Linear Algebra: Linear Combinations and Span

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Linear combination

A *linear combination* is a sum of vectors $v_1 \dots v_n \in \mathbb{R}^m$, with each multiplied by constants $c_1 \dots c_n \in \mathbb{R}$. For example, say we have two vectors:

$$\hat{\mathbf{a}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}}$$
$$\hat{\mathbf{b}} = 3\hat{\mathbf{j}}$$

For example, here are a few valid linear combinations of these vectors.

$$0\hat{\mathbf{a}} + 0\hat{\mathbf{b}} = \mathbf{0}$$
$$3\hat{\mathbf{a}} + (-2)\hat{\mathbf{b}} = 3\hat{\mathbf{i}}$$

Span

The *span* is the set of all the vectors you can represent by adding and subtracting vectors.

$$\text{span}(v_1, v_2, \dots v_n) = \{c_1 v_1 + c_2 v_2 \dots c_n v_n \mid c_i \in \mathbb{R}\}$$

For example with vectors $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$:

$$\text{span}(\hat{\mathbf{a}}, \hat{\mathbf{b}}) = \mathbb{R}^m$$

If two vectors are collinear, their span is **not** all of $\mathbb{R}^m$! It is $c\hat{\mathbf{a}}$.

$$\text{span}(\mathbf{0}) = \mathbf{0}$$

The most familiar vectors that span the $\mathbb{R}^2$ vector space are the unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$. These form the *basis* of $\mathbb{R}^2$.