Connecting the three dots: A HSB Model

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9/25/2019

This handout shows a model, the lmer command, and the output for the model discussed in class in Lecture 2.4.

The model Level 1 models:

$$\begin{aligned} y_{ij} &= \beta_{0j} + \beta_{1j} ses_{ij} + \beta_2 female_{ij} + \epsilon_{ij} \\ \epsilon_{ij} &\sim N(0, \sigma_y^2) \end{aligned}$$

Level 2 models:

$$\begin{aligned} \beta_{0j} &= \gamma_{00} + \gamma_{01} sector_j + \gamma_{02} meanSES_j + u_{0j} \\ \beta_{1j} &= \gamma_{10} + \gamma_{11} sector_j + u_{1j} \end{aligned}$$

with

$$\begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} \sim N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_{00} & \tau_{01} \\ & \tau_{11} \end{pmatrix} \right]$$

The τ_{01} is the covariance of the random intercept and random slope. We usually look at the correlation of

$$\rho = \frac{\tau_{01}}{\sqrt{\tau_{00}\tau_{11}}}$$

The estimated ρ is what R gives us in the printed output.

The derivation of the reduced form is:

$$\begin{aligned} y_{ij} &= \beta_{0j} + \beta_{1j} ses_{ij} + \epsilon_{ij} \\ &= (\gamma_{00} + \gamma_{01} sector_j + \gamma_{02} meanSES_j + u_{0j}) + (\gamma_{10} + \gamma_{11} sector_j + u_{1j}) ses_{ij} + \beta_2 female_{ij} + \epsilon_{ij} \\ &= \gamma_{00} + \gamma_{01} sector_j + \gamma_{02} meanSES_j + u_{0j} + \gamma_{10} ses_{ij} + \gamma_{11} sector_j ses_{ij} + u_{1j} ses_{ij} + \beta_2 female_{ij} + \epsilon_{ij} \\ &= \gamma_{00} + \gamma_{01} sector_j + \gamma_{02} meanSES_j + \gamma_{10} ses_{ij} + \gamma_{11} sector_j ses_{ij} + \beta_2 female_{ij} + (u_{0j} + u_{1j} ses_{ij} + \epsilon_{ij}) \end{aligned}$$

This tells us what to tell R.

The code

```
M1 = lmer( mathach ~ 1 + female + ses*sector +
             meanses + (1+ses|id),
           data = dat )
```

This code is the exact same model, using the fact that `ses*sector` means `ses + sector + ses:sector`:

```
M1 = lmer( mathach ~ 1 + sector + meanses + ses + sector:ses + female + (1+ses|id),
           data = dat )
```

The output

```
display( M1 )
```

```
## lmer(formula = mathach ~ 1 + sector + meanses + ses + sector:ses +
##     female + (1 + ses | id), data = dat)
##             coef.est coef.se
## (Intercept) 12.79     0.21
```

```
## sector       1.29     0.29
## meanses      3.04     0.37
## ses          2.73     0.14
## female      -1.18     0.16
## sector:ses  -1.31     0.21
##
## Error terms:
##  Groups   Name        Std.Dev. Corr
##  id       (Intercept) 1.45
##           ses         0.18     0.65
##  Residual             6.05
## ---
## number of obs: 7185, groups: id, 160
## AIC = 46482.9, DIC = 46445.1
## deviance = 46454.0
```

Now, using this output, we have

- $\gamma_{00} = 12.79$ - The overall average math achievement for a student with 0 ses in a public school with 0 mean SES.
- $\gamma_{01} = 1.29$ - The average difference between otherwise equivilent catholic and public schools.
- $\gamma_{02} = 3.04$ - The impact on average achievement due to mean SES of schools. Higher SES schools have higher achievement.
- $\gamma_{10} = 2.73$ - The average slope of ses vs. math achievement in public schools.
- $\beta_2 = -1.18$ - The gender gap; girls have lower math scores on average.
- $\gamma_{11} = -1.31$ - The difference in slope between public and catholic schools (catholic schools have flatter slopes).
- $\tau_{00} = 1.45^2$ - Variation in overall intercept of schools (within category of public or catholic, and beyond mean SES).
- $\tau_{11} = 0.18^2$ - The variation in the random slopes for ses vs. math achievement.
- $\rho = 0.65$ - The random intercepts are correlated with random slopes. High achievement schools have more discrepancy between low and high ses students.
- $\sigma_y = 6.05$ - The unexplained student variation within school.