CIS 621 Assignment 5

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Problem 1

For the "ski rental" problem, suppose renting the ski costs \$1 for Day 1 and buying the ski costs \$p, where $p \in \mathbb{Z}^+$ (the positive integers) and $p \gg 1$. It is already known that, if the rental price stays \$1 for every day, the best competitive ratio for any deterministic online algorithm is $c_{\mathsf{static}} = 2 - \frac{1}{p}$. Suppose the rental price can vary arbitrarily in $\mathbb{Z}^+$ since Day 2, prove that the best competitive ratio for any deterministic online algorithm is $c_{\mathsf{dynamic}} = p$.

Hint: Consider an online algorithm A_d that keeps renting the ski until buying it on the dth day, where $d \in \mathbb{Z}^+$. Think about how the "adversary" (or "environment") can respond to A_d. Maybe study the cases of $d = 1$ and $d \geq 2$, respectively, and then summarize.

Solution:
Our goal is to minimize our cost, or maximize the distance between the rental price and the price of the skis. That is

$$\max \|f(rent) - p\|$$

Conversely the renter wants to maximize the distance (maximize their profits)

$$\min \|f(rent) - p\|$$

$\therefore$ our best option is to actually buy the skis on the first day, as this maximizes our cost distance. But the optimal solution is renting for \$1 at day one. So we have

$$\begin{aligned} c_{\mathsf{dynamic}} &= \frac{\max \|f(rent) - p\|}{1} \\ &= \frac{p}{1} \\ &= p \end{aligned}$$