

Assignment 10

Design and Analysis of Algorithms

Problem 1: (20 Points) ¹You are given a digraph $H = (V, E)$, where elements of the vertex set V are 1, 2, 3, 4, 5, and 6. Members of the edge set E are (1, 2), (1, 3), (2, 3), (2, 4), (3, 4), (3, 5), (4, 6), (5, 3), (5, 4), (5, 6), and (6, 4). The lengths of these arcs are $c_{12} = 1, c_{13} = 2, c_{23} = 5, c_{24} = 7, c_{34} = 1, c_{35} = 2, c_{46} = 2, c_{53} = 8, c_{54} = 3, c_{56} = 5$, and $c_{64} = 9$.

Assume that the source-node is node 1. Using Dijkstra's algorithm, determine the distance and predecessor vectors, $\mathcal{D} = (d_1, d_2, d_3, d_4, d_5, d_6)$, and $\mathcal{T} = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6)$ respectively. Show all the iterative steps, as given in my class notes.

Problem 2: (50 Points) This problem is on enumerating all the possible spanning trees of an undirected graph with n labelled nodes.

- (a) (5 Points) What is the total number of spanning trees of a graph with 2 labelled nodes? Draw it.
- (b) (5 Points) What is the total number of spanning trees of a graph with 3 labelled nodes? Draw them.
- (c) (10 Points) What is the total number of spanning trees of a graph with 4 labelled nodes? Draw them.
- (d) (10 Points) Guess the total number of spanning trees of a graph with n labelled nodes?
- (e) (20 Points) Derive the formula in part (d).

Hint: Search the Internet for parts (d) and (e), or read a standard textbook on graph theory.

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