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Adaptive Kalman Filtering for INS/GPS

A. H. Mohamed, K. P. Schwarz

Department of Geomatics Engineering, The University of Calgary, 2500 University Drive NW, Calgary, Alberta, Canada T2N 1N4

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Abstract. After reviewing the two main approaches of adaptive Kalman filtering, namely, innovation-based adaptive estimation (IAE) and multiple-model-based adaptive estimation (MMAE), the detailed development of an innovation-based adaptive Kalman filter for an integrated inertial navigation system/global positioning system (INS/GPS) is given. The developed adaptive Kalman filter is based on the maximum likelihood criterion for the proper choice of the filter weight and hence the filter gain factors. Results from two kinematic field tests in which the INS/GPS was compared to highly precise reference data are presented. Results show that the adaptive Kalman filter outperforms the conventional Kalman filter by tuning either the system noise variance–covariance (V–C) matrix 'Q' or the update measurement noise V–C matrix 'R' or both of them.

Key words. Adaptive · Kalman filtering · GPS/INS

1 Introduction

In this section, an overview of the inertial navigation system/global positioning system (INS/GPS) integration problem is presented and the reasons for adaptive Kalman filtering in this specific case are given.

The integration of an INS with the GPS for kinematic applications in the geomatics engineering field has been implemented for almost two decades through the use of conventional Kalman filtering and fixed or semi-fixed integration algorithms; see e.g. Britting (1971), Schwarz (1983), Wong (1988), Wei and Schwarz (1990), Schwarz (1991), Knight (1996), and Chatfield (1997). The fixed integration formulation has shown success in fulfilling the accuracy requirements of many kinematic applications. There were, however, always applications where the accuracy requirements could not be fulfilled or could not be fulfilled at all times. Examples are precise engineering and cadastral applications requiring an root mean square (rms) of 5–10 cm in position and 10 arc-seconds in attitude. In these applications, the change in receiver–satellite geometry or in trajectory geometry or dynamics could be readily seen in the data. It seemed, therefore, reasonable to investigate the question whether algorithms that reflect these changes in an adaptive manner would not result in a better overall performance.

The problem of achieving better performance (reliability and accuracy) of integrated INS/GPS systems can be divided into two parts, a modeling problem and an estimation problem. While the modeling problem is concerned with developing better error models that more accurately describe the INS/GPS system, the estimation problem is concerned with achieving better trajectory and sensor error estimates through the proper use of the available process and measurement information. The optimality of the estimation algorithm in the Kalman filter setting is closely connected to the quality of the a priori information about the process noise and the update measurement noise (Kalman 1960; Gelb 1988; Brown and Hwang 1992). Conceptually, a good a priori knowledge of the process and measurement information depends on factors such as the type of application and the process dynamics, which are difficult to obtain. Also, the estimation environment in the case of INS/GPS kinematic applications is not always fixed but is subject to change.

Insufficiently known a priori filter statistics will on the one hand reduce the precision of the estimated filter states or introduce biases to their estimates (robustness) (Toda et al. 1967). In addition, wrong a priori information will lead to practical divergence of the filter. For example: if R and/or Q are too small at the beginning of the estimation process, the uncertainty tube around the true value, in a probabilistic sense, will tighten and a

Correspondence to: K.P. Schwarz. e-mail: schwarz@ensu.calgary.ca; Tel.: +1 403 220 7377; Fax: +1 403 284 1980

biased solution will result. If R and/or Q are too large, filter divergence, in the statistical sense, could result. In addition, it will result in a longer estimation transition for the filter. Also, insufficiently known a priori statistics will, in many cases, lead to an inadequate estimation of weak observable components in the filter. For the problem at hand, accelerometer biases and gyro drifts are such components. Since their estimation has a direct effect on the estimation of the main filter components (position, velocity, and attitude) through the coupling effect, this problem is serious. Insufficient a priori information and a frequently changing estimation environment affect the accuracy of the integrated INS/GPS system. This implies that using a fixed filter designed by conventional methods is a major drawback in a changing dynamics environment.

From this point of view, the fixed estimation formulation should be replaced by an adaptive estimation formulation with an adaptive integration throughout the INS/GPS trajectory estimation process. It can be expected that with the adaptive integration scheme better performance for INS/GPS systems can be achieved. The main advantage of the adaptive technique is its weaker reliance on the a priori statistical information. An adaptive filter formulation, therefore, tackles the problem of imperfect a priori information and provides a significant improvement in performance over the fixed filter through the filter learning process based on the innovation sequence (Mehra 1970; 1971). In this case, perfect knowledge of the a priori information is only of secondary importance because the new measurement and process covariance matrices are adapted according to the filter learning history. Also, the frequent adaptation of the statistical filter information, through the filter innovation sequence, goes hand in hand with the idea of having a dynamic system in a dynamic environment.

The objective of this contribution is to introduce the adaptive Kalman filter as an alternative for use with INS/GPS systems. In this contribution, the general layout of the adaptive Kalman filtering problem and its maximum-likelihood (ML) solution will be given. Results from field tests will be used to illustrate the concept and to show a case in which the adaptive Kalman filter outperforms the conventional one.

The two approaches to the adaptive Kalman filtering problem are multiple-model-based adaptive estimation (MMAE) and innovation-based adaptive estimation (IAE). While in the former a bank of Kalman filters runs in parallel under different models for the filter's statistical information, in the latter the adaptation is done directly to the statistical information matrices R and/or Q based on the changes in the innovation sequence; see Sect. 2 for details.

The MMAE has its application in the design of controllers for flexible vehicles, tracking problems, and failure and interference/jamming and spoofing detection. For example, in White (1996) the use of MMAE in detecting the interference, jamming and spoofing in a DGPS-aided inertial system is discussed (see also White et al. 1996). While in standard MMAE only constant parameters are assumed, in moving-bank MMAE time-varying adaptive parameters are permissible (Maybeck 1989). In Magill (1965), a parallel-filter scheme is suggested which is used in Girgis and Brown (1985) to classify faults in a three-phase transmission line. A similar technique is used in Levy (1996) to adapt filter parameters for the purpose of system identification.

The use of IAE, however, is more applicable to INS/GPS systems used in the geomatics field. In Salychev (1993, 1994), a scalar adaptive estimator based on the ML estimation principle is described. The resulting algorithm has been used in a real-time INS/GPS system to detect sensor failure and abrupt changes. It is also used in an INS/GPS airborne gravity system to achieve better accuracy estimating the gravity anomaly. In case of GPS only, an IAE algorithm based on the adaptation of the measurement covariance matrix is proposed in Wang et al. (1997), to improve the reliability of the phase ambiguity resolution. At the University of Calgary, a full-scale IAE for INS/GPS systems is under development and is used in estimating the trajectory for mobile georeferencing and airborne gravity systems.

The outline of the remainder of the paper is as follows. In Sect. 2, the adaptive Kalman filtering problem is overviewed and the MMAE and IAE approaches are briefly discussed. A description of the mathematical model and the development of the IAE filter used at the University of Calgary are the subject of Sect. 3. In Sect. 4, results from two field tests in a controlled environment are presented to illustrate the concept of the developed IAE filter. Section 5 contains a summary and conclusions.

2 Adaptive Kalman filtering

The emphasis of this section is on the general concept of adaptive Kalman filtering. The principles of the MMAE and IAE approaches are briefly overviewed. The mathematical notation used in this paper follows that used in Gelb (1988) with few additions.

In the context of adaptive Kalman filtering, the uncertain parameters that need to be adapted may be part of the system model through the state transition matrix Φ, the measurement design matrix H, or the statistical information through the variance–covariance (V–C) matrices R/Q. The first case is more likely to occur in problems where system design/identification is of concern. In this development, it is assumed that the used INS/GPS system model is sufficient for the intended applications. The optimization of the filter performance will be done through the adaptive estimation of the filter statistical information, the V–C matrices. Therefore, the discussion in the following will be restricted to the problem of adapting the filter V–C matrices without questioning the system modeling.

The two approaches to adaptive Kalman filtering, namely, MMAE and IAE, presented in the following, share the same concept of utilizing the new information in the innovation (or residual) sequence, but differ in their implementation. The innovation sequence v_k at epoch k in the Kalman filter algorithm is the difference between the real measurement z_k received by the filter and its estimated (predicted) value $z_k(-)$, and is computed as follows:

$$v_k = z_k - z_k(-) \tag{1}$$

The predicted measurement is computed by projecting the filter predicted states $x_k(-)$ onto the measurement space through the measurement design matrix H_k, i.e.

$$z_k(-) = H_k \hat{x}_k(-) \tag{2}$$

At the current time k, the new observation z_k does not really provide completely new information because some of the information is obtained by prediction from previous filter states, $z(-)$. On the other hand, the values of the innovation v_i at different instants are, in principle, uncorrelated. In other words, the value of the innovation v_i at the current epoch k cannot be predicted from previous values of it, and therefore each observation v_i brings *new* information. Hence, the innovation sequence represents the information content in the new observation and is considered as the most relevant source of information for the filter adaptation; the interested reader is directed to Genin (1970) and Kailath (1972, 1981) for a more detailed discussion of the innovation sequence and its use in linear filter theory.

2.1 Multiple model adaptive estimation

In the multiple model adaptive estimation (or parallel-filter) approach (Magill 1965; Maybeck 1989; Brown and Hwang 1992; Gary and Maybeck 1996; White 1996; White et al. 1996), a bank of Kalman filters runs in parallel under different models for the statistical filter information matrices, i.e. the process noise matrix Q and/or the update measurement noise matrix R. The structure of each filter in the bank of filters is depicted in Fig. 1 and the final estimate of the bank of filters is explained in Fig. 2.

In every run, each filter of the bank will have its own estimate $\hat{x}_k(\alpha_i)$. At the first epoch, the bank of filters receives the first measurement z_0, and the $P(z_0|\alpha_i)$ distribution is computed for each permissible α_i. At each recursive step the adaptive filter does three things, as follows.

1. First, each filter in the bank of filters computes its own estimate, which is hypothesized on its own model.
2. Second, the system computes the a posteriori probabilities for each of the hypotheses.
3. Finally, the scheme forms the adaptive optimal estimate of x as a weighted sum of the estimates produced by each of the individual Kalman filters as

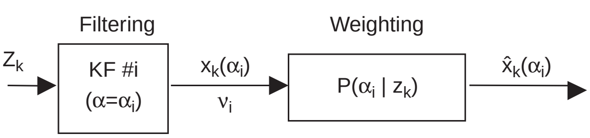

Fig. 1. The estimate of the ith filter in the MMAE

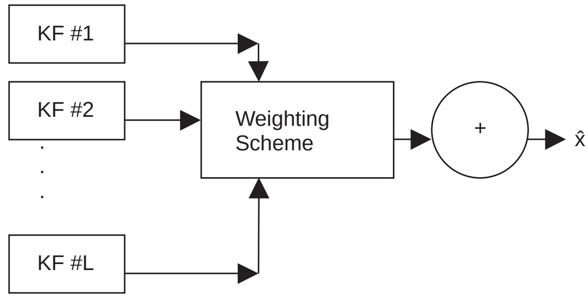

Fig. 2. The estimate of the bank of filters

$$\hat{x}_k = \sum_{i=1}^{L} \hat{x}_k(\alpha_i) P(\alpha_i|z_k) \tag{3}$$

where $P(\alpha_i|z_k)$ is the weight of the ith filter when measurements z_k up to epoch k are available; α_i is an unknown random variable with known statistical distribution $P(\alpha_i)$, which drives the adaptive process of the filter, and L is the total number of filters used.

As measurements evolve with time, the adaptive scheme learns which of the filters is the correct one, and its weight factor approaches unity while the others are going to zero. The bank of filters accomplishes this, in effect, by looking at the sums of the weighted squared measurement innovations or residuals. The filter with the smallest sum prevails.

2.2 Innovation-based adaptive estimation

In the innovation-based adaptive estimation (IAE) approach window (Mehra 1970, 1971; Kailath 1972; Maybeck 1982; Salychev 1994), the covariance matrices R_k and Q_k themselves are adapted as measurements evolve with time. Based on the whiteness of the filter innovation sequence, the filter statistical information matrices are adapted as follows:

$$\hat{R}_k = \hat{C}_{v_k} - H_k P_{k(-)} H_k^T \tag{4}$$

and

$$\hat{Q}_k = K_k \hat{C}_{v_k} K_k^T \tag{5}$$

where $P_{k(-)}$ and K_k are the predicted covariance of state matrix and gain matrix, respectively. Knowing the innovation sequence, Eq. (1), one can compute the innovation V–C matrix, $\hat{C}_{v_k}$, at epoch k, through averaging inside a moving estimation window of size N, as

$$\hat{C}_{v_k} = \frac{1}{N} \sum_{j=j_0}^{k} v_j v_j^T \tag{6}$$

where $j_0 = k - N + 1$ is the first epoch inside the estimation. In order to account for such an adaptive approach in the Kalman filter algorithm, an additional block for computing the innovation V–C matrix and both Q and R is needed, as shown in Fig. 3. Details of this approach will be discussed in Sect. 3.

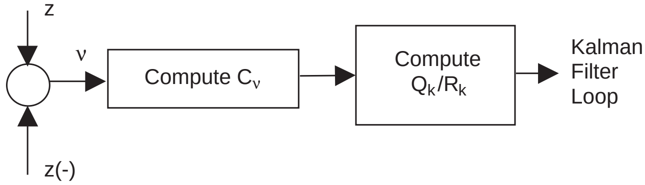

Fig. 3. The innovation process in the adaptive Kalman filter algorithm

3 Innovation-based adaptive Kalman filtering

The ML concept is used in this section to derive the innovation-based adaptive Kalman filter which is used for the development at the University of Calgary. Some development and implementation aspects are briefly discussed.

3.1 Maximum likelihood estimator of innovation-based adaptive Kalman filter

Adaptive Kalman filtering is one of the methods which is not a simple extension of conventional least-squares (LS) estimation, widely used in geomatics and in many other engineering fields. The reason for that is that LS aims at estimating and modifying the first moment information (the mean), while in adaptive Kalman filtering the adaptation of the second moment information (the variance/covariance) is also of concern. It is worth mentioning that some authors, e.g. Haykin (1996), like to classify the conventional Kalman filter among the adaptive techniques based on its property of sequentially modifying the filter states V–C matrix P which is in essence an adaptation to the filter tap weights according to Wiener theory. This, however, is not our intention in this paper. By adaptive, we mean, imposing conditions under which the filter statistical information matrices R and/or Q, which are considered constant in the conventional Kalman filter, are estimated via the available new information in the filter innovation sequence. For this reason, the formulas are derived in the ML setting, which is more suitable for the problem formulation. The suitability of the ML technique stems from the fact that for the case of independent and identically distributed measurements, an unbiased estimate with finite covariance can always be found through the ML method such that no other unbiased estimate with a lower covariance exists (Cramér 1946). The INS/GPS measurements are assumed to be independent with identical (usually Gaussian) distribution. The other attractive property of the ML estimate is its uniqueness and its consistency. Uniqueness, the first property, means that only one solution is the outcome of the ML formulation; consistency, on the other hand, means that the ML estimate converges, in a probabilistic sense, to the true value of the variable as the number of sample data grows without bound. The ML estimate, however, will in general be biased for small sample sizes. Notwithstanding, it will generally provide the unique minimum attainable variance estimate under the existence of sufficient statistics. It should be noted, here, that the minimum variance formulation not only suffers sever analytical difficulties when handling this problem, but also will, in general, result in a biased estimate for small sample sizes. The sample size puts additional restriction on the choice of the estimation window size, which will be discussed in Sect. 4.1; the interested reader is directed to Cramér (1946) and Maybeck (1982) for more details.

In this development, the specific case of a fixed-length memory (windowing) filter for INS/GPS kinematic positioning will be considered. In addition, the V–C matrices containing the statistics are to be adapted and not the filter states. Therefore, the underlying assumptions to the ML adaptive Kalman filtering problem are as follows.

1. The filter states x are independent of the adaptive parameters α, i.e. $\partial x/\partial \alpha = 0$.
2. The filter transition matrix Φ and design matrix H are time invariant and independent of α.
3. The innovation sequence is a white and ergodic sequence within the estimation window.
4. The covariance matrix C_v (through v) is the key to adaptation and hence is the α-dependent parameter.

Further, the case will be considered where the data is Gaussian distributed. According to the central limit theorem, if the random phenomenon we observe is generated as the sum of effects of many independent infinitesimal random phenomena, then the distribution of the observed phenomenon approaches a Gaussian distribution as more random effects are summed, regardless of the distribution of each individual phenomenon. Therefore, our assumption of Gaussian distribution of the data is not restricting. In this case, the probability density function of the measurements conditioned on the adaptive parameter α at the specific epoch k is

$$P_{(z|\alpha)_k} = \frac{1}{\sqrt{(2\pi)^m |C_{v_k}|}} e^{-\frac{1}{2} v_k^T C_{v_k}^{-1} v_k} \tag{7}$$

where m is the number of measurements, $|\cdot|$ is the determinant operator, and e is the natural base. To simplify the above equation, its logarithmic form is taken

$$\ln P_{(z|\alpha)_k} = -\frac{1}{2}\{m * \ln(2\pi) + \ln(|C_{v_k}|) + v_k^T C_{v_k}^{-1} v_k\} \tag{8}$$

Note that after multiplying Eq. (8) by -2, the ML criterion of maximizing P becomes the minimization of the resulting right-hand side of the same equation. Also, for a fixed-length memory filter, the innovation sequence will only be considered inside a window of size N; all innovations inside the estimation window will be summed. After multiplying them by -2, summation, and neglecting the constant term, the ML condition becomes

$$\sum_{j=j_0}^{k} \ln |C_{v_j}| + \sum_{j=j_0}^{k} v_j^T C_{v_j}^{-1} v_j = \min \tag{9}$$

It is worth mentioning here that k in the above formula represents the epoch number at which estimation takes place, while j is the moving counter inside the estimation window.

In conventional LS, only the second term of Eq. (9) is considered, which corresponds to the error norm in the L2 space. Minimizing that norm with respect to the state vector will result in the optimal states estimate (see e.g. Sorenson 1970; Swerling 1971; Kailath 1972, 1974 for a discussion of the LS method). This, however, is different for Eq. (9). The V–C matrix of the innovation sequence C_v, not the innovation sequence itself, is dependent on the adaptive parameter α, and is the key to adaptation. So, in terms of C_v, the above formula represents a condition for the decision to choose the error weight, not the state optimal estimate. In other words, while the LS problem aims at finding the smallest error norm according to a predefined weight, the above ML problem aims at finding the weight that will result in the smallest error norm. This means that the adaptive estimation of the weight is complementary to the state estimation.

The above formula, then, describes the best estimate as the one that has the maximum likelihood based on the adaptive parameter α. Matrix differential calculus will be used to obtain the derivative of Eq. (9) and equate it to zero. The formula

$$\partial P/\partial\alpha = 0$$

results in

$$\sum_{j=j_0}^{k}\left[\mathrm{tr}\left\{C_{v_j}^{-1}\frac{\partial C_{v_j}}{\partial\alpha_k}\right\} - v_j^T C_{v_j}^{-1}\frac{\partial C_{v_j}}{\partial\alpha_k}C_{v_j}^{-1}v_j\right] = 0 \tag{10}$$

where tr is the matrix trace operator. To obtain the above formula, the following two relations from matrix differential calculus have been used (Maybeck 1972; Rogers 1980; Golub and Loan 1989):

$$\frac{\partial \ln|A|}{\partial x} = \frac{1}{|A|}\frac{\partial|A|}{\partial x} = \mathrm{tr}\left\{A^{-1}\frac{\partial A}{\partial x}\right\}$$

and

$$\frac{\partial A^{-1}}{\partial x} = -A^{-1}\frac{\partial A}{\partial x}A^{-1}$$

It is clear, from Eq. (10), that the problem of adaptive Kalman filtering is reduced to the problem of determining C_v and its partial derivative with respect to α. Since there is little interest in C_v itself, but rather in R and Q, the following substitution will be made (see e.g. Gelb 1974; Brown and Hwang 1992):

$$C_{v_k} = R_k + H_k P_k(-)H_k^T \tag{11}$$

The partial derivative of Eq. (11) with respect to α yields

$$\frac{\partial C_{v_k}}{\partial\alpha_k} = \frac{\partial R_k}{\partial\alpha_k} + H_k\frac{\partial P_k(-)}{\partial\alpha_k}H_k^T \tag{12}$$

It is also known that

$$P_{k(-)} = \Phi P_{k-1^{(+)}}\Phi^T + Q_k \tag{13}$$

which after differentiation with respect to α yields

$$\frac{\partial P_{k(-)}}{\partial\alpha_k} = \Phi\frac{\partial P_{k-1^{(+)}}}{\partial\alpha_k}\Phi^T + \frac{\partial Q_k}{\partial\alpha_k} \tag{14}$$

Assuming that the process inside the estimation window is in steady state, the first term can be neglected and Eq. (14) can be rewritten as

$$\frac{\partial P_{k(-)}}{\partial\alpha_k} = \frac{\partial Q_k}{\partial\alpha_k} \tag{14a}$$

Substituting Eq. (14a) into Eq. (12) results in

$$\frac{\partial C_{v_k}}{\partial\alpha_k} = \frac{\partial R_k}{\partial\alpha_k} + H_k\frac{\partial Q_{k-1}}{\partial\alpha_k}H_k^T \tag{15}$$

Now, substitute Eq. (15) into Eq. (10) and expand it. The resulting expression, Eq. (16), is the ML equation for the adaptive Kalman filter

$$\sum_{j=j_0}^{k}\mathrm{tr}\left\{\left[C_{v_j}^{-1} - C_{v_j}^{-1}v_j v_j^T C_{v_j}^{-1}\right]\left[\frac{\partial R_j}{\partial\alpha_k} + H_j\frac{\partial Q_{j-1}}{\partial\alpha_k}H_j^T\right]\right\} = 0 \tag{16}$$

Equation (16) shows that both R and Q can be adapted based on α.

3.2 Adaptive estimation of the measurement noise matrix R

In order to obtain an explicit expression for R, it is assumed that Q is completely known and independent of α. The case where $\alpha_i = R_{ii}$ will be considered, where i is the matrix row or column index; i.e. the adaptive parameters are the variances of the update measurements. This is a situation frequently encountered in practice. In this specific case, the adaptive Kalman filter, Eq. (16), reduces to

$$\sum_{j=j_0}^{k}\mathrm{tr}\left\{\left[C_{v_j}^{-1} - C_{v_j}^{-1}v_j v_j^T C_{v_j}^{-1}\right][I+0]\right\} = 0$$

which after expansion becomes

$$\sum_{j=j_0}^{k}\mathrm{tr}\left\{C_{v_j}^{-1}\left[C_{v_j} - v_j v_j^T\right]C_{v_j}^{-1}\right\} = 0 \tag{17}$$

From the above formula and under the assumption of an ergodic innovation sequence inside the estimation window, the expression for the estimated V–C matrix of the innovation sequence as in Eq. (6) can be obtained. Substituting C_v from Eq. (6) into Eq. (11), the innovation-based adaptive estimate of R of Eq. (4) is obtained. It is repeated here for convenience.

$$\hat{R}_k = \hat{C}_{v_k} - H_k P_k(-)H_k^T$$

A similar expression using the residual sequence instead of the innovation sequence can also be derived. It is computed as follows (see Appendix A for derivation):

$$\hat{R}_k = \hat{C}_{v_k} + H_k P_k(+) H_k^T \quad (18)$$

where

$$\hat{C}_{v_k} = \frac{1}{N} \sum_{j=j_0}^{k} v_j \, v_j^T \quad (19)$$

and the residual sequence

$$v_k = z_k - z_k(+) \quad (20)$$

where $z_k(+)$ is the predicted measurement based on the updated filter states and is computed as follows:

$$z_k(+) = H\hat{x}_k(+) \quad (21)$$

Judging by the results presented in Sect. 4, this estimator of R has proven to be numerically more suitable for the case of INS/GPS systems.

3.3 Adaptive estimation of the system noise matrix Q

The same strategy used for R will also be used to obtain an estimate of Q. In Eq. (16), R will be considered to be completely known and independent of α, i.e. its partial derivative with respect to α vanishes. Taking $\alpha_i = Q_{ii}$, as in the case of R, Eq. (16) reduces to

$$\sum_{j=j_0}^{k} \mathrm{tr}\{H_j^T [C_{v_j}^{-1} - C_{v_j}^{-1} v_j v_j^T C_{v_j}^{-1}] H_j\} = 0 \quad (22)$$

which is transformed (see Appendix 2 for a proof) to

$$\hat{Q}_k = \frac{1}{N} \sum_{j=j_0}^{k} \Delta x_j \Delta x_j^T + P_k(+) - \Phi P_{k-1}(+) \Phi^T \quad (23)$$

where Δx is the state correction sequence (the difference between the state before and after updates) and is computed as

$$\Delta x_k = \hat{x}_k(+) - \hat{x}_k(-) \quad (24)$$

In steady state, considering only its first term and the relation

$$\Delta x_k = K_k v_k \quad (25)$$

Eq. (23) can be approximated by Eq. (5).

4 Tests and results

Two tests along well controlled trajectories are discussed and analyzed in this section to compare the performance of the developed adaptive Kalman filter with the conventional Kalman filter. The analysis in this section is meant to illustrate the adaptive Kalman filtering concept.

The reference data was obtained from a kinematic measurement base, called Anorad AG12-84. The base provides precision position and velocity data for a platform moving, under computer control, forward and backward along a 2-m track (see the test setup in Fig. 4 and the base specifications in Table 1). In this case, the kinematic trajectory is generated by mounting the INS/GPS system on top of the moving base. The platform, then, goes back and forth along its track according to a preloaded program to the servo control unit. The calibration of the Anorad system, which is done by comparing the actual trajectory implemented by the system to the nominal trajectory, shows an accuracy of better than 0.1 mm (rms). So, results can be compared to a tenth of a millimeter; this accuracy is more than an order of magnitude better than that expected from the integrated INS/GPS.

Each of the two trajectories generated and used in this study consists of three static periods and a kinematic one, as depicted in Fig. 5. The three static data sets were collected at the center and at both ends of the track. The static data set at the track center was used to resolve the GPS phase ambiguity. The three static data sets were used to orient the data to the WGS-84 reference system, and then to a local TM coordinate system.

The dynamics of the test is shown in Fig. 6. One complete cycle to travel from a certain point on the track and come back takes 35 s. The cycle in a time-position axis, the first subplot of Fig. 6, is a sinusoidal wave of 1000 incremental distances. To accomplish this trajectory, the platform is accelerated and decelerated in a sinusoidal fashion according to the profile shown in the third subplot with a value ranging from zero to a maximum of 0.032 m/s^2. The resulting velocity profile is a co-sinusoidal wave and is shown in the second subplot and its value ranges from zero to a maximum of 0.179 m/s.

By up-sampling the 10-Hz base-logged data to 100 Hz and correlating the result with a 100-Hz nominal sinusoid, it was found that the base-generated sinusoid has a synchronization error of 70 ms; see Fig. 7. This error results in a periodic residual error with a maximum value of 0.0125 m in the difference sequences. The synchronization error will be removed from the results. What remains afterwards represents the actual errors plus a residual synchronization effect.

Fig. 4. Test setup, INS/GPS on Anorad platform

Table 1. Anorad AG12-84 manufacturer's specification. (Anorad 1993)

Controller length	2 m
Position resolution	1 count (16 000 000 counts/m)
Position range	±999 999 999 counts
Position accuracy	Within 1 count
Velocity range	±16 000 000 counts/s
Acceleration range	1000 to 127 000 000 counts/s^2

The synchronization error can be treated as a random error with uniform distribution in the region of interest. If T is the synchronization resolution possible, the synchronization error is $-T/2 < e < T/2$. The uniform probability density is then $1/T$. The resulting root-mean-square error (rmse) can be calculated as

$$\mathrm{rmse}_T^2 = \frac{1}{T}\int_{-T/2}^{T/2} e^2\,\mathrm{d}e = \frac{T^2}{12}$$

For the 1-Hz data rate, the expected synchronization rmse is about 300 ms, while it is 30 ms for the 10-Hz data rate. The first synchronization error corresponds to a systematic position error of 0.054 m, while the latter corresponds to 0.0054 m, under the aforementioned test dynamics.

4.1 Position error results

Figure 8 shows the position error of the kinematic part of the INS/GPS trajectory when using the conventional Kalman filter. It is worth noting here that the trajectory contains a large number of sharp peaks which are regularly distributed. Each peak coincides with one end of the track and corresponds to a half-cycle period of 17 epochs. The maximum errors occur at these turns.

Before discussing the results of the adaptive filter, the effect of the estimation window size on the adaptive filter performance will be discussed. A window of the same size as the data length is essentially converting the adaptive filter into a conventional filter, since adaptation will take place only once. The following three cases lead

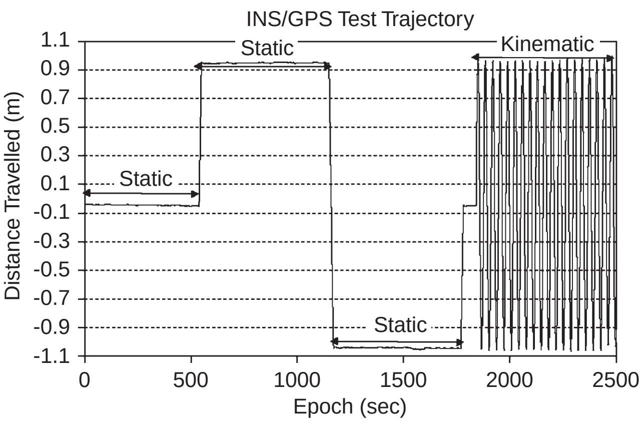

Fig. 5. INS/GPS test trajectory

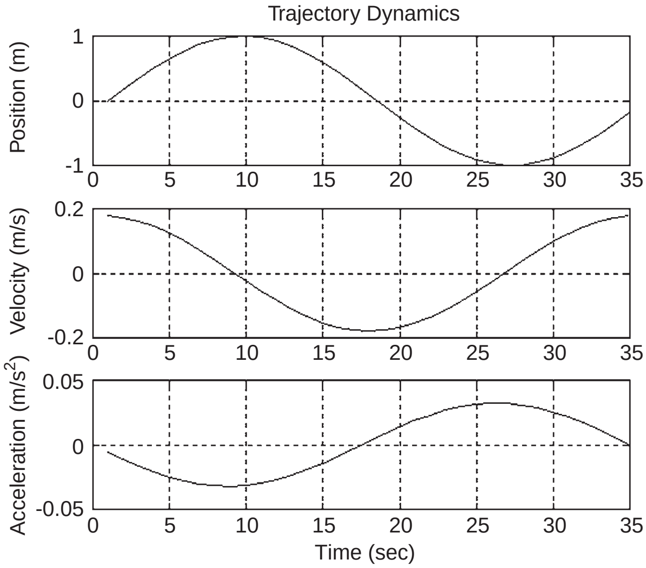

Fig. 6. INS/GPS trajectory dynamics

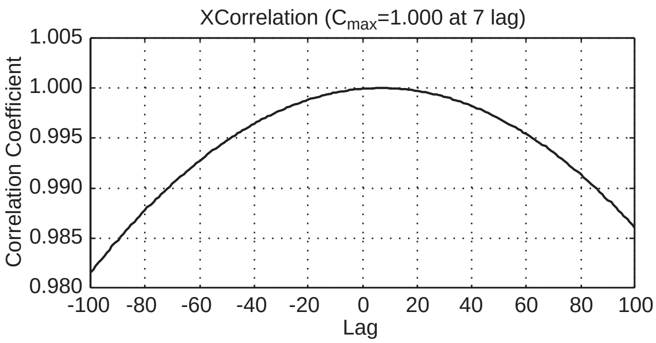

Fig. 7. Synchronization error between the base logged data and the nominal trajectory

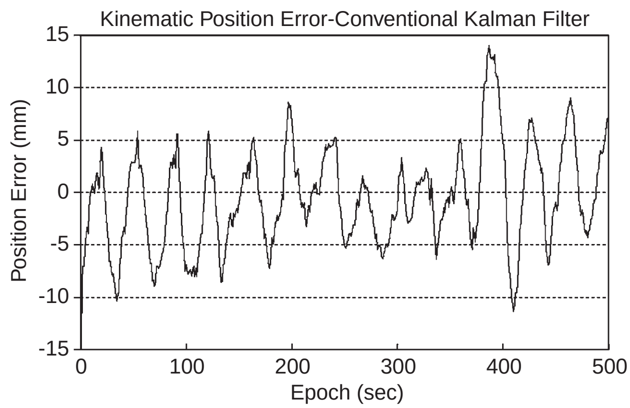

Fig. 8. Position error from conventional Kalman filter

to destabilization of the filter and to the problem of filter divergence in practice.

1. A window size smaller than the number of update measurements when adapting R.
2. A window size smaller than the number of filter states when adapting Q.

3. A window size smaller than the sum of update measurements and filter states when adapting both R and Q simultaneously.

The divergence in any of the previous cases occurs because the number of equations required to estimate the unknown adaptive parameters, is smaller than the number of unknowns themselves.

Referring to the discussion of Sect. 3.1, the ML estimate, in general, will be biased for small sample sizes. This suggests an additional constraint on the choice of the estimation window size. The larger the estimation window, the less unlikely the biasness of the estimate. However, the large estimation window reduces the ability of the algorithm to correctly trace high-frequency changes of the trajectory, e.g. turns. Therefore, a trade-off between the biasness and the tractability of the estimate according to the application at hand should be taken into account. In addition, the proper choice of the window size depends very much on the trajectory dynamics. Since the dynamics encountered in the two tests is benign, the number of states of the INS/GPS filter is small, 15, and the update measurements vary between 5 and 8, a window of 100 epochs was chosen for the following tests.

As can be seen from Figs. 9 and 10, the error amplitude in the adaptive case, and hence the rmse, is reduced to about one half of its previous size. The most likely reason for this improvement in performance is the use of the proper weights. At turns, one can clearly see that the adaptation of Q produces an error pattern that is more random (and in fact a flatter spectrum, see Fig. 13) and hence a better filter performance. In general, one can state that the adaptation of either R or Q produces better filter performance than the use of constant R and Q.

The error spectra for the previous three cases are shown in Figs. 11–13. A spike at frequency 0.0285 Hz ($\sim$35 s), corresponding to the system motion period, appears with different power densities in the three spectra. This gives an indication that the remaining synchronization effect is still contained in the error spectrum. The error spectrum in the Q-only case is, however, flatter than in the other two cases.

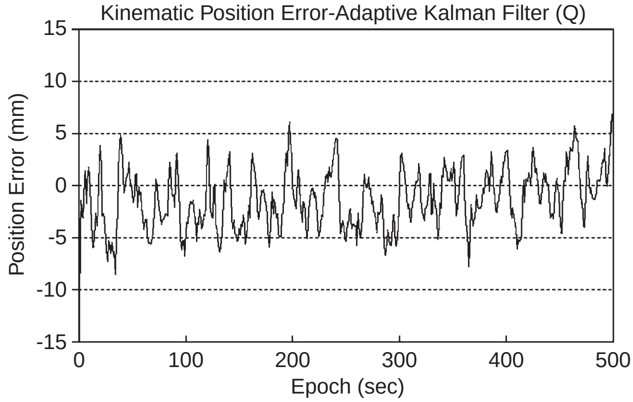

Fig. 10. Position error from adaptive Kalman filter (Q)

4.2 Velocity and attitude error results

With the velocity and attitude errors (not shown), the pattern of performance is very much the same as with the position error. As expected, the adaptive filter outperforms the conventional one in velocity as well. The adaptive filter also dramatically improves the azimuth estimates when adapting the system noise matrix Q; compare the relative results in the last row of Table 2.

5 Summary and conclusions

The integration of INS and GPS is generally implemented through a conventional Kalman filter. In this paper, an adaptive Kalman filter, based on the filter innovation sequence, is introduced as an alternative for integrating INS/GPS systems. The problem of adaptive Kalman filtering is overviewed and the choice of the specific filter algorithm used in this research is discussed. It is shown that the problem of adaptive Kalman filtering is complementary to the problem of filter state estimation. While in the latter the filter states are of concern, the determination of proper error weights is the concern of the adaptive Kalman filtering problem. This

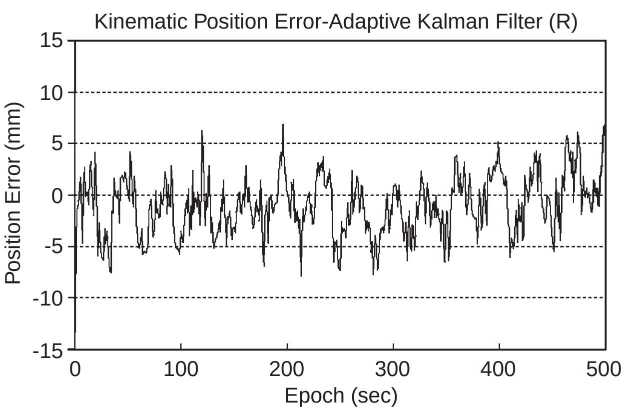

Fig. 9. Position error from adaptive Kalman filter (R)

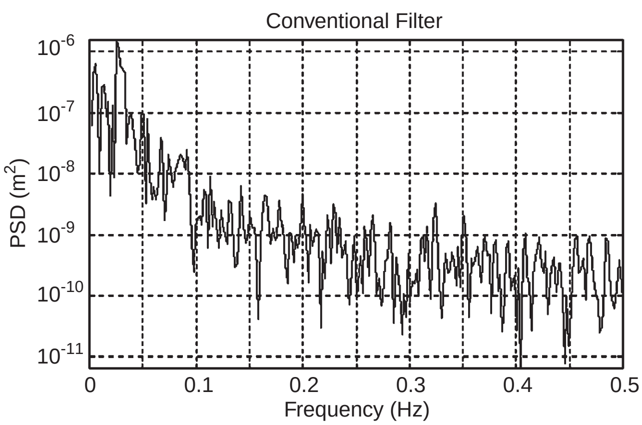

Fig. 11. Position error spectrum – conventional

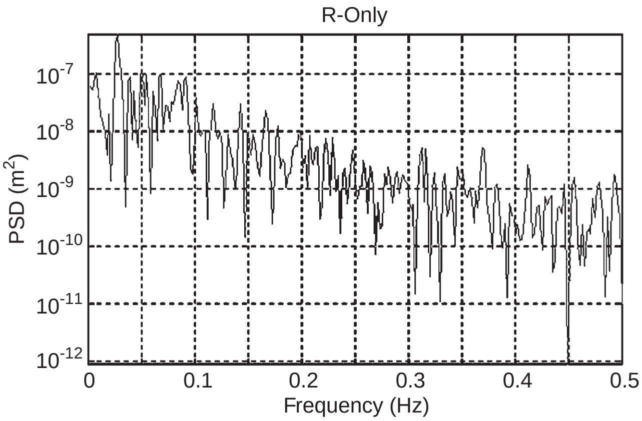

Fig. 12. Position error spectrum – *R* Only

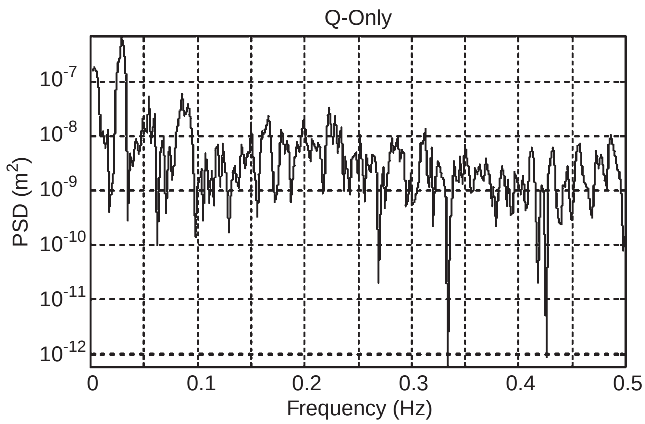

Fig. 13. Position error spectrum – *Q* Only

Table 2. Performance of conventional (non-adaptive) vs adaptive Kalman filters

	Kinematic accuracy (rms)		
	Non adaptive	Adaptive	
		R	*Q*
Position (mm)	5	3	3
Velocity (mm/s)	6	3	3
Pitch & roll (arcsec)	40	35	35
Azimuth (arcmin)	18	12	4

is efficiently accomplished by adapting the matrices R and/or Q which are kept constant in the conventional Kalman filter. The derivation of the adaptive Kalman filter along with efficient computational formulas is given in detail. Two tests in a controlled environment are presented to illustrate the concept.

In kinematic applications, neither the trajectory geometry nor the trajectory dynamics remain constant. Therefore, there is a major drawback in using constant filter statistical information as in the conventional Kalman filter. This becomes evident when analyzing the errors in the kinematic results at the turns. The adaptive filter showed a major improvement over the conventional one through the adaptation of R/Q. The performance of the adaptive Kalman filter, for most of the navigation parameters used in this study, is improved by almost 50% or more when compared to that of the conventional filter. The drawback of the adaptive Kalman filter is a more complex algorithm which leads to an additional estimation block in the Kalman filter algorithm. This drawback is acceptable in cases where highest accuracy is required. For INS/GPS integration, such cases are direct georeferencing of airborne remote-sensing systems and airborne gravimetry.

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Appendix A

Derivation of the residual-based adaptive R matrix

The point of departure here is Eq. (17), which reads

$$\sum_{j=j_0}^{k} \mathrm{tr}\{C_{v_j}^{-1}[C_{v_j} - v_j v_j^T]C_{v_j}^{-1}\} = 0$$

From Kalman filtering theory, one has

$$C_v^{-1} v = R^{-1} v \tag{A1}$$

Substitute Eq. (A1) into Eq. (17) to obtain a similar expression in terms of the residual sequence

$$\sum_{j=j_0}^{k} \mathrm{tr}\{R_j^{-1}[R_j C_{v_j}^{-1} R_j - v_j v_j^T]R_j^{-1}\} = 0 \tag{A2}$$

Also, from Kalman filtering theory

$$P(-)H^T C_v^{-1} = P_{(+)} H^T R^{-1} \tag{A3}$$

Multiply both sides of Eq. (A3) by H and use Eq. (11) to obtain

$$(C_v - R)C_v^{-1} = HP_{(+)}H^T R^{-1} \tag{A4}$$

Multiply both sides of Eq. (A4) by matrix R and rearrange the terms

$$RC_v^{-1}R = R - HP_{(+)}H^T \tag{A5}$$

Now, substitute Eq. (A5) into Eq. (A2)

$$\sum_{j=j_0}^{k} \mathrm{tr}\{R_j^{-1}[R_j - H_j P_{j(+)} H_j^T - v_j v_j^T]R_j^{-1}\} = 0 \tag{A6}$$

The solution of Eq. (A6) yields the required expression for the residual-based R matrix

$$\hat{R}_k = \hat{C}_{v_k} + H_k P_{k(+)} H_k^T$$

Appendix B

Proof of Eq. (23) from Eq. (22)

Starting from Eq. (22)

$$\sum_{j=j_0}^{k} \mathrm{tr}\{H_j^T[C_{v_j}^{-1} - C_{v_j}^{-1} v_j v_j^T C_{v_j}^{-1}]H_j\} = 0$$

the Kalman gain matrix K_k at epoch k is computed as follows:

$$K_k = P_k(-)H_k^T C_{v_k}^{-1} \tag{B1}$$

see e.g. Gelb (1988), and Brown and Hwang (1992). From the above expression, the following expression can be deduced:

$$H_k^T C_{v_k}^{-1} = P_k^{-1}(-)K_k \tag{B2}$$

which after transposing becomes

$$C_{v_k}^{-1} H_k = K_k^T P_{k(-)}^{-1} \tag{B3}$$

Rewriting Eq. (22) in explicit form as

$$\sum_{j=j_0}^{k} \mathrm{tr}\{H_j^T C_{v_j}^{-1} H_j - H_j^T C_{v_j}^{-1} v_j v_j^T C_{v_j}^{-1} H_j\} = 0 \tag{B4}$$

and substituting Eqs. (B2) and (B3) into Eq. (B4), one obtains

$$\sum_{j=j_0}^{k} \mathrm{tr}\{P_j^{-1}(-)K_j H_j - P_{j(-)}^{-1} K_j v_j v_j^T K_j^T P_{j(-)}^{-1}\} = 0 \tag{B5}$$

Equation (B5) can be further rearranged to

$$\sum_{j=j_0}^{k} \mathrm{tr}\{P_j^{-1}(-)(K_j H_j P_j(-) - K_j v_j v_j^T K_j^T)P_j^{-1}(-)\} = 0 \tag{B5a}$$

From Kalman filter theory, it is well known that the V–C matrix of the predicted states $P(-)$ should at least be positive semi-definite. Hence, Eq. (B5a) only vanishes when

$$\sum_{j=j_0}^{k} \mathrm{tr}\{K_j H_j P_j(-) - K_j v_j v_j^T K_j^T\} = 0 \tag{B6}$$

Also, from Kalman filtering theory, one has

$$\Delta x_k = K_k v_k \tag{B7}$$

and

$$P_k(+) = P_k(-) - K_k H_k P_k(-) \tag{B8}$$

from which

$$K_k H_k P_k(-) = P_k(-) - P_k(+) \tag{B8a}$$

Substituting Eqs. (B7) and (B8a) into Eq. (B6), one obtains

$$\sum_{j=j_0}^{k} \mathrm{tr}\{P_j(-) - P_j(+) - \Delta x_j \Delta x_j^T\} = 0 \tag{B9}$$

The V–C matrix of the predicted states $P(-)$ is computed as in Eq. (13) by propagating V–C matrices of the previous epoch. Thus

$$P_k(-) = \Phi P_{k-1}(+)\Phi^T + Q_k \tag{B10}$$

Substituting Eq. (B10) into Eq. (B9) and moving Q to the left-hand side, Eq. (23) for the adaptive V–C matrix of the system noise Q can be obtained, i.e.

$$\hat{Q}_k = \frac{1}{N}\sum_{j=j_0}^{k} \Delta x_j \Delta x_j^T + P_k(+) - \Phi P_{k-1}(+)\Phi^T.$$

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