

# CMSC 441: Homework #6 Solutions

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## Exercise 7.1–1

Show result of partition for array  $A = [13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21]$ .

**Solution:**

Pivot element is 21. Thus A remains the same after the PARTITION operation.  $A = [13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21]$ .

## Exercise 7.1–4

Change the line 4 of PARTITION from  $A[j] \leq x$  to  $A[j] \geq x$ .

## Exercise 7.2–1

Use substitution method to prove that the recurrence  $T(n) = T(n-1) + \Theta(n)$  has the solution  $T(n) = \Theta(n^2)$ .

**Solution:**

We guess that  $T(n) \leq O(n^2)$ .

$$\begin{aligned} T(n) &\leq c_1(n-1)^2 + \Theta(n) \\ &\leq c_1(n-1)^2 + c_0n \\ &\leq c_1(n^2 - 2n + 1) + c_0n \\ &\leq c_1n^2 - (2c_1 - c_0)n + c_1 \\ &\leq c_1n^2 \text{ for } n_0 \geq 1 \text{ and } c_0 > c_1 \end{aligned}$$

Thus  $T(n) \in O(n^2)$ . Similarly, we can prove that  $T(n) \in \Omega(n^2)$ . Consequently,  $T(n) \in \Theta(n^2)$ .

## Exercise 7.4–1

Show that in the recurrence

$$T(n) = \max_{0 \leq q \leq n-1} (T(q) + T(n-q-1)) + \Theta(n)$$

,

$$T(n) = \Omega(n^2)$$

**Solution:**

We guess that  $T(n) \geq cn^2$  for some constant  $c$ . Substituting, we get

$$\begin{aligned} T(n) &\geq \max_{0 \leq q \leq n-1} (cq^2 + c(n-q-1)^2) + \Theta(n) \\ &= c \cdot \max_{0 \leq q \leq n-1} (q^2 + (n-q-1)^2) + \Theta(n) \end{aligned}$$

The pure quadratic  $q^2 + (n - q - 1)^2$  achieves its maximum at two end-points,  $0, n - 1$ . We choose  $q = 0$ .  
Thus  $\max_{0 \leq q \leq n-1} (q^2 + (n - q - 1)^2) = (n - 1)^2 = n^2 - 2n + 1$ .

$$\begin{aligned} T(n) &\geq cn^2 - c(2n - 1) + \Theta(n) \\ &\geq cn^2, \text{ where } c \text{ is chosen to dominate } \Theta(n). \end{aligned}$$

## Exercise 8.2–1

Step (a)

A: 6,0,2,0,1,3,4,6,1,3,2

C: 2,2,2,2,1,0,2

Step (b)

C: 2,4,6,8,9,9,11

Step (c)

B: -, -, -, -, 2, -, -, -, -

C: 2,4,5,8,9,9,11

Step (d)

B: -, -, -, -, 2, -, 3, -, -, -

C: 2,3,5,7,9,9,11

Step (e)

B: -, -, -, 1, -, 2, -, 3, -, -, -

C: 2,3,5,7,9,9,11

Step (f)

B: -, -, -, 1, -, 2, -, 3, -, -, 6

C: 2,3,5,7,9,9,10

Step (g)

B: -, -, -, 1, -, 2, -, 3, 4, -, 6

C: 2,3,5,7,8,9,10

B: -, -, 1, 1, -, 2, 3, 3, 4, -, 6

C: 2,2,5,6,8,9,10

Step (i)

B: -, -, 1, 1, -, 2, 3, 3, 4, -, 6

C: 2,2,5,6,8,9,10

Step (j)

B: -, 0, 1, 1, -, 2, 3, 3, 4, -, 6

C: 1,2,5,6,8,9,10

Step (k)

B: -, 0, 1, 1, 2, 2, 3, 3, 4, -, 6

C: 0,2,4,6,8,9,10

Step (l)

B: 0,0,1,1,2,2,3,3,4, -, 6

C: 1,2,4,6,8,9,10

Step (m)

B: 0,0,1,1,2,2,3,3,4,6,6

C: 0,2,4,6,8,9,9

## Exercise 8.3–1

Radix-sort on the list COW, DOG, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX.

COW	SEA	TAB	BAR
DOG	TEA	BAR	BIG
SEA	MOB	EAR	BOX
RUG	TAB	TAR	COW
ROW	DOG	SEA	DIG
MOB	RUG	TEA	DOG
BOX	DIG	DIG	EAR
TAB	BIG	BIG	FOX
BAR	BAR	MOB	MOB
EAR	EAR	DOG	NOW
TAR	TAR	COW	ROW
DIG	COW	ROW	RUG
BIG	ROW	NOW	SEA
TEA	NOW	BOX	TAB
NOW	BOX	FOX	TAR
FOX	FOX	RUG	TEA

## Exercise 8.4–1

Illustrate the operation of Bucket-Sort on the array  $A = [.79.13.16.64.39.20.89.53.71.42]$ .

**Solution:**

.13	.16
.20	
.39	
.42	
.53	
.64	
.71	.79
.89	