CIS 510 Assignment 1

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Problem 1

Implement the MILP to compute an SSE using Cplex.
Instrcution: the input of your program is two CSV files: *param.csv* and *payoff.csv*. The format of the *param.csv* file is *num_of_targets, number_of_defender_resources*. In the *payoff.csv* file each line consists of five numbers: *target_id, def_payoff_cov, def_payoff_uncover,att_payoff_cov, att_payoff_unciver*. The **output** of your program is a CSV file named *SSE.csv*. Each line of the output file is in the format of *target_id, def_coverage_probability*. A sample of the three files are provided.
Submission must include: (i) source codes; (ii) documentary including description of your program and instructions to run it. Your program will be tested based on different games.

Documentation

Note: This code uses Cplex, which does not work on python 3.7. This code was written with python 3.6.5 and will be assumed that the user is using a similarly compatible python version.

Problem 2

Part 1)

Consider a security game with four targets. The payoffs are given in the following table. In each cell, the first number is the defender's payoff and the second is the attacker's.

	t1	t2	t3	t4
covered	(1,0)	(3,0)	(8,0)	(8,-1)
uncovered	(-1,1)	(0,2)	(0,4)	(-4,4)
variable	w	z	y	x

For a single resource we can see that there are two options that maximize the defender's utility: $t3, t4$.

One Resource

If we have one resource then the first step is to solve the following

$$4(1-x)-x=4(1-y)=2$$

We can trivially see that the solution is

$$x=\frac{2}{5}$$

$$y=\frac{1}{2}$$

At this point our total resources used is $\frac{9}{10}$ leaving us with $\frac{1}{10}$ resources.

$$4(1-x)-x=4(1-y)-y=2(1-z)=1$$

Solving we get

$$x=\frac{3}{5}$$

$$y=\frac{3}{4}$$

$$z=\frac{1}{2}$$

Unfortunately this does not work $\because$ $x+y+z=\frac{37}{20}>1$! So we need to do the following

$$\begin{aligned}4(1-x)-x&=4(1-y)\\4-5x&=4-4y\\5x&=4y\\y&=\frac{5}{4}x\end{aligned}$$

$$\begin{aligned}4(1-x)-x&=2(1-z)\\4-5x&=2-2z\\2-5x&=-2z\\z&=\frac{5}{2}x-1\end{aligned}$$

We can now solve the following equation:

$$\begin{aligned}\sum c_i&=1\\x+y+z&=1\\x+\frac{5}{4}x+\frac{5}{2}x-1&=1\\\frac{x}{8}(8+10+20)&=2\\x&=\frac{8}{19}\end{aligned}$$

$$x = \tfrac{8}{19} \quad y = \tfrac{10}{19} \quad z = \tfrac{1}{19}$$

And thus we have a result of
Which we can see that the sum here is 1.

Two Resources

We can cheat a little because of the work we did in the previous example. We know that we only use up one resource once we consider equality across x, y, z. $\therefore$ we can start at that point. We already calculated out that for these to all be equal then we need $\frac{37}{20}$ resources. While this is > 1 it is < 2 and $\therefore$ we need to do equality across all 4 variables. Leveraging our previous work we need to write w in terms of x.

$$\begin{aligned} 4(1-x) - x &= 1 - w \\ 4 - 5x &= 1 - w \\ w &= 5x - 3 \end{aligned}$$

We will now sum up everything and set the r.h.s. to 2, our number of resources.

$$\begin{aligned} x + \frac{5}{4}x + \frac{5}{2}x - 1 + 5x - 3 &= 2 \\ 39x &= 24 \\ x &= \frac{24}{39} \end{aligned}$$

Thus we get the result Here we can see that the sum is 2 and we are done.

$$x = \tfrac{24}{39} \quad y = \tfrac{10}{13} \quad z = \tfrac{7}{13} \quad w = \tfrac{1}{13}$$

Three Resources

Again we don't have to start over. We can start right here

$$\begin{aligned} x + \frac{5}{4}x + \frac{5}{2}x - 1 + 5x - 3 &= 3 \\ 39x &= 28 \\ x &= \frac{28}{39} \end{aligned}$$

Checking that the sum is 3 we can verify that we are done.

$$x = \tfrac{28}{39} \quad y = \tfrac{35}{39} \quad z = \tfrac{31}{39} \quad w = \tfrac{23}{39}$$