Initialization

Run the following code to import the modules you'll need. After your finish the assignment, **remember to run all cells** and save the note book to your local machine as a PDF for gradescope submission.

In [1]:
```
import time
import os
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.patches as patches
```

Download data

In this section we will download the data and setup the paths.

In [2]:
```
# Download the data
if not os.path.exists('./content/aerialseq.npy'):
    !wget https://www.cs.cmu.edu/~deva/data/aerialseq.npy -O ./content/aeri
alseq.npy
if not os.path.exists('./content/antseq.npy'):
    !wget https://www.cs.cmu.edu/~deva/data/antseq.npy -O ./content/antseq.
npy
```

```
--2024-02-15 20:58:22--  https://www.cs.cmu.edu/~deva/data/aerialseq.npy
Resolving www.cs.cmu.edu (www.cs.cmu.edu)... 128.2.42.95
Connecting to www.cs.cmu.edu (www.cs.cmu.edu)|128.2.42.95|:443... connected.
HTTP request sent, awaiting response... 200 OK
Length: 92160128 (88M)
Saving to: ‘./content/aerialseq.npy’

./content/aerialseq 100%[===================>]  87.89M   357KB/s    in 3m 57s

2024-02-15 21:02:19 (380 KB/s) - ‘./content/aerialseq.npy’ saved [92160128/92
160128]

--2024-02-15 21:02:20--  https://www.cs.cmu.edu/~deva/data/antseq.npy
Resolving www.cs.cmu.edu (www.cs.cmu.edu)... 128.2.42.95
Connecting to www.cs.cmu.edu (www.cs.cmu.edu)|128.2.42.95|:443... connected.
HTTP request sent, awaiting response... 200 OK
Length: 65536128 (62M)
Saving to: ‘./content/antseq.npy’

./content/antseq.np 100%[===================>]  62.50M   376KB/s    in 3m 6s

2024-02-15 21:05:26 (344 KB/s) - ‘./content/antseq.npy’ saved [65536128/65536
128]
```

Q4: Efficient Tracking

Q4.1: Inverse Composition (15 points)

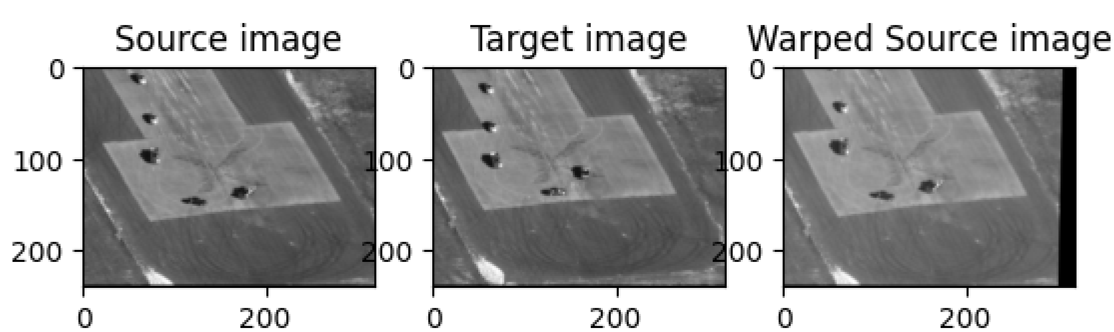
In [28]:
```python
from scipy.interpolate import RectBivariateSpline

def InverseCompositionAffine(It, It1, threshold, num_iters):
    """
    :param It        : (H, W), current image
    :param It1       : (H, W), next image
    :param threshold : (float), if the length of dp < threshold, terminate the
optimization
    :param num_iters : (int), number of iterations for running the optimizatio
n

    :return: M       : (2, 3) The affine transform matrix
    """
    # Initial M
    M = np.array([[1.0, 0.0, 0.0], [0.0, 1.0, 0.0]])
    T_rbs = RectBivariateSpline(np.arange(It.shape[0]), np.arange(It.shape
[1]), It)
    It1_rbs = RectBivariateSpline(np.arange(It1.shape[0]), np.arange(It1.shape
[1]), It1)
    x_grid, y_grid = np.meshgrid(np.arange(It.shape[1]), np.arange(It.shape
[0]))
    old_coords = np.vstack([x_grid.flatten(), y_grid.flatten(), np.ones_like(x
_grid.flatten())])
    template = It
    temp_grad_x = np.gradient(template, axis=1).flatten()
    temp_grad_y = np.gradient(template, axis=0).flatten()
    steepest_descent = np.vstack([temp_grad_x.flatten() * x_grid.flatten(),
                                  temp_grad_x.flatten() * y_grid.flatten(),
                                  temp_grad_x.flatten(),
                                  temp_grad_y.flatten() * x_grid.flatten(),
                                  temp_grad_y.flatten() * y_grid.flatten(),
                                  temp_grad_y.flatten()]).T
    inverse_hessian = np.linalg.inv(steepest_descent.T @ steepest_descent)
    # ===== your code here! =====
    for ii in range(num_iters):
        new_coords = M @ old_coords
        mask = ((new_coords[0,:] >= 0) &
                (new_coords[0,:] < It.shape[1]) &
                (new_coords[1,:] >= 0) &
                (new_coords[1,:] < It.shape[0]))
        It1_warp = It1_rbs.ev(new_coords[1], new_coords[0]).reshape(It.shape)
        error = It1_warp - template
        dp = inverse_hessian @ steepest_descent.T @ error.flatten()
        if np.linalg.norm(dp) < threshold:
            break
        dM = np.array([[1 + dp[0], dp[1], dp[2]], [dp[3], 1 + dp[4], dp[5]],
[0, 0, 1]])
        M = M @ np.linalg.inv(dM)
    # ===== End of code =====
    return M
```

Debug Q4.1

Feel free to use and modify the following snippet to debug your implementation. The snippet simply visualizes the translation resulting from running LK on a single frame. When you warp the source frame using the obtained transformation matrix, it should resemble the target frame.

In [29]:
```
import cv2

num_iters = 100
threshold = 0.01
seq = np.load("./content/aerialseq.npy")
It = seq[:,:,0]
It1 = seq[:,:,10]

# Source frame
plt.figure()
plt.subplot(1,3,1)
plt.imshow(It, cmap='gray')
plt.title('Source image')

# Target frame
plt.subplot(1,3,2)
plt.imshow(It1, cmap='gray')
plt.title('Target image')

# Warped source frame
M = InverseCompositionAffine(It, It1, threshold, num_iters)
warped_It = cv2.warpAffine(It, M,(It.shape[1],It.shape[0]))
plt.subplot(1,3,3)
plt.imshow(warped_It, cmap='gray')
plt.title('Warped Source image')
```

Out[29]:
```
Text(0.5, 1.0, 'Warped Source image')
```



Q4.2 Tracking with Inverse Composition (10 points)

Re-use your impplementation in Q3.2 for subtract dominant motion. Just make sure to use InverseCompositionAffine within.

In [30]:

```
import numpy as np
from scipy.ndimage import binary_erosion
from scipy.ndimage import binary_dilation
from scipy.ndimage import affine_transform
import scipy.ndimage
import cv2

def SubtractDominantMotion(It, It1, num_iters, threshold, tolerance):
    """
    :param It        : (H, W), current image
    :param It1       : (H, W), next image
    :param num_iters : (int), number of iterations for running the optimizatio
n
    :param threshold : (float), if the length of dp < threshold, terminate the
optimization
    :param tolerance : (float), binary threshold of intensity difference when
computing the mask
    :return: mask    : (H, W), the mask of the moved object
    """
    mask = np.ones(It.shape, dtype=bool)

    # ===== your code here! =====
    M = InverseCompositionAffine(It, It1, threshold, num_iters)
    warped_It = cv2.warpAffine(It, -M, It.shape)
    # ===== End of code =====
    mask = np.abs(It1 - warped_It.T) > tolerance
    mask = binary_erosion(mask)
    mask = ~binary_dilation(mask)
    mask[-1,:] = ~mask[-1,:]
    return mask
```

Re-use your implementation in Q3.3 for sequence tracking.

In [31]:

```python
from tqdm import tqdm

def TrackSequenceAffineMotion(seq, num_iters, threshold, tolerance):
    """
    :param seq        : (H, W, T), sequence of frames
    :param num_iters : int, number of iterations for running the optimization
    :param threshold : float, if the length of dp < threshold, terminate the optimization
    :param tolerance : (float), binary threshold of intensity difference when computing the mask
    :return: masks    : (T, 4) moved objects for each frame
    """
    H, W, N = seq.shape

    rects =[]
    It = seq[:,:,0]
    masks = []
    # ===== your code here! =====
    for i in tqdm(range(1, seq.shape[2])):
        masks.append(SubtractDominantMotion(seq[:,:,i-1], seq[:,:,i], num_iters, threshold, tolerance))

    # ===== End of code =====
    masks = np.stack(masks, axis=2)
    return masks
```

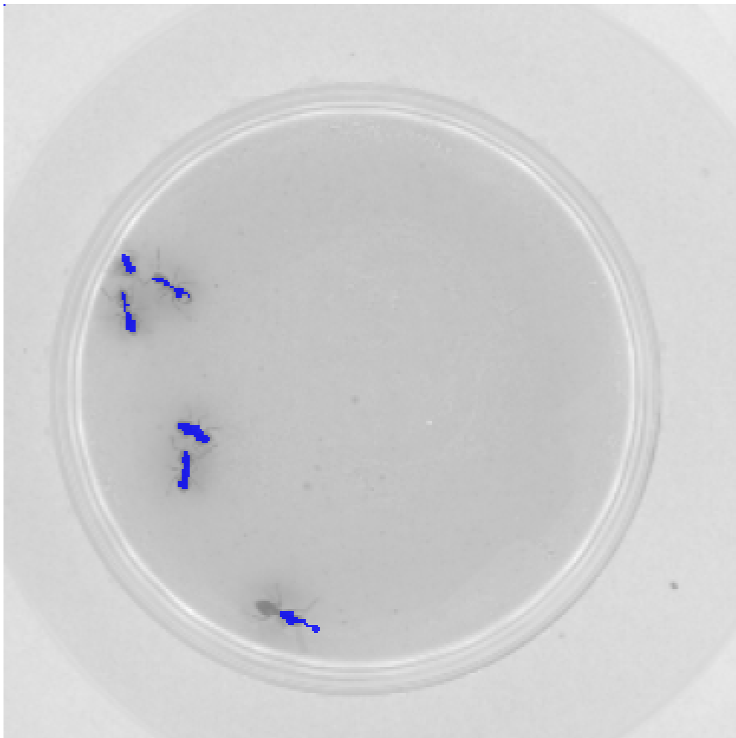
Track the ant sequence with inverse composition method.

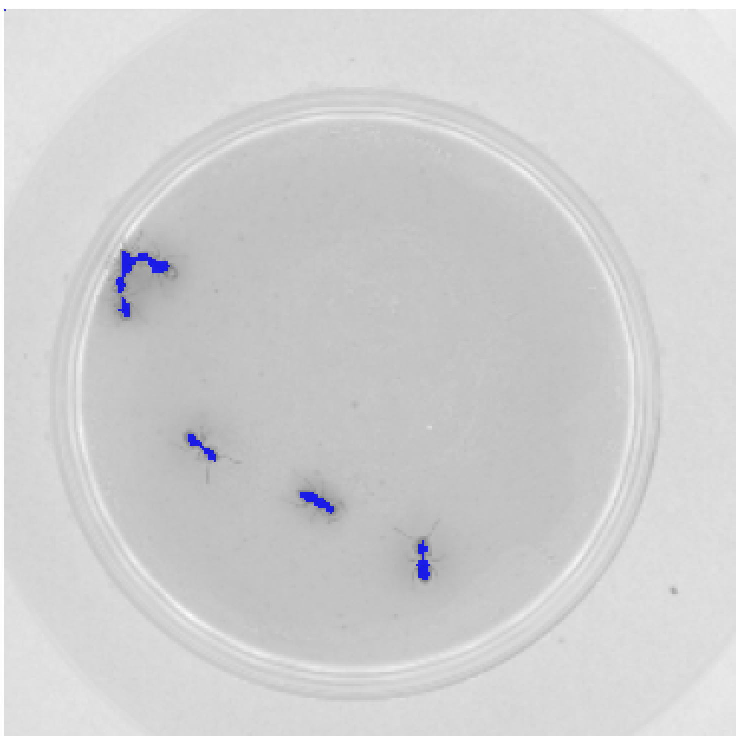
In [32]:

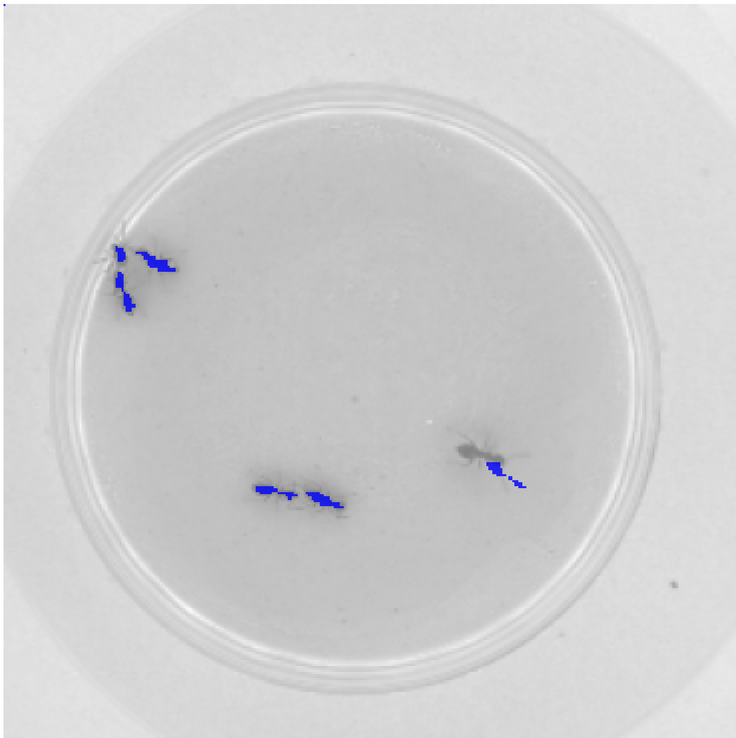
```python
seq = np.load("./content/antseq.npy")

# NOTE: feel free to play with these parameters
num_iters = 1000
threshold = 0.01
tolerance = 0.2

tic = time.time()
masks = TrackSequenceAffineMotion(seq, num_iters, threshold, tolerance)
toc = time.time()
print('\nAnt Sequence takes %f seconds' % (toc - tic))
```

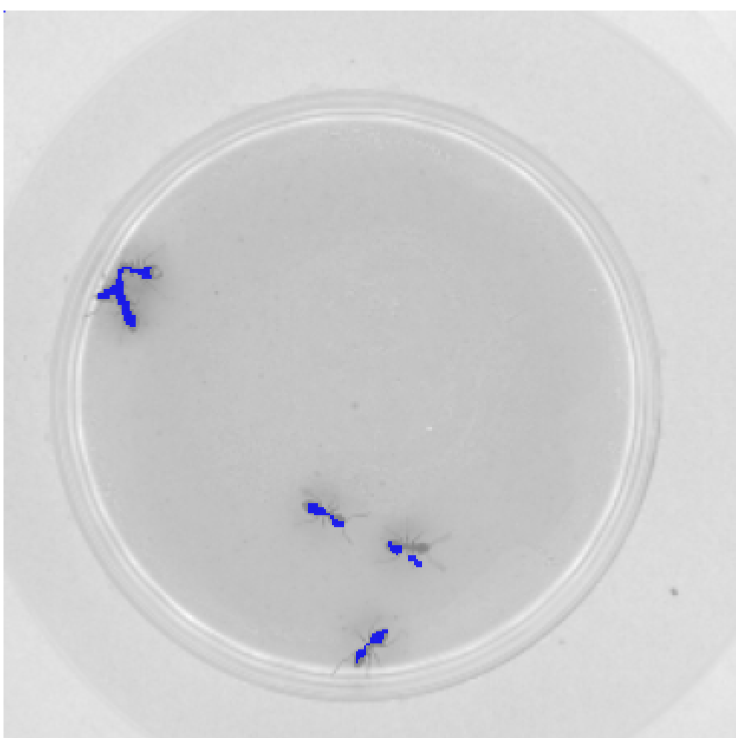
```
  0%|          | 0/124 [00:00<?, ?it/s]100%|██████████| 124/124 [00:13<00:00, 9.27it/s]

Ant Sequence takes 13.389612 seconds
```

In [33]:
```
frames_to_save = [29, 59, 89, 119]

# TODO: visualize
for idx in frames_to_save:
    frame = seq[:, :, idx]
    mask = masks[:, :, idx]

    plt.figure()
    plt.imshow(frame, cmap="gray", alpha=0.5)
    plt.imshow(np.ma.masked_where(np.invert(mask), mask), cmap='winter', alpha
=0.8)
    plt.axis('off')
```









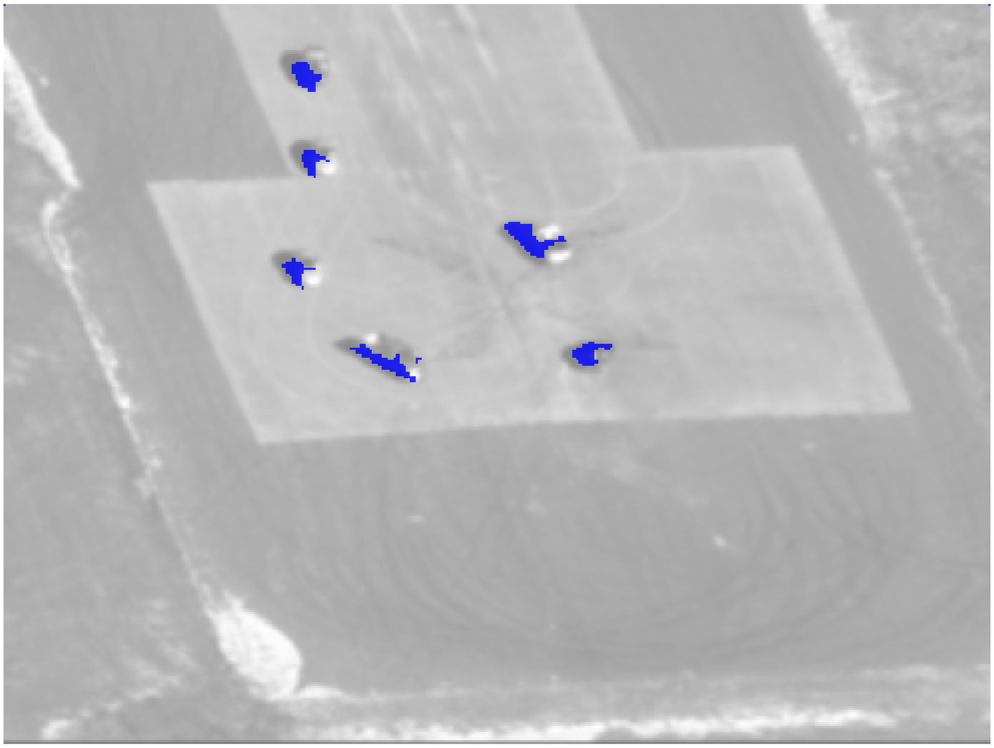
Track the aerial sequence with inverse composition method.

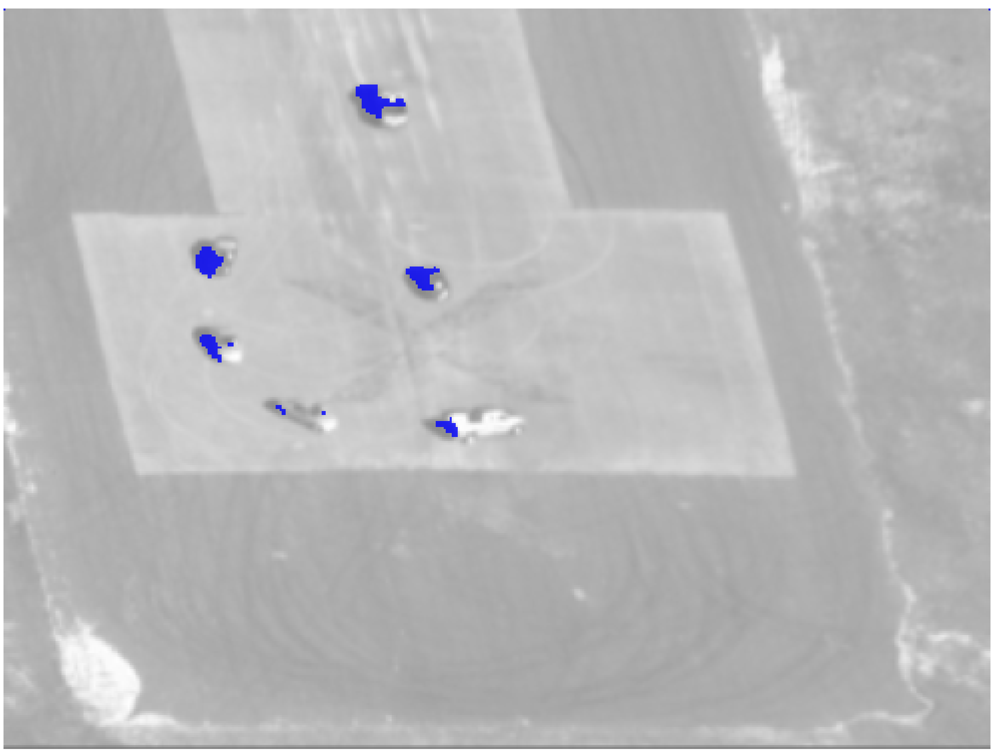
In [36]:

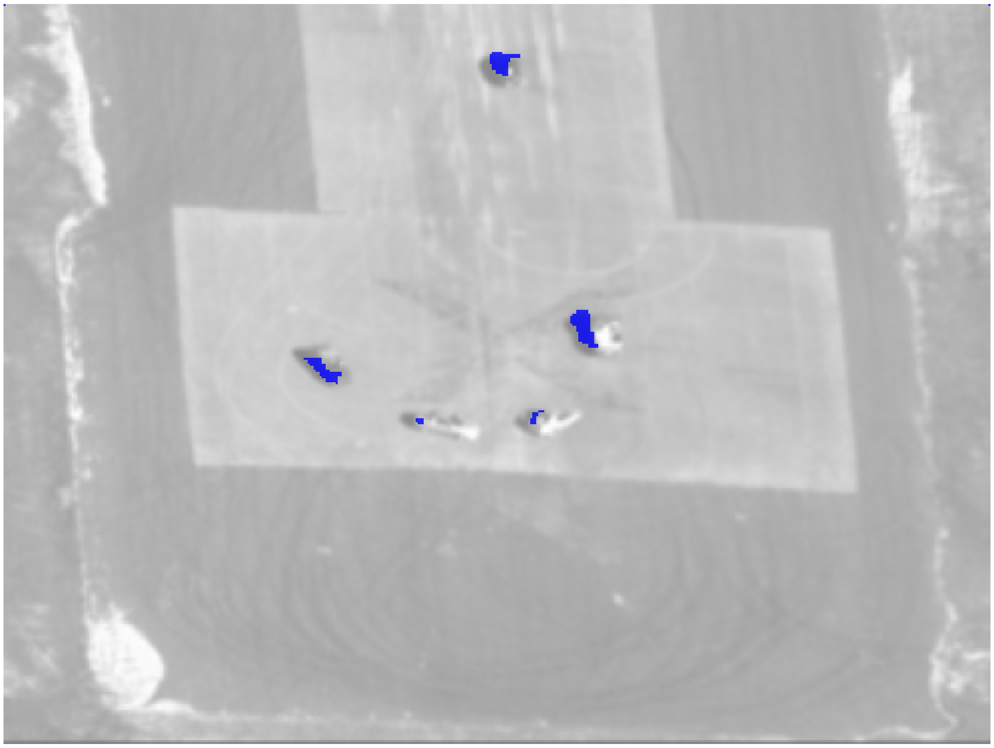
```python
seq = np.load("./content/aerialseq.npy")

# NOTE: feel free to play with these parameters
num_iters = 1000
threshold = 0.01
tolerance = 0.3

tic = time.time()
masks = TrackSequenceAffineMotion(seq, num_iters, threshold, tolerance)
toc = time.time()
print('\nAnt Sequence takes %f seconds' % (toc - tic))
```

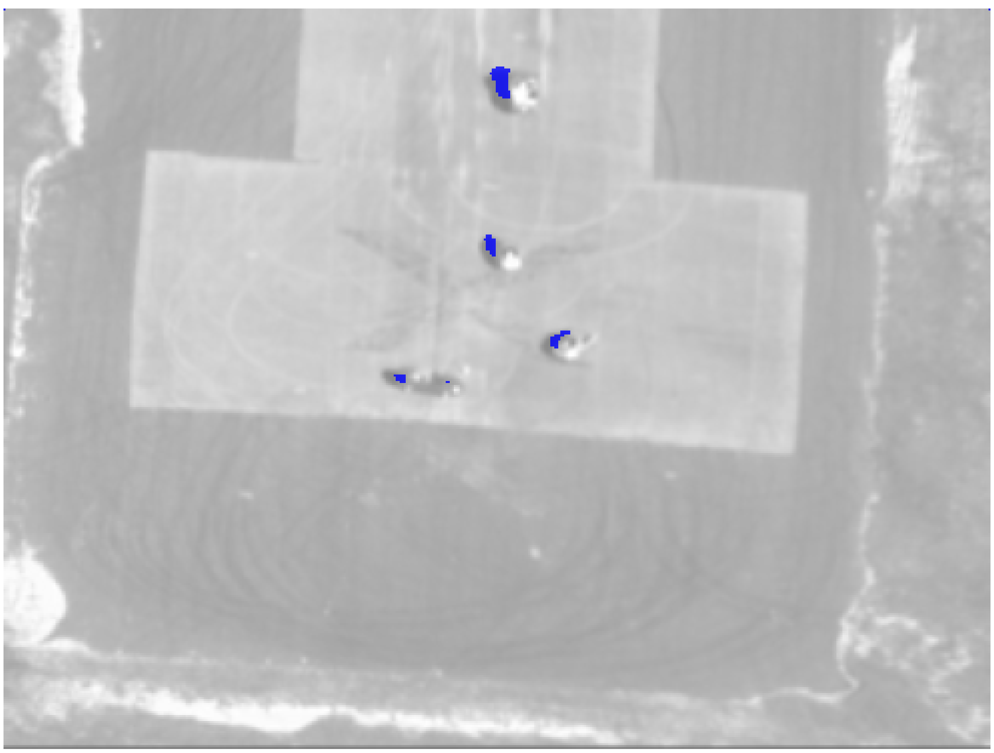
```
100%|██████████| 149/149 [00:34<00:00,  4.27it/s]

Ant Sequence takes 34.889022 seconds
```

In [37]:
```
frames_to_save = [29, 59, 89, 119]

# TODO: visualize
for idx in frames_to_save:
    frame = seq[:, :, idx]
    mask = masks[:, :, idx]

    plt.figure()
    plt.imshow(frame, cmap="gray", alpha=0.5)
    plt.imshow(np.ma.masked_where(np.invert(mask), mask), cmap='winter', alpha
=0.8)
    plt.axis('off')
```









Q4.2.1 Compare the runtime of the algorithm using inverse composition (as described in this section) with its runtime without inverse composition (as detailed in the previous section) in the context of the ant and aerial sequences:

===== your answer here! =====

Sequence	LK Algorithm	Inverse Composition Algorithm
ant	16.857641 s	13.389612 s
aerial	42.298250 s	34.889022 s

===== end of your answer ====

Q4.2.2 In your own words, please describe briefly why the inverse compositional approach is more computationally efficient than the classical approach:

===== your answer here! =====

The inverse composition algorithm computes the hessian ahead of time (before the loop) as opposed to within the optimization routine (loops) like in the classic LK algorithm. By finding the gradients of the template (unwarpped) and forming a Hessian without the explicit warp params (with ∇T and $\frac{\partial W}{\partial p}$), this method (inverse) is more efficient.

===== end of your answer ====