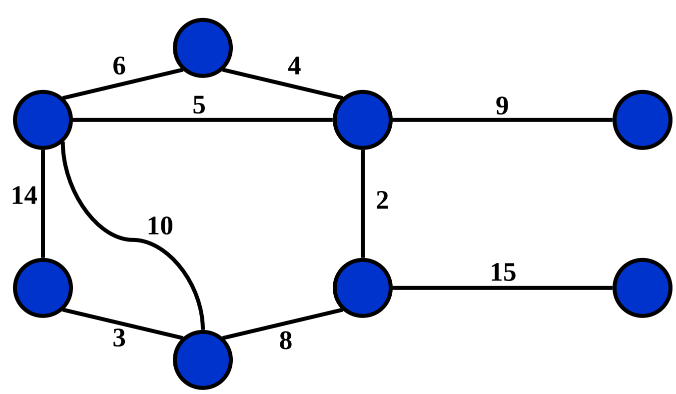
Minimum Spanning Trees

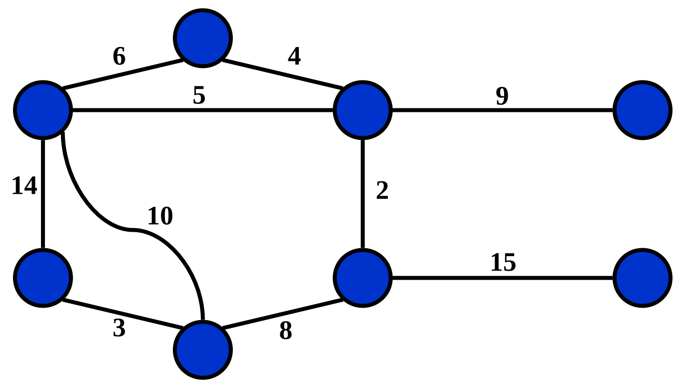
Minimum Spanning Tree

- Problem: given a connected, undirected, weighted graph:



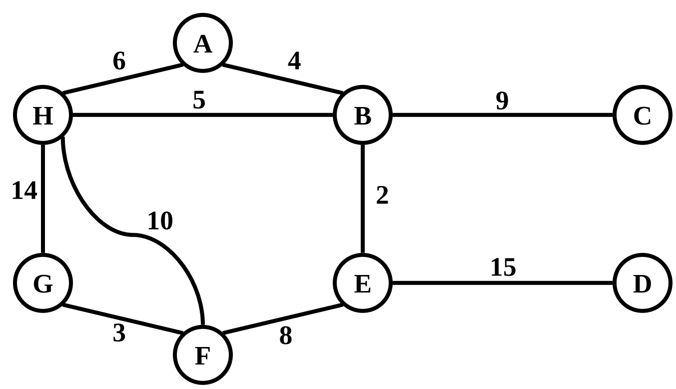
Minimum Spanning Tree

- Problem: given a connected, undirected, weighted graph, find a *spanning tree* using edges that minimize the total weight



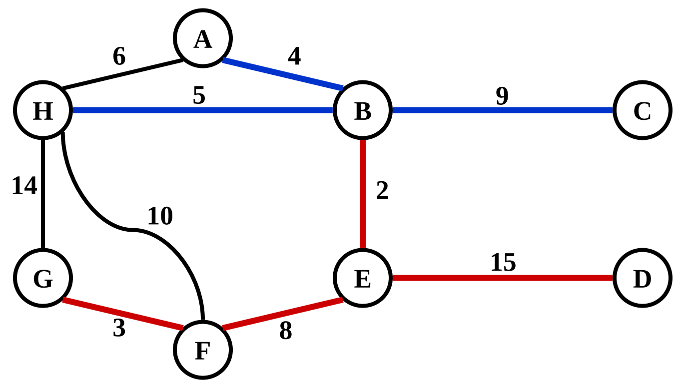
Minimum Spanning Tree

- *Which edges form the minimum spanning tree (MST) of the below graph?*



Minimum Spanning Tree

- Answer:



Minimum Spanning Tree

- MSTs satisfy the *optimal substructure* property: an optimal tree is composed of optimal subtrees
  - Let T be an MST of G with an edge (u,v) in the middle
  - Removing (u,v) partitions T into two trees T_1 and T_2
  - Claim: T_1 is an MST of $G_1 = (V_1,E_1)$, and T_2 is an MST of $G_2 = (V_2,E_2)$ (*Do V_1 and V_2 share vertices? Why?*)
  - Proof: $w(T) = w(u,v) + w(T_1) + w(T_2)$ (There can't be a better tree than T_1 or T_2, or T would be suboptimal)

Minimum Spanning Tree

- Theorem:
  - Let T be MST of G, and let A $\subseteq$ T be subtree of T
  - Let (u,v) be min-weight edge connecting A to V-A
  - Then $(u,v) \in$ T

Minimum Spanning Tree

- Theorem :
  - Let T be MST of G, and let A $\subseteq$ T be subtree of T
  - Let (u,v) be min-weight edge connecting A to V-A
  - Then $(u,v) \in$ T
- Proof: in book (see Theorem 23.1)