Linear Algebra: Null Space and Column Space

Steven Schmatz, University of Michigan: College of Engineering

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stevenschmatz@gmail.com

Matrix vector products

If you have an $m \times n$ matrix, there are simply m rows and n columns:

$$\mathbf{A} = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{bmatrix}$$

If the number of components in a vector is equal to the number of *columns* in a matrix, then you can multiply matrices and vectors.

$$\mathbf{A}\hat{x} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \dots a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots a_{mn}x_n \end{bmatrix} = \hat{b}$$

It's essentially the dot product of each row vector with the matrix $\hat{x}$! For example,

$$\begin{bmatrix} -3 & 0 & 3 & 2 \\ 1 & 7 & -1 & 9 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} -6 + 0 + 12 - 2 \\ 2 - 21 - 4 - 9 \end{bmatrix} = \begin{bmatrix} 4 \\ -32 \end{bmatrix}$$