

12. $f(n) = \sqrt{n}$, $g(n) = 5^{\log_2 n}$.

Solution: $g(n) = 5^{\log_2 n} = n^{\log_2 5}$

$$L = \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{n^{1/2}}{n^{\log_2 5}} = \lim_{n \rightarrow \infty} \frac{n^{1/2}}{n^{2.25}} = \lim_{n \rightarrow \infty} \frac{1}{n^{1.75}} = 0$$

$\therefore f(n) = o(g(n))$.

13. $f(n) = n 2^n$; $g(n) = 3^n$.

Solution: $L = \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{n 2^n}{3^n}$

$a^n \gg n \gg \log n$
 $a > 1$, as $n \rightarrow \infty$
 important for exam.

$$= \lim_{n \rightarrow \infty} n \left(\frac{2}{3} \right)^n = \lim_{n \rightarrow \infty} \frac{n}{\left(\frac{3}{2} \right)^n}$$

$$= \lim_{n \rightarrow \infty} \frac{n}{1.5^n} = 0$$

$\therefore f(n) = o(g(n))$

14. $f(n) = (\log n)^{\log n}$; $g(n) = 2^{(\log_2 n)^2}$

SOLUTION: $g(n) = 2^{(\log_2 n)^2} = 2^{(\log_2 n)(\log_2 n)}$
 $= n^{(\log_2 n) \log_2 n} = n^{\log_2 n}$

$$L = \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{(\log n)^{\log n}}{n^{\log_2 n}} = \lim_{n \rightarrow \infty} \left(\frac{\log n}{n} \right)^{\log n}$$

$$= 0$$

$\therefore f(n) = o(g(n))$