Due Date: Mar 26, 2018 in class
How to submit: Hard copy in the Class

3.1 Consider the optimization problem

$$\begin{array}{ll} \text{minimize} & f_0(x_1, x_2) \\ & 2x_1 + x_2 \geq 1 \\ \text{subject to} & x_1 + 3x_2 \geq 1 \\ & x_1 \geq 0, x_2 \geq 0 \end{array}$$

Make a sketch of the feasible set. For each of the following objective functions, give the optimal set and the optimal value. Then use CVX to verify the optimal values you obtained.

(a) $f_0(x_1, x_2) = x_1 + x_2$
(b) $f_0(x_1, x_2) = -x_1 - x_2$
(c) $f_0(x_1, x_2) = x_1$
(d) $f_0(x_1, x_2) = \max\{x_1, x_2\}$
(e) $f_0(x_1, x_2) = {x_1}^2 + 9{x_2}^2$

3.2 Solve the optimal activity level problem described in exercise 4.17 in Convex Optimization, for the instance with problem data

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 3 & 1 & 1 \\ 2 & 1 & 2 & 5 \\ 1 & 0 & 3 & 2 \end{bmatrix}, c^{\max} = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 100 \end{bmatrix}, p = \begin{bmatrix} 3 \\ 2 \\ 7 \\ 6 \end{bmatrix}, p^{\text{disc}} = \begin{bmatrix} 2 \\ 1 \\ 4 \\ 2 \end{bmatrix}, q = \begin{bmatrix} 4 \\ 10 \\ 5 \\ 10 \end{bmatrix}$$

You can do this by forming the LP you found in your solution of exercise 4.17, or more directly, using CVX. Give the optimal activity levels, the revenue generated by each one, and the total revenue generated by the optimal solution. Also, give the average price per unit for each activity level, i.e., the ratio of the revenue associated with an activity, to the activity level. (These numbers should be between the basic and discounted prices for each activity.) Give a very brief story explaining, or at least commenting on, the solution you find. **You also need to submit your CVX matlab code.**

3.3 The illumination problem. In lecture 1 we encountered the function

$$f(p) = \max_{i=1,\ldots,n} \left| \log a_i^T p - \log I_{des} \right|$$

where $a_i \in \mathbf{R}^m$, and $I_{des} > 0$ are given, and $p \in \mathbf{R}_+^{\boldsymbol{m}}$

(a) Show that $\exp f$ is convex on $\{p | a_i^T p > 0, i = 1, \ldots, n\}$.

(b) Show that the constraint 'no more than half of the total power is in any 10 lamps' is convex (i.e., the set of vectors p that satisfy the constraint is convex).

(c) Show that the constraint 'no more than half of the lamps are on' is (in general) not convex.