Homework 2

Deadline: May 17th, 2019

Instruction: You may discuss these problems with classmates, but please complete the write-ups individually. Your answers must be **typewritten**, except for figures, which may be hand-drawn. Please submit your answers (pdf format only for non-programming assignments) on **Canvas**.

P1. Programming [50 points]

Undergrads: Implement the computation of Shapley values.

Grads: Implement the dynamic programming algorithm to find the optimal coalition structure.

Instruction: The **input** of your program is a text file: *gameShapley.txt* (for computing Shapley values) or *gameCS.txt* (for computing aoptimal coalition structure) which encodes a coalitional game. The first line is the number of players in the game. Players' IDs are $\{1, 2, \ldots, \#players\}$. Each other line has the following format: $\{id_1, id_2, \ldots, id_k\}, value$ in which $\{id_1, id_2, \ldots, id_k\}$ is a coalition of k players with IDs: $id_1, id_2, \ldots, id_k$.
Output:

- Undergrads: the output is a text file, *Shapley.txt*. Each line of the output file is in the format of *id,payoff* in which *id* is a player's ID and *payoff* is the Shapley value of that player.
- Grads: the output is a text file, named *optimalCS.txt*. The first line is the value of the optimal coalitional structure. Each other line is a coalition in the optimal coalitional structure, with the format: $\{id_1, id_2, \ldots, id_k\}$ which consists of all IDs of its members.

Your **submission** must include: (i) source codes; (ii) documentary including description of your program and instruction to run it. Your program will be tested based on different games.

Problem Solving [50 points]

Q1. Perfect information EFG [20 points]. Consider the "cross-out game." In this game, one writes down the numbers 1, 2, 3. Person 1 starts by crossing out any one number or any two adjacent numbers: for example, person 1 might cross out 1, might cross out 1 and 2, or might cross out 2 and 3. Then person 2 also crosses out either one number or two adjacent numbers. For example, starting from 1, 2, 3, say person 1 crosses out 1. Then person 2 can either cross out 2, cross out 3, or cross out both 2 and 3. Play continues like this. Once a number is crossed out, it cannot be crossed out again. Also, if for example person 1 crosses out 2 in her first move, person 2 cannot then cross out both 1 and 3, because 1 and 3 are not adjacent (even though 2 is crossed out). The winner is the person who crosses out the last number.

1. Model this as an extensive form game. Show a subgame perfect Nash equilibrium of this game by drawing appropriate arrows in the game tree.
2. Now instead of just three numbers, say that you start with m numbers. In other words, you have the numbers 1, 2, 3,..., m. Can person 1 always win this game? (Hint: look at m = 4, m = 5,etc. first to get some ideas.)

Q2. Imperfect information EFG. Consider the following imperfect information extensive form game with two players: player 1 and player 2.

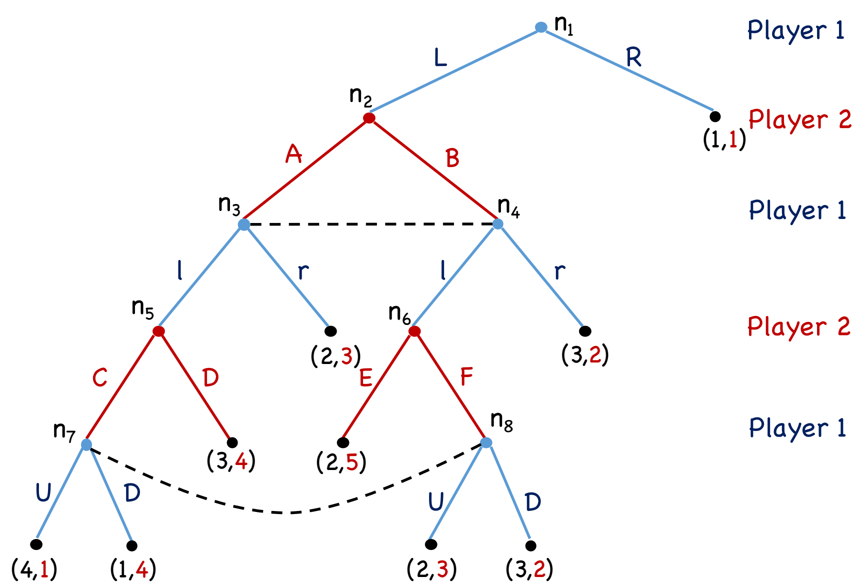

Figure 1: Caption

1. (10 points) Provide an example of a realization plan for each player.
2. (10 points))Find the behavioral strategy of each player corresponding to the provided realization plan.
3. (Grads only) (10 points) Given the provided realization plan of player 1, write down the linear program to compute an optimal realization plan of player 2 (Reference: textbook).

Q3. Coalition structure generation (undergrads only) (10 points). Consider the following coalitional game with 5 players. Compute an optimal coalition structure.
$v(\{1\}) = 30$, $v(\{2\}) = 40$, $v(\{3\}) = 25$, $v(\{4\}) = 45$, $v(\{5\}) = 35$
$v(\{1,2\}) = 50$,$v(\{1,3\}) = 60$, $v(\{1,4\}) = 80$, $v(\{1,5\}) = 70$, $v(\{2,3\}) = 55$, $v(\{2,4\}) = 70$,

$v(\{2,5\}) = 50$, $v(\{3,4\}) = 80$, $v(\{3,5\}) = 65$, $v(\{4,5\}) = 85$
$v(\{1,2,3\}) = 90$, $v(\{1,2,4\}) = 120$, $v(\{1,2,5\}) = 115$, $v(\{1,3,4\}) = 100$, $v(\{1,3,5\}) = 90$, $v(\{1,4,5\}) = 125$, $v(\{2,3,4\}) = 115$, $v(\{2,3,5\}) = 85$, $v(\{2,4,5\}) = 130$, $v(\{3,4,5\}) = 100$
$v(\{1,2,3,4\}) = 140$, $v(\{1,2,3,5\}) = 165$,$v(\{1,2,4,5\}) = 130$, $v(\{1,3,4,5\}) = 175$, $v(\{2,3,4,5\}) = 160$
$v(\{1,2,3,4,5\}) = 200$