Assignment 5

CIS 621: Algorithms and Complexity

Problem (10 points) For the "ski rental" problem, suppose renting the ski costs 1 \$ for Day 1, and buying the ski costs p \$, where $p \in \mathbb{Z}_+$ (i.e., positive integers) and $p \gg 1$. It is already known that, if the rental price stays 1 \$ for every day, the best competitive ratio for any deterministic online algorithm[1] is $c_{static} = 2 - \frac{1}{p}$. Now, suppose the rental price can vary arbitrarily in $\mathbb{Z}_+$ since Day 2, prove that the best competitive ratio for any deterministic online algorithm is $c_{dynamic} = p$.

Hint: Consider an online algorithm A_d that keeps renting the ski until buying it on the dth day, where $d \in \mathbb{Z}_+$. Think about how the "adversary" (or "environment") can respond to A_d. Maybe study the cases of $d = 1$ and $d \geq 2$, respectively, and then summarize.

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[1]Note that in some literature, it can be $c_{static} = 2$ regardless of p, rather than $c_{static} = 2 - \frac{1}{p}$. This is due to a slightly different description of the ski rental problem and/or the algorithm, *i.e.*, using A_d to mean keeping renting until buying the ski on the dth day *versus* keeping renting until (and including) the dth day and afterwards buying the ski.