CS502)

Computer Science: is the science of
- algorithm processing
- representation
- storage and
- transmission

of information.

What is algorithm?

Def: an algo is well defined computational procedure that converts input into output.

Input → Algo → Output

E.g: a seq of n real numbers.

Why study Algorithms?

Algorithm address issues related to: feasibility, effeciency & performance, & scalability.

- Study of algo enble us to determine, if a computer prog is feasible or infeasible
- Effecient algo lead to a effecient computer program, & effecient use of hardware resources.
- Algo help us to understand issues related to scalability.
- Analysys of algo provides a language for talking abt program behavior.

However, we should understand that computer program effeciency is only certain facet of overall computer resource usage.

Examples:

1. gcd = greatest common divisor.

$\gcd(2,4) = 2$

$\gcd(10,50) = 10$

$\gcd(27,112) = 1$

$\gcd(56432, 92431) = ?$

$\gcd(256, 384) = 128$

Algo:

256) 384 (1
256
128) 256 (2
256
0

gcd

$\gcd(361, 190) = 19$

190) 361 (1
190
171) 190 (1
171
19) 171 (9
171
0

gcd

⇒ This is Euclidean algorithm.
(2000 yrs old).

Examples

$2 \times 2 = 2^2 = 4$ 1 MULTIPLY.

$2 \times 2 \times 2 = 2^3 = 8$ 2 "

$$\underbrace{2\times\ldots\times 2}_{53 \text{ times}} = 2^{53}$$ 52 MULTIPLIES

$$\underbrace{2\times\ldots 2}_{6 \text{ times}} = 2^{6}$$

$2 \times 2 \times \ldots 2 = 2^8 = 256$ 7 MULTIPLIES

$2 \times 2 = 2^2 = 4$ 1 MULTIPLY.

$2^2 \times 2^2 = 2^4$ 1 "

$2^4 \times 2^4 = 256 = 2^8$ 1 "

3 " (instead of 7 MUL)

$2^{32} \longrightarrow$ 31 MULTIPLIES.

another way $2^2, 2^4, 2^8, 2^{16}, 2^{32}$ 5 MULTIPLIES

Algo is called. METHOD OF REPEATED SQUARING

NOTATION:

$\mathbb{P} = \{1, 2, 3 \ldots\}$ = Set of positive numbers.

$\mathbb{N} = \{0, 1, 2 \ldots\}$ = Set of natural numbers

$\mathbb{Z} = \{\ldots -1, 0, 1\}$ = Set of pos & neg & 0.

$\mathbb{R} = (-\infty, +\infty)$ = set of real numbers.

$\mathbb{R}_0^+ = [0, +\infty)$ = set of real numbers ≥ 0.

$\mathbb{R}^+ = (0, +\infty)$ = set of pos real no.

$\mathbb{Q} = \left\{ \frac{m}{n} \mid m, n \in \mathbb{Z};\ n \neq 0 \right\}$ = set of rational numbers

$\mathbb{C} = \{ a + ib \mid a, b \in \mathbb{R},\ i = \sqrt{-1} \}$ = set of complex nos.

SOME IMPORTANT FACTS

1) Logarithms:

(i) $\log xy = \log x + \log y$.

(ii) $\log_a b \, \log_b a = 1$,

(iii) $\log_a x^y = y \log_a x$

(iv) $a^{\log_b n} = n^{\log_b a}$.

2) i). $S = a + ar + ar^2 + \dots + ar^{n-1}$

$= \frac{a.(1-r^n)}{1-r}$; $r \neq 1$

$S = na$ if $r = 1$.

ii). $S = a + ar + ar^2 + \dots$

$= \frac{a}{1-r}$ if $|r| < 1$.

GP

3) i) $S = 1 + 2 + \dots + n = \frac{n(n+1)}{2}$.

ii) $S = a + (a+d) + (a+2d) + \dots + (a+(n-1)d)$

$= [a + (a+(n-1)d)] \frac{n}{2}$

AP

4) Calculus.

i). $\frac{d}{dx} x^n = n x^{n-1}$

ln = natural log.

Base = e = $1 + \frac{1}{1!} + \frac{1}{2!} + \dots$

ii) $$\frac{d}{dx} a^x = a^x \ln a = a^x \ln a$$

MATHS INTERLUDE:

① $$S = 1 + x + x^2 + \dots = \frac{1}{(1-x)} ; \quad |x| < 1.$$

$$S = \sum_{i=0}^{\infty} x^i = \frac{1}{1-x}, \quad |x| < 1$$

② $$\frac{dS}{dx} = 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$\frac{dS}{dx} = \frac{1}{(1-x)^2}$$

$$\therefore \quad \frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$\frac{x}{(1-x)^2} = x + 2x^2 + 3x^3 + 4x^4 + \dots = \sum_{i=1}^{\infty} i x^i$$

$$\boxed{\sum_{i=1}^{\infty} i x^i = \frac{x}{(1-x)^2}, \quad |x| < 1}$$

EXAMPLE:

Worst case running time as a function of n is.

$$4n^2 - 3n \log n + 17.5n - 43n^{2/3} + 75 = f(n).$$

LET $\boxed{n = \omega^7}$

(i) $$4n^2 = 4 \times (\omega^7)^2 = 4 \times \omega^{14}$$ highest ii). $3n.\log n = 3(10^7).\log_{10} 10^7 = 21 \times 10^7$

iii) $17.5n = 17.5 \times 10^7$

iv) $43n^{2/3} < 43n = 43 \times 10^7$

v) $75 < 10^7$

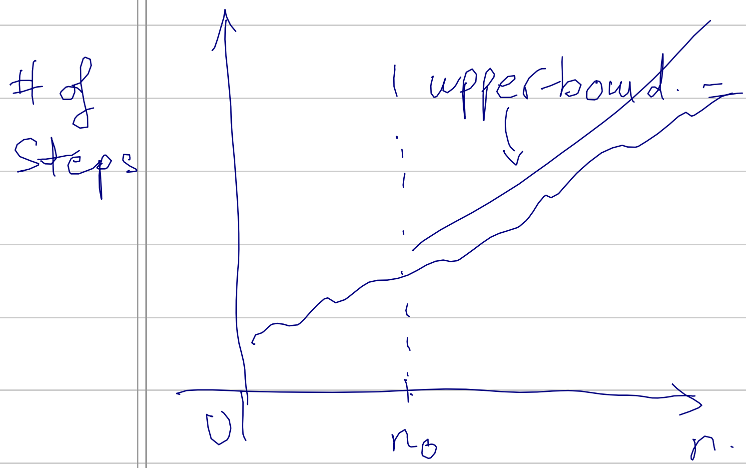
for $n \to \infty$ $f(n) \simeq 4n^2 = O(n^2)$ Big OH

ignore 4.

$\Rightarrow$ $\boxed{f(n) \text{ is } \propto n^2 \text{ for large } n.}$

proportional. to

ASYMTOTIC NOTATION

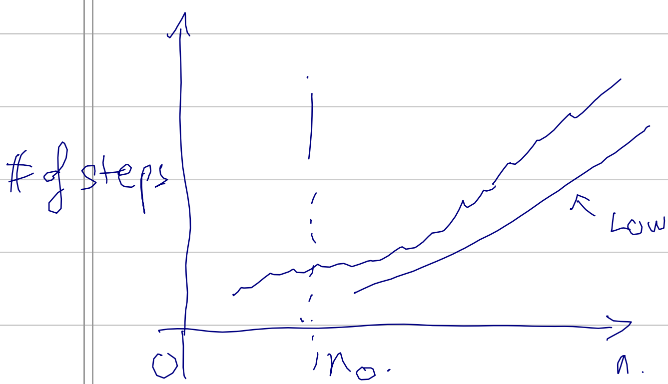


$= cg(n)$.

$c > 0$.

$f(n) = O(g(n))$

Big OH.

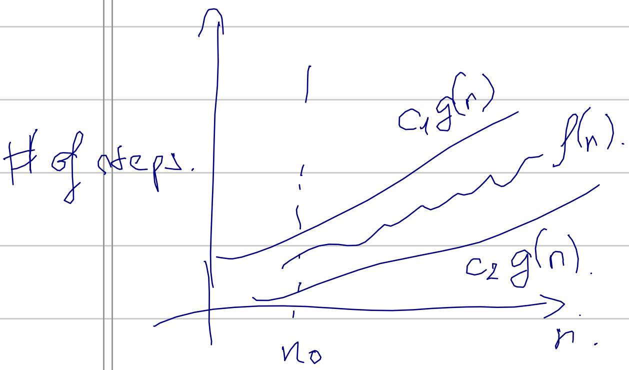


Big OMEGA.

$f(n) = \Omega(g(n))$

Lowerbound. $= cg(n)$

$c > 0$.



$c_1, c_2 > 0.$

$f(n) = \Theta(g(n)).$

BIG THETA

$f(n) :=$ # of steps (EXACT) in the algorithm.

n is important parameter of algorithm.

LET $L = \lim\limits_{n \to \infty} \dfrac{f(n)}{g(n)}$

	L	
O	$0 \leq L < \infty$	BIG OH
Ω	$0 < L \leq \infty$	BIG OMEGA
Θ	$0 < L < \infty$	BIG THETA
o	$L = 0$	SMALL OH
ω	$L = +\infty$	SMALL OMEGA

$\boxed{\text{IMPORTANT FOR EXAMS}}$

PROBLEMS: Indicate whether

$$f = O(g),\ f = \Omega(g),\ f = \Theta(g),\ f = o(g) \text{ or } f = \omega(g).$$

1) $f(n) = (n - 100)$; $g(n) = n - 200$

SOLUTION: $L = \lim\limits_{n \to \infty} \dfrac{f(n)}{g(n)} = \lim\limits_{n \to \infty} \dfrac{n - 100}{n - 200} = 1 = \text{CONST}$

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assignmen	35%
midterm	25%.
final	35%
attendance	5%

$\therefore \quad f(n) = \Theta(g(n))$

2) $f(n) = \sqrt{n} \quad ; \quad g(n) = n^{2/3}$

SOLUTION:

$$L = \text{limit} \frac{f(n)}{g(n)} = \frac{\sqrt{n}}{n^{2/3}} = \frac{n^{1/2}}{n^{2/3 - 1/2}} = \lim_{n\to\infty} \frac{1}{n^{1/6}}$$

$$= 0 \quad \text{(small oh)}$$

$$\therefore f(n) = o(g(n))$$

3) $f(n) = 100n + \log n \quad ; \quad g(n) = n + (\log n)^2$

$$L = \lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{100n + \log n}{n + (\log n)^2}$$

n is much bigger than log n.

$$= \lim_{n\to\infty} \frac{100n}{n} = 100 \quad \text{Constant}$$

ignore log n & $(\log n)^2$

$$f(n) = \Theta(g(n))$$

4) $f(n) = n \log n \quad , \quad g(n) = 10n \log(10n)$

$$L = \lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n \log n}{10n \log(10n)} = \lim_{n\to\infty} \frac{n \log n}{10n(\log 10 + \log n)}$$

$$= \lim_{n\to\infty} \frac{\log n}{10 \log 10 + 10 \log n} = \frac{1}{10} \quad \text{(constant)}$$

ignore

$$\Rightarrow f(n) = \Theta(g(n))$$

5) $f(n) = \log(2n)$, $g(n) = \log(3n)$

$f(n) = \log 2n = \log 2 + \log n$

$g(n) = \log 3n = \log 3 + \log n$

$\Rightarrow L = \lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{\log_2 + \log n}{\log 3 + \log n} = 1$

$\Rightarrow f(n) = \Theta(g(n))$

6) $f(n) = 10 \log n$, $g(n) = \log n^2$

$\rightarrow g(n) = 2\log n$

$L = \lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{10\log n}{2\log n} = 5$ constant

$\Rightarrow f(n) = \Theta(g(n))$

7) $f(n) = n^{1.01}$, $g(n) = n(\log n)^2$

$L = \lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{n^{1.01}}{n(\log n)^2} = \frac{n \times n^{0.01}}{n \cdot (\log n)^2}$

$= \lim_{n\to\infty} \frac{n^{0.01}}{(\log n)^2} = \infty$ (n is much bigger than $\log n$)

$\therefore f(n) = \omega(g(n))$

8) $f(n) = \frac{n^2}{\log n}$, $g(n) = n(\log n)^2$

$L = \lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n^2}{\log n \cdot n(\log n)^2} = \frac{n}{(\log n)^3} = \infty$

$\therefore L = w(g(n))$.

9) $f(n) = n^{0.1}$, $g(n) = (\log n)^{10}$.

$L = \lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n^{0.1}}{(\log n)^{10}} = \infty$ (n is much bigger than $\log n$).

$\Rightarrow f(n) = w(g(n))$. $\boxed{\text{USE L'HOSPITAL'S RULE}}$

LOPITAL'S RULE

10). $f(n) = (\log n)^{\log n}$, $g(n) = \frac{n}{\log n}$.

$L = \lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{(\log n)^{\log n} \cdot \log n}{n}$

$= \lim_{n\to\infty} \frac{(\log n)^{(1+\log n)}}{n} = 0$

$\Rightarrow f(n) = o(g(n))$

wrong

Sol: $n = 10^m$, $\log n = \log 10^m = m$

$f(n) = m^m$, $g(n) = \frac{n}{m} = \frac{10^m}{m}$.

$L = \lim_{n\to\infty} \frac{m^m \cdot m}{10^m} = \lim_{n\to\infty} \left(\frac{m}{10}\right)^m m \to \infty$

$f(n) = w(g(n))$.

<u>Assignment 1.</u> The following problems are from. Introduction to Algorithms, by CLRS.

Points	Second Ed.	Third Ed.
10	Page 13: 1.2-2.	Page 14: 1.2-2
10	Pag 13: 1.2-3	Page 14: 1.2-3.

<u>Hint:</u> Use EXCEL spreadsheet

<u>Answers</u>

$$\begin{cases} 2 \le n \le 43. \\ n = 15 \end{cases}$$