

# On the Modeling of Data Aggregation and Report Delivery in QoS-constrained Sensor Networks

Jin Zhu  
Industrial Tech. Dept.  
Univ. of Northern Iowa  
Cedar Falls, IA50614

Symeon Papavassiliou, Stella Kafetzoglou  
School of Electrical and Computer Engineering  
National Technical University of Athens  
Athens 15773, Greece

Jie Yang  
ECE Dept.  
NJIT  
Newark, NJ07102

## Abstract

*In this paper the problem of efficient data aggregation and report delivery in QoS-constrained distributed sensor networks is investigated. An analytical statistical model is introduced, to represent the data aggregation and report delivery process in sensor networks, with specific delivery quality requirements in terms of the achievable end-to-end delay and the successful report delivery probability. Furthermore, a lower bound approximation of the corresponding successful report delivery probability is obtained. The tradeoffs among various design parameters, as well as the impact of these parameters on the successful report delivery probability and its corresponding bound, are also discussed and evaluated.*

## 1 Introduction

A distributed sensor network is usually a self-organized system composed of large number of sensor nodes, which are used to measure different parameters that may vary with time and space, and send the corresponding data to a collector center or base station for further processing. Although several research works reported in the literature have discussed the problems of developing efficient routing and data aggregation processes mainly for energy savings/minimization in sensor networks (e.g. [1, 2, 4–9]), issues associated with the modeling of data aggregation process in order to meet specific task requirements are not yet well addressed.

In this paper, the problem of modeling the data aggregation process in a distributed, multi-hop sensor network under specific QoS constraints is investigated. Specifically, a data aggregation and report delivery process is considered, where each intermediate sensor node determines independently whether or not to perform data aggregation, according to the resource conditions and the specific task require-

ments. Based on this approach, an analytical model to represent the data aggregation and report delivery process in sensor networks, with specific delivery quality requirements in terms of the achievable end-to-end delay and the successful report delivery probability is developed. This model is utilized to gain some insight about the impact on the achievable system performance of the various design parameters and the tradeoffs involved in the process of data aggregation.

The remaining of the paper is organized as follows. First, in Section 2 the data aggregation approach in a distributed QoS-constrained sensor network is described, while in section 3 a statistical model is introduced in order to model and represent the data aggregation and report delivery process in sensor networks. This model emphasizes on the study of the performance of the data aggregation process in terms of the end-to-end delay and probability of success in the report delivery process. Furthermore a lower bound approximation of the corresponding successful report delivery probability is obtained in section 4. Finally, the tradeoffs among various design parameters, as well as the impact of these parameters on the successful report delivery probability and its corresponding bound, are discussed and evaluated in Section 5.

## 2 Data Aggregation and Processing in QoS-constrained Sensor Network

Throughout the paper we consider a multi-hop communication sensor network. We assume that the data collected and/or observed by each sensor node will be transmitted, through this multi-hop architecture, to a collector center for further processing and decision making. It should be noted that, in many applications of sensor networks the data from different neighboring sensors are usually highly correlated, and therefore using collaborative data aggregation processing may improve the overall system performance and reduce the communication load and energy consumption. Therefore in the following we assume that when a

sensor node receives a packet or message from its neighbor, it is able to either perform local processing and aggregation or just forward (relay) it, according to the QoS requirements of the corresponding applications. In this paper, the procedure that a sensor node locally generates and/or processes a measurement packet in which new data may be aggregated is referred to as reporting, while the corresponding new/updated packet is referred to as a report.

When a sensor node receives a report from its neighbor, it first determines whether or not it would perform data aggregation on the report. The following different cases may occur. a) If the delay constraint can be satisfied, the sensor node defers the report for a fixed time interval  $\tau$  with probability  $\gamma$ , during which the node processes and aggregates any reports that arrive, and generates a new report before transmitting it to the next hop. With probability of  $(1 - \gamma)$  the sensor node will directly try to forward the report without introducing any deferred period. b) If the delay constraint can be satisfied only if the report is not deferred, the sensor node simply tries to forward this report. c) If the delay constraint cannot be satisfied in any case, the sensor node will discard the report, to avoid further wasting of any additional resources.

In this paper, the considered end-to-end QoS constraint is the end-to-end latency requirement  $D$  of a report, that may aggregate other data or reports along its path from source to the collector center. If the report is delivered to the collector center within the given latency constraint  $D$  after its initial generation, it will be considered as a successful delivery. At a sensor node, for a received report, in addition to the possible deferred period  $\tau$ , there is some additional waiting time caused by the transmission of the previous report at the node. We assume that at each node there can be at most two reports: one is under transmission and the other is waiting to be transmitted. If there are report arrivals while a report is waiting in the sensor node to be transmitted, these newly arrived reports will be aggregated into the waiting report.

### 3 Data Aggregation Modeling

By using proper routing mechanisms, we assume that each report goes through each node only once, and nodes always forward the report to other nodes that are closer to the collector center. Therefore assuming that  $l$  nodes are visited from the source node to the collector center, we denote the set of these sensor nodes as  $G_l = \{s_1, s_2, \dots, s_l\}$ . Without loss of generality we assume that the distances between the sensor nodes and the collector site are arranged in decreasing order, i.e.,  $d_1 > d_2 > \dots > d_l$ , where  $d_i$  is the distance between node  $s_i$  and the collector center.

Let us also denote by  $t_i^{(R)}$  the reporting time at node  $s_i$  which includes the time period for data aggregation, by  $t_i^{(F)}$

the forwarding time at node  $s_i$  which accounts for the transmission time and is related to the report length and bandwidth, and by  $t_i^{(P)}$  the propagation time from node  $s_i$  to next node  $s_{i+1}$  which depends on the distance between the two nodes. Time periods  $t_i^{(R)}$ ,  $t_i^{(F)}$  and  $t_i^{(P)}$  are random variables and in the following their corresponding probability density functions (pdf) are denoted by  $f_i^{(R)}(t)$ ,  $f_i^{(F)}(t)$  and  $f_i^{(P)}(t)$ , respectively. Let us denote by  $t_i$  the time interval between the point that node  $s_i$  receives a report to the point that this report is delivered to node  $s_{i+1}$ . If node  $s_i$  does not perform data aggregation the corresponding time interval is  $t_i^{(F)} + t_i^{(P)}$ ; otherwise, the time interval is  $t_i^{(R)} + t_i^{(F)} + t_i^{(P)}$ ,  $i \geq 1$ , therefore:

$$t_i = \begin{cases} t_i^{(R)} + t_i^{(F)} + t_i^{(P)}, & \text{with reporting} \\ t_i^{(F)} + t_i^{(P)}, & \text{without reporting,} \end{cases}$$

and its pdf is denoted by  $f_i(t)$ . Under the assumption that a sensor node performs reporting with probability  $\gamma$ , we have

$$f_i(t) = f_i(t|\text{with reporting})\gamma + f_i(t|\text{without reporting})(1 - \gamma).$$

Let us also assume that the time periods are independent of each other, and denote by  $F_i^{(R)}(s)$ ,  $F_i^{(F)}(s)$  and  $F_i^{(P)}(s)$  the Laplace transforms of  $f_i^{(R)}(t)$ ,  $f_i^{(F)}(t)$  and  $f_i^{(P)}(t)$ , respectively. Applying the Laplace transform to  $f_i(t)$ , we have [3]

$$\begin{aligned} F_i(s) &= E[e^{-st_i}] = \int_0^\infty f_i(t)e^{-st}dt \\ &= F_i^{(F)}(s) \cdot F_i^{(P)}(s) \cdot [\gamma F_i^{(R)}(s) + (1 - \gamma)] \end{aligned} \quad (1)$$

In the following, we first assume that no reports will be discarded due to the delay constraint, and obtain the end-to-end delay distribution, which can be used to obtain the probability  $P_{succ}$  that the report is delivered to the collector center within the delay constraint  $D$ . Then the probability that the report is discarded due to unsatisfactory end-to-end delay performance can be obtained as  $(1 - P_{succ})$ . Let us

denote the end-to-end delay of a report by  $T_L = \sum_{i=1}^L t_i$  and its corresponding pdf by  $f_{T_L}(t)$ , where the random variable  $L$  is the number of hops that are involved in the transmission of a report from the source node to the collector center (including the source node). Thus, the Laplace transform of  $f_{T_L}(t)$ , denoted by  $F_{T_L}(s)$ , is given by

$$F_{T_L}(s) = E[e^{-s(t_1+t_2+\dots+t_L)}] = \sum_{l=1}^N p_L(l) \prod_{i=1}^l F_i(s),$$

where  $p_L(l)$  is the probability mass function of  $L$ , where the random variable  $L$  represents the number of hops that are

involved in the transmission of the report from the source node to the collector center (including the source node). The pdf of  $T_L$  can be obtained by using the inverse Laplace transform of  $F_{T_L}(s)$ , i.e.,

$$f_{T_L}(t) = \mathcal{L}^{-1}\{F_{T_L}(s)\} = \sum_{l=1}^N p_L(l) \mathcal{L}^{-1}\left\{\prod_{i=1}^l F_i(s)\right\}.$$

When  $f_{T_L}(t)$  is obtained, the successful probability  $P_{succ}$  that a report can reach the collector center within the delay constraint  $D$  is given by

$$P_{succ} = P[T_L \leq D] = \int_0^D f_{T_L}(t) dt. \quad (2)$$

In general, it is difficult to obtain an analytical expression for  $P_{succ}$  in practice, since the distribution of  $T_L$  is generally unknown. Therefore, in the next section we use a probabilistic model to lower-bound the probability  $P_{succ}$ .

#### 4 Lower bound on $P_{succ}$

The end-to-end delay of an independent report that meets the delay constraint and passes through  $l$  hops can be represented by

$$T_l = \sum_{i=1}^l t_i^{(R)} + \sum_{i=1}^l t_i^{(F)} + \sum_{i=1}^l t_i^{(P)} \leq D, \quad (3)$$

where in general  $t_i^{(F)}$  can be upper-bounded based on the largest report length and the corresponding data rate of the sensor network, and  $t_i^{(P)}$  can be upper-bounded by the range of the sensor network and the longest distance between two sensor nodes. Thus, we can assume that  $D$  is decomposed as

$$D = D_r(l) + D_f(l) + D_p(l), \quad (4)$$

where  $D_r(l)$ ,  $D_f(l)$ , and  $D_p(l)$  are the upper bounds on the end-to-end reporting time, forwarding time and propagation time, respectively, when the report needs to be delivered to the collector center using  $l$  hops. As a result, in our study the constraint that needs to be satisfied regarding the reporting time can be represented as

$$\sum_{i=1}^l t_i^{(R)} \leq D_r(l). \quad (5)$$

Noted that  $t_i^{(R)}$  is a function of  $\tau$  and  $\gamma$ , and (5) provides a worst-case bound on the reporting time under the constraint (3). Therefore, the lower bound on the probability  $p_{succ}(l)$  that a specific independent report is delivered to the collector center within the end-to-end constraint, when the distance between the source node and the collector center is  $l$

hops, is lower-bounded by

$$p_{succ}(l) = P[S_L(t) < D | L = l] \geq P\left(\sum_{i=1}^l t_i^{(R)} \leq D_r(l)\right). \quad (6)$$

Thus the probability of success  $P_{succ}$  under delay constraint (3) is lower-bounded by

$$P_{succ} = \sum_{l=1}^N p_L(l) p_{succ}(l) \geq \sum_{l=1}^N p_L(l) P\left(\sum_{i=1}^l t_i^{(R)} \leq D_r(l)\right). \quad (7)$$

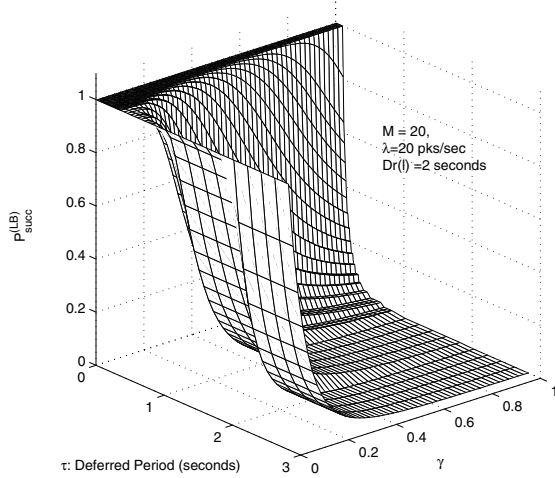
Note that (7) provides a lower bound to the probability of a successful report delivery within the QoS constraint for sensor networks with and without data aggregation schemes. When  $\gamma = 0$ ,  $P_{succ}$  is reduced to the probability of a successful delivery in a sensor network without any data aggregation scheme, in which each received report will be forwarded as is (without any deferred period). Here we assume that if there is no data aggregation scheme deployed in the sensor network, the report can be delivered to the collector center within its end-to-end delay constraint  $D$ , through the use of the deployed routing algorithm. Otherwise the sensor node will not participate in the specific measurement task. That is, we assume  $P_{succ, non-aggregation} = 1$ . Furthermore, we assume that when aggregation is performed, the generated reports will follow the same path as in the case without data aggregation.

If data aggregation is not performed at node  $i$ , the reporting time  $t_i^{(R)} = 0$  while if data aggregation is performed with probability  $\gamma$ ,  $t_i^{(R)} = \tau$ . It is clear that the longest delay that a report may experience due to data aggregation is  $l\tau$ , when the number of hops between the source to the collector center is  $l$ . If  $l\tau \leq D_r(l)$ , the end-to-end delay can be guaranteed even if at each node data aggregation is performed, i.e.,  $\gamma = 1$ . Thus we can have  $p_{succ}(l) = 1$  when  $l\tau \leq D_r(l)$ . When  $l\tau > D_r(l)$ , if all the intermediate nodes perform data aggregation and reporting with a deferred period  $\tau$ , the end-to-end delay of a report may exceed the delay constraint. The maximum number of data aggregation and reporting that can be performed to guarantee the delay constraint, determined by the upper bound on the reporting time  $D_r(l)$ , is given by

$$C(l) = \left\lfloor \frac{D_r(l)}{\tau} \right\rfloor. \quad (8)$$

That is, the lower bound of  $P_{succ}$  is equal to the probability that a report experiences at most  $C(l)$  times of data aggregations and reporting along its path. Therefore, the probability  $p_{succ}(l)$  is lower-bounded by

$$p_{succ}(l) \geq p_{succ}^{(LB)}(l) \triangleq \begin{cases} \sum_{k=0}^{C(l)} \binom{l}{k} \gamma^k (1-\gamma)^{l-k}, & C(l) < l \\ 1, & C(l) \geq l. \end{cases} \quad (9)$$



**Figure 1. Probability of successful report delivery as a function of  $\gamma$  and  $\tau$ .**

Consequently, the probability of success  $P_{succ}$  under delay constraint is lower-bounded by

$$P_{succ} \geq P_{succ}^{(LB)} \triangleq \sum_{l=1}^N p_L(l) p_{succ}^{(LB)}(l). \quad (10)$$

## 5 Numerical Results and Discussions

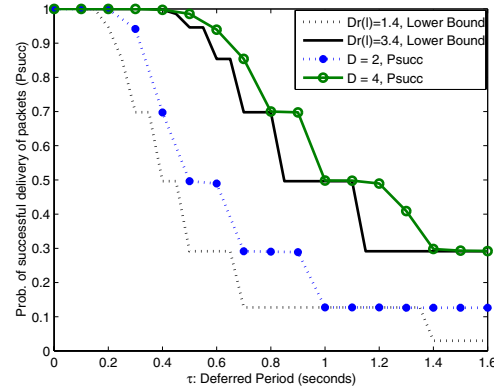
In this section, based on the developed models, we study the impact of parameters  $\gamma$  and  $\tau$  on the data aggregation model and the performance of the data aggregation approach. Among our objectives is to identify the various trade-offs that these parameters present, in order to provide guidelines to choose the appropriate values of these design parameters that achieve the desired performance. In the following, let us consider a sensor network where the sensor nodes are uniformly distributed in a disk area with radius  $R$ , and each node has a fixed limited transmission range  $r$ . We assume that each node always transmits a report as far as possible within its transmitting range, and therefore the maximum number of hops is  $M = \lceil \frac{R}{r} \rceil$ .

If the total number of sensor nodes  $N$  is large, the probability  $p_L(l)$  can be approximated by

$$p_L(l) = \begin{cases} \frac{2l-1}{M^2}, & 1 \leq l \leq M \\ 0, & l > M. \end{cases} \quad (11)$$

Let us assume the report arrival process at each sensor node follows Poisson distribution with rate  $\lambda$ .

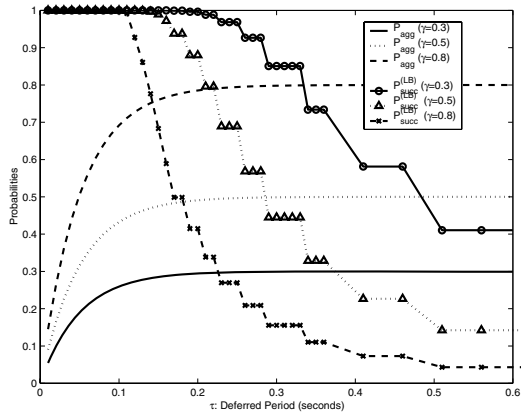
In the first numerical example presented here, our objective is to demonstrate how the lower bound approximation



**Figure 2. Lower bound approximation of  $P_{succ}$  ( $P_{succ}^{(LB)}$ ) and actual  $P_{succ}$  for  $\gamma = 0.5$  and different delay constraints  $D$ , as a function of deferred period  $\tau$ .**

of  $P_{succ}$ , given by (10), is affected by different values of  $\gamma$  and  $\tau$ . Specifically, Fig. 1 shows the lower bound  $P_{succ}^{(LB)}$  for different values of  $\gamma$  and  $\tau$ , for the case with  $\lambda = 20$ ,  $M = 20$ , and  $D_r(l) = 2$  seconds. It can be seen from this figure that  $P_{succ}^{(LB)}$  decreases with  $\gamma$  and  $\tau$ , since larger values of  $\gamma$  will result in more frequent data aggregation and reporting during the delivery of the report, and larger values of  $\tau$  will increase the end-to-end delay. As these values both increase, there is higher probability that the end-to-end delay is larger than the constraint, which results in the decrease of  $P_{succ}$ .

As shown before, expression (10) provides a simple lower bound on the successful report delivery probability  $P_{succ}$ . In the next experiment we discuss and evaluate the relation between this lower bound approximation and the actual value of  $P_{succ}$ . In Fig. 2 the curves of the corresponding probabilities are plotted as functions of the deferred period  $\tau$ , for a sensor network with  $M = 10$ ,  $\lambda = 20$ , and  $\gamma = 0.5$ . In this figure the lower bound approximation  $P_{succ}^{(LB)}$  is obtained by using (9) and (10), and the actual  $P_{succ}$  is obtained from (2) for Poisson report arrivals. In this figure we plot four different curves, which represent the corresponding probabilities for delay constraints  $D = 4$  and  $D = 2$  seconds respectively. Correspondingly in the lower bound calculation, we assume the upper bound for packets transmission and propagation is 0.6 time units through their delivery from the source to the collector center and thus in the corresponding lower bound approximation, parameters  $D_r(l) = 3.4$  and  $D_r(l) = 1.4$  are used. The results in Fig. 2 demonstrate that, in general smaller  $D$  will result in



**Figure 3. The relationship between  $P_{succ}^{(LB)}$  and  $P_{agg}$ .**

lower  $P_{succ}$ , while the lower bound approximation demonstrates similar trend with the actual performance of  $P_{succ}$ . Furthermore, it can be seen that the accuracy of the lower bound approximation increases as the value of  $\tau$  decreases. Based on these results we conclude that  $P_{succ}^{(LB)}$  provides an accurate lower bound approximation of the probability of successful report delivery for all values of  $\tau$ .

As can be seen from the above discussions, for a given sensor network with specific delay requirement  $D$ ,  $P_{succ}$  depends on parameters  $\tau$ ,  $\gamma$  and the arrival rate  $\lambda$ . From Figs. 1 and 2 it becomes clear that in order to meet the required quality objectives, there are many different choices for parameters  $\tau$  and  $\gamma$ , while  $\lambda$  is determined by the nature of the measurement task. Furthermore, it can be noted that if a report is deferred at a node for the time period  $\tau$ , while no other reports arrive during that period, and as a result no aggregation is actually performed, we do not get any benefits from such an approach, although  $P_{succ}$  decreases. In order to enhance the efficiency of the deployed quality constrained data aggregation approach and maximize its benefits, when determining the optimal  $\tau$  and  $\gamma$ ,  $P_{succ}$  can be specified as a QoS requirement of the task or application, together with the delay constraint  $D$ . Then the objective function can be to maximize  $P_{agg}$ , the probability that a node determines to perform data aggregation and the data aggregation occurs during the deferred period  $\tau$ . The optimal values of  $\tau$  and  $\gamma$  can be determined by

$$(\tau_{opt}, \gamma_{opt}) = \arg \max_{P_{succ} \geq P_{req}} (P_{agg}) \quad (12)$$

where  $P_{req}$  is the minimum required probability of successful report delivery to the collector center within the end-to-end delay requirement  $D$ . When the report arrival process

follows Poisson distribution with rate  $\lambda$ , the probability that there is at least one report arrival during the deferred period  $\tau$  is  $1 - e^{-\lambda\tau}$ . In this case, the probability that data aggregation occurs during the deferred period  $\tau$  is given by

$$P_{agg} = \gamma (1 - e^{-\lambda\tau}). \quad (13)$$

Fig. 3 shows the lower bound  $P_{succ}^{(LB)}$ , and  $P_{agg}$ , for different combinations of  $\tau$  and  $\gamma$  under the assumption of Poisson report arrivals with  $\lambda = 20$ . Given  $P_{req}$ , we can choose the set of  $(\tau, \gamma)$  that can provide the maximum  $P_{agg}$  with  $P_{succ}^{(LB)} \geq P_{req}$ . It should be noted that since  $\gamma \in [0, 1]$ , from (13), we have  $P_{agg} \leq 1 - e^{-\lambda\tau}$ . It can be observed that  $P_{agg}$  approaches  $\gamma$  and is insensitive to  $\tau$  for large values of  $\tau$ . Finally although the objective function considered in this study is  $P_{agg}$  so that aggregation efficiency can be maximized, other objective functions, such as the number of reports aggregated, can be considered, depending on the metrics of interest.

## References

- [1] K. Akkaya and M. Younis. An energy-aware QoS routing protocol for wireless sensor networks. In *Proc. 23rd Int. Conf. On Distributed Computing Systems Workshops*, pages 710–715, 2003.
- [2] C. L. Barrett, S. J. Eidenbenz, L. Kroc, M. Marathe, and J. P. Smith. Routing, coverage, and topology control: Parametric probabilistic sensor network routing. In *Proc. 2nd ACM Int. Conf. on Wireless Sensor Networks and Applications*, pages 122–131, Sept. 2003.
- [3] I. S. Gradshteyn and I. M. Ryzhik. *Table of Integrals, Series, and Products*. Academic Press, New York, 1980.
- [4] W. Heinzelman, A. Chandrakasan, and H. Balakrishnan. Energy-efficient communication protocol for wireless microsensor networks. In *Proc. 33rd Hawaii Int. Conf. on System Sciences*, pages 3005–3014, Jan. 2000.
- [5] C. Intanagonwiwat, R. Govindan, and D. Estrin. Directed diffusion: A scalable and robust communication paradigm for sensor networks. In *Proc. 6th Annual ACM/IEEE Int. Conf. on Mobile Computing and Networking (Mobicom 2000)*, pages 56–67, Aug. 2000.
- [6] B. Krishnamachari, D. Estrin, and S. Wicker. The impact of data aggregation in wireless sensor networks. In *Proc. IEEE 22nd Int. Conf. on Distributed Computing Systems Workshop*, pages 575–578, July 2002.
- [7] J. Mirkovic, G. Venkataramani, S. Lu, and L. Zhang. A self-organizing approach to data forwarding in large-scale sensor networks. In *Proc. IEEE Int. Conf. on Communications (ICC 2001)*, volume 5, pages 1357–1361, June 2001.
- [8] S. Olariu, Q. Xu, and A. Zomaya. An energy-efficient self-organization protocol for wireless sensor networks. In *Proc. Intelligent Sensors Sensor Networks and Information Processing Conf.*, pages 55–60, Dec. 2004.
- [9] I. Stojmenovic and X. Lin. Power-aware localized routing in wireless networks. *IEEE Transactions on Parallel and Distributed Systems*, 12(11):1122–1133, Nov. 2001.