CIS 551: Assignment 5

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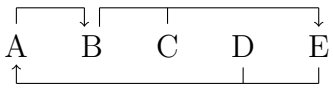
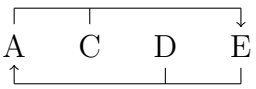
Problem 1: 19.2

Consider a relation R with five attributes $ABCDE$. You are given the following dependencies.



Part 1)

List all keys for R
We can see that $A \to B$ which means we can rewrite the relationships as $ACDE$. Giving us



With A and C we are given E. Thus we have

A C D

And we no longer have any relationships.
Similarly we can get the other keys. Giving us the answer:
ACD, BCD, CDE

Part 2)

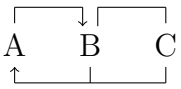
Is R in 3NF?
Yes. B,E,A (the right sides from the FDs) are all part of keys.

Part 3)

Is R in BCNF?
No. None of the left sides of the FDs contain a key and there is no super key.

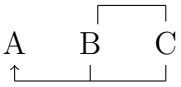
Problem 2: 19.6

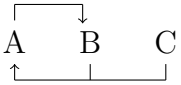
Suppose that we have the following three tuples in a legal instance of a relation scheme S with three attributes ABC (listed in order): $(1, 2, 3), (4, 2, 3),$ and $(5, 3, 3)$.

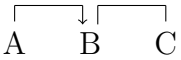


Part 1)

Which of the following dependencies can you infer does *not* hold over scheme S?
(a) $A \to B$, (b) $BC \to A$, (c) $B \to C$
We'll look at the following three pictures







We see that the third diagram is the only legal dependency, which is where the dependency $BC \to A$ is removed. $\therefore$ (b) does **not** hold.

Part 2)

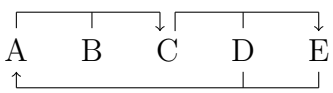
Can you identify any dependencies that hold over S?
We are only given an instance of S and can't determine what relationships hold for **all** instances of S.

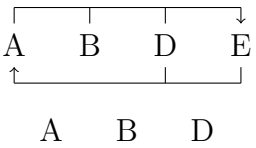
Problem 3

Removed from assignment

Problem 4

Consider relation $R = ABCDE$ with the below relationships. Convert to BCNF.





Problem 5: 19.13

Consider a relation R with five attributes $ABCDE$

Part 1)

For each of the following instances of R, state whether it violates (a) the FD $BC \rightarrow D$ and (b) the MVD $BC \rightarrow\rightarrow D$

(a) $\{\}$ No.

(b) $\{(a, 2, 3, 4, 5), (2, a, 3, 5, 5)\}$ $BC \rightarrow D$ is violated if $a = 2$

(c) $\{(a, 2, 3, 4, 5), (2, a, 3, 5, 5), (a, 2, 3, 4, 6)\}$ $BC \rightarrow D$ and $BC \rightarrow\rightarrow D$ are both violated if $a = 2$. We need another tuple that ends in $5, 6$.

(d) $\{(a, 2, 3, 4, 5), (2, a, 3, 4, 5), (a, 2, 3, 6, 5)\}$ $BC \rightarrow D$ is violated because if $a = 2$ then there is a duplicate row.

(e) $\{(a, 2, 3, 4, 5), (2, a, 3, 7, 5), (a, 2, 3, 4, 6)\}$ Both $BC \rightarrow D$ and $BC \rightarrow\rightarrow D$ are violated. $BC \rightarrow D$ is violated because of the second tuple. $BC \rightarrow\rightarrow D$ is violated because we must have another tuple that pairs with the second tuple.

(f) $\{(a, 2, 3, 4, 5), (2, a, 3, 4, 5), (a, 2, 3, 6, 5), (a, 2, 3, 6, 6)\}$ $BC \rightarrow D$ is violated because there is not a unique value defined by BC. $BC \rightarrow\rightarrow D$ is violated because the entry for the Dth column in the 4^{th} tuple isn't the same as in the 1^{st} tuple.

(g) $\{(a, 2, 3, 4, 5), (a, 2, 3, 6, 5), (a, 2, 3, 6, 6), (a, 2, 3, 4, 6)\}$ Just $BC \rightarrow D$ is violated because the third tuple's last entry isn't also a 5.

Part 2)

If each instance for R listed above is legal, what can you say about the DF $A \rightarrow B$?
We cannot say anything, because there are not unique relationships.

Problem 6

Let R be a relation, X a set of attributes of R, and A an attribute of R. (Also denote that XA the result of adding A to X) Define the support of X, written as $\#X$, as the number of distinct tuples in $R|X$ (R restricted to, or projected onto, the attributes of X). Prove that if $X \rightarrow A$, then $\#X = \#XA$.
Proof by contradiction:
Let $X = BC$ such that $BC \rightarrow A$.

A	B	C
1	2	3
4	5	6
7	2	3

We see here that there is a violation in the third row because $\#X = 2$ and $\#XA = 3$. This is not a legal relationship. We note that we only have legal schemas if there is an injective relationship. If we have an injective relationship (onto) then we would have the relationship $\#X \leq \#XA$ (allows for surjection). For strict equality to hold, there needs to be a bijective relationship.

Problem 7

Prove that if R is in 3NF and has only one candidate key, then R is in BCNF.

Problem 8

Insert the following values into an initially empty B+ tree with parameter $d = 2$ and values $17, 11, 50, 22, 5, 35, 42, 60, 15, 30, 25, 27, 37, 40, 20$.

Problem 9

Repeat the previous problem with $d = 3$

Problem 10: 10.8 part 1

Assume that you have just built a dense $B+$ tree index using Alternative (2) on a heap file containing 20,000 records. The key field for this $B+$ tree index is a 40-byte string, and it is a candidate key. Pointers (le., record ids and page ids) and (at most) 10-byte values. The size of one disk page is 1000 bytes. The index was built in a bottom-up fashion using the bulk loading algorithm, and the nodes at each level were filled up as much as possible.

Part 1)

How many levels does the resulting tree have?