CIS 621 Assignment 4

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Problem 1

For the graph $G = (\mathcal{U}, \mathcal{E})$, where $\mathcal{U}$ is the set of vertices and $\mathcal{E}$ is the set of edges, we define the following nonlinear integer program, where $w_{i,j} \geq 0, \forall (i,j) \in \mathcal{E}$ and k is a nonnegative integer:

$$\begin{aligned} &\sup \sum_{(i,j)\in\mathcal{E}} w_{i,j}(x_i + x_j - 2x_i x_j) \\ &s.t. \sum_{i\in\mathcal{U}} x_i = k \\ &\quad x_i \in \{0,1\}, \forall i \in \mathcal{U} \end{aligned}$$

Show that the following linear program is a relaxation of the above problem:

$$\begin{aligned} &\sup \sum_{(i,j)\in\mathcal{E}} w_{i,j} z_{i,j} \\ s.t. \quad & z_{i,j} \leq x_i + x_j, \forall (i,j) \in \mathcal{E} \\ & z_{i,j} \leq 2 - x_i - x_j, \forall (i,j) \in \mathcal{E} \\ & \sum_{i\in\mathcal{U}} x_i = k \\ & 0 \leq x_i \leq 1, \forall i \in \mathcal{U} \\ & 0 \leq z_{i,j} \leq 1, \forall (i, jP \in \mathcal{E} \end{aligned}$$

Also, let $F(x) = \sum_{(i,j)\in\mathcal{E}} w_{i,j}(x_i + x_j - 2x_i x_j)$ be the objective function of the nonlinear integer program. Show that for any (x, z) that is feasible to the linear program, $F(x) \geq \frac{1}{2} \sum_{(i,j)\in\mathcal{E}} w_{i,j} z_{i,j}$

Part 1)

If we remember the definition of linear relaxation we see that it is $x_i \in \{0,1\} \mapsto 0 \leq x_i \leq 1$