

Math 61

Dichen Yin

June 8, 2015

Formulas/Properties

Euler's Formula. (Proved in class by induction) This formula states that the sum of number of vertices $|V|$ and the number of faces $|F|$ subtracted by the number of edges $|E|$ is equal to 2.

$$|V| - |E| + |F| = 2$$

Catalan's number. Catalan's number is a number in the sequence defined by

$$C_n = \binom{2n}{n} - \binom{2n}{n-1} = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{n!(n+1)!}$$

It is often interpreted as the number of shortest grid walks on or above the diagonal of a $n \times n$ grid.

Hand Shake Lemma

Trees and stuff

A Tree is a graph that is connected but contains no cycle.

If $T = (V, E)$ is a tree, then:

1. T is bipartite. Why? Because $G(V, E)$ is bipartite iff there is no odd cycle.
Tree does not contain cycles \rightarrow no odd cycles \rightarrow bipartite.
2. (proved by induction): $|V| = |E| + 1$
3. there exist at least two vertices v, w such that $\{v, w | \deg(v) = \deg(w) = 1\}$

Cayley's formula Number of spanning trees in $K_n = n^{n-2}$

Ex: What is the probability of random $n-1$ edges chosen from K_n forming a spanning tree in K_n ?

$$\frac{n^{n-2}}{\binom{n}{n-1}}$$

Interesting Counting Examples

C_r in K_{nn} .

of Hypercubes H_n in K_m

$$\frac{\binom{m}{2^n} 2^n!}{2^n n!}$$