Dynamic Programming - MATRIX MULTIPLICATION.

Optimal way to compute.

$\boxed{\text{ANALYSYS.}}$

$$\begin{array}{cccccc} {}_{r_1}\overset{c_1}{A_1} & {}_{r_2}\overset{c_2}{A_2} & {}_{r_3}\overset{c_3}{A_3} & A_4 & A_5 \ldots & A_n \end{array}$$

$B[1,n]$

$B[1,1]$	$B[1,2]$	$B[1,3]$	$B[1,n-1]$
$+ B[2,n]$	$+ B[3,n]$	$+ B[4,n]$	$\ldots + B[n,n]$
$+ r_1 \times c_1 \times c_n$	$+ r_1 \times c_2 \times c_n$	$+ r_1 c_3 c_n$	$+ r_1 c_{n-1} c_n$

cost of multiply

find MIN

$$B(i,i) = 0$$

$$B(1,n) = \min \begin{cases} B(1,1) + B(2,n) + r_1 c_1 c_n \\ B(1,2) + B(3,n) + r_1 c_2 c_n \\ \vdots \\ B(1,n-1) + B(n,n) + r_1 c_{n-1} c_n \end{cases}$$

Generalize.

$$B(i,j) = \begin{cases} 0 \text{ if } i = j \\ \min\limits_{k=i}^{j-1} \{ B[i,k] + B[k+1,j] + r_i c_k c_j \end{cases}$$

Example:

	A_1	A_2	A_3	A_4	A_5	A_6
	30×35	35×15	15×5	5×10	10×20	20×25

$$m(1,2) = m(1,1) + m(2,2) + 30\times35\times15 = 0 + 0 + 15{,}750$$

6					5,000	0
5				1000	0	
4			750	0		
3	7,875	2,625	0			
2	15,750	0				
1	0					
	1.	2	3	4	5	6.

$$m(2,3) = m(2,2) + m(3,3) + 35\times15\times5$$

$$m(1,3) = \min\begin{cases} m(1,1) + m(2,3) + 30\times35\times5 = 0 + 2{,}625 + 5{,}250 & \xleftarrow{} 7{,}875 \\ m(1,2) + m(3,3) + 30\times15\times5 = 15{,}750 + 0 + 2{,}250 & \xleftarrow{} 18{,}000 \end{cases}$$

$= 7,875$ etc.

Example 2: (SVU)

	M_1	M_2	M_3	M_4
Dimension	2×3	3×4	4×2	2×4

Ans

4	52	48	32	0
3	36	24	0	
2	24	0		
1	0			
	1	2	3	4

$$m(1,2) = m(1,1) + m(2,2) + 2\times3\times4 = 24$$

$$m(2,3) = m(2,2) + m(3,3) + 3\times4\times2 = 24$$

$$m(3,4) = m(3,3) + m(4,4) + 4\times2\times4 = 32$$

$$m(1,3) = \min\begin{cases} \underbrace{m(1,1)}_{0} + \underbrace{m(2,3)}_{24} + \underbrace{2\times3\times2}_{12} = 36 \\ \underbrace{m(1,2)}_{24} + \underbrace{m(3,3)}_{0} + \underbrace{2\times4\times2}_{16} = 40 \end{cases}$$

$= 36$

$$m(2,4) = \min\begin{cases} \underbrace{m(2,2)}_{0} + \underbrace{m(3,4)}_{32} + \underbrace{3\times4\times4}_{48} = 80 \\ \underbrace{m(2,3)}_{24} + \underbrace{m(4,4)}_{0} + \underbrace{3\times2\times4}_{24} = 48 \end{cases}$$

$= 48$

$$m(1,4) = \min\begin{cases} \underbrace{m(1,1)}_{0} + \underbrace{m(2,4)}_{48} + \underbrace{2\times3\times4}_{24} = 72 \\ \underbrace{m(1,2)}_{24} + \underbrace{m(3,4)}_{32} + \underbrace{2\times4\times4}_{32} = 88 \\ \underbrace{m(1,3)}_{36} + \underbrace{m(4,4)}_{0} + \underbrace{2\times2\times4}_{16} = 52 \end{cases}$$

$= 52$