Telecommunications System Modeling

Mathematical representation of networks

Problems in Telecommunications Networks

- Some high level problems in networks, eg
- 1 Topology planning
- 2 Dimensioning
- 3 Routing
- 4 Traffic engineering

Example

- You have been given the job of designing a network to provide Internet connectivity to the Illawarra
- The data you have is in terms of population numbers, geographical distribution and behaviour

Example

- Private dwellings which are connected use the internet about 4 hours per week on the average, after 6pm. Average usage is increasing 5% per annum. Connections are increasing 15% per annum
- Enterprises use it from 9am to 5pm. Average usage is increasing 10% per annum

Example: Demand

- Given by marketing as behaviour of users
- We would prefer a matrix of offered traffic between every source and every destination
- This matrix should be time-varying over 24 hours

Supply - Demand

- We do not know how fast users will connect to our service
- If our price is low, the uptake will be fast and our profit will suffer
- If our price is high, the uptake will be slow and we may not make enough connections

Topology Planning

- Where do we place our network nodes?
- How many do we use?
- How do we connect the various nodes together?
- How far ahead should we plan?
- What is the lowest cost network possible?

Dimensioning

- Given the topology, what should the capacity of the links be?
- What should the capacity of the switches be?
- What allowances should we make for future expansion?

Routing

- If we have the network, and we know what the existing traffic pattern is, what is the best path for new traffic to follow when it needs to get from a given source to its destination?
- The answer to this question will vary over 24 hours

Traffic Engineering

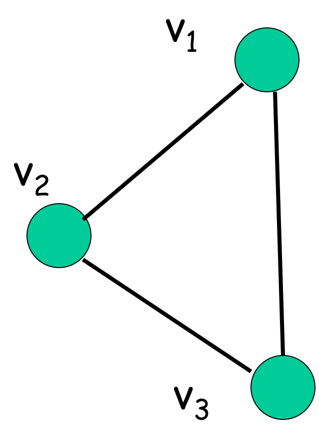
- If we know what traffic demand to expect, what paths should we prefer to use through the network?
- How should the network be upgraded to give the greatest improvement for the smallest cost?

Graph Theory

- Provides a useful method of describing many different networks, eg road or rail networks, electric circuits, urban traffic flows, and telecommunications networks
- A graph is a combination of nodes and links

Graph with 3 Nodes

- Vertices (nodes) are v_1, v_2 and v_3
- Edges are the links between vertices
- Graph, G has the set of vertices, V and edges, E



Order and Size

- The number of vertices is called the "order" of G
- The number of edges is called the "size" of G
- We use the notation of set theory to discuss graphs as follows

Set Notation

- $V = \{v_1, v_2, v_3\}$
- $R = \{(v_1,v_2), (v_2, v_3), (v_3, v_1)\}$
- R is a set of directed arcs on V
- The set of edges is a set of symmetric pairs of vertices
- $E = \{\{(v_1, v_2), (v_2, v_1)\}, \{(v_2, v_3), (v_3, v_2)\}, \{(v_3, v_1), (v_1, v_3)\}\}$

Set Notation

- An edge between vertices u and v, is denoted e_uv
- We describe a graph by using its sets of vertices and edges
- G = <V,E>
- We use upper case letters to denote sets, and lower case for members

Set Notation

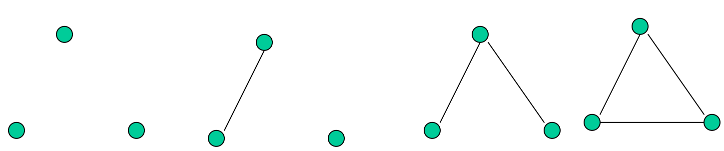
- Examples: $V = \{v_1, v_2, v_3\}$, $E = \{v_1v_2, v_1v_3\}$
- Equivalently: V = {1,2,3}, E = {e_12, e_23}
- The set of edges, E, may be empty
- However, if V is empty, we do not have a graph at all

Set Notation

- If e_uv is in E, we say that it joins the vertices together
- Or, u and v are "adjacent"
- If e_uv is not in E, then u and v are nonadjacent vertices
- If e_uv is in E, then u and v are both "incident" to e_uv

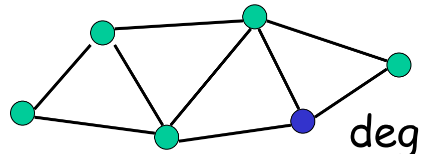
Set Notation

- If e-uv and e-uw are distinct edges in E, then these are "adjacent" edges
- Question: if G has order 3, what are the possible sizes of G?
- Answer; 0, 1, 2, 3



Graph Theory

- If v is a vertex of G, then the number of edges incident with v is called the degree of v. It is denoted by "deg_G v" or simply "deg v" if G is implied



deg v = 3

Graph Theory

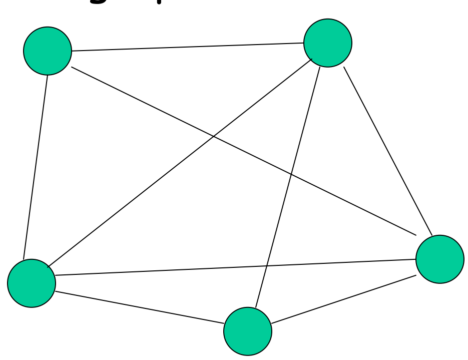
- A (p, q) graph is a graph with order P and size q. The sum of the degrees of the vertices of a graph is always 2q
- It is obvious that this is so, since we consider each edge twice when summing the degrees of the vertices

Complete Graph

- A graph is "complete" if every pair of vertices is adjacent
- A complete graph of order p has the same degree (p – 1) for every vertex
- This is also called a "fully meshed" graph in telecommunications
- No of edges = m = n (n – 1)/2

Complete Graph

- Complete graph with five vertices

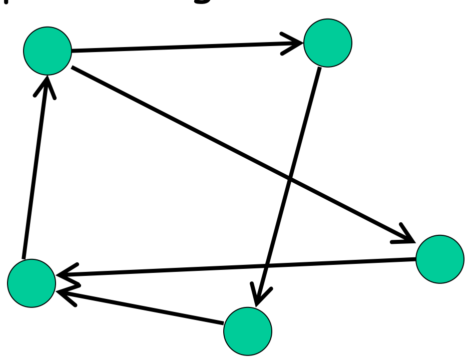


Digraph

- A directed graph (or digraph) is a graph in which the edges have a sense of direction
- The relationship, R, is not symmetrical
- The edges are called “arcs” or “directed edges”

Digraph

- Digraph showing directional edges



Real World Graphs

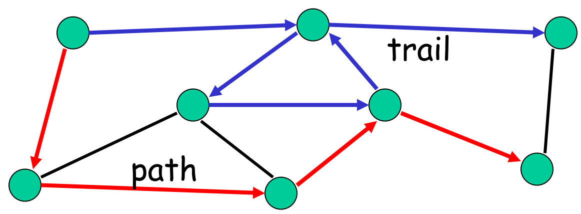
- Graphs can be used to model many different systems in the real world
- Eg, street maps, road maps, electric circuits, communications networks
- Numbers are associated with links, eg to indicate bandwidth, utilisation, distance, etc and nodes to indicate population, connections, etc

More on Graphs

- Let G be a graph. A graph, H, is a subgraph of G if V(H) is a subset of V(G) and E(H) is a subset of E(G)
- A u-v "walk" is a way through a graph. We specify a walk by listing the vertices in order. The edges are then implied (if there is only one edge between any two vertices)

More on Graphs

- A u-v "trail" is a u-v walk which does not repeat any edge
- A u-v "path" is a u-v walk which does not repeat any vertices

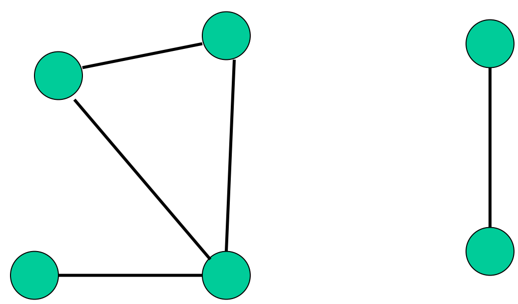


Connected Graphs

- Two vertices, u and v ($u \neq v$) are "connected" if a u-v path exists in *G*
- A graph, *G*, is connected if every two vertices of *G* are connected. Otherwise it is disconnected

Connected Graphs

- This is a disconnected graph

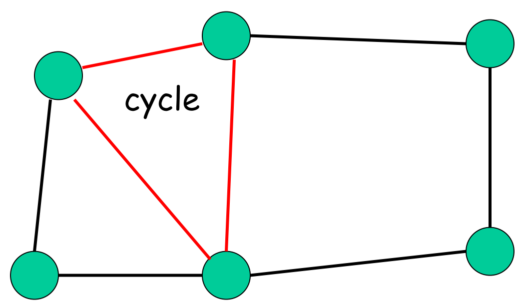


More on Graphs

- A u-v trail in which u = v and which contains at least three edges is called a "circuit".
- A circuit which does not repeat any vertices (except u and v) is called a "cycle"

More on Graphs

- Minimum requirement for a cycle

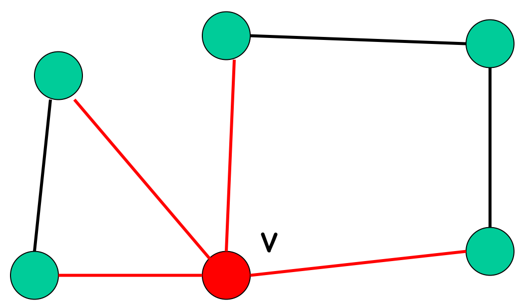


More on Graphs

- If e is an edge in G, then G – e is a subgraph of G, without the edge, e
- If v is a vertex in G, then G – v is a subgraph of G without v in the vertex set, and with all edges incident with v removed from the edge set

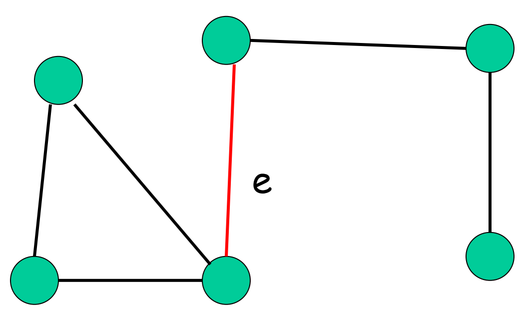
More on Graphs

- A vertex, v, in G is called a "cut-vertex" if G – v is disconnected



More on Graphs

- An edge, e, in G is called a "bridge" if G – e is disconnected

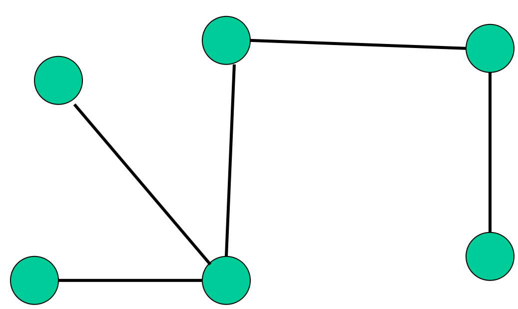


Graphs and Trees

- Let G be a connected graph. An edge, e, of G is a bridge if and only if e does not lie on any cycle of G
- A "tree" is a graph that does not have any cycles
- If u and v are in a tree, G, then there is exactly one u-v path in G

Trees

- Example of a tree



Trees

- We can think of a tree being built up, starting from a single vertex. The tree grows by adding a link and a vertex at every step
- So if *G* is a tree of order p, and size q, then q = p – 1

Spanning Tree

- If *G* is a connected network, then we can construct a tree, T, such that T is a subgraph of *G* and T contains every vertex of *G*
- T is then called a "spanning tree" of *G*
- In many cases we want to construct a spanning tree so that the total cost is minimum