Tracking fast and slow changes in synaptic weights from simultaneously observed pre- and postsynaptic spiking

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Todo:

Make sure any acronyms are spelled out on the first use

Avoid saying “this plot/figure shows” when you can it’s better to say “We find that Y varies with X (Fig 1)” or whatever the main interpretation is

Double-check that we are careful to distinguish between STP/LTP and short-term/long-term effects within the model. STP and LTP have somewhat specific meanings for experimentalists. Also need to be careful when using LTP, since the “P” stands for “potentiation” not “plasticity” it is limited to cases where the synapse gets stronger

# Abstract

Synapses change on multiple timescales, ranging from milliseconds to minutes, due to a combination of both short- and long-term plasticity. Here we develop an extension of the common Generalized Linear Model to infer both short- and long-term changes in the coupling between a pre- and post-synaptic neuron based on observed spiking activity. We model short-term synaptic plasticity using additive effects that depend on the presynaptic spike timing, and we model long-term changes in both synaptic weight and baseline firing rate using point process adaptive smoothing. Using simulations, we first show that this model can accurately recover time-varying synaptic weights 1) for both depressing and facilitating synapses, 2) with a variety of long-term changes (including realistic changes, such as due to STDP), 3) with a range of pre- and post-synaptic firing rates, and 4) for both excitatory and inhibitory synapses. We also show the importance of tracking fast changes in synaptic weights, slow changes in synaptic weights, and unexplained variations in baseline firing simultaneously. Omitting any one of these factors can lead to spurious inferences for the others. Altogether, this model provides a flexible framework for tracking short- and long-term variation in spike transmission.

# 1 Introduction

Detecting and characterizing synaptic connections and how they change over time is a major experimental challenge, especially in behaving animals. The extracellular spiking activities of neurons provide a way for synapses detection, and it is usually done by analyzing the cross-correlogram (Barthó et al., 2004; Fetz et al., 1991). The cross-correlogram is a histogram of postsynaptic spiking times forward and backward to presynaptic spikes, which provides an estimate of cross-correlation between pre- and post-synaptic neurons (Perkel et al., 1967). When a monosynaptic occurs, the postsynaptic spiking probability will briefly increase or decrease after a presynaptic spike, for an excitatory or inhibitory synapse, respectively. This corresponds to a transient and short-latency peak or trough in the cross-correlogram. However, investigating the cross-correlogram directly may not be enough for modern neural data analysis. By using multielectrode arrays, the extracellular spiking of hundreds of neurons can be recorded simultaneously. These large-scale recorded neurons can have tens of thousands of potential synapses between them, but both spiking and synapses can be sparse. This makes it difficult for synapses detection by simply plotting the cross-correlograms. Some recent research tried to solve this problem by modeling spike cross-correlations to handle the sparse information for detection (Kobayashi et al., 2019; Ren et al., 2020).

In cases where synapses can be reliably identified from spikes, one possibility is that these recordings can be used to examine changes in synaptic strength over time. Changes in synaptic strength occur over multiple timescales and due to different biophysical mechanisms (Zucker & Regehr, 2002). For instance, on timescales of a few milliseconds (short-term synaptic plasticity, STSP), the synaptic strength may decrease by consumption of neurotransmitters (depression), or may increase by influx of calcium into presynaptic axon terminal, which increases neurotransmitters release probability (facilitation). In contrast, synaptic strength may change on timescales of minutes to hours (long-term synaptic plasticity, LTSP) due to increases (long-term potentiation, LTP) or decreases (long-term depression, LTD) in receptor density or structural changes. To understand learning rules and to make sure our models generalize it is important to separate the short- and long-timescale effects. It is also important to note that fluctuations in the presynaptic rate can create fluctuations in synaptic strength on much longer timescales. For instance, higher presynaptic rate will produce a weaker synaptic strength, if the synapse has short-term depression (Swadlow & Gusev, 2001).

The short- and long-term dynamics in synapses can be roughly studied by examining the cross-correlations corresponding to different presynaptic patterns, such as inter-spike intervals (ISI) (Carandini et al., 2007; Csicsvari et al., 1998; Fujisawa et al., 2008; Mantel & Lemon, 1987; Swadlow & Gusev, 2001; Usrey et al., 2000) or different recording times. Several authors have proposed models for estimating either long-term (Linderman et al., 2014; Stevenson & Kording, 2011) or short-term (Chan et al., 2008; English et al., 2017; Ghanbari et al., 2017) changes in synaptic weights. Unlike English et al. (English et al., 2017), the short-term plasticity estimation is conditioned on the previous presynaptic inter-spike intervals (ISIs) in Ghanbari et al. (Ghanbari et al., 2020), and this provides only an incomplete picture for synaptic strength inference. Here we introduce a statistical model that estimates both long- and short-term changes simultaneously. The unexplained changes in postsynaptic neuron are treated as the baseline firing rate. The long-term changes in both synaptic weight and baseline is estimated by the point process adaptive smoothing (Eden et al., 2004), while the short-term plasticity is modeled by additive effects that depend on the presynaptic spike timing(Stevenson & Kording, 2011). By several simulations, we show that this model provides a flexible framework for tracking short- and long-term variation in spike transmission, and it’s necessary to estimate both long-term and short-term effects simultaneously.

# 2 Methods

Here we introduce an extension of a Poisson GLM that aims to describe the coupling between a pre- and postsynaptic neuron (Brillinger, 1988, 1992). While many previous studies have modeled static coupling between neurons (Harris et al., 2003; Okatan et al., 2005; Pillow et al., 2008; Truccolo et al., 2005), our goal here is to describe a time-varying synaptic strength with both short- and long-term changes.

We model the postsynaptic spiking in discrete time as a doubly stochastic Poisson process with time-varying parameters. Partitioning the total recording time T into evenly-spaced bins , such that with time steps , we denote the total number of presynaptic spikes in as , and represents the number of spikes observed in . and denote the same calculations for the postsynaptic neuron’s spikes. For small enough both and take values of 0 or 1 and can be viewed as spike indicators for time bin .

Previous models of static coupling between neurons typically include the recent spiking history of both the presynaptic input (the coupling effect) and the postsynaptic neuron itself.

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|  |  | (2.1) |
|  |  | (2.2) |

where is the conditional intensity of the postsynaptic neuron at , given the recent spiking of the postsynaptic neuron and presynaptic neuron , steps into the past. For the model parameters, defines the fixed baseline firing rate and the coupling and history effects are weighted by and **,** for the post- and pre-synaptic neurons respectively.

Here we extend this static model to include fast and slow dynamics in the coupling effect and model the postsynaptic neuron’s firing rate as

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| (2.3) |
|  |  | (2.4) |

In our model, the conditional intensity of postsynaptic neuron is captured by an unstructured time-varying baseline , a history effect , and a time-varying coupling term . In the static coupling model, the main goal is to accurately infer the shapes of the filters, here we assume that the shape is fixed , but is weighted by an additional factor that varies over time. Additionally, we aim to partition the variations in into a long-term component and a short-term component that scales the long-term synaptic weight multiplicatively. Here we model transient increases/decreases in as a function of the presynaptic spike timing to account for short-term synaptic facilitation/depression. In the absence of presynaptic activity, returns to a base value of . To simplify the notation, we write , , and as , , and below.

## Estimating the Synaptic Filter

Here we estimate the shape of the static presynaptic filter by directly modeling the cross-correlogram between the pre- and postsynaptic spikes, similar to Ren et al. (Ren et al., 2020). Briefly, we assume that the shape of the synaptic connection is described by an alpha function , with a latency and time constant . These parameters are then estimated by modeling the cross-correlogram as a combination of a slow background correlation and a fast, transient effect of the synaptic connection:

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|  |  | (2.5) |
|  |  | (2.6) |
|  | and | (2.7) |

where denotes the expected rate of coincidences from the cross-correlogram with baseline and a linear combination of smooth basis functions accounting for slow changes in cross-correlation, at time bin . Here we use cubic B-spline bases with equally spaced knots for the smooth basis and model the time-range in bins. The fast effect is described as where is the connection strength from pre-synaptic neuron to post-synaptic neuron, and the alpha function is convolved with the auto-correlation of the presynaptic neuron to account for the effects of presynaptic dynamics. The parameters are estimated by maximizing the Poisson log-likelihood. We use random restarts, since the objective function is non-convex in the latency and time-constant parameters.

After fitting the latency and time constant of the synaptic filter to the cross-correlogram we assume that these parameters are fixed when modeling the long- and short-term changes in synaptic strength. This simplifying assumption allows us to model rescaling of the basic presynaptic input, given by and avoid a computationally intensive non-convex optimization of the full likelihood with respect to and .

## Estimating long-term changes in baseline firing rate and synaptic weight

To estimate the time-varying baseline firing rate and the effects of synaptic plasticity (, and ) we use two distinct strategies. For the long-term effects we estimate and by point process adaptive smoothing (Eden et al., 2004; Rauch et al., 1965), and for the short-term effects we estimate using an additive model that depends on the presynaptic inter-spike intervals (ISIs) (Ghanbari et al., 2017). To estimate all of the effects together we use an alternating optimization – we hold the long-term effects constant while updating the short-term effects then hold the short-term effects constant while updating the long-term effects and repeat this alternating pattern until convergence.

### Estimating Long-Term Effects

To model the long-term changes in baseline firing and synaptic strength we assume that the parameters evolve over time with noisy, linear dynamics. Denoting the parameters as a vector , we assume that the model parameters evolve over time following

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|  |  | (2.8) |

where is a system evolution matrix and represents Gaussian noise with covariance at . The conditional intensity can be re-written as where . We then use adaptive smoothing to track estimates of the parameters given the observed spiking of the pre- and postsynaptic neurons. During a forward step, we first approximate the distribution using adaptive filtering (Eden et al., 2004). Then, during a backward step, we approximate the distribution using Rauch-Tung-Striebel (RTS) smoothing (Rauch et al., 1965). In both cases, we approximate the distribution over using a multivariate Gaussian.

For adaptive filtering, we assume an initial mean and covariance and . We first propagate the estimated mean and covariance forward in time according to the process model

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|  |  | (2.9) |
|  |  | (2.10) |

Here and denote the predicted mean and covariance given observations up to time . We then update the mean and covariance based on the observed spiking at time .

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|  |  | (2.11) |
|  |  | (2.12) |
|  |  | (2.13) |

Here and are the resulting mean and covariance after incorporating the observation at . These equations are a special case of updates previously derived for the general Poisson adaptive filtering model (Eden et al., 2004).

Given the estimates from adaptive filtering we then step backwards to find smooth estimates of the parameters. Here we use updates based on the RTS method

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|  |  | (2.14) |
|  |  | (2.15) |
|  |  | (2.16) |

where and denote smoothed estimates for the mean and covariance at , and to make the algorithm numerically stable, we use an equivalent update

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|  |  | (2.17) |

In the results that follow we assume that the process covariance is constant , and that the parameter evolution is a random walk .

The performance of adaptive smoothing is highly affected by , and choosing improperly can prevent the algorithm from converging. Estimating from the data itself using the EM algorithm (Ananthasayanam et al., 2016) is notoriously slow, even with an accelerator (Du & Varadhan, 2020). Here we choose to estimate by maximizing prediction likelihood, i.e. likelihood under . To simplify, we assume that is diagonal with independent noise for () and ().

Besides estimating by direct two-dimensional optimization, the optimized can be approximated by sequential one-dimensional optimization. Since estimation of depends on and vice versa, the order of sequential one-dimensional approximation can influence the results. Since the synaptic effect is sparse () and the time-averaged coupling effect is typically small relative to the baseline, the value of has negligible influence on estimation. Based on this observation, we do one-dimensional approximation by first fixing and finding the MLE then fixing as and finding the MLE .

### Estimating Short-Term Effects

In addition to the slow changes in baseline firing and the synaptic weight, we also aim to model fast changes in synaptic weights due to short-term synaptic plasticity. These changes occur on timescales much faster than the typical postsynaptic ISI (~10 ms), and cannot be accurately tracked by adaptive smoothing. Rather than using smoothing, we thus model short-term synaptic plasticity using a parametric model previously introduced in Ghanbari et al. (Ghanbari et al., 2017). Namely, we model the short-term synaptic weight

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|  |  | (2.18) |
|  |  | (2.19) |

Here is the presynaptic spike time and the denotes the inter-spike interval (ISI) between the and presynaptic spikes. is a nonlinear function of presynaptic ISI, which describes how the synaptic strength increases or decreases following a pair of presynaptic spikes with a specific ISI. decreases synaptic strength, mimicking short-term synaptic depression, while creates increases in synaptic strength akin to facilitation. The cumulative effects of STSP, , are modeled by a convolution of with the presynaptic spikes, and we assume that the effects decay exponentially with rate . We parametrize the shape of using a linear combination of raised-cosine basis functions .Note that, since we can rewrite the short-term synaptic effect , optimizing (when the long-term effects are fixed) is simply a matter of fitting a GLM.

Additionally, assuming that the long-term effects are fixed allows us to approximate standard errors for the cumulative effects of STSP and the modification function (defined as )

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|  |  | (2.20) |
|  |  | (2.21) |

To summarize, altogether, we have the following parameters: , , , , and , with hyper-parameters defining the raised-cosine basis and timescale for short-term modifications, and the long-term weight process covariance . To fit these parameters we first estimate the synaptic latency and time-constant directly from the cross-correlogram. Then we optimize the remaining parameters by alternating between fitting the short-term parameters assuming , fixed and fitting the long-term parameters , assuming fixed. Here we assume that is a fixed, known refractory effect (exponential filter with time constants ranging from 5ms to 10ms), but, in practice, it could easily be estimated alongside the short-term parameters.

## Simulations

Here we validate the model using simulated pre- and postsynaptic spiking. If not otherwise specified, presynaptic spike times are generated by a homogeneous Poisson process with a firing rate of 5Hz. The postsynaptic neuron is simulated as a conditionally Poisson process defined in equation 2.3 and 2.4. The observed time length is 20min if not specified, and we use bin size 1ms, throughout.

For simulating spike-timing-dependent plasticity (STDP) we use a long-term modification function that depends on the relative timing of pre- and postsynaptic spikes. Here we use a double-exponential modification function, based on the STDP observed in cortical and hippocampal slices (Abbott & Nelson, 2000). In this case, each pair of pre- and post-synaptic spikes modifies the synapse by

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| (2.22) |
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And we set , , and . We further add an additional long-term decay that pushes the synaptic weights back to 1, as in Stevenson and Kording (Stevenson & Kording, 2011). Namely,

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|  |  | (2.23) |

where is the indicator function, and we set .

In the following results, 5 raised-cosine bases with non-linear stretching peaks are used to model STSP modification function within [0, 600ms] in 1ms bins. When fitting the model we typically assume , except in the section “Selection of Hyper-parameter in Adaptive Smoothing.”

Code for the model and all simulations is available at http://github.com/weigcdsb/GBLM\_SMOOTH/.

# 3 Results

Here we present simulation results illustrating how this model performs, how the hyperparameters can be efficiently optimized, and why tracking *both* short- and long-term effects can prevent spurious inference of synaptic dynamics. If not otherwise specified, the simulated recording lengths are 20min and presynaptic firing rates are 5Hz.

## Simultaneous short-term and long-term changes in synaptic weights

Although most experimental paradigms aim to manipulate and/or observe only a single synaptic timescale, short-term synaptic dynamics (depression and facilitation) and long-term synaptic dynamics (LTD and LTP) coexist and have distinct physiological mechanisms. To illustrate how these dynamics can coexist we simulate an excitatory synaptic connection between a pre- and postsynaptic neuron where the synaptic weight increases instantaneously midway through a 20min recording, while also having short-term synaptic depression on fast timescales (Fig 1). Presynaptic spike timing is generated using a homogeneous Poisson process, while postsynaptic spike times are generated by our full model. The overall cross-correlogram between the pre- and postsynaptic shows a short latency, fast onset peak where the probability of postsynaptic spiking increases following each presynaptic spike (Fig 1B, right). Qualitatively, this peak is similar to putative excitatory connections observed in large-scale multielectrode recordings (Barthó et al., 2004).

Several previous studies have characterized the short-term and long-term dynamics of spike transmission by partitioning or splitting the overall cross-correlogram to use only a subset of presynaptic spikes. For instance, comparing the cross-correlogram generated using only the presynaptic spikes before the change-point to the cross-correlogram generated using the spikes after the change-point reveals the increase in synaptic strength (Fig 1B). By using presynaptic spikes that are preceded by a specific inter-spike interval we can also observe the effects of short-term synaptic depression (Fig 1D). In this case, presynaptic spikes that occur recently after another spike tend to have lower efficacy compared to presynaptic spikes that occur following a long period of silence, where the synaptic resources have had an opportunity to recover (Fig 1C). Although partitioning the cross-correlogram can reveal clear evidence of long- and short-term changes in spike transmission, it is often unclear what the best partitioning should be and how to combine evidence from multiple types of partitions (e.g. time-based and ISI-based) into a single description. It is also important to note that in the simulation here the firing rate for the presynaptic neuron is set to 5Hz and the firing rate of postsynaptic neuron is ~15Hz. In cases where firing rates are lower and/or when synaptic efficacy is weaker the partitioned cross-correlograms may be too noisy to obtain meaningful estimates of the synaptic weight.

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| **Fig 1. Characterizing simultaneous short-term and long-term plasticity from spikes.** Here we simulate the spiking of two neurons connected by an excitatory, depressing synapse where the synaptic strength increases abruptly after a change-point **(A)**. We can assess the change in synaptic strength by partitioning the correlogram before (blue) and after (orange) the change-point **(B)**. Here the histograms indicate the observed counts of postsynaptic spiking relative to presynaptic spiking, and the curves denote the corresponding cross-correlation from the model postsynaptic rate. **(C)** By partitioning the correlograms by presynaptic ISI we can also assess the potential influence of short-term synaptic dynamics. Here, shorter presynaptic ISIs correspond to lower postsynaptic efficacies, since the synapse simulated here is depressing. Dots indicate the observed efficacy after splitting the distribution of pre-synaptic ISIs by quantile. Lines indicate the average simulated efficacy from the model itself. **(D)** By partitioning the cross-correlogram based on both the change-point and the presynaptic ISI (median split shown here), we can quantify long-term and short-term synaptic changes simultaneously. |

Using a model-based approach the short- and long-term synaptic weights can be estimated simultaneously to create a unified description of postsynaptic spiking. Here we track the long-term changes in synaptic weight using adaptive smoothing, and we fit the short-term changes in synaptic weights using a modification function that describes the additive effects for different presynaptic ISIs (see Methods).

Using adaptive smoothing, the model simply updates its estimate of the synaptic weight based on the observed spike transmission at each time. If postsynaptic spikes follow presynaptic spikes more than expected the parameter for the long-term synaptic weight increases, and if postsynaptic spikes follow presynaptic spikes less than expected the parameter decreases. The hyperparameter provides a constraint on how fast the long-term synaptic weight parameter can change. This unstructured approach allows the model to track a wide variety of patterns (Fig 2A). Since the model estimates a synaptic effect for every presynaptic spike, it also provides efficacy estimates for arbitrary partitions of the cross-correlogram (Fig 2B).

For the short-term synaptic weight, we use a more structured approach that depends on the specific pattern of presynaptic ISIs. Following each presynaptic spike, we assume that the short-term synaptic weight increases or decreases by an amount that depends on the inter-spike interval (ISI) preceding that spike. In the absence of presynaptic spikes, we assume that the short-term weight decays (exponentially) back to a baseline weight. These changes can occur rapidly but are constrained by the presynaptic spike timing. Within the model, presynaptic spikes following an interval of 100ms, for instance, will always modify the short-term synaptic weight by the same amount. However, these structured short-term changes allow a range of behaviors, including short-term facilitation and depression-like effects (Fig 2C). These effects occur simultaneously with any long-term changes in synaptic weight, and, again, allow for efficacy estimates with arbitrary partitions of the cross-correlogram (Fig 2D).

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| **Figure 2. Tracking long- and short-term plasticity with a model-based approach.** Here we simulate several combinations of short- and long-term changes in synaptic weights. **(A)** shows results from an excitatory, depressing synapse that undergoes long-term changes without a specific change-point. Slow linear (top left) or sinusoidal (bottom left) changes in synaptic weight can be accurately tracked from pre- and postsynaptic spiking alone, and the modification function used to generate short-term dynamics can be accurately reconstructed (right). Here the pre- and postsynaptic baseline firing rates are constant, and dashed lines denote standard error. **(B)** The long-term changes are also apparent when the cross-correlogram is partitioned by recording time. Here the histograms indicate the observed counts of postsynaptic spiking relative to presynaptic spiking, and the curves denote the corresponding cross-correlation from the postsynaptic rates estimated by the model. **(C)** shows results from excitatory synapses that strengthen following a change-point, but with three distinct short-term dynamics: facilitating (top), depressing (middle), or without plasticity (bottom). **(D)** Here the differences in short-term dynamics are apparent when the cross-correlogram is partitioned by presynaptic ISI. |

During natural ongoing spiking activity there are unlikely to be large shifts in long-term synaptic strength like Fig 1 and 2. In the absence of external stimulation or task demands, long-term synaptic changes are more likely to result from ongoing patterns of activity. Here we simulate a synapse with spike-timing-dependent plasticity (STDP). The modification function is a traditional double-exponential function, where the synapse is strengthened when postsynaptic spikes follow presynaptic spikes and weakened when presynaptic spikes follow postsynaptic spikes. With the presynaptic spike timing again coming from a homogeneous Poisson process, STDP induces slow fluctuations in the synaptic strength of the full model (Fig 3A). These changes are accurately tracked by the adaptive smoother, even though the smoother does not model pre-post spike timing explicitly. As before, these slow changes also exist simultaneously with short-term synaptic depression (Fig 3B). Although the short-term synaptic effects are visible when partitioning the cross-correlograms (Fig 3C), it is unclear how the long-term fluctuations in STDP could be similarly partitioned.

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| **Figure 3. Tracking naturalistic long-term synaptic dynamics.** Here we simulate an excitatory, depressing synapse with long-term changes generated by spike-timing-dependent plasticity (STDP). The model can accurately track the slow fluctuations in synaptic weight induced by a double-exponential STDP learning rule **(A)**, as well as the modification function that generates short-term dynamics **(B)**. As before, the short-term dynamics are also apparent when the cross-correlogram is partitioned by presynaptic ISI. Here the baseline firing rate for postsynaptic neuron is set to ~15Hz, and dashed lines denote standard error. |

The previous simulations are all for excitatory synapses. However, the model works similarly for inhibitory synapses. In practice, the “sign” of the synapse is determined by the signs of the long-term and short-term synaptic effects. For simplicity, we assume that the short-term synaptic effect is positive and decays to a baseline value of such that the sign of the long-term effect effectively determines whether the synapse is excitatory or inhibitory . As before, both short- and long-term synaptic effects can be estimated when simulating from the full model (Fig 4).

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| **Figure 4. Tracking the long- and short-term dynamics of an inhibitory synapse.** Here we simulate the spiking of pre- and postsynaptic neurons connected by an inhibitory synapse with depressing short-term dynamics. The firing rate for the presynaptic neuron is 10Hz and the rate for the postsynaptic neuron is ~160Hz. The model accurately tracks the long-term synaptic weight as it weakens following a change-point **(A)**, and accurately fits the modification function that governs short-term depression **(B)**. Dashed lines show standard error. The long- and short-term dynamics are apparent when splitting the cross-correlogram by recording time **(C)** or presynaptic ISIs **(D)**, respectively. |

## Variation in pre- and postsynaptic firing rates

In addition to modeling fast and slow changes in synaptic strength we also model slow, unexplained fluctuations in the baseline postsynaptic firing rate. Here we include the baseline parameter in the adaptive smoother, and, as with the long-term synaptic weight, it is constrained to vary slowly using the hyperparameter .

To illustrate the effects of a time-varying baseline we simulate, again, a presynaptic neuron with homogeneous Poisson spiking that provides excitatory input to a postsynaptic neuron simulated by the full model. Here the synapse undergoes short-term synaptic depression as well as long-term changes in strength, as before, and, in addition, the firing rate of the postsynaptic neuron fluctuates (Fig 5). In general, the baseline is estimated more accurately and with higher certainty than the long-term synaptic weight. Information about the presence or absence of postsynaptic spikes is always available, but information about the synaptic weight is only available in the short time window following each presynaptic spike. This difference also creates a kind of separability where estimates of the baseline are not particularly influenced by the changes in synaptic weight and estimates of synaptic weight are not influenced by the changing baseline (Fig 5A and C).

As in the previous examples, since there is an estimate of the postsynaptic rate at every time, using a model-based approach allows for reconstruction of arbitrary partitions of the cross-correlogram. When the baseline fluctuates it can also have an influence on the cross-correlogram (Fig 5B and D). Note that here, the baseline affects both the base level of the cross-correlogram as well as the magnitude of the synaptic effect. With the exponential output nonlinearity, the time-varying baseline acts like a time-varying gain and may be useful to mimic slow fluctuations in excitability due to neuromodulation or brain state.

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| **Figure 5. Accounting for time-varying baseline postsynaptic firing rates.** In addition to tracking variation in synaptic strength, accurately fitting cross-correlograms may also require tracking the postsynaptic rate. Here we simulate an excitatory, depressing synapse that undergoes slow changes in both synaptic strength and postsynaptic rate. The model accurately tracks changes in baseline with sudden **(A)** and continuous **(C)** changes in synaptic strength. Dashed lines denote standard error. In this case, splitting the cross-correlograms by recording time reveals changes in both the baseline correlation and in the amplitude of the peak, and these are well captured by the model (orange lines). |

Since information about the synaptic strength is only available when there is a presynaptic spike, the presynaptic firing rate influences how accurately the synaptic weight can be estimated. Higher presynaptic firing rates allow the long-term synaptic strength to be estimated more accurately and with less uncertainty (Fig 6). To illustrate this feature of the model we simulate presynaptic spiking with a Poisson process whose rate changes abruptly, either increasing (Fig 6A) or decreasing (Fig 6B). The postsynaptic neuron spikes according to the full model, in this case, with short-term synaptic facilitation. The baseline postsynaptic rate and long-term synaptic weight are both constant in this simulation. However, the accuracy and precision of the adaptive smoothing estimates for the long-term weight change substantially when the presynaptic rate changes. Estimates for the short-term modification function will also depend on the presynaptic spike rate to some extent, but the impact on accuracy is not as strong, since the short-term model pools information from presynaptic spikes across the entire recording depending on their ISIs.

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| **Figure 6. Presynaptic firing rates influence estimation accuracy.** Here we simulate an excitatory, facilitating synapse with a constant long-term synaptic strength and postsynaptic firing rate (~30Hz). To illustrate how presynaptic firing rates influence the model, the recording time is divided into three time periods with an increase (A) or decrease (B) in presynaptic rate occurring during the second period. When the presynaptic firing rate increases during the second period (3Hz-15Hz-3Hz), the estimation of the synaptic strength is more precise. In contrast, when the presynaptic firing rate decreases during the second period (5Hz-0Hz-5Hz), the lack of the presynaptic spikes leads to high uncertainty in the synaptic strength. Dashed lines show standard error. |

## Omitted variable bias

In the examples above we have shown how estimates of the baseline and synaptic weight are largely separable, since the synaptic effects are constrained to follow presynaptic spikes. Moreover, estimates of the short-term and long-term synaptic effects are largely separable, since short-term effects are constrained to be ISI-dependent. However, omitting components of the model can result in spurious estimates for the long-term synaptic weight.

To illustrate how omitting effects can bias estimation, we again simulate a presynaptic neuron with Poisson spiking that provides synaptic input to a postsynaptic neuron whose firing is determined by the full model. Here we simulate a synapse with short-term synaptic depression and a constant long-term weight. Although in the previous examples the presynaptic rate has been held constant, in this case we simulate a fluctuating baseline for both the pre- and postsynaptic neurons. Slow fluctuations in the presynaptic rate induce slow changes in the synaptic weight that are purely due to short-term synaptic effects (Fig 7). For the depressing synapse simulated here, a high presynaptic rate causes the synapse to be in a chronically depleted state, while a lower presynaptic rate allows the synapse to recover from depression. In this case, the short-term synaptic weight is negatively correlated with the presynaptic rate.

These slow fluctuations in the short-term synaptic weight are not necessarily problematic and can be accurately estimated with the full model (Fig 7A). However, if we instead fit a model that only includes a long-term synaptic weight and omits the short-term effect, the slow fluctuations are misattributed to the long-term weight (Fig 7B). The adaptive smoother effectively tracks these changes when they are not otherwise explained. Since natural neural activity contains these types of slow fluctuations in presynaptic rate (as a function of brain state, for instance), there are likely to be slow fluctuations in synaptic strength that can be explained away as byproducts of short-term synaptic mechanisms. Modeling short- and long-term effects simultaneously can allow these “explainable” fluctuations to be separated from other phenomena, such as LTP/LTD or STDP.

Omitting the fluctuations in the baseline can also result in spurious estimates of the synaptic strength (Fig 7C). Here, when the model is fit while assuming that the baseline is constant, the adaptive smoother aims to account for higher/lower than expected postsynaptic firing rates using the long-term synaptic weight. Although the synaptic effect is limited to the brief interval following each presynaptic spike, misattributing the rate changes to fluctuations in the synaptic weight improves the overall likelihood. In this case, there is a strong positive correlation between the (omitted) baseline fluctuations and the misestimated long-term synaptic weight. Tracking unexplained variation in the postsynaptic rate is thus likely to be important for accurately tracking synaptic weights in experimental data.

In these scenarios, partitions of the cross-correlogram may serve as a useful check on how well the model describes spike transmission in specific time-periods or as a function of ISI. The model that omits STSP, for instance, will fail to explain the observed depression as a function of ISI, while the model that assumes a constant baseline will fail to model the changing base level of the cross-correlograms when they are partitioned over time.

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| **Figure 7. Omitted variable bias in long-term synaptic weights.** Here we simulate spiking neurons connected by an excitatory, depressing synapse, in this case, with fluctuations in both the baseline postsynaptic firing rate (average 20Hz) and the presynaptic firing rate (average 8Hz). The variation in the presynaptic rate causes slow fluctuations in short-term synaptic strength, but, when fitting the full model **(A)**, these fluctuations are properly attributed to short-term effects. We then consider how omitting variables from the model might influence inference. When all three effects (baseline, LTSP and STSP) are estimated simultaneously they are accurately estimated **(A)**, but when the short-term effects are omitted **(B)** or when the baseline is omitted **(C)** there is substantial misestimation. We find that when the short-term effects are omitted from the model, slow variation in the short-term weight is misattributed to the long-term synaptic effects (B, middle). When the baseline postsynaptic rate is assumed to be constant, the fluctuation in the baseline is also misattributed to long-term synaptic effects (C, middle). Dashed lines denote standard error. |

## Optimizing the adaptive smoother

For the baseline and long-term synaptic weight, the process noise covariance defines the timescales for tracking. In the examples above we use a fixed during inference. In general, however, should be matched to the underlying timescale of the process. If is too small the estimated baseline and synaptic strength will be oversmoothed and potentially meaningful changes may be underestimated. On the other hand, if is too large the estimated baseline and synaptic strength will be undersmoothed and may reflect noise. Here we assume that is diagonal and show how the variance of the baseline and synaptic weight can be optimized by maximizing the prediction likelihood (see Methods).

To illustrate the process of optimizing we simulate from the full model and sample the baseline and long-term synaptic weight using a Gaussian random walk with a ground truth . Although the full likelihood always improves with larger values of , the prediction likelihood has a single maxima as a function of and (Fig 8A). We find that changes in have a much larger effect on the prediction likelihood than changes in , presumably due to the fact that the synaptic weight only influences the likelihood in the short interval following each presynaptic spike. Values of have negligible influence on prediction likelihood under fixed but not vice versa. Full 2D optimization of the prediction likelihood (bounded gradient descent in this example) accurately recovers the true process noise, as does a simple 1D optimization scheme where we first optimize with then optimize using the optimized in the first step. As expected, when is too high or too low, the estimated baseline and long-term synaptic weights are under- and oversmoothed, respectively (Figure 8B).

Additionally, we find that in a series of short, 10min simulations with pre- and postsynaptic rates set to 5Hz and ~15Hz, respectively, the true values of can be accurately recovered (Fig 8C) by maximizing the prediction likelihood. As before, the accuracy in tracking the baseline is typically higher than the accuracy in tracking the synaptic weight. The accuracy in recovering the baseline hyperparameter is also typically higher than the accuracy in recovering the synaptic weight hyperparameter.

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| **Figure 8. Selection and influence of in point process adaptive smoothing. (A)** Here we simulate an excitatory, depressing synapse with random walk fluctuations in both the long-term synaptic strength and the postsynaptic baseline rate. The heatmap shows the prediction log-likelihood under different values of . The red triangle and dashed lines denote the true, simulated . The orange dot represents the maximum prediction likelihood estimate (MLE) after optimizing the full model. For reference, we also show results when is under-estimated (blue, ) and over-estimated (green ). A fast, one-dimensional approximation to the MLE is also shown (blue triangle). **(B)** shows the corresponding estimates for the baseline and long-term synaptic strength under (top), (bottom), and (middle). Dashed lines denote standard error. When is too small, the estimates are over-smoothed; when is too large, the estimates are too noisy. We then run many simulations with different values of . We find that the MLE (2D) can accurately recover the simulated hyper-parameters over a wide range of values **(C)**. Each combination has 5 replicates and a simulated recording time of 10 min. |
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# 4 Discussion

Here we introduce a statistical model that aims to simultaneously track short- and long-term changes in synaptic weights from spike observations. Using simulations, we show that this model can successfully recover time-varying synaptic weights for both excitatory and inhibitory synapses with a variety of short-term and long-term plasticity patterns. The accuracy in estimating the long-term synaptic weight depends heavily on the presynaptic firing rate – the more the presynaptic neuron fires, the more accurately synaptic weights can be estimated from spikes. In naturalistic settings, long-term changes in synaptic weights also exist alongside short-term changes in synaptic strength, due to depression or facilitation, and fluctuations in the pre- and postsynaptic rates, due to changes in stimuli, behavior, or brain state. Here we demonstrate why long-term weights, short-term weights, and variations in baseline should all be estimated simultaneously to avoid misinterpretation. If we omit any one of these factors, inference for others will generally be biased.

Including additional covariates can often improve spike prediction accuracy in GLM-like models (Harris et al., 2003; Stevenson, 2018; Truccolo et al., 2005). In these cases, previously unexplained fluctuations in the postsynaptic rate can be explained by an observed covariate such as the firing of simultaneously observed neurons (Harris et al., 2003; Okatan et al., 2005; Pillow et al., 2008), local field potentials (Kelly et al., 2010) or the neuron’s own history (Pillow & Simoncelli, 2003; Truccolo et al., 2005). However, once these covariates are taken into account the baseline firing rate itself is assumed to be constant. Here we consider a general approach where slow fluctuations in the baseline firing rate are directly tracked with adaptive filtering/smoothing. By using adaptive smoothing to track the changing postsynaptic rate, inference of synaptic strengths will be less influenced by unexplained variation caused by slow unobserved variables.

It is important to note, however, that adaptive smoothing cannot track arbitrarily fast changes in the underlying synaptic weight or postsynaptic baseline. In fact, the exact firing rate of a neuron is not necessarily identifiable from single trials (Amarasingham et al., 2015). While adaptive filtering has previously been applied to tracking nonstationary place fields from neurons in the hippocampus (Brown et al., 2001) and for assessing the stability of tuning curves in motor cortex (Stevenson et al., 2011), there may be cases where the assumption of linear dynamics is not appropriate. Additionally, while here we assume a Poisson noise model, spiking activity can be more or less variable than Poisson. With stationary models, non-Poisson spiking is typically not a major source of estimation error (Stevenson, 2016). However, the adaptive filtering updates may be sensitive to misspecified noise, and it may be useful to extend the framework here with other noise models (Gao et al., 2015; Pillow & Scott, 2012).

Additional covariates can also easily be incorporated within the extended-GLM framework used here. Given the increasing scale of simultaneous multi-electrode recordings, it would be particularly interesting to model multiple synaptic inputs simultaneously. Extending the model to allow short- and long-term changes in multiple synaptic inputs would be relatively straightforward. Given presynaptic neurons, the conditional intensity can be written . The long-term weights can be tracked using adaptive smoothing with parameters: , and the short-term effects can be estimated with a modification function for each synapse. Combining information from a population of spiking neurons, using a state-space model (Paninski et al., 2010; Smith & Brown, 2003), could also potentially allow for fluctuations in the baseline postsynaptic rate to be estimated at timescales faster than what can be estimated from a single neuron alone.

Previous models have aimed to describe short-term changes in synaptic strength from spikes, without modeling long-term effects or baseline fluctuations (Chan et al., 2008; English et al., 2017; Ghanbari et al., 2017) or to describe long-term changes (Linderman et al., 2014; Robinson et al., 2016; Stevenson & Kording, 2011). Both?

(If I understand correctly: In the Chapter of Song et al., they do things very similar to ours… The difference is that they model STSP by Volterra kernels, which is time-invariant. Then based on that, they further make the kernels time-varying, which is estimated by adaptive filtering.

The difference (improvement)? 1) STSP is now modeled by ISI, but I think these 2 approaches should give similar results; 2) Add baseline to avoid bias)

While the adaptive smoother simply tracks an estimated synaptic weight, it may be preferable to more directly describe the long-term synaptic dynamics with an explicit learning rule based on pre- and postsynaptic spike timing or underlying Calcium dynamics (Graupner & Brunel, 2012). In previous work, we found that a biophysical model of short-term plasticity could, in some cases, account for the short-term dynamics of spike transmission *in vivo* (Ghanbari et al., 2020). Incorporating biophysical models of STSP (Costa et al., 2013) or LTP/LTD (Graupner & Brunel, 2012) is possible with additional computational costs. Recent studies (Deperrois & Graupner, 2020) have also shown that short-term and long-term synaptic plasticity can act together tune sensitive range of synaptic plasticity, and this suggest the necessity for considering effects in different timescale at the same time.

(In this paper, they model LTSP by Calcium-based model, with STD included (and also without STD for comparison). They found that at different STD, the synaptic changes occur in different firing rate ranges.)

The model introduced here aims to track time-varying synaptic weights from simultaneous extracellular recordings from a pre- and postsynaptic neuron. For the sake of simplicity, we assume that monosynaptic connections can be accurately identified. However, detecting synaptic connections from large-scale multielectrode recordings is not necessarily straightforward, particularly for weak connections or short recording times. Assuming that synaptic connections can be accurately identified, however, this model extends static models of coupling between neurons to account for potential plasticity. Applying this model to experimental recordings may lead to new insights into how synapses vary on both short- and long- timescales during ongoing behavior.

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