CIS 510 Assignment 1

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Problem 1

Implement the MILP to compute an SSE using Cplex.
Instrcution: the input of your program is two CSV files: *param.csv* and *payoff.csv*. The format of the *param.csv* file is *num_of_targets, number_of_defender_resources*. In the *payoff.csv* file each line consists of five numbers: *target_id, def_payoff_cov, def_payoff_uncover,att_payoff_cov, att_payoff_unciver*. The **output** of your program is a CSV file named *SSE.csv*. Each line of the output file is in the format of *target_id, def_coverage_probability*. A sample of the three files are provided.
Submission must include: (i) source codes; (ii) documentary including description of your program and instructions to run it. Your program will be tested based on different games.

Documentation

Note: This code uses Cplex, which does not work on python 3.7. This code was written with python 3.6.5 and will be assumed that the user is using a similarly compatible python version. 3.5 and 3.6 should be compatible with this code though it is not tested.
To see options in the program run The delimiter option is the only option that is

```
python MILP.py --help
usage: MILP.py -p <parameter file> -i <payoff file> -o <output file>
-p, --params      sets the parameter file
-i, --payoff      sets the payoff file
-o, --output      sets the output file. Defaults to out.csv
-d, --delimiter   sets the delimiter of ALL files. Defaults to csv
```

optional. Params and payoff are required.
The code can be run with the following command:

```
python MILP.py -p param.csv -i payoff.csv
```

Problem 2

Part 1)

Consider a security game with four targets. The payoffs are given in the following table. In each cell, the first number is the defender's payoff and the second is the attacker's.

	t1	t2	t3	t4
covered	(1,0)	(3,0)	(8,0)	(8,-1)
uncovered	(-1,1)	(0,2)	(0,4)	(-4,4)
variable	w	z	y	x

For a single resource we can see that there are two options that maximize the defender's utility: $t3, t4$.

One Resource

If we have one resource then the first step is to solve the following

$$4(1-x) - x = 4(1-y) = 2$$

We can trivially see that the solution is

$$x = \frac{2}{5}$$

$$y = \frac{1}{2}$$

At this point our total resources used is $\frac{9}{10}$ leaving us with $\frac{1}{10}$ resources.

$$4(1-x) - x = 4(1-y) - y = 2(1-z) = 1$$

Solving we get

$$x = \frac{3}{5}$$

$$y = \frac{3}{4}$$

$$z = \frac{1}{2}$$

Unfortunately this does not work $\because x + y + z = \frac{37}{20} > 1$! So we need to do the following

$$\begin{aligned} 4(1-x)-x &= 4(1-y) \\ 4-5x &= 4-4y \\ 5x &= 4y \\ y &= \frac{5}{4}x \end{aligned} \qquad \begin{aligned} 4(1-x)-x &= 2(1-z) \\ 4-5x &= 2-2z \\ 2-5x &= -2z \\ z &= \frac{5}{2}x-1 \end{aligned}$$

We can now solve the following equation:

$$\begin{aligned} \sum c_i &= 1 \\ x+y+z &= 1 \\ x+\frac{5}{4}x+\frac{5}{2}x-1 &= 1 \\ \frac{x}{8}(8+10+20) &= 2 \\ x &= \frac{8}{19} \end{aligned}$$

And thus we have a result of
Which we can see that the sum here is 1.

$$x = \tfrac{8}{19}, \quad y = \tfrac{10}{19}, \quad z = \tfrac{1}{19}$$

Two Resources

We can cheat a little because of the work we did in the previous example. We know that we only use up one resource once we consider equality across x, y, z. $\therefore$ we can start at that point. We already calculated out that for these to all be equal then we need $\frac{37}{20}$ resources. While this is > 1 it is < 2 and $\therefore$ we need to do equality across all 4 variables. Leveraging our previous work we need to write w in terms of x.

$$\begin{aligned} 4(1-x)-x &= 1-w \\ 4-5x &= 1-w \\ w &= 5x-3 \end{aligned}$$

We will now sum up everything and set the r.h.s. to 2, our number of resources.

$$\begin{aligned} x+\frac{5}{4}x+\frac{5}{2}x-1+5x-3 &= 2 \\ 39x &= 24 \\ x &= \frac{24}{39} \end{aligned}$$

Thus we get the result Here we can see that the sum is 2 and we are done.

$$x = \frac{24}{39}, \quad y = \frac{10}{13}, \quad z = \frac{7}{13}, \quad w = \frac{1}{13}$$

Three Resources

Again we don't have to start over. We can start right here

$$\begin{aligned} x + \frac{5}{4}x + \frac{5}{2}x - 1 + 5x - 3 = 3 \\ 39x = 28 \\ x = \frac{28}{39} \end{aligned}$$

Checking that the sum is 3 we can verify that we are done.

$$x = \frac{28}{39}, \quad y = \frac{35}{39}, \quad z = \frac{31}{39}, \quad w = \frac{23}{39}$$

Part 2)

oger has invited Caleb to his party. Roger must choose whether or not to hire a clown. Simultaneously, Caleb must decide whether or not to go the party. Caleb likes Roger but he hates clowns. Calebs payoff from going to the party is 4 if there is no clown, but 0 if there is a clown there. Calebs payoff from not going to the party is 3 if there is no clown at the party, but 1 if there is a clown at the party. Roger likes clowns (he especially likes Calebs reaction to them but does not like paying for them). Rogers payoff if Caleb comes to the party is 4 if there is no clown, but 8 x if there is a clown (x is the cost of a clown). Rogers payoff if Caleb does not come to the party is 2 if there is no clown, but 3 x if there is a clown there.
1. Write down the payoff matrix of this game
2. Find any dominated strategies and the Nash Equilibrium of the game (with explanation) when (i) $x = 0$; (ii) $x = 2$; (iii) $x = 3$; (iv) $x = 5$.

Solution

Let's summarize first:
Action sets:

$$\begin{aligned} Roger_{action\_set} &= \{clown, \neg clown\} \\ Caleb_{action\_set} &= \{go, \neg go\} \end{aligned}$$

Payoff:

$$\begin{aligned}
Caleb(go|\neg clown) &= 4\\
Caleb(go|clown) &= 0\\
Caleb(\neg go|\neg clown) &= 3\\
Caleb(\neg go|clown) &= 1\\
Roger(Caleb(go)|\neg clown) &= 4\\
Roger(Caleb(go)|clown) &= 8 - x\\
Roger(Caleb(\neg go)|\neg clown) &= 2\\
Roger(Caleb(\neg go)|clown) &= 3 - x
\end{aligned}$$

Where x is the cost of the clown
We can now easily write the payoff matrix. We order the payoff as $(Roger, Caleb)$

	go	¬ go
clown	$(8 - x, 0)$	$(3 - x, 1)$
¬ clown	$(4, 4)$	$(2, 3)$

(i)
Now let's let $x = 0$ and find the Nash Equilibrium If $Roger(clown)$ then Caleb's

	go	¬ go
clown	$(8, 0)$	$(3, 1)$
¬ clown	$(4, 4)$	$(2, 3)$

best decision is to not go, gaining utility of 1. If $Roger(\neg clown)$ then Caleb's best decision is to go, gaining utility of 4. We'll create a small table of expected utilities

	Roger	Caleb	
clown	12	go	4
¬ clown	6	¬ go	4

$\therefore$ Roger should get a clown and Caleb should not go. Nash equilibrium is (3,1). Roger's strategy dominates because he still gets a clown regardless of Caleb's choice. But Caleb does not have a dominating strategy because he would switch based on Roger's choice.

(ii)
Now let's let $x = 2$ and find the Nash Equilibrium

	go	¬ go
clown	$(6, 0)$	$(1, 1)$
¬ clown	$(4, 4)$	$(2, 3)$

	Roger	Caleb	
clown	7	go	4
¬ clown	6	¬ go	4

There is no pure Nash Equilibrium in this case, so we need to solve.

$$
\begin{aligned}
< C_{\neg g} > &= p(0) + (1-p)4 \\
&= 4 - 4p \\
< C_g > &= p(1) + (1-p)3 \\
&= 3 - 2p \\
4 - 4p &= 3 - 2p \\
p &= \frac{1}{2} \\
< R_c > &= q(6) + (1-q)(1) \\
&= 1 + 5q \\
< R_{\neg c} > &= q(4) + (1-q)(2) \\
&= 2 + 2q \\
1 + 5q &= 2 + 2q \\
q &= \frac{1}{3}
\end{aligned}
$$

This gives us the results $R(\frac{1}{2}c, \frac{1}{2}\neg c), C(\frac{1}{3}g, \frac{2}{3}\neg g)$
There are no dominating strategies but Roger has a higher expected payout.

(iii)
Now let's let $x = 3$ and find the Nash Equilibrium

	go	¬ go
clown	$(5, 0)$	$(0, 1)$
¬ clown	$(4, 4)$	$(2, 3)$

	Roger	Caleb	
clown	5	go	4
¬ clown	6	¬ go	4

There again is no pure Nash Equilibrium so we need to find a mixed strategy. "Cheating" a little we can see that Caleb's payoffs don't change so we can know that Roger will have the same mixed strategy. So we'll just calculate utility of Roger.

$$\begin{aligned}
< R_c > &= q(5) + (1-q)(0) \\
&= 5q \\
< R_{\neg c} > &= q(4) + (1-q)(2) \\
&= 2 + 2q \\
5q &= 2 + 2q \\
q &= \frac{2}{3}
\end{aligned}$$

So we get the Mixed Nash Equilibrium of $R(\frac{1}{2}c, \frac{1}{2}\neg c), C(\frac{2}{3}g, \frac{1}{3}\neg g)$
There are no dominating strategies but Roger has a higher expected payout.

(iv)
Now let's let $x = 5$ and find the Nash Equilibrium

	go	¬ go
clown	$(3, 0)$	$(-2, 1)$
¬ clown	$(4, 4)$	$(2, 3)$

	Roger	Caleb	
clown	1	go	4
¬ clown	6	¬ go	4

∴ Roger really shouldn't get a clown and Caleb should go. Nash Equilibrium is (4,4). Roger dominates this time with $Roger(\neg clown)$ because he chooses that regardless of Caleb's decision. Caleb's best strategy is still dependent on Roger's.