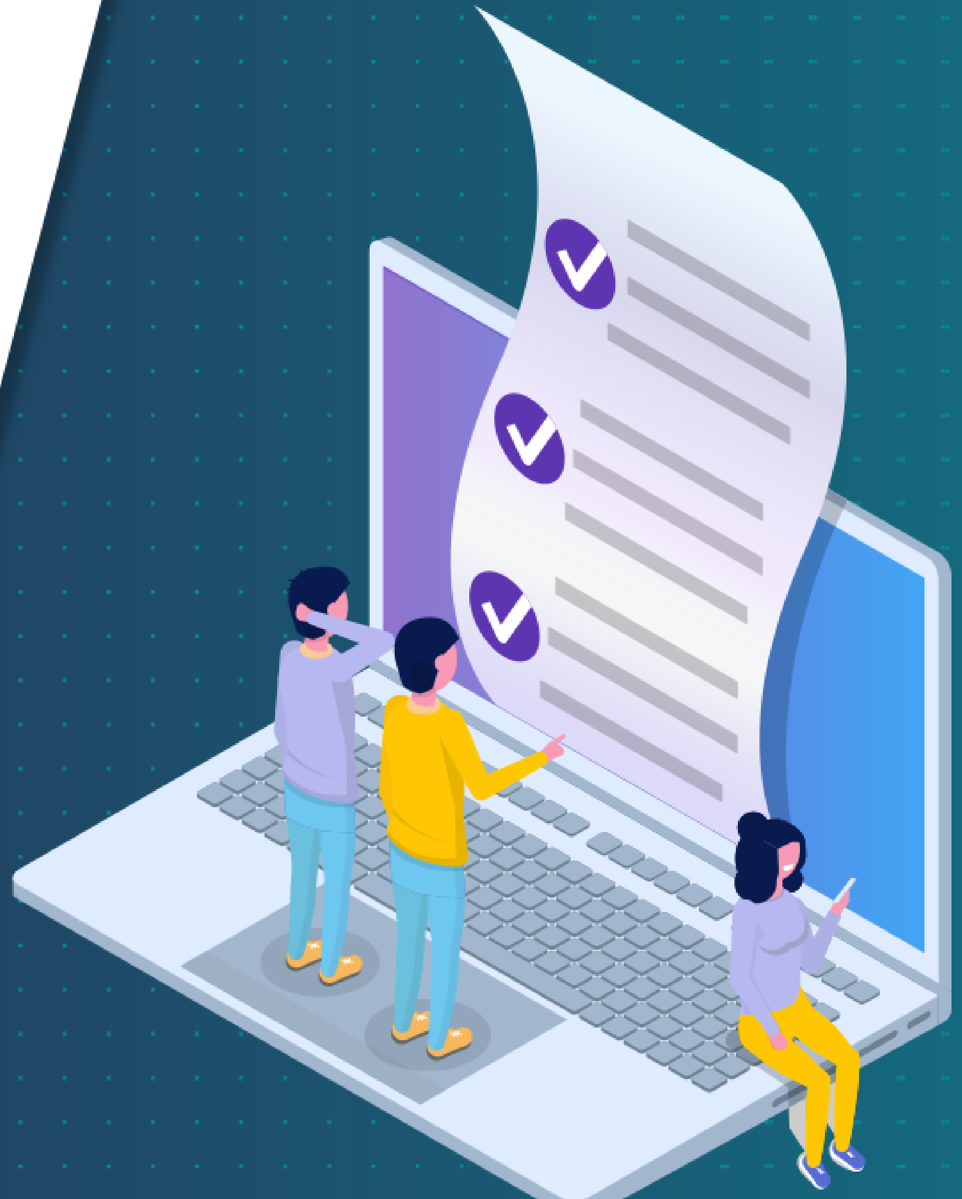
Lean Six Sigma Green Belt Certification Course

Probability and Statistics

Learning Objectives

By the end of this lesson, you will be able to:

- Outline probability basics
- Explain permutations and combinations
- List the addition and multiplication rules



Roles of Probability and Statistics

Business

Business phrase

3 out of 4 dentists choose whitening toothpaste

Economics

To understand

the relationship between supply and demand

Banking

To estimate

number of people making deposits vs. number of people requesting loans

Government

To design federal budgets

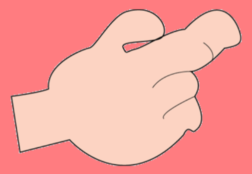
based on statistical data to estimate expected expenditure and revenue

Probability Basics and Statistical Rules

Probability

Probability refers to the chance of something occurring or happening.

Random Experiment



- A chance event
- A single coin toss

Elementary Outcomes

All possible results
- Heads
- Tails

Sample Space

{ H T }

Set of elementary outcomes

Probability: Characteristic Properties

PROPERTY 2
Impossible event

PROPERTY 3
Certain event

0

1

PROPERTY 1
Probability of an event
0 to 1

!

The total probability of the sample space or all elementary outcomes must be 100% or 1.

Probability of an Event

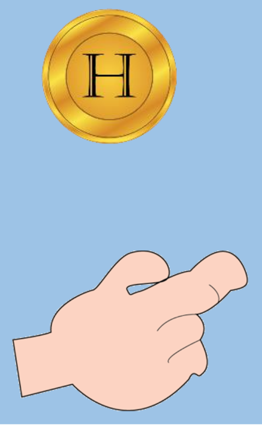
For a particular event, if:

N – possible outcomes that are equally likely

f – a specific type of event or outcome

Then, $$\text{Probability of the event} = \frac{f}{N}$$

Example:



In an event of a coin toss, what is the probability of getting a “head”?

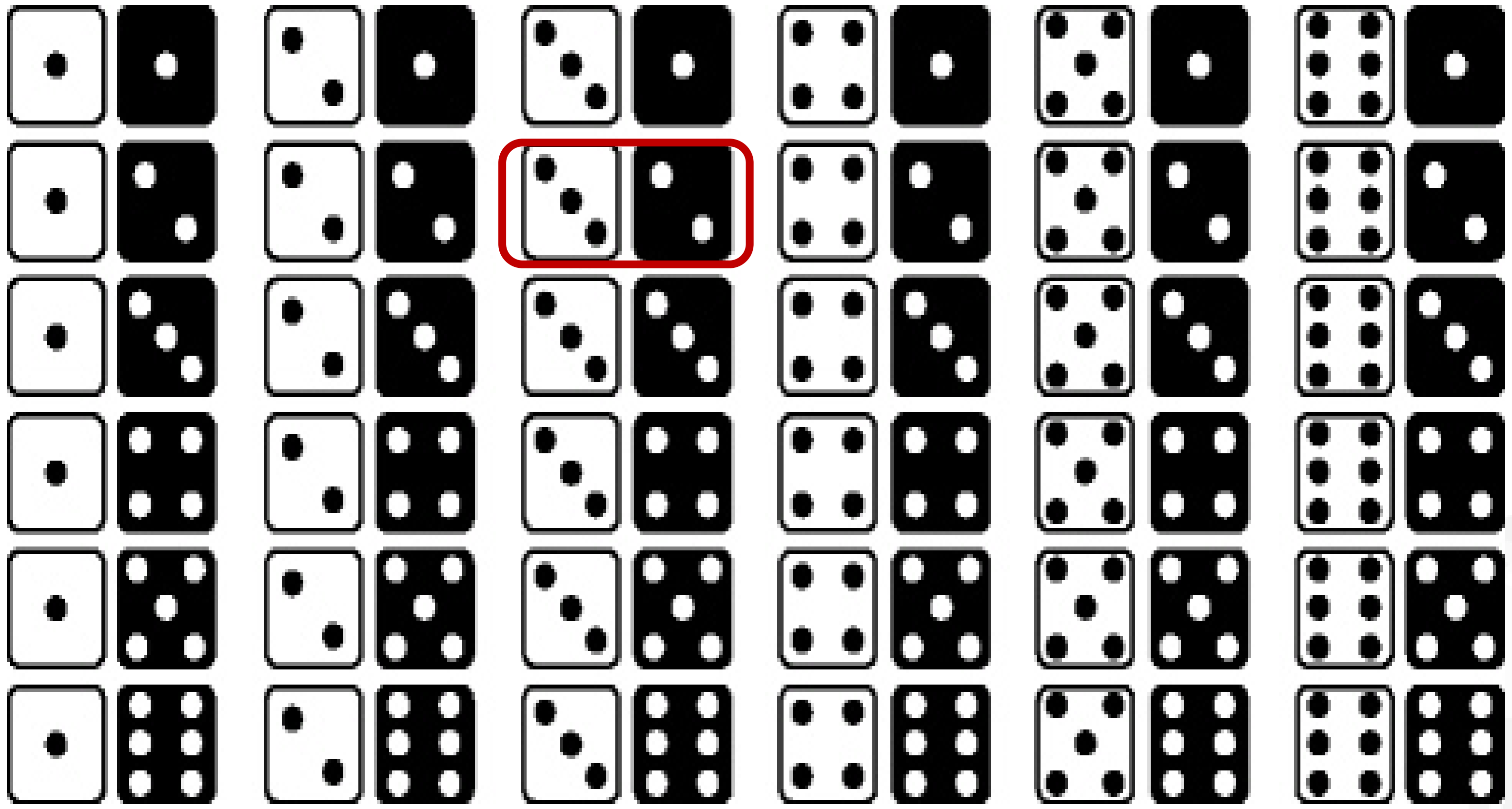
- There is a probability of one of two possible outcomes, head or tail.

$$P(H) = \frac{f}{N} = \frac{1}{2}$$

Probability: Example

What is the probability of getting a three followed by two when the dice are thrown twice?

Experiment: Rolling 1 dice twice



Desired event: roll 1 = 3; roll 2 = 2

The probability of getting a 3 followed by 2 can happen only in one way:

$$\text{Probability of an event} = \frac{\text{Total number of events}}{\text{Total number of } outcomes} = \frac{1}{36} = 2.7\%$$

Probability: Basic Operations

An Event is a set of elementary outcomes.

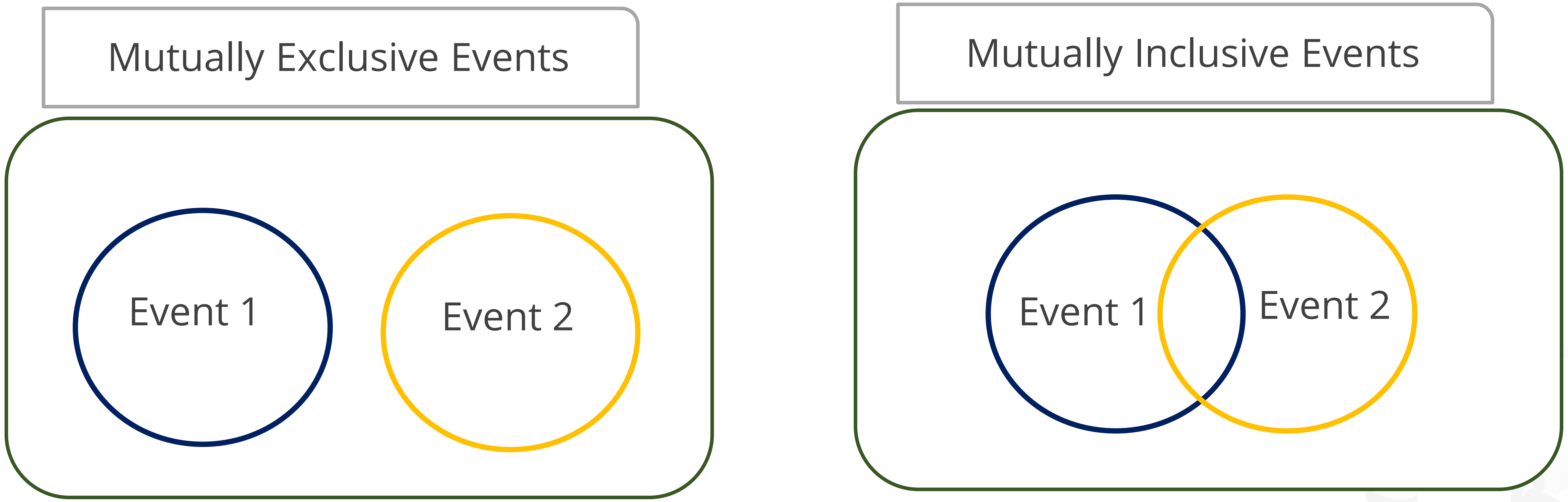
The Probability of an Event = Sum of the Probabilities of the Outcomes in the Set

Example: Some events of rolling a dice twice.

Event	Elementary Outcomes	Probability
A: Dice add up to 3	{ (1,2),(2,1) }	P(A) = 2/36 = 5.5%
B: Second roll is a 2	{ (1,2), (2,2), (3,2), (4,2), (5,2), (6,2) }	P(B) = 6/36 =16.7%

We can combine events to make other events using AND, OR, or NOT.

Probability Concepts

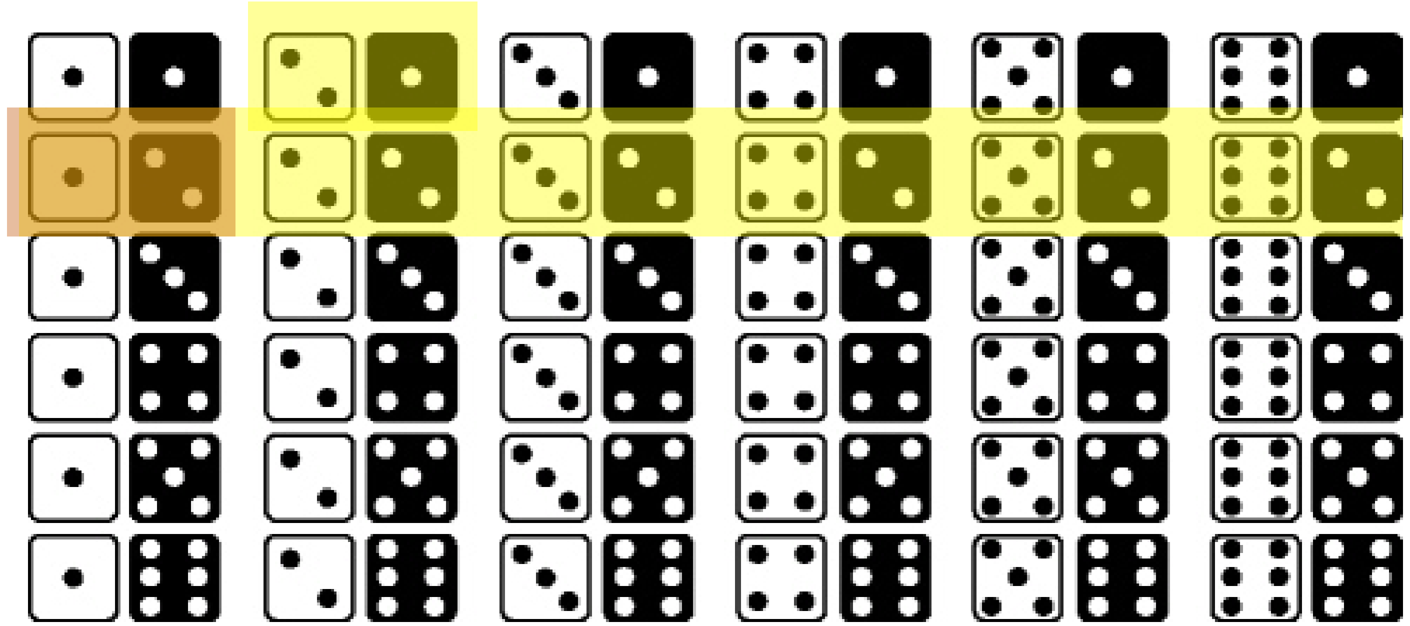


Relates to the Addition or OR rule

Addition Rule for Mutually Inclusive Events

In the scenario from the previous screen:

Event A: Dice add up to 3 ; Event B: Second roll is a 2.



- **A OR B** is the entire shaded area where either event occurs.
- **A AND B** is the darker shaded area where both shaded areas overlap (dice add up to 3 and second roll is a 2).

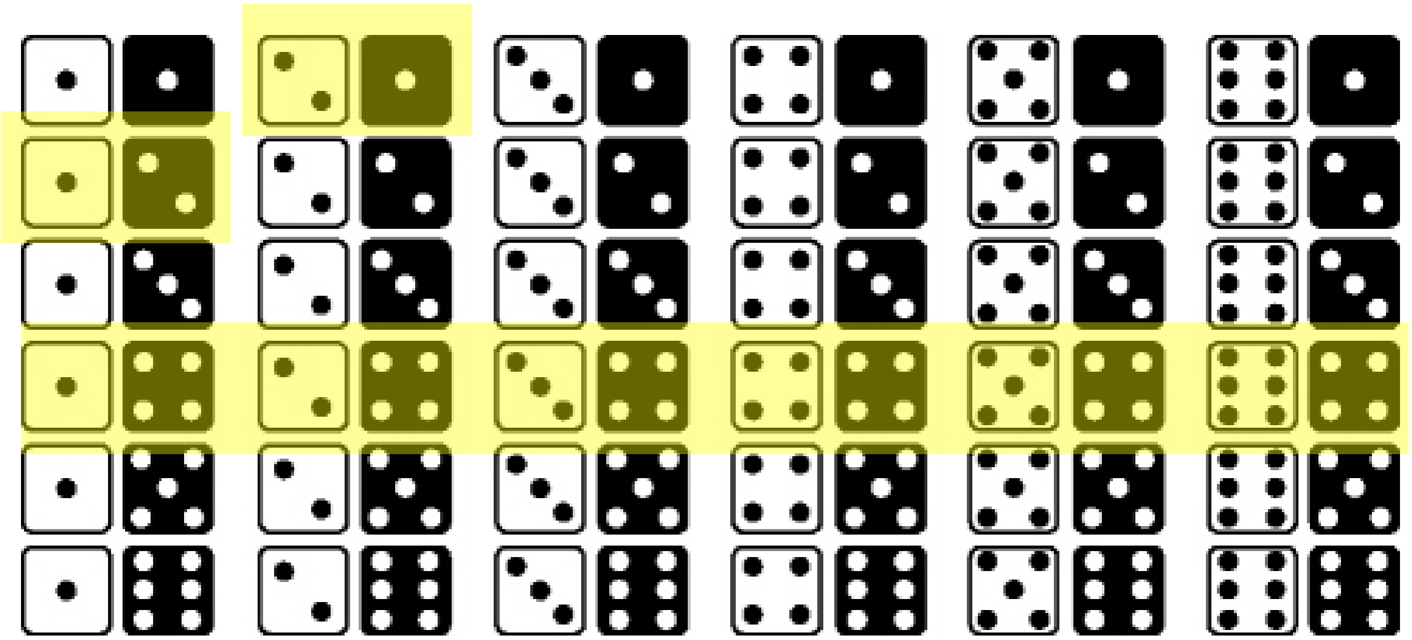
P(A or B) = P(A) + P(B) – P (A and B)

ADDITION RULE for Mutually Inclusive Events

2/36 + 6/36 – (1/36) = 7/36 = 19.4%

Addition Rule for Mutually Inclusive Events

Sometimes, there is no overlap shared between outcomes between A and B.
Event A: Dice add up to 3 ; Event B: Second roll is a 4.



- A OR B is the entire shaded area where either event occurs.

P(A or B) = P(A) + P(B)

ADDITION RULE for Mutually Exclusive Events

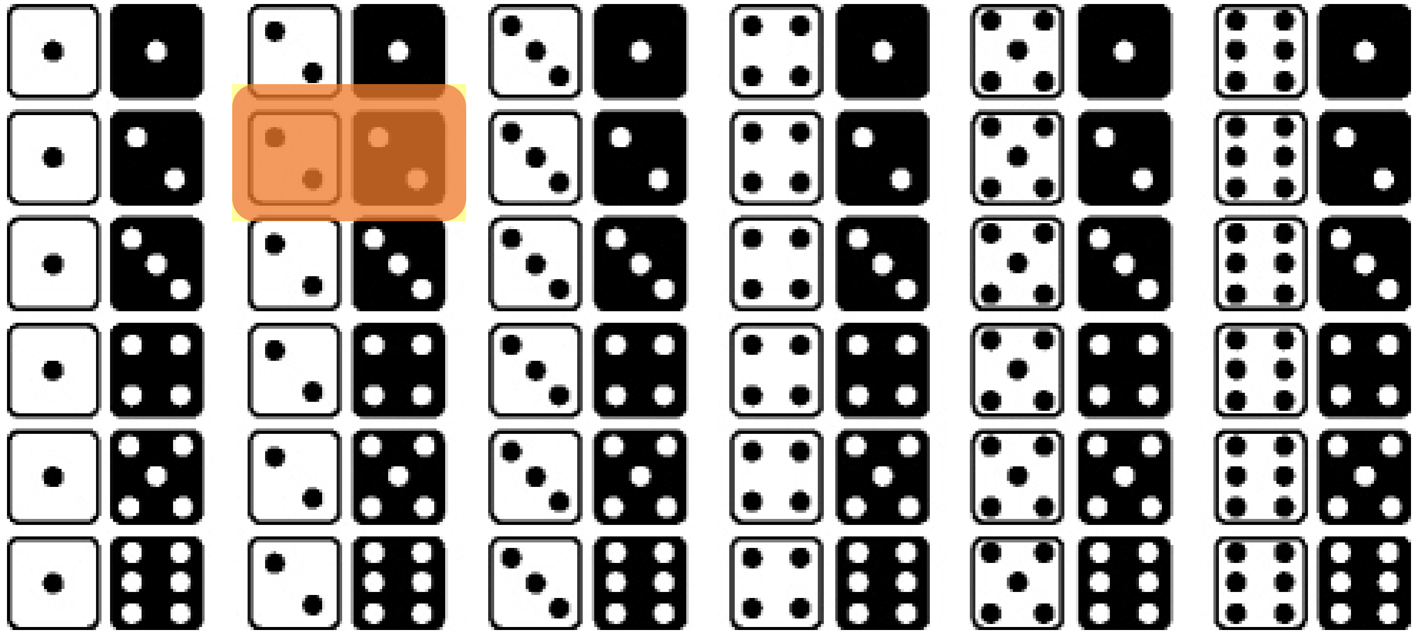
2/36 + 6/36 = 8/36 = 22.2%

simplilearn

Subtraction Rule Or Not Rule

Used when it's easier to calculate the opposite probability

Event A: Double 2's NOT thrown (The event "NOT A" is a Double 2 is thrown.)

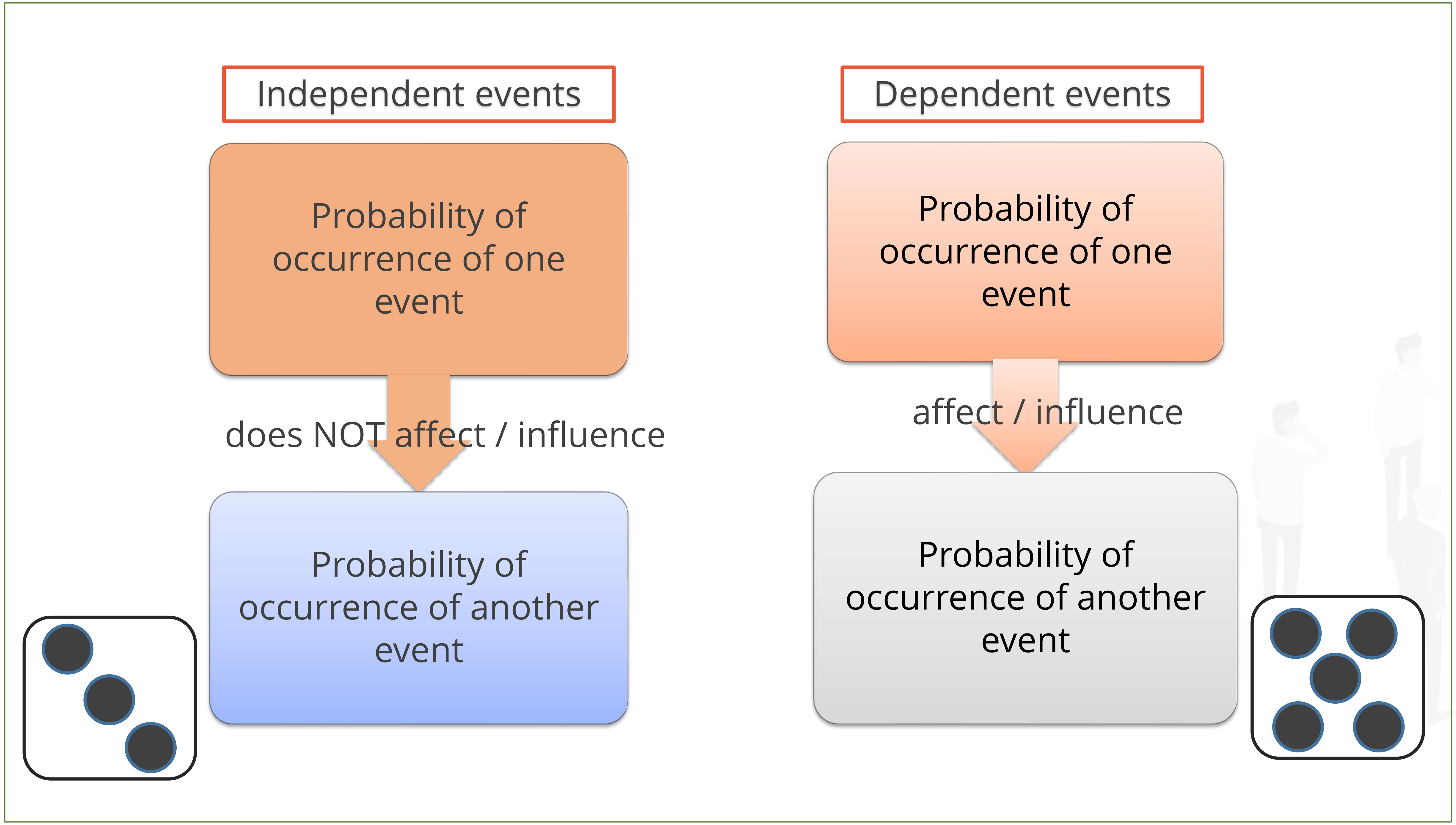


P(A) = 1 - P(NOT A)

Subtraction RULE

1 – 1/36 = 35/36 = 97.2%

Probability Concepts



Relates to the Multiplication or AND rule

Multiplication Rules for Independent Events

The Multiplication Rules or AND rules depend on the event dependency.

Example

If events A, B, C, . . . are independent

Special multiplication rule is applied

$$P\ (A\ \&\ B\ \&\ C\ \&\ \ldots) = P(A) * P(B) * P(C)\ldots$$

Multiplication Rules for Independent Events: Example

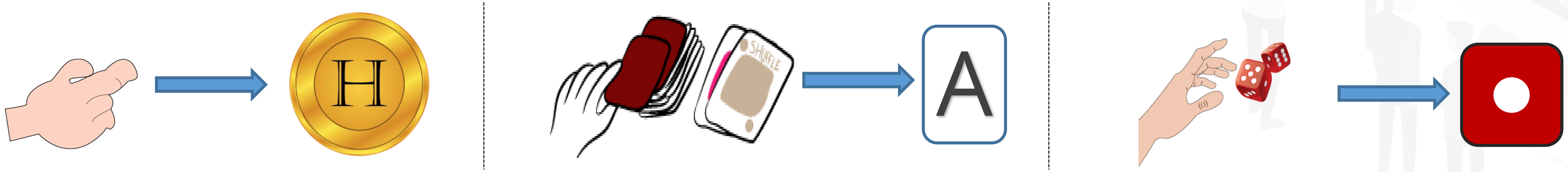
What is the probability of occurrence of all the following events?

Flip a coin and get a head, draw a card and get an ace, and throw a dice and get a 1.

P(A & B & C) = P(A) ∗ P(B) ∗ P(C)

$$= \frac{1}{2} * \frac{1}{13} * \frac{1}{6}$$

$$= 0.0064 = 0.64\%$$



Second Rule For Dependent Variables

Example

If A and B are two dependent events

General multiplication rule is applied

$$P(A \cap B) = P(A) * P(B|A)$$

$$P(B|A) = P(B \text{ and } A)/ P(A)$$

Second Rule For Dependent Variables: Example

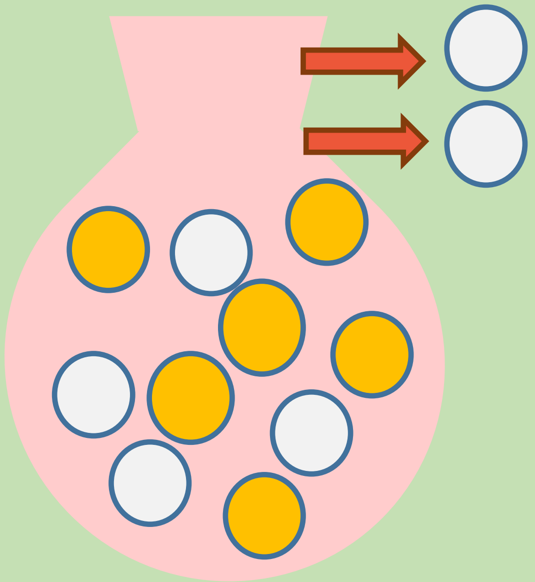
A bag contains 6 golden coins and 4 silver coins. Two coins are drawn without replacement from the bag. What is the probability that both of the coins are silver?

$P(A) = \frac{4}{10}$; $P(B|A) = \frac{3}{9}$

$P(A \cap B) = P(A) * P(B|A)$

$= \frac{4}{10} * \frac{3}{9} = \frac{12}{90}$

$= 0.1334 = 13.34\%$



Permutation

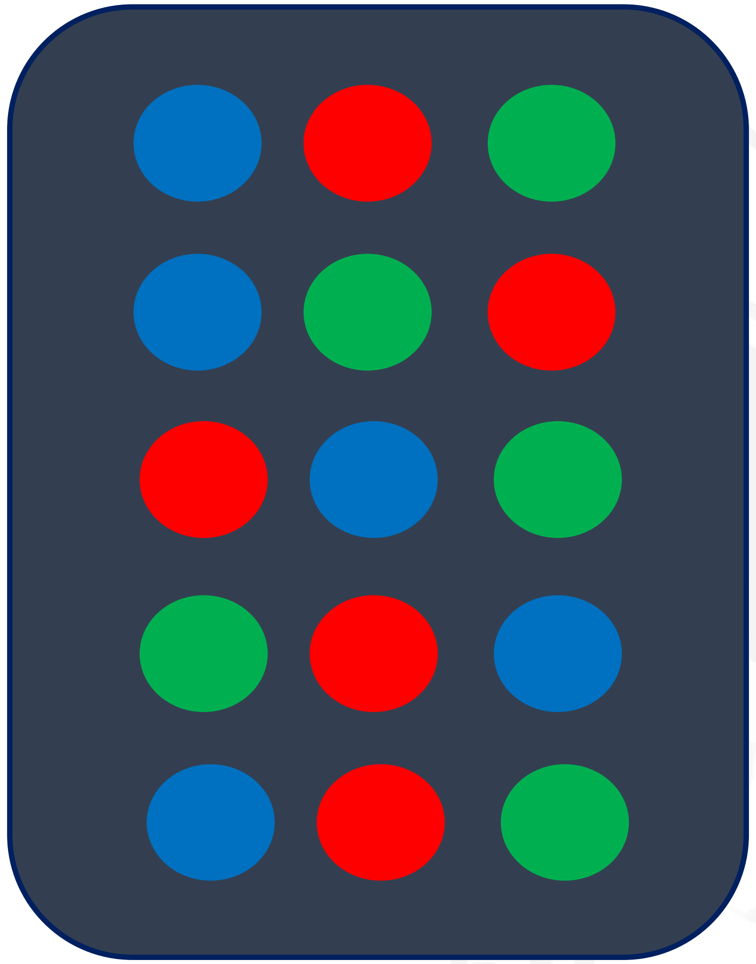
Permutation is the total number of ways in which a set, group, or number of things can be arranged.

$${}^{n}P_{r} = p\,(n, r) = \frac{n!}{(n-r)!}$$

n = number of total items

r = number of selections or group size

x! = x * (x-1) *(x-2)*…*(x-(x-1))



Combination

The unordered arrangement of a set, group, or number of things is known as combination.

$^{n}C_{r} = c(n, r) = \frac{n!}{r!\,(n-r)!}$

n = number of total items

r = number of selections or group size

x! = x * (x-1) *(x-2)*…*(x-(x-1))

Permutation and Combination: Example

Q

From a group of 10 employees, a company has to select 4 for a particular project. In how many ways can the selection happen, given the following conditions?

a. The arrangement of employees needs to be different due to 4 different team roles.

b. The arrangement of employees need not be different because each member will have the same role.

A

Here, n = 10 and r = 4

a. From a group of 10 employees, 4 employees need to be selected. The arrangement needs to be different.

$${}^{n}P_{r} = p(n, r) = \frac{n!}{(n-r)!} = {}^{10}P_{4} = p(10, 4) = \frac{10!}{(10-4)!} = 5040 \text{ ways}$$

[EXCEL "=PERMUT(10,4)" =5040]

b. From a group of 10 employees, 4 have to be selected. The arrangement of these 4 employees need not be different.

$${}^{n}C_{r} = c(n, r) = \frac{n!}{r!\,(n-r)!} = {}^{10}C_{4} = c(10, 4) = \frac{10!}{4!\,(10-4)!} = 210 \text{ ways}$$

[EXCEL "=COMBIN(10,4)" =210]

Use Excel functions "= PERMUT( )" and "= COMBIN( )"

Key Takeaways

- Probability refers to the chance of something occurring or happening.
- *P(E) = "Probability of an event"=* f/N
- When the probability of occurrence of an event does not affect the probability of the occurrence of another event, then the two events are said to be independent.
- When the probability of one event occurring influences the likelihood of another event, the events are said to be dependent.
- Permutation is the total number of ways in which a set, group, or number of things can be arranged.
- The unordered arrangement of a set, group, or number of things is known as combination.

Knowledge Check

Knowledge Check
1

You have two cars. The probability of each one starting is 60%. What is the likelihood that at least one car starts?

A. 36%

B. 64%

C. 84%

D. 24%

Knowledge Check 1	You have two cars. The probability of each one starting is 60%. What is the likelihood that at least one car starts?

A. 36%

B. 64%

C. 84%

D. 24%

The correct answer is **C**

P(A) is car 1 starting and P(B) is car 2 starting.

P (A OR B) = P(A) + P(B) – P(A AND B) = .6 + .6 – (.36) = 1.2 - .36 = .84 = 84%

Knowledge Check

2

You want to list three desserts from a menu of nine. How many different ways can this be done?

A. 504

B. 84

C. 27

D. 12

Knowledge Check 2

You want to list three desserts from a menu of nine. How many different ways can this be done?

A. 504

B. 84

C. 27

D. 12

The correct answer is **B**

This a combination problem since the order in which the desserts are listed is not of concern. Therefore, using the EXCEL function "=COMBIN(9,3)", the resulting value is 84.

Knowledge Check

3

You want to list your three favorite desserts in order from a menu of 11. How many different ways can this be done?

A. 990

B. 165

C. 33

D. 14

Knowledge Check
3

You want to list your three favorite desserts in order from a menu of 11. How many different ways can this be done?

A. 990

B. 165

C. 33

D. 14

The correct answer is **A**

This a permutation problem because the order that the desserts are listed in is of concern. Therefore, the EXCEL function to be used is "=PERMUT(11,3)", which results in the value 990.