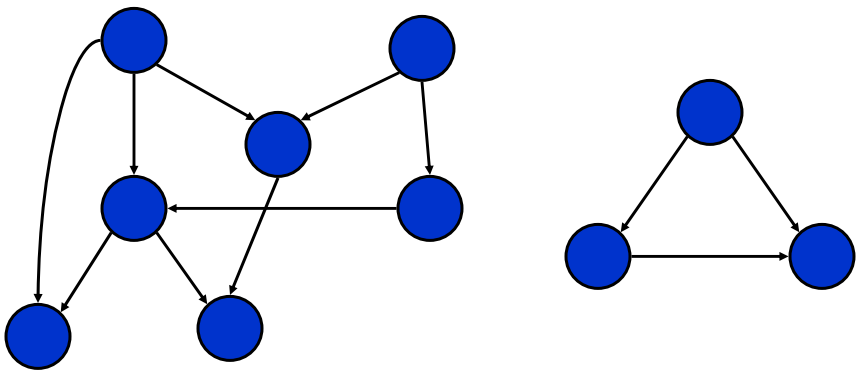
Elemetary Graph Algorithms – contd.

Directed Acyclic Graphs

- A *directed acyclic graph* or *DAG* is a directed graph with no directed cycles:



DFS and DAGs

- Argue that a directed graph G is acyclic iff a DFS of G yields no back edges:
  - Forward: if G is acyclic, will be no back edges
    - Trivial: a back edge implies a cycle
  - Backward: if no back edges, G is acyclic
    - Argue contrapositive: G has a cycle $\Rightarrow \exists$ a back edge
      - Let v be the vertex on the cycle first discovered, and u be the predecessor of v on the cycle
      - When v discovered, whole cycle is white
      - Must visit everything reachable from v before returning from DFS-Visit()
      - So path from u$\rightarrow$v is yellow$\rightarrow$yellow, thus (u, v) is a back edge

Topological Sorting

•A *directed acyclic graph* (DAG) is a directed graph with no directed cycles.

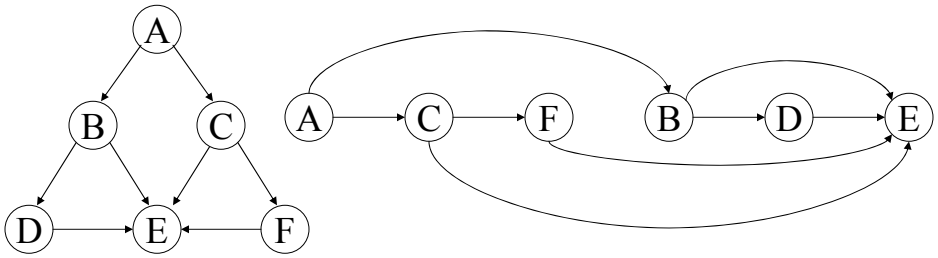
•A *topological sort* of a DAG is an ordering of nodes where all edges go from left to right. That is:

Linear ordering of all vertices in graph G such that vertex u comes before vertex v if edge $(u, v) \in$ G

•Topological sort is not quite a sort, in the usual sense of sorting, and its relation to topology (in the sense understood by topologists) is obscure.

•How can BFS or DFS help us topologically sort a directed graph (or determine the graph is not a DAG)?

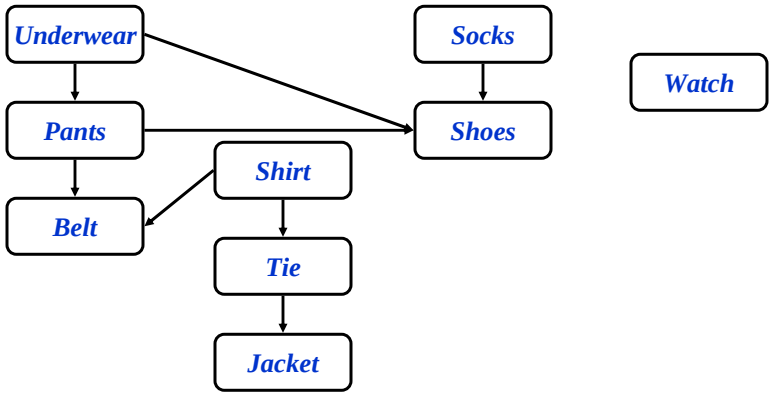
Topological Sorting – contd.



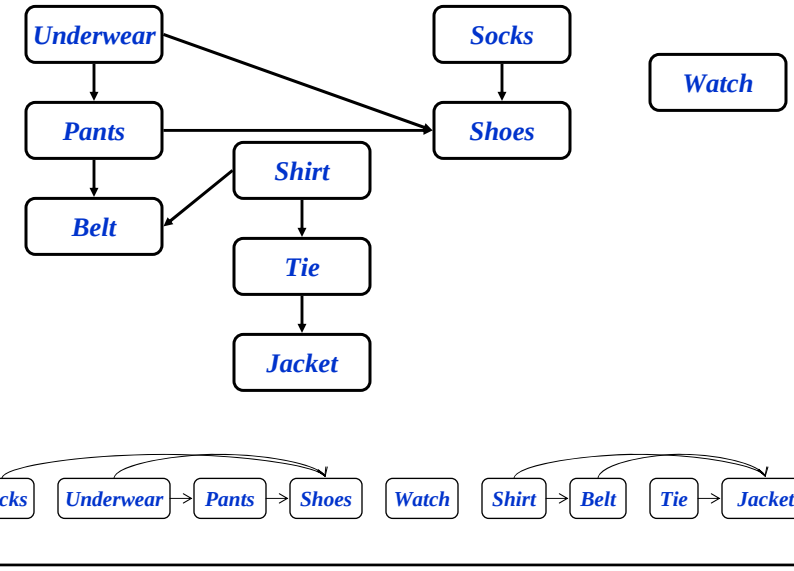
Topological Sorting – contd.

- Real-world example: getting dressed

Getting Dressed



Getting Dressed



Topological Sort Algorithm

```
Topological-Sort()
{
   Run DFS
   When a vertex is finished, output it
   Vertices are output in reverse
     topological order
}
```

- Time: O(V+E)
- Correctness: Want to prove that
  $(u,v) \in G \Rightarrow u{\rightarrow}f > v{\rightarrow}f$

Correctness of Topological Sort

- Claim: $(u,v) \in G \Rightarrow u{\rightarrow}f > v{\rightarrow}f$
  - When (u,v) is explored, u is yellow
    - v = yellow $\Rightarrow$ (u,v) is back edge. Contradiction (*Why?*)
    - v = white $\Rightarrow$ v becomes descendent of u $\Rightarrow$ $v{\rightarrow}f < u{\rightarrow}f$ (since must finish v before backtracking and finishing u)
    - v = black $\Rightarrow$ v already finished $\Rightarrow$ $v{\rightarrow}f < u{\rightarrow}f$