CMSC 441: Homework #4 Solutions

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The regularity condition is always satisfied for all the problems in this homework where case 3 applies, because $a\ f(n/b) = a\ n^k/b^k = (a/b^k)n^k = (a/b^k)f(n)$, and a/b^k is a constant strictly less than 1 for all the problems on this homework.

Exercise 4.3–1

(a)

$T(n) = 4T(n/2) + n$
Solution: $a = 4, b = 2, f(n) = n$. Case 1 applies, thus $T(n) = \Theta(n^2)$.

(b)

$T(n) = 4T(n/2) + n^2$
Solution: $a = 4, b = 2, f(n) = n^2$. Case 2 applies, thus $T(n) = \Theta(n^2\ lg\ n)$.

(c)

$T(n) = 4T(n/2) + n^3$
Solution: $a = 4, b = 2, f(n) = n^3$. Case 3 applies, thus $T(n) = \Theta(n^3)$.

Problem 4.1

(a)

$T(n) = 2T(n/2) + n^3$
Solution: $a = 2, b = 2, f(n) = n^3$. Case 3 applies, thus $T(n) = \Theta(n^3)$.

(b)

$T(n) = T(9n/10) + n$
Solution: $a = 1, b = 10/9, f(n) = n$. Case 3 applies, thus $T(n) = \Theta(n)$.

(c)

$T(n) = 16T(n/4) + n^2$
Solution: $a = 16, b = 4, f(n) = n^2$. Case 2 applies, thus $T(n) = \Theta(n^2\ lg\ n)$.

(d)

$T(n) = 7T(n/3) + n^2$
Solution: $a = 7, b = 3, f(n) = n^2$. Case 3 applies, thus $T(n) = \Theta(n^2)$.

(e)

$T(n) = 7T(n/2) + n^2$
Solution: $a = 7, b = 2, f(n) = n^2$. Case 1 applies, thus $T(n) = \Theta(n^{lg\ 7})$.

Problem 4.1 continued on next page...

(f)

$T(n) = 2T(n/4) + \sqrt{n}$
Solution: $a = 2, b = 4, f(n) = \sqrt{n}$. Case 2 applies, thus $T(n) = \Theta(\sqrt{n}\ lg\ n)$.

(g)

$T(n) = T(n-1) + n$
Solution: Applying recursion tree, $T(n) = n \cdot (n-1) \cdot (n-2) \cdots 1 = n(n-1)/2 = \Theta(n^2)$.
We must prove the $T(n) = \Omega(n^2)$ by induction. The inductive hypothesis is $T(n) = cn^2$ for some constant $c > 0$.

$$\begin{aligned} T(n) &= T(n-1) + n \\ &\geq c(n-1)^2 + n \\ &= cn^2 - 2cn + c + n \\ &\geq cn^2 \end{aligned}$$

We can prove the $T(n) = O(n^2)$ by similar arguments.

(h)

$T(n) = T(\sqrt{n}) + 1$
Solution: Let $m = lg\ n$ and $S(m) = T(2^m)$. Thus, $T(2^m) = T(2^{m/2}) + 1$, so $S(m) = S(m/2) + 1$. Case 2 applies and $S(m) = \Theta(lg\ m)$. Therefore, $T(n) = \Theta(lg\ lg\ n)$.