CMSC 441: Homework #5 Solutions

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Exercise 5.2–1

In HIRE–ASSISTANT, assuming that the candidates are presented in a random order, what is the probability that you will hire exactly one time? What is the probability that you will hire exactly n times?
Solution:
The probability that you will hire exactly one time corresponds to the event that the strongest candidate is interviewed first. The probability of this event is $1/n$.
The probability that you will hire all n candidates corresponds to the event that the candidates are interviewed in weakest–to–strongest order. The probability of this event is $\frac{1}{n!}$.

Exercise 5.2–3

Use indicator random variables to compute the expected value of the sum of n dice. **Solution:**
Let $X_1, X_2, \ldots, X_6$ be the random varibles which count number of times faces $1, 2, \ldots, 6$ come up. Let The X be the random variable corresponding to sum of n dice rolls. $E[X] = 1E[X_1] + 2E[X_2] + \ldots, +6E[X_6]$. Expected value $E[X_i]$, is $n/6$.

$$\begin{aligned}
E[X] &= \sum_{i=1}^{6} iE[X_i] \\
&= \sum_{i=1}^{6} i(n/6) \\
&= (n/6)\sum_{i=1}^{6} i \\
&= (21/6)n \\
&= 3.5n
\end{aligned}$$

Exercise 5.3–2

Professor Kelp decides to write a procedure that will produce at random any permutation besides the identity permutation. The identity permutation is $(1, 2, 3, \ldots, n)$, i.e. the numbers 1 to n in their natural order. He proposes the following procedure:

Algorithm 1 PERMUTE-WITHOUT-IDENTITY

```
1: n ← length[A]
2: for i ← 1 to n do
3:     swap A[i] ↔ A[RANDOM(i + 1, n)]
4: end for
```

Does this code do what Professor Kelp intends.
Solution:
This code always swaps 1 with a randomly chosen value from 2 through n. Thus permutations such as $(1, 3, 2, \ldots)$ will never be produced. This is not what Professor Kelp intended.

Exercise 5.4–1

(a)

How many people must there be in a room before the probability that someone has the same birthday as you do is at least 1/2?

Solution:

Probability that someone in the room has the same birthday as me, denoted by $P(B)$ is $1-$ probability that no one in the room has the same birthday as me. $P(B) = 1 - (\frac{364}{365}^n)$. We wish $P(B) \geq 1/2$, thus $1 - (\frac{364}{365})^n \geq 1/2$. Taking logs,

$$\begin{aligned}
log(1/2) &\geq log(\frac{364}{365})^n \\
-log\ 2 &\geq n\ log(\frac{364}{365}) \\
log\ 2 &\leq n\ log(\frac{365}{364}) \\
253 &\leq n
\end{aligned}$$

(b)

How many people must there be in a room before the probability that at least two people have a birthday on July 4 is greater than 1/2?

Solution:

Probability that at least two people have a birthday on July 4, denoted by $P(J)$ is 1 - probability that exactly one person in the room has a birthday on July 4 - probability that no one in the room has a birthday on July 4. $P(J) = 1 - \binom{n}{1}(\frac{1}{365})(\frac{364}{365})^{n-1} - \binom{n}{0}(\frac{364}{365})^n \geq 1/2$. Any value ≥ 613 works.