

Assignment 1 Report

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Task 1. Problem Formulation

Environment: A matrix of maze

State Space: $5 \times 5, 7 \times 7, 9 \times 9$ or 11×11 array(int)

Actions: Visualize by number of moves(either move up ,right ,left or down according to the number in the current position)

Perception/Observations: state of numbers in the matrix

Transition Function: add steps in the matrix

Evaluation Metric: positive numbers for visited states

X for unvisited states

0 for start state

G for target state

Task 2. Puzzle Representation

The software is able to receive as input the size of the puzzle n and then generate a random puzzle that contains only legal moves, and the "G" is the goal. For example:

```
Enter 5, 7, 9, or 11
5
The original matrix is:
2 4 1 1 1
1 3 3 1 1
1 1 2 1 2
3 2 3 3 3
3 3 4 2 G
```

```
Enter 5, 7, 9, or 11
7
The original matrix is:
2 6 6 1 1 3 2
2 2 2 3 4 3 6
4 1 2 1 4 5 2
4 1 4 1 4 1 3
6 4 1 3 1 4 6
2 4 4 5 3 1 3
5 3 6 3 4 4 G
```

Task 3. Puzzle Evaluation

In this task we compute and visualize on our GUI the minimum distance to each cell from the start cell. On one hand, if there is no path, the evaluation function value will be $-k$, where k is the number of cells not reachable from the start. On the other hand, the evaluation function value will be the value of the puzzle to the minimum distance from the start to the goal.

Example 1_1: ($n = 5$, solvable)

Example 1_2: ($n = 5$, not solvable)

```
Enter 5, 7, 9, or 11
5
Task2: The original matrix is:
1 2 4 3 3
4 3 1 1 3
1 3 1 1 4
3 3 1 3 3
4 1 4 1 G

Task 3: Function evaluation
0 1 X 2 X
1 3 X X 2
X 2 X X 3
X X X 3 X
X X X X 3
The value function is 3
```

```
Enter 5, 7, 9, or 11
5
Task2: The original matrix is:
1 4 2 3 1
4 1 1 3 4
4 1 1 1 4
1 1 2 3 1
4 4 4 3 G

Task 3: Function evaluation
0 1 X X X
1 X X X 2
X X X X X
X X X X X
X 2 X X X
The value function is -20
```

Example 2_1: (n = 7, solvable)

```

Enter 5, 7, 9, or 11
7
Task2: The original matrix is:
2 1 6 3 4 4 4
2 2 2 2 4 4 1
3 3 2 2 3 5 4
4 2 4 3 1 4 6
5 2 3 1 3 1 2
1 3 2 5 2 4 6
4 2 4 1 4 5 6

Task 3: Function evaluation
0 X 1 3 X X 4
X X X X X X X
1 3 3 2 4 3 X
X X X 4 X X X
3 X 4 3 4 4 X
2 3 X 4 4 X X
3 X 2 X 4 X 3
The value function is 3

```

Example 2_2: (n = 7, not solvable)

```

Enter 5, 7, 9, or 11
7
Task2: The original matrix is:
1 6 1 3 6 5 2
2 5 2 2 1 1 1
4 1 4 4 4 1 2
4 4 3 2 2 3 4
3 1 1 3 4 3 3
5 4 5 3 4 3 4
5 2 2 4 4 5 6

Task 3: Function evaluation
0 1 4 5 4 5 6
1 5 2 4 3 4 5
5 X X 4 4 5 6
2 4 3 5 3 4 4
4 3 4 5 X X 6
5 4 5 6 4 5 7
5 2 4 3 5 5 X
The value function is -5

```

Example 3_1: (n = 9, solvable)

```

Enter 5, 7, 9, or 11
9
Task2: The original matrix is:
6 1 7 6 1 7 5 7 2
6 4 5 3 2 3 2 4 4
5 6 2 3 4 3 4 2 5
5 3 1 4 3 2 1 1 4
4 7 6 2 1 4 4 1 7
6 5 3 4 3 2 5 6 5
4 6 5 1 5 4 3 1 6
2 3 3 2 3 5 6 6 6
6 2 4 4 6 7 3 1 6

Task 3: Function evaluation
0 2 3 6 5 6 1 7 7
7 3 4 6 3 4 4 5 5
2 X 4 5 5 3 6 6 4
X 5 X 6 4 5 5 5 6
X 6 5 7 10 5 6 6 6
4 3 X 5 11 4 2 5 6
1 6 5 7 2 X 7 6 7
3 8 4 7 9 5 X 7 5
X X X 8 X 6 7 9 10
The value function is 10

```

Example 3_2: (n = 9, not solvable)

```

Enter 5, 7, 9, or 11
9
Task2: The original matrix is:
3 1 8 1 1 7 1 6 1
1 1 3 1 5 6 6 5 3
3 4 3 2 6 1 1 3 5
8 5 1 1 5 3 4 2 8
6 6 1 5 3 3 4 5 5
3 6 4 4 4 1 4 2 7
6 7 5 6 6 2 4 5 1
7 4 5 6 7 7 3 5 8
5 3 8 7 8 7 7 4 6

Task 3: Function evaluation
0 9 2 1 2 3 9 10 X
7 8 3 2 3 4 6 8 X
8 4 6 3 5 4 5 6 X
1 6 5 6 7 5 6 8 2
X 5 4 4 X 7 8 6 5
9 8 5 10 9 8 6 7 7
X 5 8 7 4 6 X 7 6
8 X 9 8 X 4 7 8 7
10 7 3 X 6 7 9 X X
The value function is -12

```

Example 4_1: (n = 11, solvable)

```
Enter 5, 7, 9, or 11
11
Task2: The original matrix is:
2 10 7 6 6 9 9 1 5 8 7
10 5 4 5 4 3 3 9 2 4 5
9 7 4 2 6 4 4 7 1 2 9
9 7 5 5 5 3 4 4 6 2 2
4 5 1 1 1 1 4 6 5 7 1
2 2 8 6 6 4 6 2 1 8 6
2 4 3 4 2 3 4 4 8 8 6
8 3 4 5 7 2 3 1 6 4 8
7 6 5 5 8 3 1 3 6 9 6
8 9 6 5 8 2 5 6 3 2 4
5 1 10 3 6 5 5 9 6 1 6

Task 3: Function evaluation
0 3 1 6 7 X 5 X X 2 8
5 X 8 8 X 7 7 8 6 6 6
1 6 X 5 X 6 X 3 7 2 7
7 8 3 5 7 8 X 4 6 5 6
X 5 4 5 6 7 4 X X 3 5
6 7 5 6 7 6 5 6 X 6 6
X X 8 7 8 7 X 6 8 X 7
7 8 2 4 X 5 3 5 5 4 X
4 X 4 6 6 6 5 5 7 3 6
X 5 X 7 X 6 6 4 7 X 6
5 4 5 8 X 6 4 7 X X 7
The value function is 7
```

Example 4_2: (n = 11, not solvable)

```
Enter 5, 7, 9, or 11
11
Task2: The original matrix is:
1 4 5 2 1 8 5 7 2 2 4
5 9 1 9 1 7 4 7 7 2 6
1 9 8 4 6 2 3 3 3 2 4
3 5 4 3 5 7 1 2 5 6 5
5 9 4 6 3 5 5 6 4 8 5
10 7 5 3 5 3 2 2 8 4 7
3 4 3 4 6 2 5 4 1 5 7
6 3 4 7 6 6 5 5 7 8 2
6 5 1 8 4 7 3 4 4 7 4
6 8 2 2 3 1 5 5 3 3 8
6 4 2 8 9 7 9 10 3 9 6

Task 3: Function evaluation
0 1 9 5 7 2 9 6 8 8 9
1 6 13 7 6 2 12 7 X 8 5
9 9 14 4 6 8 6 5 9 7 6
3 12 7 4 X 6 5 6 X 7 X
5 2 X X 8 4 5 8 9 8 3
9 X 8 8 8 7 7 6 8 7 7
2 8 X 3 7 9 X 4 X 7 7
X 8 6 X 9 X 7 7 7 X 6
8 11 10 9 7 3 9 8 8 10 X
3 5 5 X 6 5 4 7 X 6 4
10 7 9 4 9 6 11 5 X X X
The value function is -20
```

Task 4. Hill Climbing

In this task, we use hill climbing algorithm to compute the hardest $n \times n$ puzzle ($n=5,7,9,11$). In this report, we will report the statistics of how evaluation function changes as the number of iterations increases (ps: based on the same original generated random matrix from task2, and just change the number of iterations)

Example: $n = 5$ (5×5 matrix)

```
Enter 5, 7, 9, or 11
5
Task2: The original matrix is:
2 1 2 2 1
4 1 1 2 1
3 1 2 2 2
3 1 2 2 3
4 4 2 3 6

Task 3: Function evaluation
0 4 1 3 2
5 4 5 4 3
1 3 2 2 3
5 4 5 5 X
4 5 3 3 4
The value function is 4
```

```
Number of iterations: 50
4 4 2 4 1
4 3 3 2 1
3 2 2 2 1
1 3 3 3 2
1 4 2 3 6

0 3 6 2 1
X 4 X 3 2
3 5 7 4 3
2 3 5 4 4
1 2 8 3 9
The value function is 9
```

```
Number of iterations: 100
2 2 4 2 3
1 2 1 1 1
3 2 1 2 2
1 1 1 1 2
1 1 1 4 6

0 4 1 3 7
X 6 5 6 6
1 3 4 2 7
5 4 3 4 5
4 3 2 3 8
The value function is 8
```

```

Number of iterations: 200
4 4 1 1 3
4 2 3 3 3
4 1 2 3 3
1 3 2 2 3
1 1 3 3 6

0 2 7 8 1
8 6 4 7 9
3 5 6 X 4
2 3 X X 2
1 2 3 8 10
The value function is 10

```

```

Number of iterations: 400
3 1 3 4 4
2 1 3 3 1
1 2 2 1 1
3 2 3 3 4
3 1 1 3 6

0 10 8 1 14
4 9 5 3 13
13 10 12 11 12
1 8 7 2 13
3 7 6 2 15
The value function is 15

```

```

Number of iterations: 800
4 1 2 4 2
2 3 3 1 4
3 3 2 2 3
3 3 1 1 2
2 3 3 3 6

0 12 2 4 1
7 13 3 5 6
2 3 3 3 2
8 11 10 9 10
1 14 2 4 15
The value function is 15

```

```

Number of iterations: 1600
4 1 2 4 1
4 3 2 1 4
3 3 1 3 1
3 3 3 2 4
3 1 2 3 6

0 14 6 2 1
2 15 4 3 2
5 8 7 4 9
11 13 5 12 10
1 16 17 2 18
The value function is 18

```

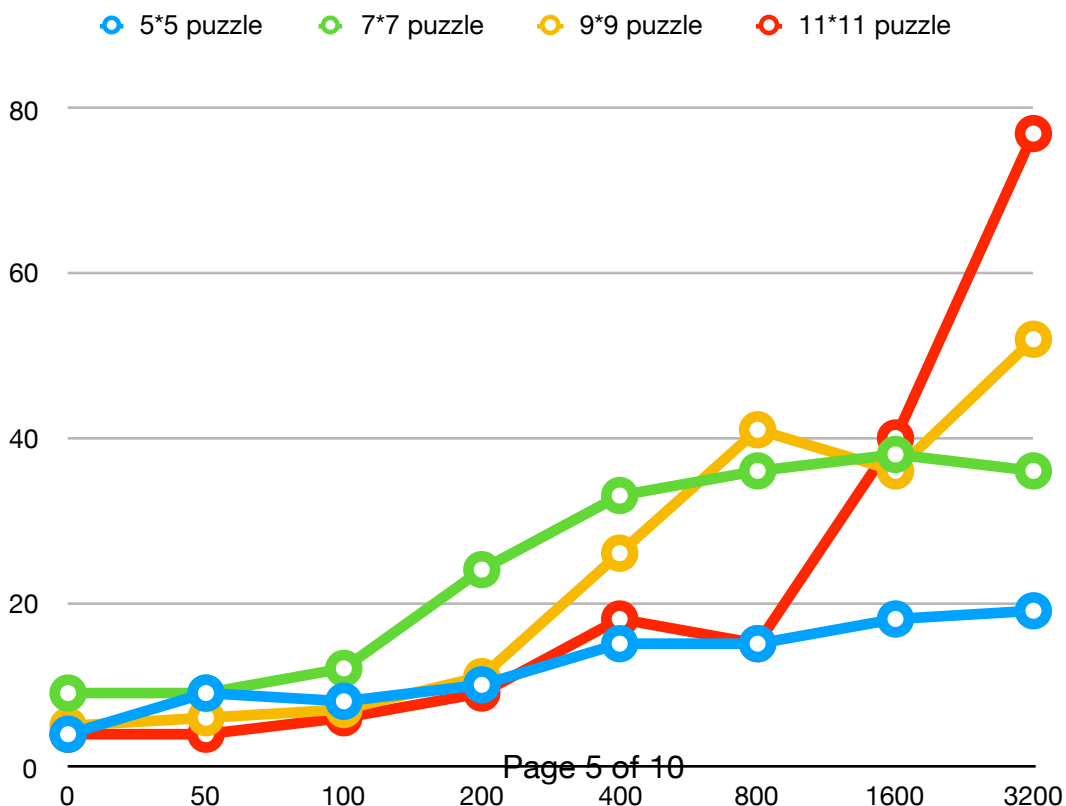
```

Number of iterations: 3200
3 3 1 4 2
4 3 2 3 2
2 1 1 3 1
3 3 3 3 4
3 1 2 3 6

0 9 8 1 10
4 16 6 3 5
13 15 14 12 11
1 10 7 2 6
3 17 18 2 19
The value function is 19

```

For a 5*5 puzzle example shows above, the number of iterations changes to 50, 100, 200, 400, 800, 1600, 3200 numbers of iterations. The function value increase from 4 to 19. The statistic graph and chart below shows 4 lines indicates that 5*5, 7*7, 9*9 and 11*11 puzzles. The x axis is the number of iterations and y axis is the value of difficulties (evaluation function value)



		0	50	100	200	400	800	1600	3200
	5*5 puzzle	4	9	8	10	15	15	18	19
	7*7 puzzle	9	9	12	24	33	36	38	36
	9*9 puzzle	5	6	7	11	26	41	36	52
	11*11 puzzle	4	4	6	9	18	15	40	77

Basically, as the number of iteration increase, the puzzle will become more difficult to solve since the function value is also increasing respectively. The interesting thing is as the dimensions of the puzzle change, the original function value will not become harder since the increase of the dimensions. But, since the number of iterations increase in task 4 : hill climbing, more dimensions the puzzle is, more difficult to solve. For example, when the number of iterations is 0, the function values of 5*5, 7*7, 9*9, 11*11 are 4, 9, 5, 4 and when the number of iterations is 3200, the function values become 19, 36, 52, and 77 respectively.

Task 5. Shortest path first algorithm

As the size of the matrix grows bigger, the run time gets bigger.

Task 6. A*

In this case, we set the heuristic as “adding the current number into its row or column can reach the target’s row or column”. From the runtime we can tell that A* algorithm works better than Dijkstra in this case.

Task 7. Propose and implement a population-based approach

In this task, we set the population set to be 10, so there will be 10 matrix to mutate together, in order to faster create a harder maze. We set 5 ways of how they combine with each other. We first create a dice, and when the dice is on

0 or 1, [A,B],[C,D],[E,F],[G,H],[I,J] will combine
 2 or 3, [A,C],[D,F],[G,I],[H,J],[B,E] will combine
 4 or 5, [A,D],[E,H],[B,I],[C,F],[G,J] will combine
 6 or 7, [A,E],[F,J],[B,G],[C,H],[D,I] will combine
 8 or 9, [A,J],[B,I],[C,H],[D,G],[E,F] will combine

the way of how they going to mutate is completely random, two dices are going to determine the number of row and column to switch each time.

As the population grows, the time to mutate decreases. However, this can still be a really random process comparing to hill climbing. Since hill climbing is more like a scientific way, to specifically go through the particle one by one, genetic algorithm is more like a mimic of natural processing, which won’t make the difficulty of the maze change that drastically. However, as the size of the matrix increases, if we are running same iteration on hill climbing and genetic algorithm, genetic algorithm actually gives a harder maze. We believe that’s because hill climbing go throw particles one by one, and it definitely will take a longer time to give a harder maze. However, since genetic algorithm change more than one particle at a time, a harder maze can be generated as the size of the matrix increases.

```
Task2: The original matrix is:
4 4 4 1 2
4 2 2 1 1
3 1 2 1 4
3 3 2 3 4
1 4 1 1 G

Task 3: Function evaluation
0 3 2 4 1
6 X 5 5 6
3 X 5 4 2
2 X 4 3 5
1 2 3 4 5
The value function is 5
```

```
Task 7: Matrix After genetic 50
3 2 2 2 4
3 1 3 2 2
2 2 2 2 3
4 2 3 2 4
1 1 1 4 G

Solve Genetic
0 2 7 1 8
7 6 7 7 X
9 3 8 2 9
1 5 6 6 2
5 4 5 3 9
```

```
Task 7: Matrix After genetic 100
2 3 4 3 1
2 2 2 3 2
3 1 2 3 4
3 2 1 3 1
2 2 1 4 G

Solve Genetic
0 X 1 4 X
7 5 8 6 9
1 4 4 2 5
5 4 3 4 10
X 3 2 3 11
```

```
Task 7: Matrix After genetic 200
3 3 3 3 2
2 2 3 1 3
3 2 1 3 2
2 1 3 3 4
3 2 1 2 G

Solve Genetic
0 7 3 1 8
2 X 3 X X
7 6 10 6 9
1 8 2 2 X
X 5 4 5 10
```

```
Task 7: Matrix After genetic 400
2 1 1 3 3
4 3 4 3 4
1 1 1 1 1
2 1 3 3 2
2 4 2 3 G

Solve Genetic
0 2 1 2 X
2 3 2 5 3
1 2 3 4 5
2 3 3 3 6
7 4 8 6 9
```

```
Task 7: Matrix After genetic 800
3 4 3 3 1
2 2 1 1 2
1 3 4 3 2
2 3 2 3 3
1 4 4 1 G

Solve Genetic
0 5 4 1 4
2 4 3 4 5
6 7 4 5 8
1 4 2 2 3
X 6 X X 9
```

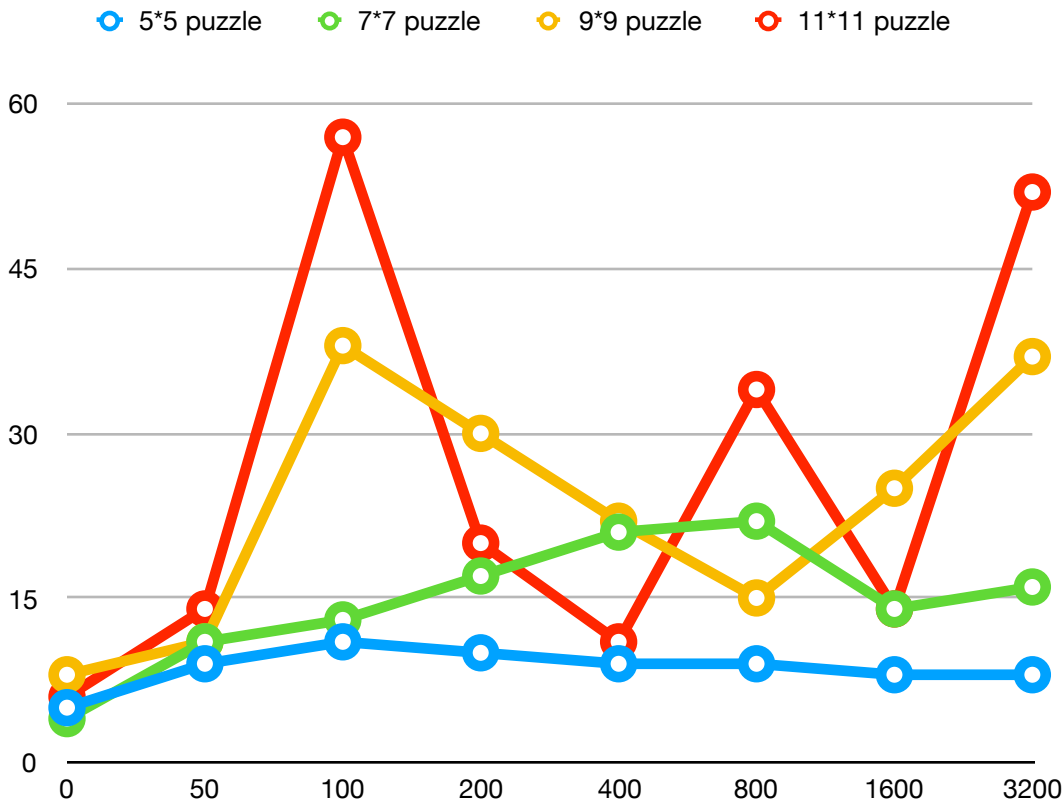
```
Task 7: Matrix After genetic 1600
3 4 2 3 4
4 1 2 1 4
4 3 1 3 1
3 2 3 2 1
4 1 4 3 G

Solve Genetic
0 5 6 1 7
5 4 4 3 4
5 5 7 4 6
1 3 5 2 7
7 6 7 X 8
```

```
Task 7: Matrix After genetic 3200
3 3 3 2 2
2 3 3 3 2
3 1 1 3 3
4 2 2 1 3
3 4 2 4 G

Solve Genetic
0 2 4 1 3
X 4 6 X 5
3 5 6 2 4
1 3 5 4 2
8 5 7 5 8
```

		0	50	100	200	400	800	1600	3200
	5*5 puzzle	5	9	11	10	9	9	8	8
	7*7 puzzle	4	11	13	17	21	22	14	16
	9*9 puzzle	8	11	38	30	22	15	25	37
	11*11 puzzle	6	14	57	20	11	34	14	52

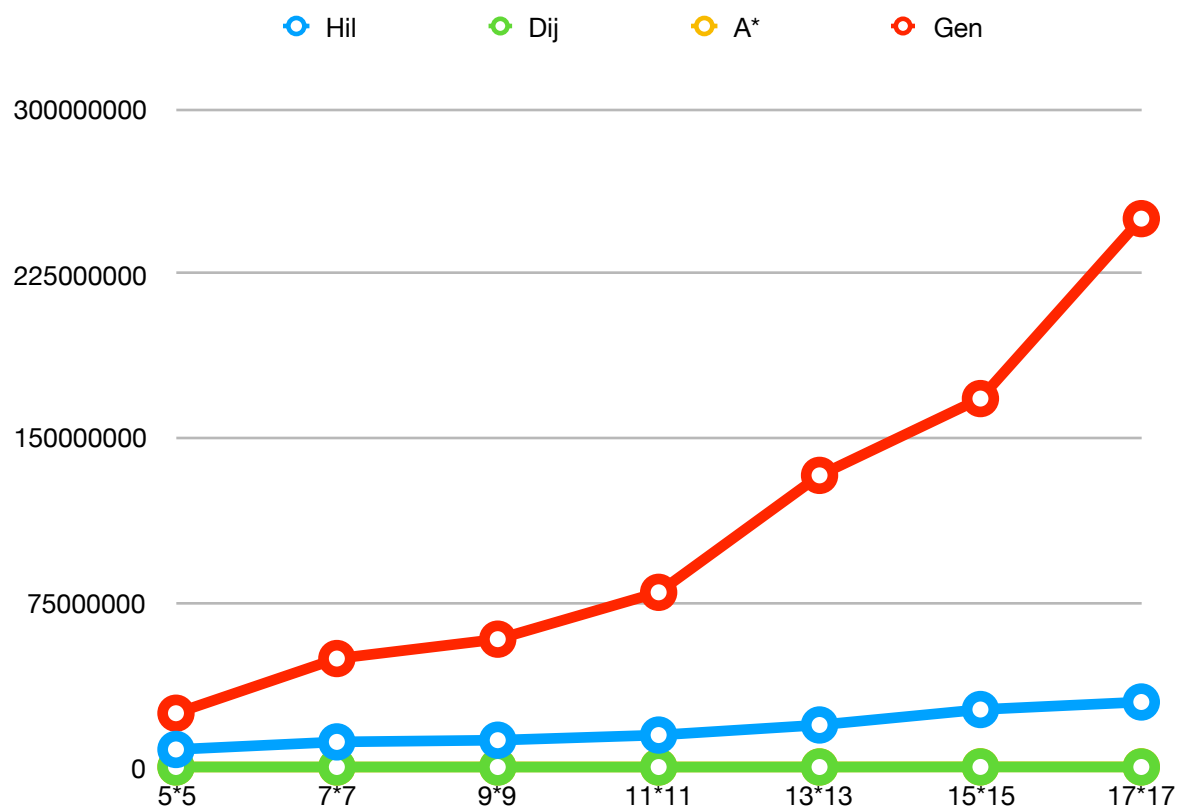


Task 8. Evaluation

For genetic, it works pretty good when the matrix size is big; however, its run-time can be much larger than hill climbing, since more than one item is changed every time. Hill climbing works fine when the size of matrix is relatively small comparing to the iteration time; however, since it has to run through every single particle to out put the hardest maze, it won't work that well when iteration is small corresponding to the size of the matrix. For A*, if the heuristic works for the case, it will perform better than Dijkstra. For Dijkstra, it increases run-time as the puzzle becomes larger.

The chart and graph below compare the run-time between hill-climbing algorithm(task 4), dijkstra algorithm (task 5), A* algorithm (task 6), and genetic algorithm (task 7). The x axis means different size of puzzles, and the y axis means the run-time in nanosecond.

		5*5	7*7	9*9	11*11	13*13	15*15	17*17
	Hil	8087169	11538278	12315871	14649495	19248341	26290716	29722700
	Dij	96846	100682	74521	77694	86452	130125	100007
	A*	131229	84113	144601	122876	134456	172956	152178
	Gen	24602711	49524341	58355050	79703038	133000000	168000000	250000000



Task 9: The most difficult and largest puzzle you can define and solve

The hardest puzzle (25*25) is:

18	18	19	7	18	22	20	11	19	9	21	17	16	21	17	16	1	17	15	24	19	3	23	24	4
13	14	10	7	22	13	20	17	9	14	23	9	22	14	14	23	16	3	17	9	19	13	7	12	15
15	9	20	2	9	17	18	1	22	13	11	9	19	14	3	11	13	13	14	1	16	21	18	17	6
21	13	5	15	14	20	5	19	18	12	14	21	11	13	17	16	17	4	8	14	16	13	2	18	23
8	20	3	17	15	10	19	11	14	4	13	12	5	4	8	19	20	20	17	5	10	20	6	11	11
17	16	10	19	10	4	3	19	1	11	5	15	3	11	6	16	9	15	1	2	16	15	1	10	14
2	20	22	17	12	18	12	3	1	14	9	2	18	18	10	6	11	12	18	17	14	18	8	23	19
11	19	3	7	9	10	12	15	15	14	17	10	12	1	15	17	1	14	17	12	8	17	16	20	15
7	7	12	8	13	15	5	14	10	15	4	13	2	2	16	16	7	8	8	5	13	16	1	6	13
18	5	12	14	6	14	7	10	15	11	9	6	7	8	9	1	6	6	3	14	13	21	2	20	20
24	12	4	13	16	13	2	4	5	4	13	11	2	13	11	7	15	4	11	14	11	15	11	3	24
21	14	12	17	20	14	7	16	2	15	12	1	1	3	5	12	11	7	1	19	8	17	14	18	1
18	13	6	5	8	18	12	14	11	4	12	2	9	4	14	11	4	10	13	10	18	20	15	9	14
10	19	4	7	7	1	13	1	14	15	12	5	2	8	13	2	13	11	1	13	11	9	22	13	7
16	3	11	14	20	11	15	1	15	6	13	8	14	14	12	6	10	17	3	12	12	17	11	18	16
19	17	21	9	17	8	1	15	4	8	3	11	12	12	3	9	3	15	17	9	20	19	18	3	18
24	1	21	18	16	13	13	14	8	14	13	10	14	10	4	11	11	6	18	16	12	19	19	9	2
9	14	20	18	13	17	17	1	2	1	16	6	8	6	6	12	17	14	3	3	20	19	7	10	12
24	12	6	14	19	16	17	18	18	13	18	14	17	6	8	8	7	17	14	11	14	14	18	23	20
6	20	15	16	8	6	7	11	17	11	12	13	5	4	13	4	17	13	7	5	16	11	22	3	20
15	3	20	17	6	8	8	12	13	17	3	10	5	20	17	8	1	9	15	12	6	21	18	11	4
11	7	14	6	20	5	19	18	17	14	21	18	3	6	17	20	14	4	20	8	20	7	17	23	14
19	21	9	6	9	4	12	18	13	6	10	22	16	7	13	5	4	14	9	16	21	21	7	1	12
10	20	4	22	1	22	7	1	14	23	10	18	14	22	9	5	1	7	11	7	2	5	19	18	6
6	20	18	5	16	4	6	10	3	17	17	14	21	16	13	19	20	11	21	3	16	6	6	16	6

And the solution of this puzzle is:

0	479	475	2	212	330	109	26	136	243	3	436	93	143	499	209	208	184	1	104	478	476	78	412	477
165	19	464	81	523	17	114	459	321	461	68	463	98	127	257	20	209	256	18	285	170	320	542	462	460
564	561	505	560	119	333	116	458	138	273	562	504	95	120	503	565	559	182	118	284	173	563	274	415	117
130	68	472	341	360	15	195	146	132	240	66	49	91	129	66	372	556	186	299	281	359	131	541	46	67
164	21	468	343	362	367	192	148	134	366	176	235	178	365	191	368	8	179	20	363	175	22	54	177	364
403	424	405	537	313	423	536	216	315	311	422	439	406	420	314	22	215	404	533	534	312	535	538	419	421
163	292	159	294	205	161	296	290	316	238	291	237	89	122	204	375	206	254	297	158	295	293	539	162	160
491	488	469	3	494	328	486	484	317	87	4	492	490	495	496	329	511	187	487	483	489	493	485	64	86
164	197	473	82	521	167	196	165	198	358	200	83	407	124	201	125	512	386	199	280	168	166	543	414	84
552	548	508	231	546	101	549	246	229	244	547	233	100	550	498	554	555	232	553	101	245	551	544	230	545
36	263	470	261	391	287	390	289	322	393	409	51	408	125	260	371	262	388	288	286	392	389	52	410	35
31	427	465	429	33	29	305	27	304	204	302	432	431	306	203	428	307	301	300	30	430	32	303	28	34
42	40	516	379	377	44	111	38	380	382	36	433	378	383	41	376	514	384	43	381	515	39	37	45	35
222	571	507	339	336	335	108	219	220	573	223	337	569	567	570	566	338	181	529	107	572	568	221	411	574
6	426	471	4	10	14	8	218	12	394	X	434	92	307	502	395	7	5	530	283	172	9	540	13	11
156	3	153	345	151	327	326	149	323	241	158	234	324	128	258	374	513	183	2	157	155	152	150	154	325
57	69	62	60	211	55	194	457	213	272	65	236	94	121	64	210	214	387	56	103	212	61	59	63	58
450	70	74	72	361	329	453	456	452	451	67	437	448	455	500	71	207	185	531	282	449	73	75	454	447
1	480	517	342	3	332	113	25	135	310	159	338	97	481	259	370	309	255	19	482	174	24	77	4	2
354	350	467	347	251	334	355	247	137	357	352	249	252	356	349	373	558	253	248	348	358	351	353	416	250
402	441	474	340	276	141	110	278	400	239	277	440	398	142	65	396	557	399	532	279	171	401	275	397	575
47	425	509	344	522	54	115	145	133	88	4	48	96	144	190	23	510	188	21	106	169	189	53	46	85
268	264	506	270	525	331	527	147	318	271	267	435	88	526	497	266	308	305	528	269	246	319	265	417	87
223	442	466	80	524	16	193	227	228	242	224	438	99	126	501	369	444	445	226	102	225	443	79	418	446
113	22	518	346	520	140	112	217	139	141	5	50	90	123	202	21	8	100	298	105	519	23	76	413	576

which need at least “576” steps from start to goal.