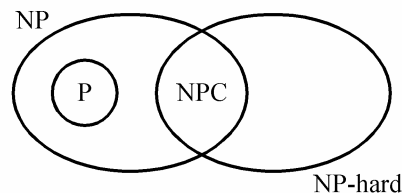# The Theory of NP-Completeness

- **P**: the class of problems which can be solved by a deterministic polynomial algorithm.
- **NP** : the class of decision problem which can be solved by a non-deterministic polynomial algorithm.
- **NP-hard**: the class of problems to which every NP problem reduces.
- **NP-complete (NPC)**: the class of problems which are NP-hard and belong to NP.

NP

NPC

P

P?NP
=

NP: Non-deterministic Polynomial

P: Polynomial

NPC: Non-deterministic Polynomial Complete

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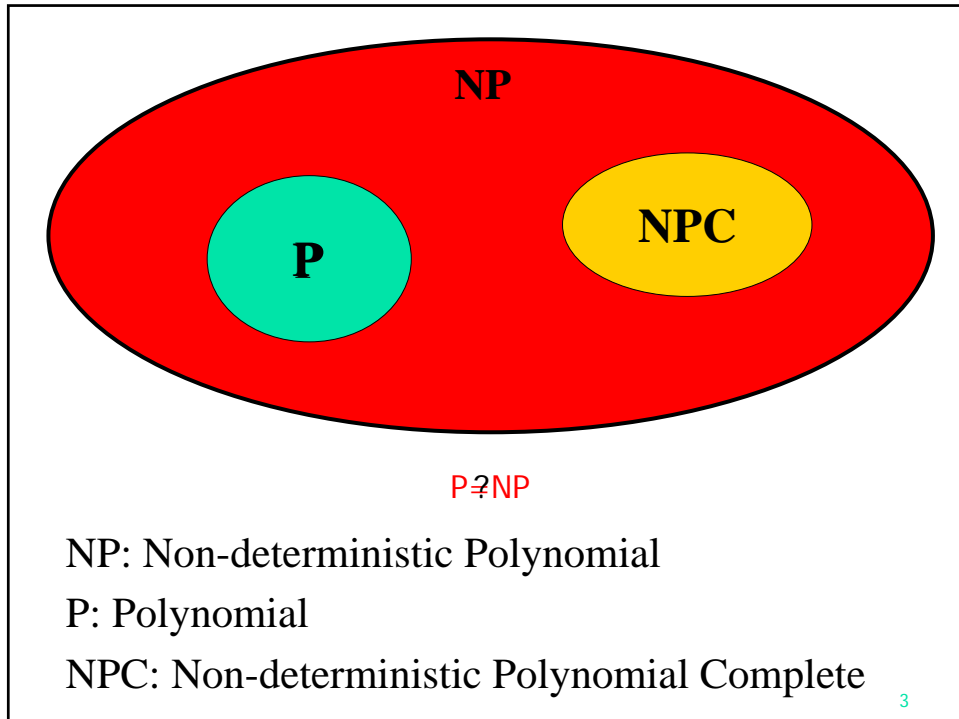
Decision problems

- The solution is simply "Yes" or "No".
- Optimization problem : more difficult
  Decision problem
- E.g. the traveling salesperson problem
  - Optimization version:
    Find the shortest tour
  - Decision version:
    Is there a tour whose total length is less than or equal to a constant C ?

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Nondeterministic algorithms

- A nondeterministic algorithm is an algorithm consisting of two phases: guessing and checking.
- Furthermore, it is assumed that a nondeterministic algorithm always makes a correct guessing.

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Nondeterministic algorithms

- They do not exist and they would never exist in reality.
- They are useful only because they will help us define a class of problems: NP problems

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NP Algorithm

- If the checking stage of a nondeterministic algorithm is of polynomial time-complexity, then this algorithm is called an NP (nondeterministic polynomial) algorithm.

NP Problem

- If a decision problem can be solved by a NP algorithm, this problem is called an NP (nondeterministic polynomial) problem.
- NP problems : (must be decision problems)