CIS 510 Assignment 2

Steven Walton

May 12, 2019

Problem P1

Program expects an input where the first line is the number of players and each subsequent line is the grand coalition of a subset of players surrounded by curly braces and comma separated with the value of that coalition. An example input file is shown here

```
4
{1},30
{2},40
{3},25
{4},45
{1,2},50
{1,3},60
{1,4},80
{2,3},55
{2,4},70
{3,4},80
{1,2,3},90
{1,2,4},120
{1,3,4},100
{2,3,4},115
{1,2,3,4},140
```

The program can be run with the following options. Not that only the input file is required, but it is suggested to specify the output file.

```
python coalition.py --help
usage: coalition.py -i <input file> -o <output file>
-h, --help        prints this message
-i, --input       sets the input file
-o, --output      sets the output file: defaults to optimalCS.txt
```

Problem Q1

Consider the “cross-out” game. In this game one writes down ”1,2,3”. Player 1 can cross out a single number or any 2 adjacent number (12,23). Player 2 then gets to make the same type of action. The winner is the one who crosses out the last number.

Part Q1.1)

Part Q1.2)

This is a solved game where player 1 can always win.

To determine this let’s first not consider when players can cross out more than one number (they can only cross out one number). This game is also solved, if it is an odd sized game then player 1 can always win, otherwise player 2 can always win. This is because if the size is odd then player 1 can place a binary partition and split the game into any two sub games. Player 2 will then be the first player on one sub game and player 1 will be the second player in the second sub game. If the second player wins any even game, then we can see that player 1 will always win this sub game (because it is even and they are the second player for an even sized game).

Now let’s consider where a player can cross out any number of adjacent numbers. Firstly there is the trivial case where player 1 can cross out all the numbers (I’m assuming this wasn’t meant to be possible). Similarly Player 1 can cross out all but two non-adjacent numbers, then this is an even game where a player may only cross out a single number (we have reduced the game to the single cross out game discussed above). Thinking about this from subgames, we can see that Player 1 can always reduce any subgame into the single cross out game. By doing this Player 1 can always put themselves to be in the first or second position of the new subgames.

Part Q2.1)

Realization Plan Player 1

$$
\begin{aligned}
r_1(\oslash) &= r_1(L) + r_1(R)\\
r_1(L) &= r_1(Ll) + r_1(Lr)\\
r_1(R) &= r_1(Rl) + r_1(Rl)\\
r_1(Ll) &= r_1(LlU) + r_1(LlD)\\
r_1(Lr) &= r_1(LrU) + r_1(LrD)\\
r_1(Rl) &= r_1(RlU) + r_1(RlD)\\
r_1(Rr) &= r_1(RrU) + r_1(RrD)\\
r_1(\oslash), r_1(L), r_1(R), r_1(Ll), r_1(Lr), r_1(Rl), r_1(Rr) &\geq 0
\end{aligned}
$$

Realization Plan Player 2

$$
\begin{aligned}
r_2(\oslash) &= r_2(A) + r_2(B)\\
r_2(A) &= r_2(AC) + r_2(AD)\\
r_2(B) &= r_2(BC) + r_2(BD)
\end{aligned}
$$