Approximation Algorithms

Approximation Algorithm

- We say an approximation algorithm for the problem has a ratio bound of $\rho(n)$ if for any input size n, the cost C of the solution produced by the approximation algorithm is within a factor of $\rho(n)$ of the *C** of the optimal solution:

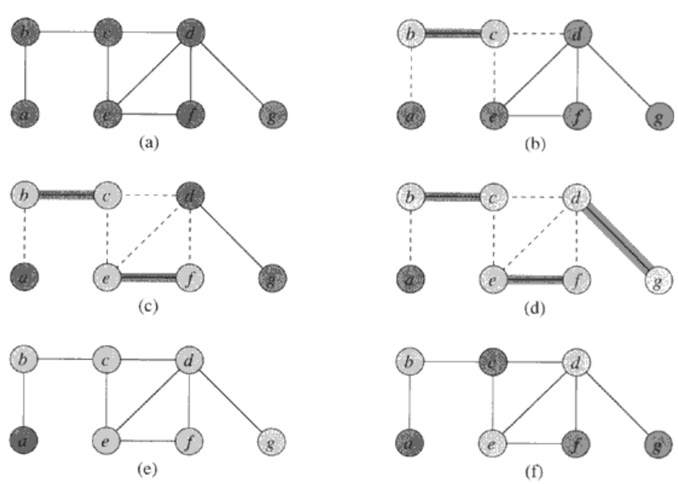
$$\max\{\frac{C}{C^*}, \frac{C^*}{C}\} \le \rho(n)$$

- This definition applies for both minimization and maximization problems.

The Vertex-cover Problem

```
APPROX-VERTEX-COVER(G)
C ← ∅
E′ ← E[G]
while E′ ≠ ∅
      do let (u, v) be an arbitrary edge of E′
         C ← C ∪ {u, v}
         remove from E′ every edge incident on either u or v
return C
```


Example



Proof

Theorem 35.1 APPROX_VERTEX_COVER has ratio bound of 2.

Proof.

Let *A* be the set of selected edges.

$$|C| = 2|A|$$
$$|A| \le |C^*|$$

$$\Rightarrow |C| \le 2|C^*|$$

The Traveling-salesman Problem

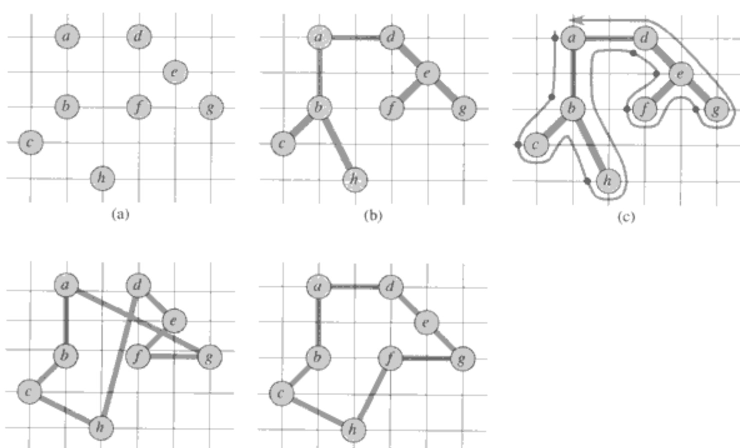
Triangle inequality

$$c(u,w) \le c(u,v) + c(v,w) \quad \forall u,v,w$$

Approximation Method

```
APPROX-TSP-TOUR(G, c)
    select a vertex r ∈ V[G] to be a "root" vertex
    compute a minimum spanning tree T for G from root r
            using MST-PRIM(G, c, r)
    let L be the list of vertices visited in a preorder tree walk of T
    return the hamiltonian cycle H that visits the vertices in the order L
```


Example



(a) (b) (c)

(d) (e)

Proof

Theorem 35.2. APPROX_TSP_TOUR is an approximation algorithm with ratio bound of 2 for TSP with triangular inequality.

Proof.

$$c(T) \le c(H^*)$$

$$c(W) = 2c(T) \le 2c(H^*)$$

$$c(H) \le c(W)$$ **Triangle inequality**

$$\Rightarrow c(H) \le 2c(H^*)$$