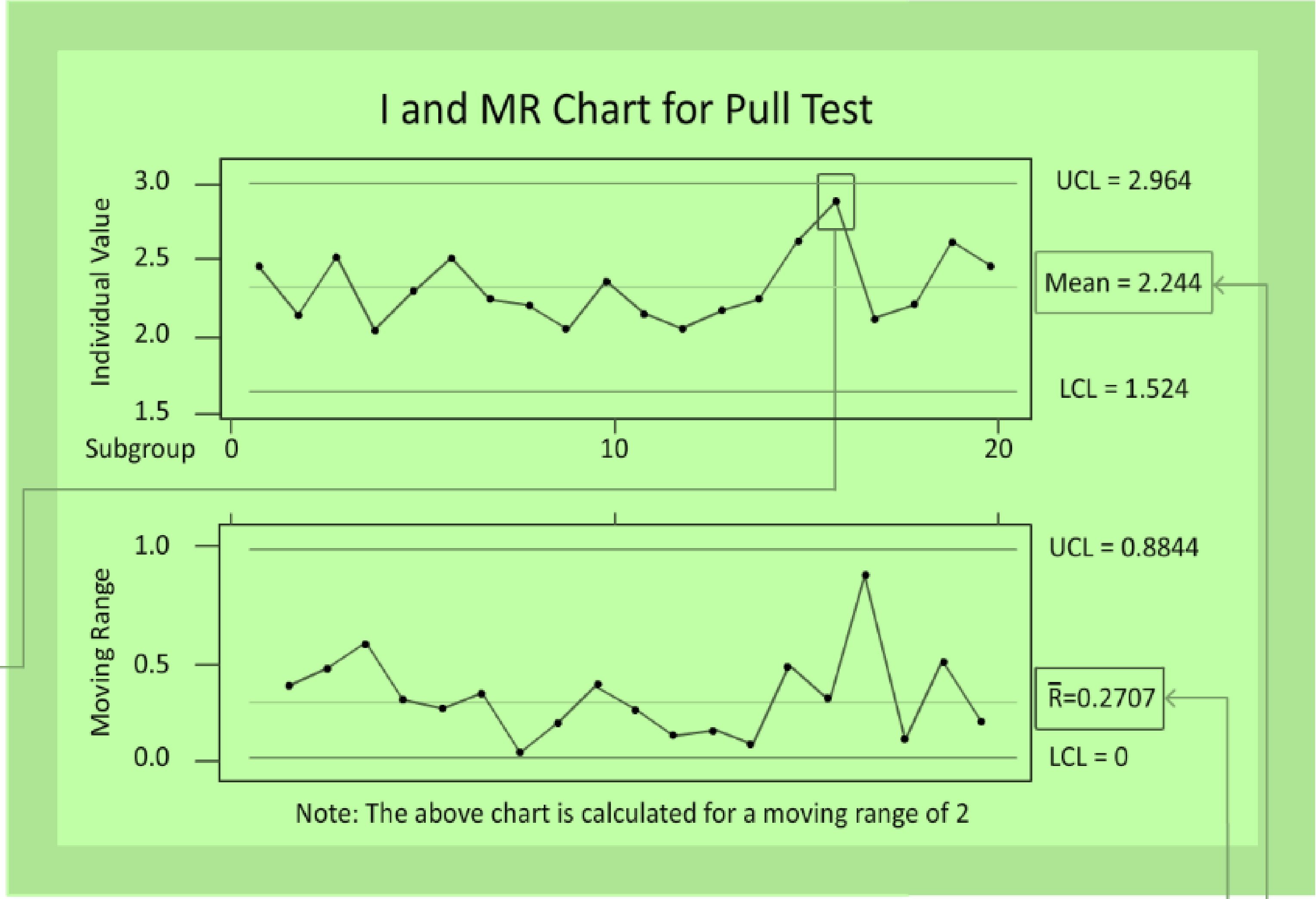
Pull Test
2.38
2.06
2.46
1.96
2.22
2.44
2.16
2.13
1.97
2.29
2.07
1.97
2.09
2.16
2.57
2.83
2.04
2.13
2.55
2.39

I-MR and Individual Data: Constructing Chart

A

- Moving range is the absolute value of difference between the last two data points.
- In I-chart, point 16 is close to the upper limit (analysis required).
- No points are out of control in the process.

	Pull Test	MR (2)
1	2.38	
2	2.06	0.32
3	2.46	0.40
4	1.96	0.50
5	2.22	0.26
6	2.44	0.22
7	2.16	0.28
8	2.13	0.03
9	1.97	0.16
10	2.29	0.32
11	2.07	0.22
12	1.97	0.10
13	2.09	0.12
14	2.16	0.07
15	2.57	0.41
16	2.83	0.26
17	2.04	0.79
18	2.13	0.09
19	2.55	0.42
20	2.39	0.16
Ave	2.244	0.270



I-MR Chart: IT/ITES Example

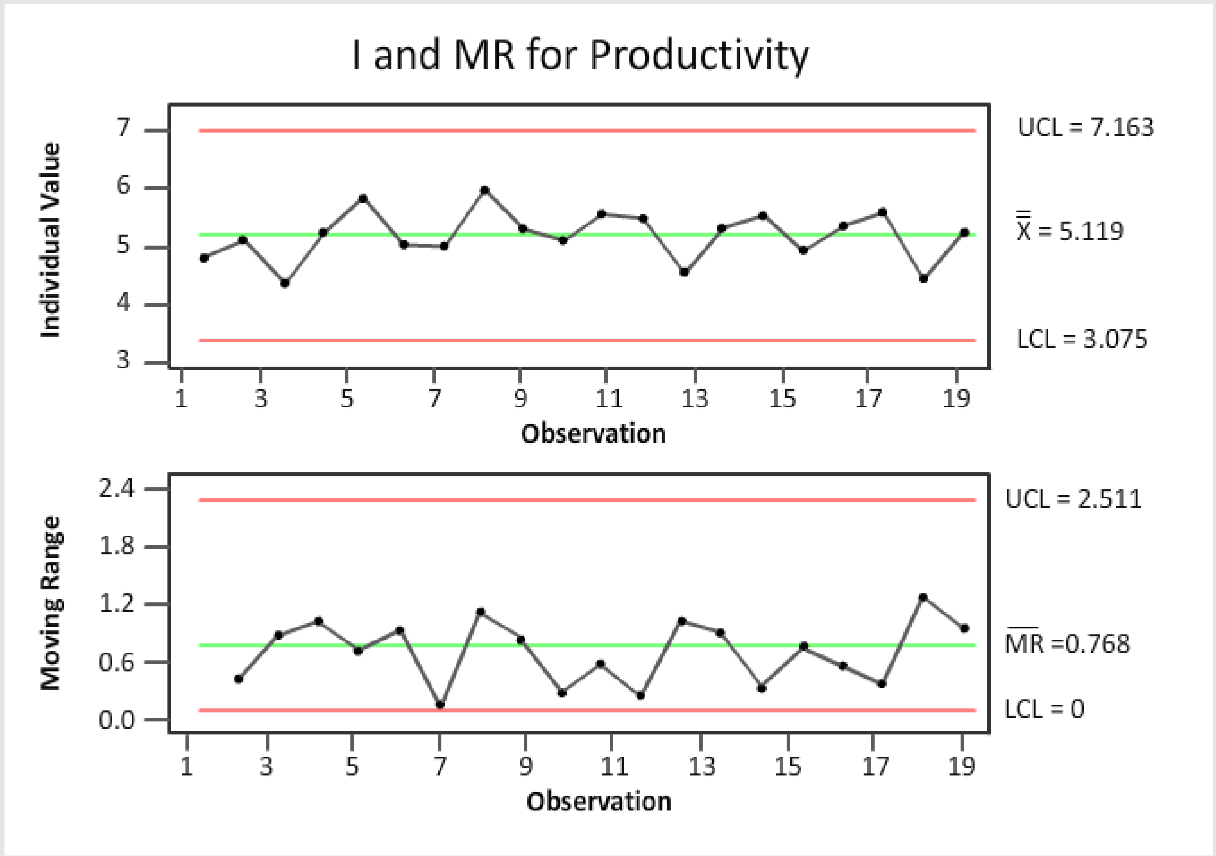
Call Center

Given is the data used to study the number of calls handled per hour in call center operations. This data was studied using I-MR charts to check if the process is in control. The data, I-MR chart, and analysis are as follows:

Data:

Productivity	Range
4.61	
5.02	0.41
4.02	1
5.2	1.18
5.99	0.79
4.92	1.07
4.87	0.05
6.19	1.32
5.24	0.95
5.02	0.22
5.61	0.59
5.45	0.16
4.26	1.19
5.29	1.03
5.57	0.28
4.75	0.82
5.34	0.59
5.67	0.33
4.13	1.54
5.23	1.1

I-MR Chart:



Analysis of I-chart

- All points are closer to mean values
- Process is well within control

Analysis of MR-chart

- Few points are closer to LCL.
- No points are outside of the control limits

Control Charts for Attribute Data

Based on sample size and data type (defects or defectives), the following types of control charts can be selected:

- ***np* chart**: If the sample size is consistent and the data type is defectives
- ***c* chart**: If the sample size is consistent and the data type available changes from defectives to defects
- ***p* chart**: If the sample size is inconsistent and the data type is defectives
- ***u* chart**: If the sample size is inconsistent and the data type is defects

Control limits may be constant, such as $\bar{X}$ and *R* charts (for *np* and *c* charts), or vary depending on sample size (for *p* and *u* charts.)

np Chart Principles

The *np* chart is used to measure the non-conforming proportions or number of defectives within a standardized group size. Some of the principles of *np* chart are as follows:

- The expectation is that the same proportion exists in each group
- The np chart follows binomial distribution
- Large subgroups are required (50 minimum) for this chart
- Subgroup size must be constant
- Control limits will be constant for an np chart

np Chart: Formulae

Important formulae of *np* chart are as follows:

- Proportion of p = $\frac{\mathrm{D}}{\mathrm{n}}$
- *np* = n $*$ $\frac{\mathrm{D}}{\mathrm{n}}$ = D
- Control Limits = $\overline{\mathrm{np}} \pm 3\sqrt{\overline{\mathrm{np}}\,(1-\overline{p})}$

where, D = Defectives

np Charts and Uniform Subgroup Size

Q

Day	Orders	Errors
1	125	14
2	125	5
3	125	7
4	125	17
5	125	4
6	125	3
7	125	14
8	125	5
9	125	10
10	125	6
11	125	5
12	125	26
13	125	6
14	125	14
15	125	6
16	125	7
17	125	8
18	125	11
19	125	13
20	125	10

The sourcing department at Java Coffee House Worldwide measures 125 purchase orders daily and records the number of entry errors in them. The tabulated data is given here. Is the order entry process in control?

- Since the data has a constant subgroup size (orders processed) of defectives, an *np* chart will be used.
- Assumption is that there is only one error per order possible.

Purchase Order

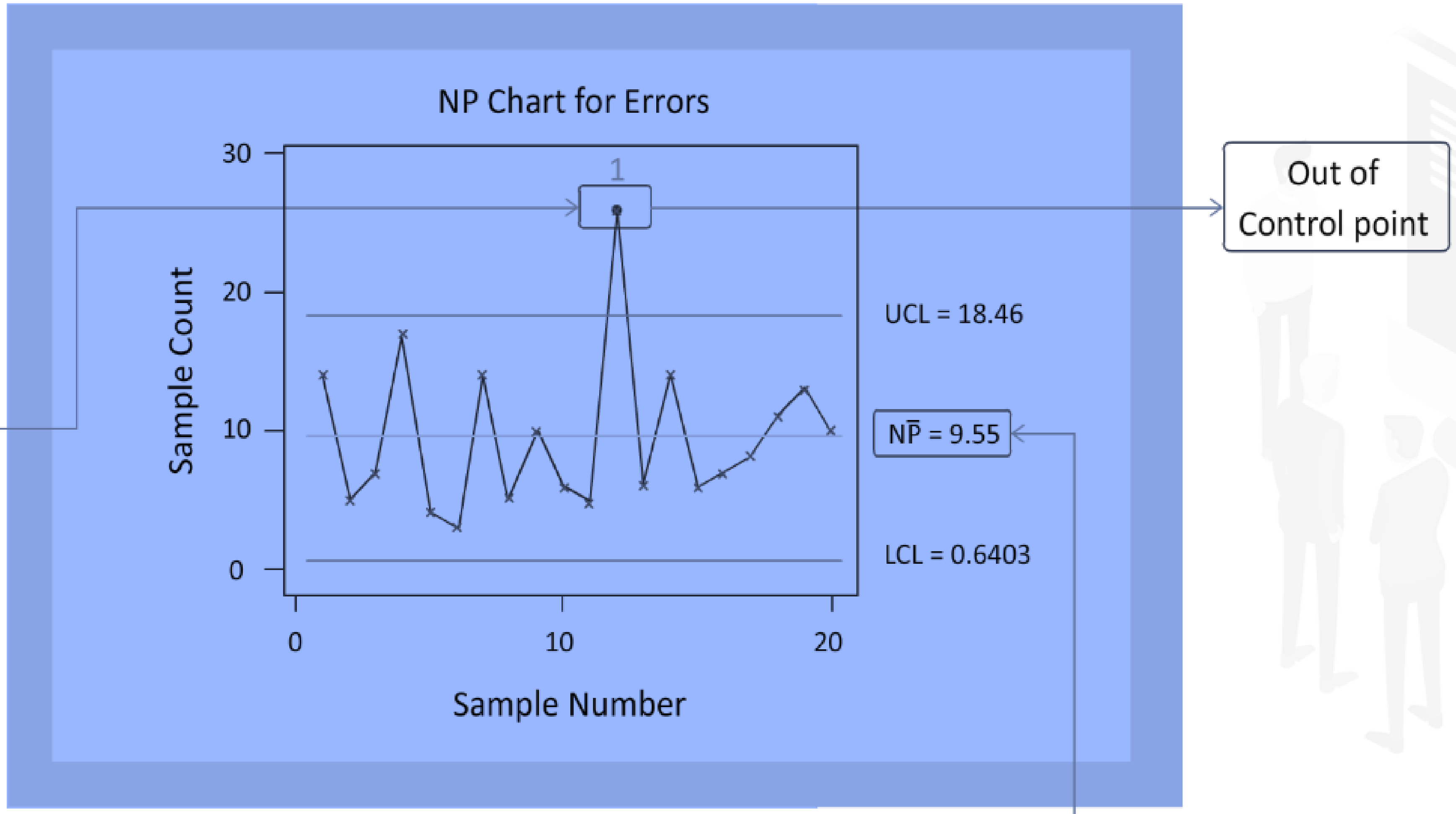
-----

u Chart: Constructing Chart

A

- In np chart, point 12 is beyond the control limit of three standard deviations. Analysis must be done to find the reason and take corrective action if necessary.
- Hence, point 12 is out of control in the process.

Day	Orders	Errors	Prop	np
1	125	14	0.112	14
2	125	5	0.040	5
3	125	7	0.056	7
4	125	17	0.136	17
5	125	4	0.032	4
6	125	3	0.024	3
7	125	14	0.112	14
8	125	5	0.040	5
9	125	10	0.080	10
10	125	6	0.048	6
11	125	5	0.040	5
12	125	26	0.208	26
13	125	6	0.048	6
14	125	14	0.112	14
15	125	6	0.048	6
16	125	7	0.056	7
17	125	8	0.064	8
18	125	11	0.088	11
19	125	13	0.104	13
20	125	10	0.080	10
Total		191		
		pbar	0.076	
		npbar	9.55	



p Chart Principles

The *p* chart is used to measure the non-conforming proportion or defectives. Some of the principles of *p* chart are as follows:

- The expectation is that the same proportion exists in each group
- The p chart follows binomial distribution
- The subgroup size should at least be 50
- Subgroup size need not be constant
- Control limits may vary from subgroup to subgroup based on the subgroup size.

p Chart: Formulae

Important formulae of *p* chart are as follows:

$$\text{Control Limits} = \bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Note: When n changes, control limit also changes.

p Charts and Varying Subgroup Size

Q

Day	Orders	Errors
1	123	14
2	102	5
3	87	7
4	119	17
5	88	4
6	72	3
7	100	14
8	94	5
9	111	10
10	103	6
11	92	5
12	155	26
13	47	6
14	116	14
15	97	6
16	102	7
17	117	8
18	101	11
19	89	13
20	103	10

The sourcing department in Java Coffee House Worldwide measures the number of entry errors on a daily basis. The tabulated data is presented here. Is the order entry process in control?

- Since the data has varying subgroup sizes (orders processed) of defectives, a *p* chart will be used

Java House Coffee

Order Entry

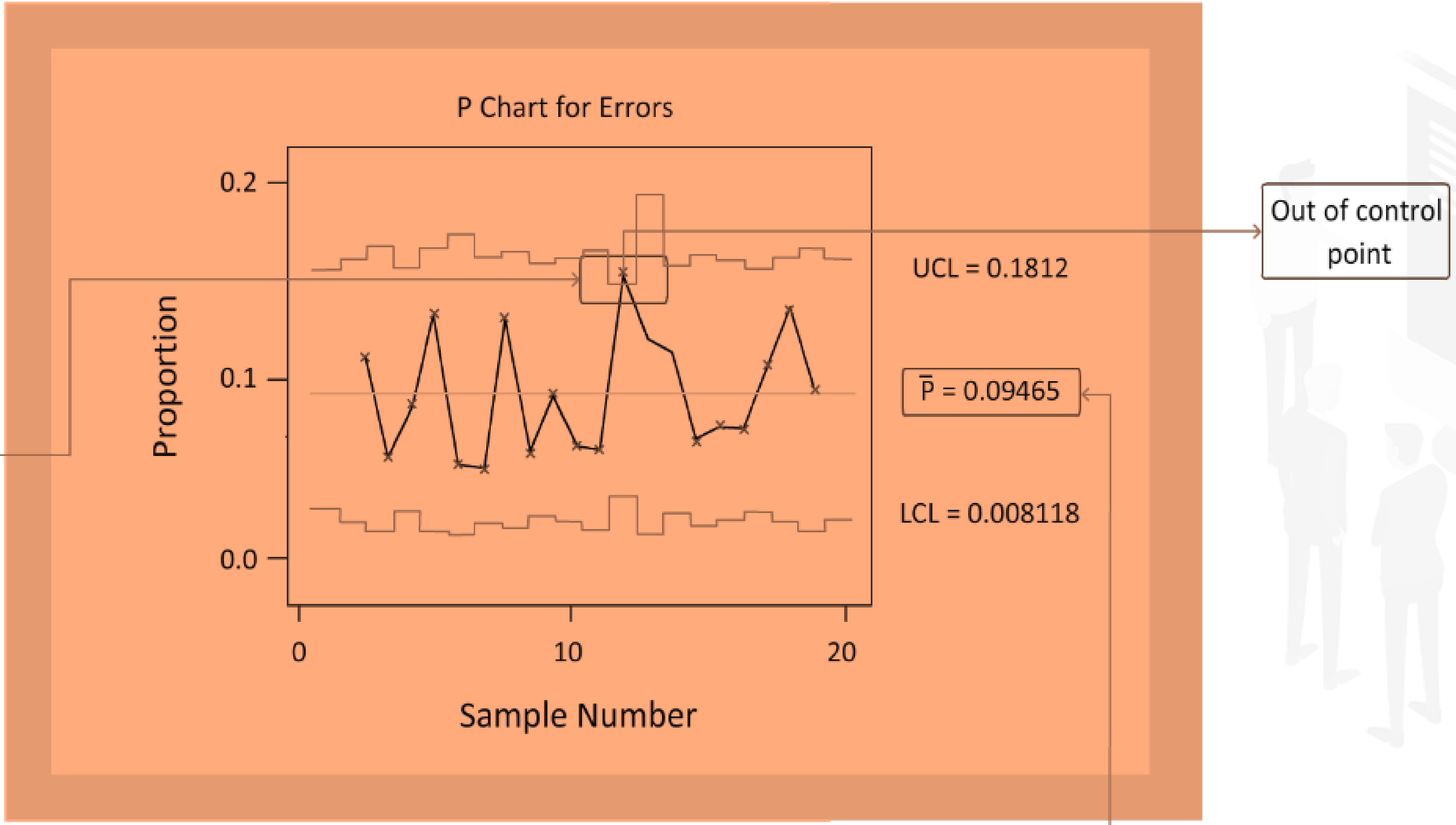
----

p Charts and Varying Subgroup Size

A

- In a *p* chart, point 12 has gone beyond the limit of 3 sigma level. Analysis must be done to find the reason and take corrective action if necessary.
- Hence, point 12 is out of control in this process.

Day	Orders	Errors	Prop
1	123	14	0.114
2	102	5	0.049
3	87	7	0.080
4	119	17	0.143
5	88	4	0.045
6	72	3	0.042
7	100	14	0.140
8	94	5	0.053
9	111	10	0.090
10	103	6	0.058
11	92	5	0.054
12	155	26	0.168
13	47	6	0.128
14	116	14	0.121
15	97	6	0.062
16	102	7	0.069
17	117	8	0.068
18	101	11	0.109
19	89	13	0.146
20	103	10	0.097
Total	**2018**	**191**	**0.09465**



simplilearn

c Chart Principles

To form a *c* chart, measure the number of occurrences of non-conforming defects. Some of the principles of the *c* chart are as follows:

- The *c* chart follows a Poisson distribution.
- The sample size is fixed or the area of opportunity is constant
- Used to identify attribute data for the sample
- Each count is a subgroup of samples
- The control limits will be constant
- The subgroup size should at least be 20

!

Control Limits = $\bar{c} \pm 3\sqrt{\bar{c}}$

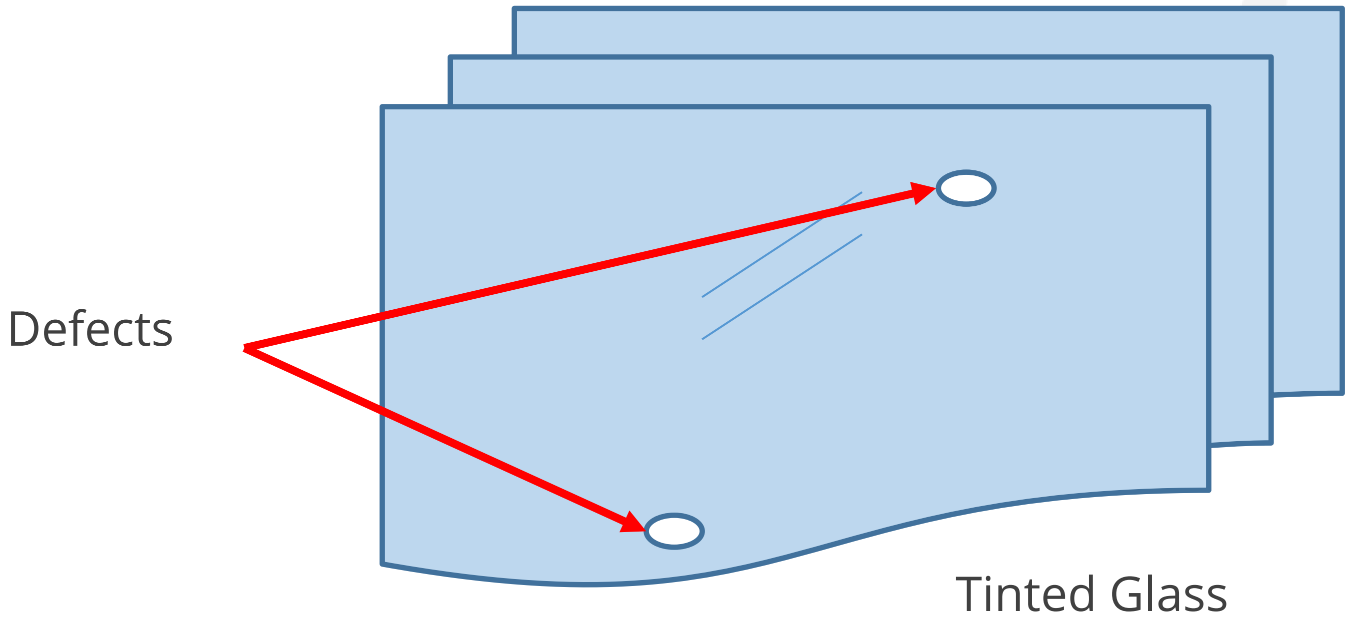
c Chart Principles

Q

Pane No	White Specs
1	31
2	39
3	38
4	5
5	22
6	34
7	10
8	23
9	11
10	36
11	25
12	4
13	4
14	11
15	25
16	4
17	38
18	36
19	36
20	17
Average	**22.45**

Final inspection grades the tinted glass on the number of white specs. The product is priced by grade. White specs are defects, not defectives, and are measured over a constant sample area; so *c* chart will be used. Is the process in control?

- Since the data is for defects, *c* chart will be used.

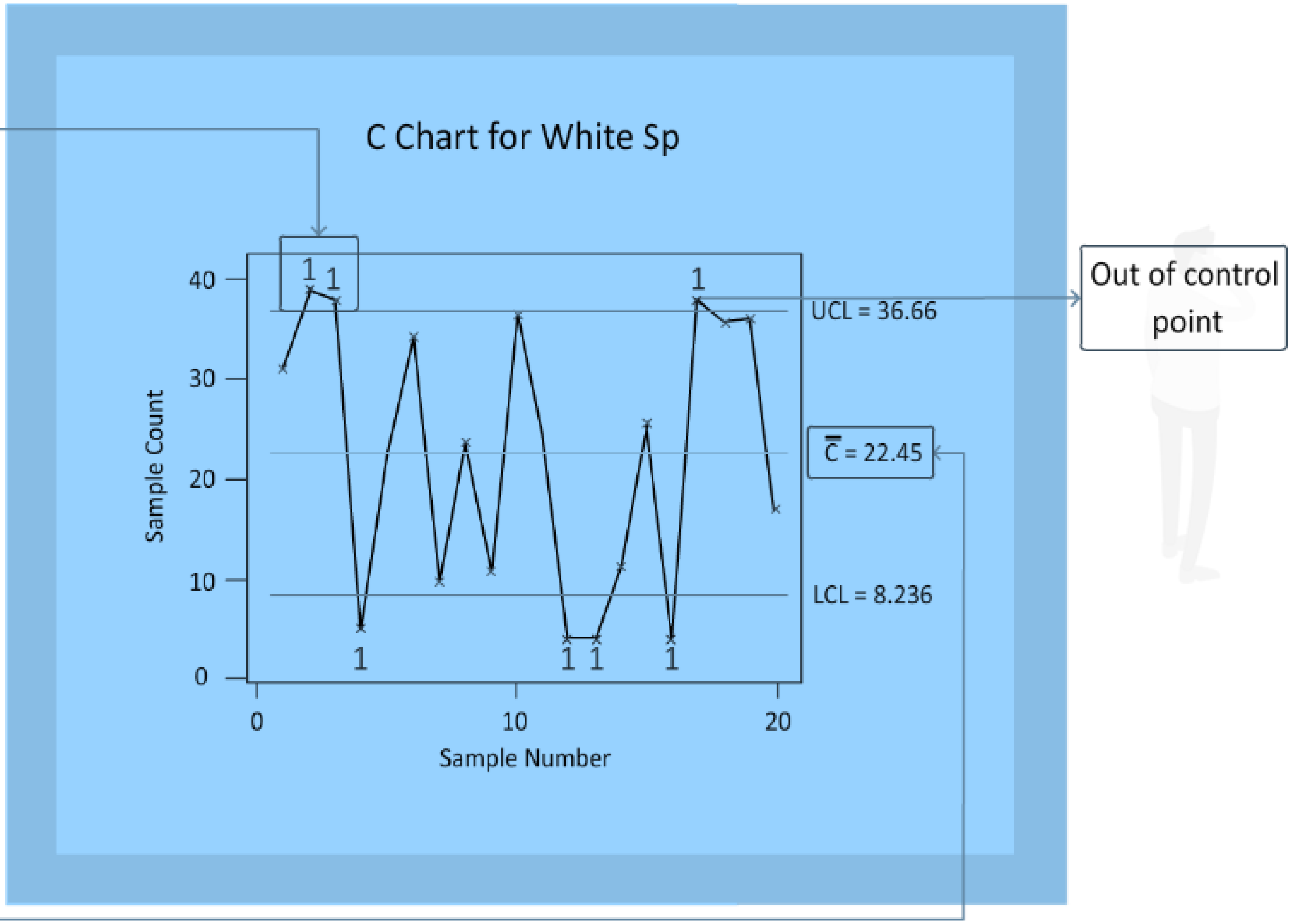


c Chart: Constructing Chart

A

- Points 2, 3, 4, 12, 13, 16, and 17 are out of control in this process; additionally, points 7, 9, 18, and 19 break rule #4.
- In this *c* chart, the process is not stable and many points go beyond 3 sigma control levels. Analysis must be done to find the reason and take corrective action.
- The process is not in control.

Pane No	White Specs
1	31
2	39
3	38
4	5
5	22
6	34
7	10
8	23
9	11
10	36
11	25
12	4
13	4
14	11
15	25
16	4
17	38
18	36
19	36
20	17
Average	22.45



u Chart Principles

The *u* chart is used to measure the non-conforming proportion or defectives. Some of the principles of *u* chart are as follows:

- The *u* chart follows a Poisson distribution.
- Used to identify attribute data for the sample
- Sample size is not fixed
- Control limits may vary
- The subgroup size should at least be 20

u Chart: Formulae

Important formulae of *u* chart are as follows:

Control limits = $\bar{u} \pm 3\sqrt{(\frac{\bar{u}}{a})}$

Where, a = area of opportunity

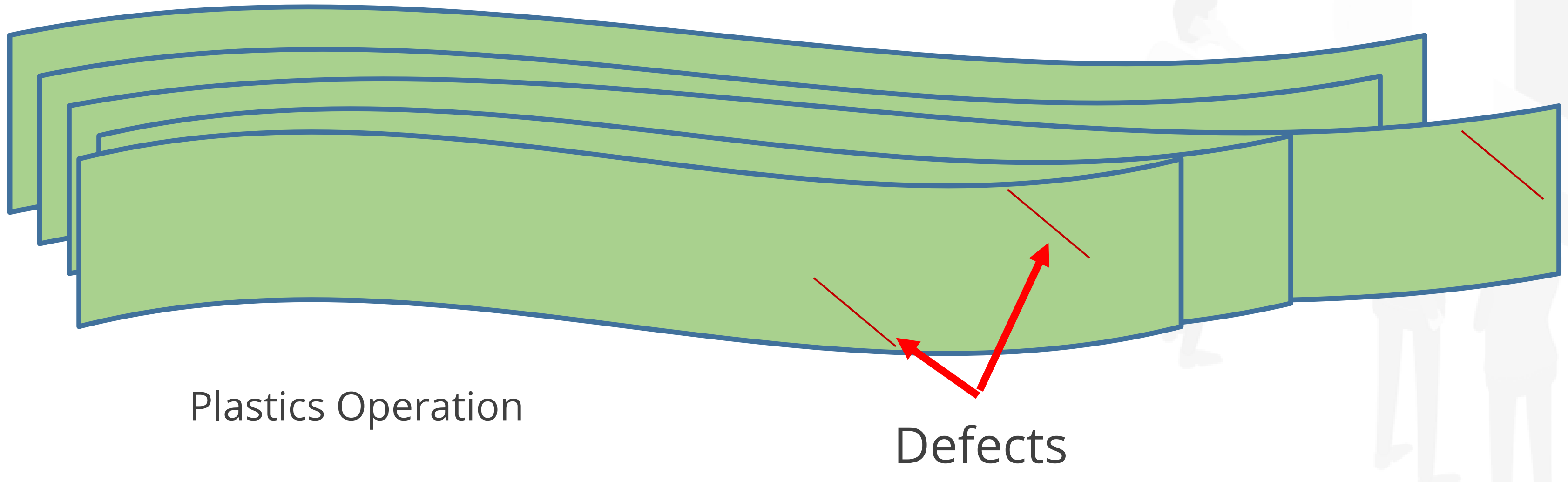
u Chart

Q

Run	Produced	Count of Defects
1	159	17
2	138	6
3	120	9
4	148	21
5	127	5
6	98	4
7	136	17
8	134	6
9	139	12
10	127	7
11	125	6
12	161	23
13	75	7
14	161	17
15	139	7
16	143	9
17	163	10
18	192	32
19	119	16
20	134	12
Total	**2738**	**243**
	Ubar	**0.088751**

The plastics operation counts defects after a “run” which is undetermined in length (once started, it continues until all material is used). Is the process in control?

Since the count of defects has a varying area of opportunity and the length of runs is not constant, *u* chart will be used.

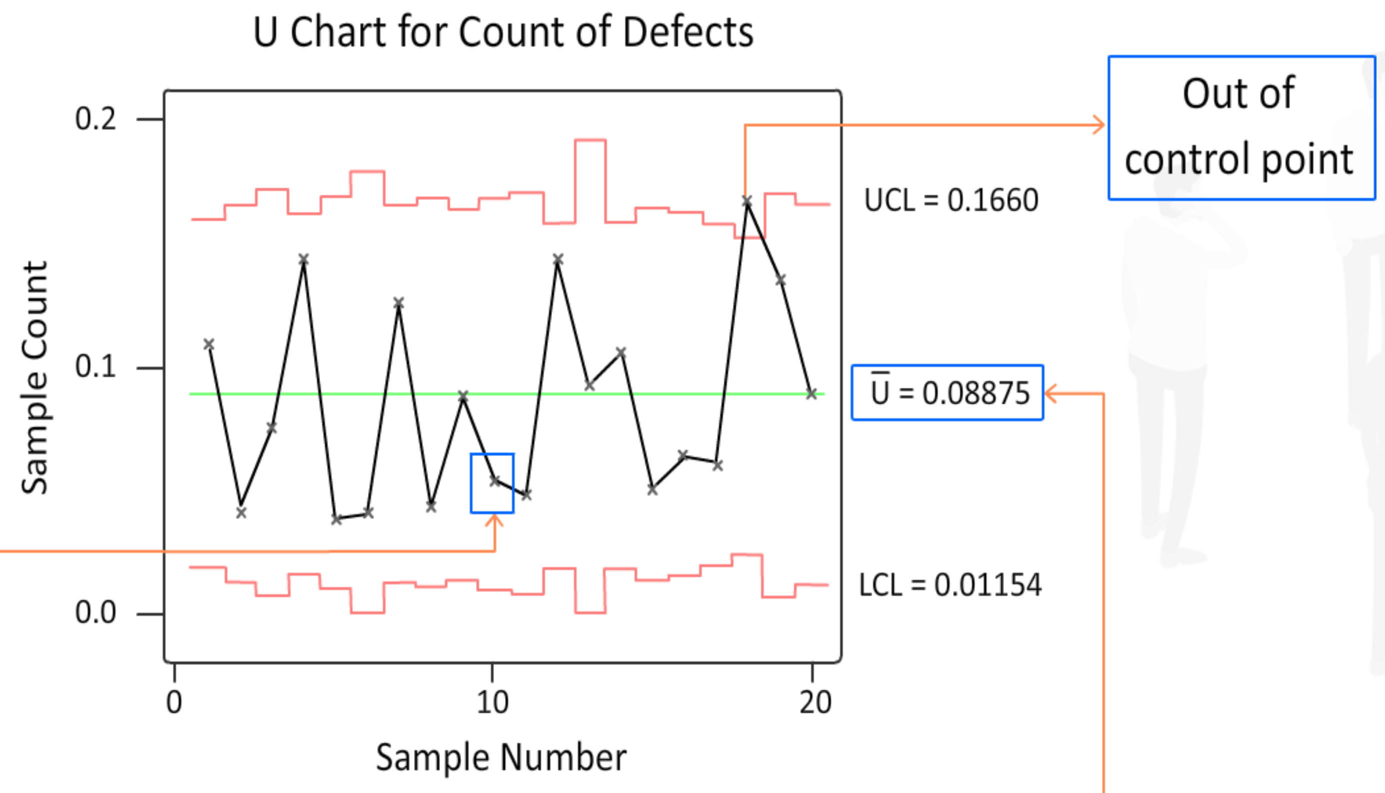


u Chart: Constructing Chart

A

- In this *u* chart, point 18 has gone beyond the 3 sigma level. Analysis must be done to find the reason and corrective action must be taken if necessary.
- Point 18 is out of control in this process.

Run	Produced	Count of Defects	Count/ area
1	159	17	0.107
2	138	6	0.043
3	120	9	0.075
4	148	21	0.142
5	127	5	0.039
6	98	4	0.041
7	136	17	0.125
8	134	6	0.045
9	139	12	0.096
10	127	7	0.055
11	125	6	0.048
12	161	23	0.143
13	75	7	0.093
14	161	17	0.106
15	139	7	0.050
16	143	9	0.063
17	163	10	0.061
18	192	32	0.167
19	119	16	0.134
20	134	12	0.090
Total	2738	243	
	Ubar	0.088751	



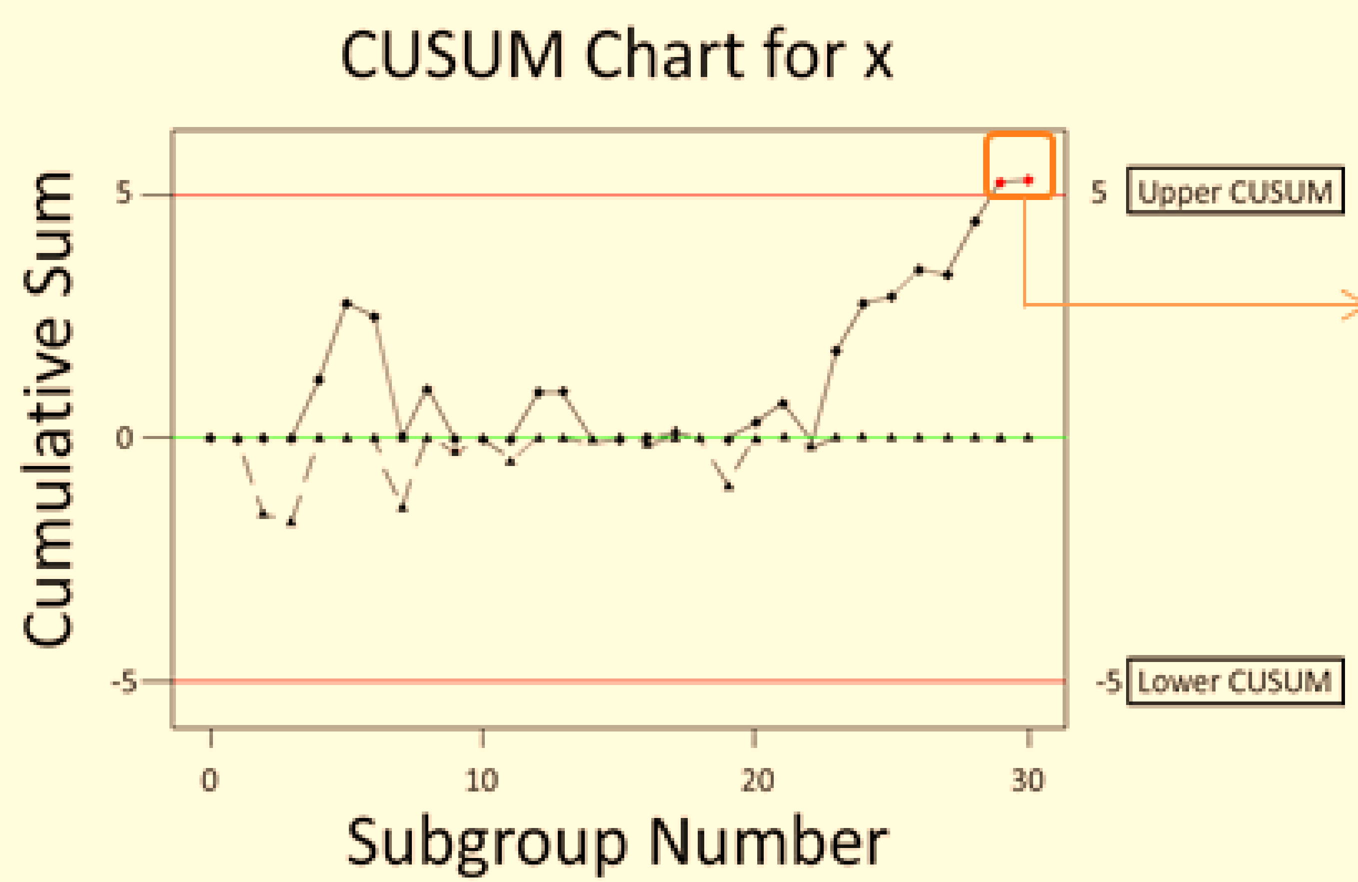
CuSum and EWMA Charts

CuSum Charts

If μ_0 → the target for the process mean
$\bar{X}_j$ → the average of the jth sample

Then, the cumulative sum control chart is formed by plotting the quantity as follows:

$$C_i = \sum_{j=1}^{i} (\bar{X} - \mu_0)$$



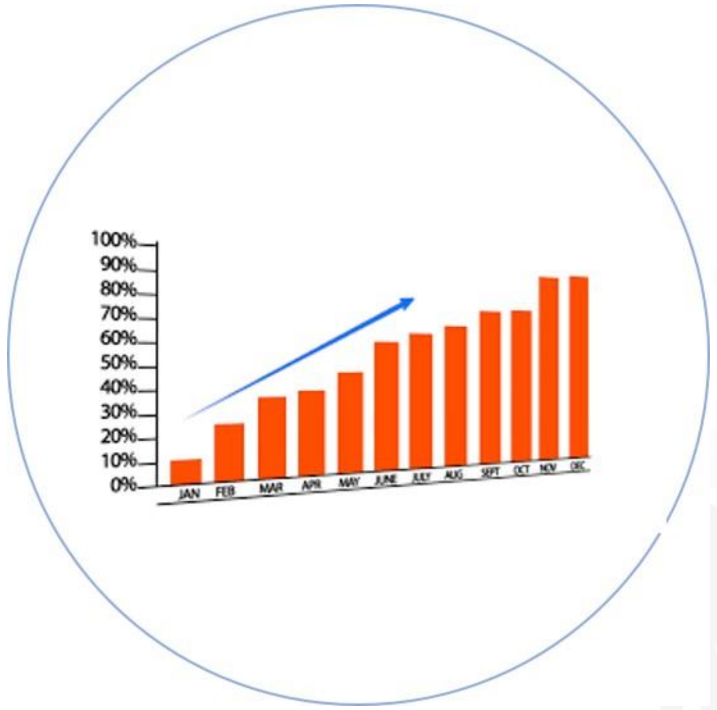
The data goes out of the limit for the 30th sub group number.

EWMA Charts

EWMA chart – time weighted control chart

- To plot data to detect small shifts over a small period of time
- To monitor process mean or variance
- Predict performance in the next period of change or instability
- More weight on recent observations

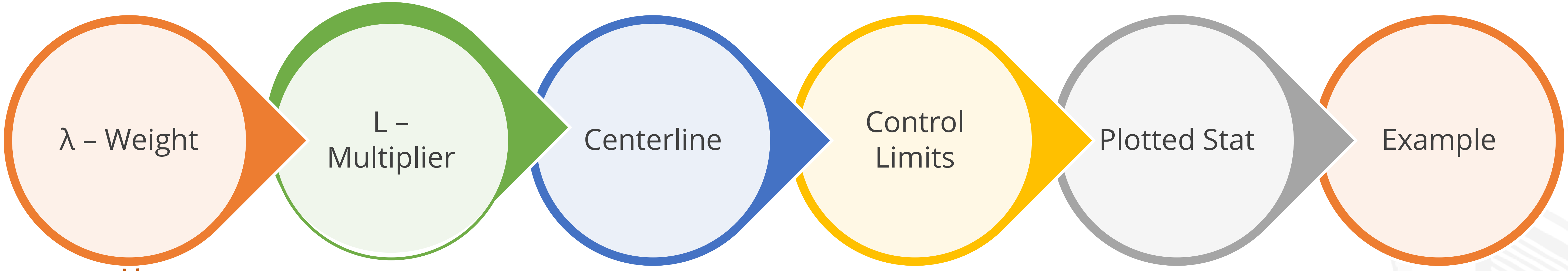
Example: Used in Stock Modeling Software Packages



EWMA Parameters

- λ – Weight
- L – Multiplier
- Centerline
- Control Limits
- Plotted Stat
- Example

EWMA Parameters



- Weight is applied to the most recent rational subgroup average
- λ is between 0 and 1
- Usually λ is selected between 0.05 and 0.25

EWMA Parameters

λ – Weight

L – Multiplier

Centerline

Control Limits

Plotted Stat

Example

- Multiple of the rational subgroup standard deviation to set control limits
- Typically set to 3 to match other control charts
- Can be reduced for if λ is small (if $0.1 \geq \lambda$, $2.6 < L < 2.8$)

EWMA Parameters

λ – Weight

L – Multiplier

Centerline

Control Limits

Plotted Stat

Example

This is represented by T or the target value of the quality characteristic

EWMA Parameters

λ – Weight

L – Multiplier

Centerline

Control Limits

Plotted Stat

Example

$$T \pm L\frac{S}{\sqrt{n}}\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2i}]}$$

T → the estimates of the long-term process mean
S → standard deviation established
n → the number of samples in the rational subgroup

The limits widen for each successive rational subgroup

EWMA Parameters

λ – Weight → L – Multiplier → Centerline → Control Limits → Plotted Stat → Example

$$z_i = \lambda \bar{x}_i + (1 - \lambda)\, z_{i-1}$$

Where,

$\bar{x}_i$ → current rational subgroup average

z_{i-1} → running average of all preceding observations

EWMA Parameters

λ – Weight

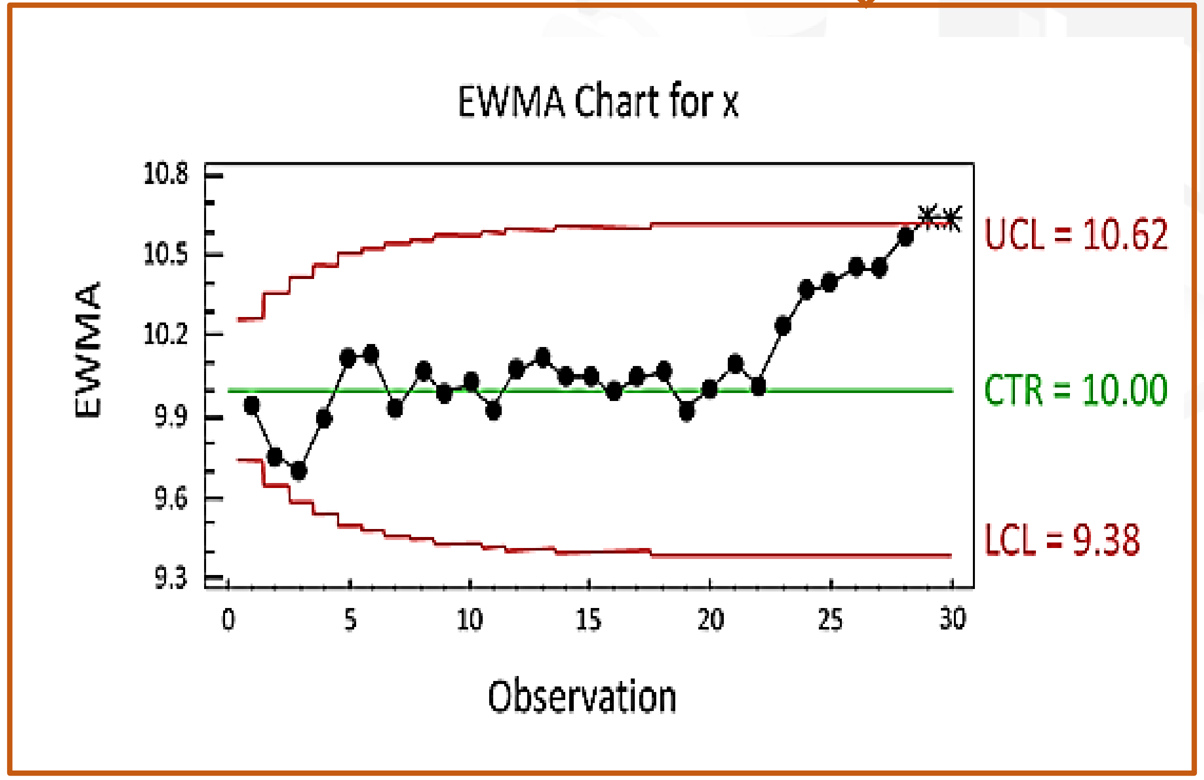
L – Multiplier

Centerline

Control Limits

Plotted Stat

Example



Key Takeaways

- SPC aids in the visual monitoring of a process and controlling its parameters
- A rational subgroup represents the process at a particular point in time
- Control chart plots and processes the input and output data over a period of time
- Out-Of-Control (OOC) condition indicates special cause for variation
- p chart is used for defectives which have a varying subgroup and is represented as a percentage of units rejected

Key Takeaways

- c chart is used for defective which have a constant subgroups size and is represented as number of defects
- u chart is used for defects that have a varying subgroup size and is represented as an average number of defects per opportunity
- An I-chart is a plot of the individual data points
- MR chart is a plot of the moving range of the previous individuals
- A CuSum chart plots the cumulative sums of the deviations of the sample values from the target value
- EWMA stands for Exponentially Weighted Moving Average Chart

Knowledge Check

Knowledge Check 1

While monitoring a process using a control chart a team observed six consecutive data points increasing. What does this mean about the process?

A. The process is not stable but it is in control

B. The process is stable and in control

C. There is not enough data to determine

D. The process is not stable and not in control

simplilearn

Knowledge Check 1

While monitoring a process using a control chart a team observed six consecutive data points increasing. What does this mean about the process?

A. The process is not stable but it is in control

B. The process is stable and in control

C. There is not enough data to determine

D. The process is not stable and not in control

The correct answer is **D**

The probability of observing 6 data points consecutively increasing is 1.6% and therefore not likely which indicates that the process is not stable or in control

Knowledge Check 2	What type of control chart should be made if measuring defectives and the subgroup size changes?

A. $\bar{X}$ and R

B. p chart

C. u chart

D. c chart

Knowledge Check 2

What type of control chart should be made if measuring defectives and the subgroup size changes?

A. $\bar{X}$ and R

B. p chart

C. u chart

D. c chart

The correct answer is **B**

If defectives are measured with a subgroup size that is not constant a p chart should be used.

Knowledge Check

3

Which of the following is NOT true of special causes of variation?

A. Difficult to eliminate

B. Occur sporadically in the process

C. Can be identified with help of a control chart

D. Is not a part of the process

Knowledge Check 3	Which of the following is NOT true of special causes of variation?

A. Difficult to eliminate

B. Occur sporadically in the process

C. Can be identified with help of a control chart

D. Is not a part of the process

The correct answer is **A**

It takes less investment to remove special cause variations than to remove common cause variation. Hence, special cause variations are not difficult to eliminate.

Knowledge Check
4

What is true of a control chart?

A. Does not assume normality of data

B. Typically the UCL and LCL are set to 2 standard deviations away from the mean

C. The probability of having an out of control condition is 0.27%

D. Typically show spec limits

Knowledge Check 4

What is true of a control chart?

A. Does not assume normality of data

B. Typically the UCL and LCL are set to 2 standard deviations away from the mean

C. The probability of having an out of control condition is 0.27%

D. Typically show spec limits

The correct answer is **C**

From the options presented the only choice that is true about control charts is C, in which the control limits are set at 3 standard deviations from mean because it provides a 0.27% of an out of control condition.

Knowledge Check

5

A team wants to measure defective items in which the subgroup size is constant. What type of control chart could be used?

A. I-MR

B. c-chart

C. u-chart

D. np -chart

Knowledge Check 5	A team wants to measure defective items in which the subgroup size is constant. What type of control chart could be used?

A. I-MR

B. c-chart

C. u-chart

D. np-chart

The correct answer is **D**

Since we are dealing with attribute data, and looking for defectives with constant subgroup size the best chart to use would be the np-chart.

Knowledge Check

6

What is the benefit of using CuSum or EWMA charts over traditional control charts?

A. Detects large shifts

B. Detects small shifts

C. Easier to calculate

D. More accurate

Knowledge Check 6

What is the benefit of using CuSum or EWMA charts over traditional control charts?

A. Detects large shifts

B. Detects small shifts

C. Easier to calculate

D. More accurate

The correct answer is **B**

The CuSum and EWMA charts are typically used to plot the data to detect small shifts over a small period of time.