Assignment 2

CIS 621: Algorithms and Complexity

Problem 1 (5 points) For the linear program below, where $a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, d_3, e_1, e_2, e_3$ are non-zero constants, derive (1) its dual linear program and (2) the KKT conditions for the dual linear program.

$$\begin{aligned} \min_{x_1 \geq 0, x_2 \leq 0, x_3} \quad & a_1x_1 + a_2x_2 + a_3x_3 \\ s.t. \quad & b_1x_1 + b_2x_2 + b_3x_3 \leq e_1 \, , \\ & c_1x_1 + c_2x_2 = e_2 \, , \\ & d_3x_3 \geq e_3 \, . \end{aligned}$$

Problem 2 (5 points) For the following problem, where a_{ij}, b_{ij}, c_j, d_i, $i = 1, 2, ..., m$, $j = 1, 2, ..., n$ are positive constants: (1) show that it is a convex optimization problem; (2) derive its KKT conditions.

$$\begin{aligned} \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} b_{ij}\left((x_{ij}+1)\ln(x_{ij}+1) - x_{ij}\right) \\ s.t. \quad & \sum_{i=1}^{m} x_{ij} \geq c_j \, , j = 1, 2, ..., n \, , \\ & \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} + d_i \geq \sum_{j=1}^{n} x_{ij} + \sum_{j=1}^{n} c_j \, , i = 1, 2, ..., m \, , \\ & x_{ij} \geq 0 \, , i = 1, 2, ..., m \, , j = 1, 2, ..., n \, . \end{aligned}$$

Problem 3 (10 points) Given $a, x \in \mathbb{R}^n$, $B \in \mathbb{R}^{m \times n}$, $c \in \mathbb{R}^m$, we have Problem P_1

$$\begin{aligned} \min \quad & a^T x \\ s.t. \quad & Bx \preceq c \, , \\ & x_i \in \{0, 1\} \, , i = 1, 2, ..., n \, , \end{aligned}$$

Problem P_2

$$\begin{aligned} \min \quad & a^T x \\ s.t. \quad & Bx \preceq c \, , \\ & 0 \leq x_i \leq 1 \, , i = 1, 2, ..., n \, , \end{aligned}$$

and Problem P_3 (which is an equivalent reformulation of P_1)

$$\begin{aligned} \min \quad & a^T x \\ s.t. \quad & Bx \preceq c \, , \\ & x_i(1 - x_i) = 0 \, , i = 1, 2, ..., n \, . \end{aligned}$$

Note that the optimal value of P_2 is a lower bound, denoted as L_1, for the optimal value of P_1. Now, derive the dual problem, denoted as P_4, for P_3. The optimal value of P_4 is also a lower bound, denoted as L_2, for the optimal value of P_1. Are L_1 and L_2 equal? Explain why. (Hint: derive the dual problem, denoted as P_5, for P_2.)