# Information Propagation Speed
# in Mobile and Delay Tolerant Networks

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*Abstract*—The goal of this paper is to increase our understanding of the fundamental performance limits of mobile and Delay Tolerant Networks (DTNs), where end-to-end multi-hop paths may not exist and communication routes may only be available through time and mobility. We use analytical tools to derive generic theoretical upper bounds for the information propagation speed in large scale mobile and intermittently connected networks. In other words, we upper-bound the optimal performance, in terms of delay, that can be achieved using any routing algorithm. We then show how our analysis can be applied to specific mobility and graph models to obtain specific analytical estimates. In particular, when nodes move at speed $v$ and their density $\nu$ is small (the network is sparse and surely disconnected), we prove that the information propagation speed is upper bounded by $(1+O(\nu^2))v$ in the random way-point model, while it is upper bounded by $O(\sqrt{\nu v}v)$ for other mobility models (random walk, Brownian motion). We also present simulations that confirm the validity of the bounds in these scenarios.

## I. Introduction

Recent research has highlighted the necessity and the significance of mobile ad hoc networks where end-to-end multi-hop paths may not exist and communication routes may only be available through time and mobility. Depending on the context, these networks are commonly referred as Intermittently Connected Networks (ICNs) or Delay Tolerant Networks (DTNs).

While there is a large body of work on understanding the fundamental properties and performance limits of wireless networks under the assumption that connectivity must be maintained (*e.g.*, since the seminal work by Gupta and Kumar [5]), there are only few results on the properties of intermittently connected or delay tolerant networks (*e.g.*, [2], [9], [10], [12]). Most of the effort has been dedicated to the design of efficient routing protocols (see [16] for a survey) and comparative simulations, using specific mobility models or concrete traces (*e.g.,* [13], [17]). A complete understanding of what one can expect for optimal performance (*e.g.,* through theoretical bounds) is still missing for many realistic models.

In this context, the objective of the paper is to evaluate the maximum speed at which a piece of information can propagate in a mobile wireless network. A piece of information is a packet (of small size) which can be transmitted almost instantaneously between two nodes in range. If the network is connected (*i.e.*, an end-to-end multi-hop path exists) information moves at a rather high speed, which can be considered infinite compared to the mobility of the nodes.

We consider a network made of $n$ nodes moving in a square area of size $\mathcal{A}$, under the unit disk graph model (*i.e.*, nodes are neighbors when their distance is smaller than one). In order to study the properties of DTNs that are relevant to the field of applications, we are interested in very sparse networks and we are investigating the case where the node density $\frac{n}{\mathcal{A}}$ is small. Indeed, most applications for DTNs are required to work for sparse mobile ad hoc networks (*e.g.*, [13], [16], [17]), where intermittent connectivity is due to node mobility and to limited radio coverage. In these cases, the mobile network is almost always disconnected, making information propagation stall as long as the node mobility does not allow the information to jump to another connected component. The information is either transmitted or carried by a node (requiring a *store-carry-and-forward* routing model). Thus, a "path" is an alternation of packet transmissions and carriages, that connects a source to a destination, and is better referred (from now on) as a **journey**. Informally, our aim is to find the shortest journey (in time) that connects any source to any destination in a square map, in order to derive the overall propagation speed.

In terms of related work on the information propagation speed in wireless networks, the problem has been studied in static networks. Zheng [19] showed that there is a constant upper bound on the information diffusion rate in large wireless networks. Recently, Xu and Wang [14] proved that there is a unified upper bound on the maximum propagation speed in large wireless networks, using unicast or broadcast. The article [7] evaluates analytical upper bounds on the packet propagation speed using opportunistic routing. In contrast, our main focus here is to evaluate the information propagation speed in mobile and intermittently connected networks.

Taking into account the node mobility, some recent papers have presented initial results on the theoretical properties of intermittently connected networks, *e.g.*, [2], [4], [9], [10], [12], [18]. The papers [4], [18] analyze the delay of common routing schemes, such as epidemic routing, under the assumption that the inter-meeting time between pairs of nodes follows an exponential distribution. The authors of [12] took a graph-theoretical approach in order to upper bound the time it takes for disconnected mobile networks to become connected through the mobility of the nodes. This work uses an Erdös-Rényi network model, where the node connections are done independently of the actual topology of the network. In this paper, we will depart from this model in order to integrate the topological nature of the network, for an instance of $n$ mobile nodes in a square map of size $\mathcal{A}$ connected according to the unit disk graph model.

In [2], an interesting model of dynamic random geometric

graphs (based on a random walk mobility model) leads to the first precise asymptotic results on the connectivity and disconnectivity periods of the network. Unfortunately, this methodology cannot be extended to evaluate the fastest possible information propagation. In [9], [10], Kong and Yeh studied the information dissemination latency in large wireless and mobile networks, in constrained i.i.d. mobility and Brownian motion models. They showed that, when the network is not percolated (under a critical node density threshold), the latency scales linearly with the Euclidean distance between the sender and the receiver, while the latency scales sub-linearly in the super-critical case where the network is percolated. A question that remains to be answered is to find precise estimates on the constant upper bounds of the information propagation speed in intermittently connected mobile networks. In [6], the authors present an initial analytical upper bound on the achievable information propagation speed in an infinite network model. Here, we present the first analytical results in the more realistic (and significantly more difficult) case of a large scale but finite mobile network model, in order to prove rigorous upper bounds on the maximum achievable information propagation speed. Moreover, we derive our theoretical bounds on a more general mobility model than those used in the literature, while we also compare our analytical results with simulations.

More precisely, our main contributions are the following:

- we present a new probabilistic model of space-time journeys of packets of information in delay tolerant networks;
- we upper bound the optimal performance that can be achieved using any routing algorithm in finite two-dimensional mobile networks and we derive theoretical bounds on the information propagation speed, depending on the node density and the network mobility;
- we verify the accuracy of our bounds via simulations.

The rest of the paper is organized as follows. In Section II, we introduce the network and mobility model, we define the information propagation speed metric, and we discuss our main results and the methodology. In Section III, we present the detailed analysis and the proof of our theoretical upper bounds. We derive asymptotic estimates for the propagation speed in sparse networks in Section IV. We illustrate the behavior of the bounds depending on the network and mobility parameters (such as the node density and change of direction rate) in Section V. We compare the analytical bounds with simulation measurements in Section VI. We conclude and propose some directions for further research in Section VII.

## II. MODEL AND OVERVIEW OF MAIN RESULTS

### A. Mobile Network Model

We consider a network of $n$ nodes in a square area of size $\mathcal{A} = L \times L$. The nodes are enumerated from 1 to $n$. In the next section, we will analyze the case where both $n, L \to \infty$ such that the node density $\nu = \frac{n}{\mathcal{A}}$ tends to a (small) constant.

Initially, the nodes are distributed uniformly at random. Every node follows an i.i.d. random trajectory, reflected on the borders of the square like billiard balls. The nodes change direction at Poisson rate $\tau$ and keep a uniform speed between direction changes. The motion direction angles are uniformly

distributed between 0 and $2\pi$. When $\tau > 0$, we have a random walk model; when $\tau \to \infty$ we are on the Brownian limit; when $\tau \to 0$ we are on a random way-point-like model, since nodes travel a distance of order $L$ before changing direction.

The billiard model is equivalent to considering an infinite area made of mirror images of the original square: a mobile node moves in the original square while its mirror images move in the mirror squares. The fact that a node bounces on a border is strictly equivalent to crossing it without bouncing, while its mirror image enters the square. With this perspective, the trajectory of a node is equivalent to a free random trajectory in the set of mirror images of the original square, while the nodes remain distributed uniformly at random.

We adopt the unit-disk model: two nodes at a distance smaller than one can exchange information. The average number of neighbors per node is therefore smaller (or equal) than $\pi \frac{n}{\mathcal{A}}$. In [15], Xue and Kumar have shown that if the average number of neighbors is smaller than $0.074 \log n$, then the network is almost surely disconnected when $n$ is large. In order to study the properties of delay tolerant networks in the context of their applications, we need to look at sparse networks. Therefore, we assume that the number of nodes $n$ tends to infinity at the same rate as the area of the network domain square (so that the node density remains constant), and we investigate the case where the node density $\frac{n}{\mathcal{A}}$ is small.

Since we are interested in computing upper bounds on the best possible information propagation, we do not consider here the effects of buffering or congestion. Indeed, we assume that a piece of information, *i.e.*, a packet of small size can be transmitted instantaneously between two nodes in range. Even under these assumptions, we are able to derive finite bounds on the information propagation speed. We note that these assumptions do not affect the validity of our upper bounds, since they correspond to an ideal scenario with that respect; this allows us to capture the fundamental performance limit of DTNs based solely on the network mobility and topology. Moreover, in the case of very sparse mobile networks, the previous assumptions do not impact on the accuracy of our results, since information transmission occurs much faster than the speed of the mobile nodes.

### B. Information Propagation Speed and Main Results

Our main result is the evaluation of a generic upper bound of the information propagation speed (presented later in Theorem 1 in this section), which in turn allows us to obtain specific bounds for particular models.

In order to evaluate the fastest possible information propagation, we establish a probabilistic space-time model of journeys of packets of information in delay tolerant networks that contains all possible "shortest" journeys: the full epidemic broadcast. We call the information, the *beacon*. Every time a new node is in range of a node which carries a copy of the beacon, the latter node transmits another copy of the beacon to the new node. In our model, journeys are expressed as space-time trajectories, since store-carry-forward routing also implies that we must take into account the time dimension.

To prove our main theorem in Section III, we decompose the packet journeys into independent segments and we evaluate

the Laplace transform of the journey probability density. From the Laplace transform, we are able to establish an upper bound on the average number of journeys arriving to a point $\mathbf{z}$ before a time $t$, where $\mathbf{z}$ is a 2D space vector expressing the spatial distance from the source that emitted the beacon. More precisely, we are interested to find when the density of journeys becomes 0 almost surely. We notice that a zero probability of reaching a given point in space in a given amount of time implies an upper bound on the information propagation speed. In order to evaluate a constant bound, we will consider the asymptotic case where the distance from the source and the time both tend to infinity. Hence, using our approach, we obtain theoretical bounds on the information propagation speed by computing the smallest ratio of the distance over the given time, which yields a journey probability of zero. The asymptotic approach must be interpreted in the following sense: we evaluate the information propagation speed to a distance which is a large multiple of the maximum radio range. In fact, in Section VI, we will see that the propagation speed quickly converges to a constant value as soon as the distance between the source and the destination is simply larger than the radio range. Similarly, in [9], in the case of disconnected mobile networks, the authors show that the information propagation latency scales linearly with the distance in the same asymptotic setting.

Therefore, the concept of propagation speed is probabilistic. To express the previous discussion using mathematical notations, let us consider that the beacon starts at time $t = 0$ on a node at coordinate $\mathbf{z}_0 = (x_0, y_0)$. Let us initially consider (for simplicity) a destination node that stays at coordinate $\mathbf{z}_1 = (x_1, y_1)$. Let $q_\nu(\mathbf{z}_0, \mathbf{z}_1, t)$ denote the probability that the destination receives the beacon before time $t$. A scalar $s_0 > 0$ is an upper bound for the propagation speed, if for all $s > s_0$, $\lim q_\nu \left( \mathbf{z}_0, \mathbf{z}_1, \frac{|\mathbf{z}_1 - \mathbf{z}_0|}{s} \right) = 0$ when $|\mathbf{z}_1 - \mathbf{z}_0| \to \infty$, with $|.|$ denoting the Euclidean norm. For example, if we prove that $q_\nu(\mathbf{z}_0, \mathbf{z}_1, t) < \exp(-a|\mathbf{z}_1 - \mathbf{z}_0| + bt + c)$, then quantity $\frac{b}{a}$ is a propagation speed upper bound.

Using the previously described methodology, we will prove the following main theorem, which expresses our generic upper bound on the information propagation speed in terms of different values of the network and mobility parameters.

*Theorem 1:* For a network in a square area $\mathcal{A} = L \times L$, where the number of nodes $n \to \infty$ and $L \to \infty$ such that the node density $\nu = \frac{n}{\mathcal{A}}$ remains constant, an upper bound on the information propagation speed is the smallest ratio:

$$\min_{\rho, \theta > 0} \left\{ \frac{\theta}{\rho} \text{ with } \theta = \sqrt{ \rho^2 v^2 + \left( \tau + \frac{\frac{n}{\mathcal{A}} 4\pi v I_0(\rho)}{1 - \frac{n}{\mathcal{A}} \pi \frac{2}{\rho} I_1(\rho)} \right)^2 } - \tau \right\},$$

where $v$ is the node speed, $\tau$ is the node direction change rate, while $I_0()$ and $I_1()$ are *modified Bessel functions* (see [1]).

*Remark:* As we will see, quantities $\rho$ and $\theta$ correspond to the parameters of the Laplace transform of the journey probabilities. Quantity $\rho$ is expressed as an inverse of distance and quantity $\theta$ is expressed as an inverse of time, therefore the ratio $\frac{\theta}{\rho}$ has the dimension of a speed.

Since quantities $I_0(x)$ and $\frac{2}{x} I_1(x)$ are both greater than 1, the previous expression has meaning when $\frac{n}{\mathcal{A}} < \frac{1}{\pi}$. Above

this density threshold, the upper bound for the information propagation speed is infinite. Such a behavior is expected, since it is known that there exists a critical density above which the graph is fully connected or at least percolates (*i.e.,* there exists a unique infinite connected component with non-zero probability) [11]. The infinite component implies an infinite information propagation speed according to our definition. The exact value of the critical density is unknown, although there are known bounds and numerical estimates [3]. However, in the context of mobile delay tolerant networks, we are interested to analyze the sub-critical case. We note that the critical threshold obtained from our analysis is smaller than the critical percolation density.

Theorem 1 gives a concise upper bound on the information propagation speed, which we will illustrate in detail in Section V. In order to give a more intuitive understanding of the fundamental performance limits of the information propagation speed, we derive the following corollaries expressing the qualitative behavior of the upper bound when the node density tends to 0. This case models very sparse mobile wireless networks, which as discussed are of special interest in the context of delay tolerant networks.

*Corollary 1:* When nodes move at speed $v > 0$ in a random walk model (with node direction change rate $\tau > 0$), and when the square length $L \to \infty$, but such that the node density $\frac{n}{\mathcal{A}} \to 0$, the propagation speed upper bound is asymptotically equivalent to $O(\sqrt{\frac{n v}{\mathcal{A} \tau}} v)$.

It is important to notice that the speed diminishes with the square root of the density $\nu$.

A special case corresponds to $\tau = 0$, which is a pure billiard model (nodes change direction only when they hit the border).

*Corollary 2:* When nodes move at speed $v > 0$ with $\tau = 0$, and when $L \to \infty$, but with node density $\nu = \frac{n}{\mathcal{A}} \to 0$, the propagation speed upper bound is $(1 + O(\nu^2))v$.

It turns out that the propagation speed upper bound at the limit is $v$. This is rather surprising because we would expect that the propagation speed would tend to zero when $\nu \to 0$.

We note that the above results do not contradict the results of [12], although they can not be directly compared, since a unit disk graph cannot be modeled like an Erdös-Rényi graph. Indeed if nodes $A$ and $B$ are connected to a same third node $C$, then both will be connected with a much higher probability than the probability we would had if they were in an Erdös-Rényi graph. On the other hand, our analysis in fact confirms the results of [9], which imply that the information propagation speed tends to a constant and finite value in intermittently connected networks; our results give the first estimates of this finite information propagation speed.

## III. ANALYSIS (PROOF OF THEOREM 1)

### A. Methodology and Journey Analysis

Our analysis is based on a segmentation of journeys between the source and the destination. Formally, a journey is a space-time trajectory of the beacon between the source and the destination. In the following, we first decompose journeys into segments (*i.e.,* space-time vectors) which model the node trajectories and the beacon transmissions in Section III-B.

Our aim is to decompose journeys into independent segments, therefore a technical difficulty comes from the dependence in the node emissions and movements (for instance, the direction of an emission depends on the direction of the node movement). However, we see how we can use an independent segment decomposition in Section III-C, in order to upper bound the journey probabilities. We then calculate the Laplace transforms of each individual segment, and, making use of the journey decomposition, we deduce the Laplace transform of the probability density of each journey in Section III-D for a fixed length sequence of segments. Finally, an asymptotic analysis on the journey Laplace transform (for large scale networks), based on Poisson generating functions, allows us to compute when the journey probability density tends to zero, and consequently evaluate an upper bound on the information propagation speed in Section III-E.

We assume that time zero is when the source transmits, and we will check at what time $t$ the beacon is emitted at distance smaller than one to the destination at coordinate $\mathbf{z} = (x, y)$. The beacon can take many journeys in parallel, due to the broadcast nature of radio transmissions, and the fact that the beacon stays in the memory of each emitter (and therefore can be emitted several times in the trajectory of a mobile node). In a first approach and in order to simplify, we assume that the destination is fixed; however, we will later see that the destination motion does not affect our results.

We will only consider simple journeys, *i.e.,* journeys which never return twice through the same node. This restriction does not affect the analysis, since if a journey arrives to the destination at time $t$, then we can extract a simple journey from this journey which arrives at time $t$ too.

Let $\mathcal{C}$ be a simple journey. Let $Z(\mathcal{C})$ be the terminal point. Let $T(\mathcal{C})$ be the time at which the journey terminates. Let $P(\mathcal{C})$ be the probability of the journey $\mathcal{C}$. In the following, we consider a journey as a discrete event in a continuous set of all possible journeys in space-time, and we convert the probability weight $P(\mathcal{C})$ to a probability density.

Assuming that there are $n$ nodes in the network, we call $p_n(\mathbf{z}_0, \mathbf{z}_1, t)$ the density of journeys starting from $\mathbf{z}_0$ at time 0, and arriving at $\mathbf{z}_1$ before time $t$:

$$p_n(\mathbf{z}_0, \mathbf{z}_1, t) = \lim_{r \to 0} \frac{1}{\pi r^2} \sum_{|\mathbf{z}_1 - Z(\mathcal{C})| < r, T(\mathcal{C}) < t} P(\mathcal{C}) \ .$$

### B. Journey Segmentation

Let us consider a journey where the beacon is carried by $k+1$ nodes $\ell_0, \ell_1, \ell_2, \ldots, \ell_k$. The node $\ell_0$ is the source. Let $\ell_1$ be the first node that receives the beacon from the source, $\ell_2$ the node that receives the beacon from $\ell_1$, *etc.* We call the $k$-tuple $(\ell_1, \ell_2, \ldots, \ell_k)$ the journey relay sequence $N(\mathcal{C})$.

*Lemma 1:* The probability distribution of the journey $\mathcal{C}$ only depends on the cardinality $|N(\mathcal{C})|$.

*Proof:* Since node motions are i.i.d., any node in $N(\mathcal{C})$ can be interchanged with any other node. ∎

Consequently, we can split the journey $\mathcal{C}$ into segments $(s_0, s_1, s_2, \ldots, s_k)$, where the segments $s_i$ are random space-time vectors, and where $s_i$ is the space-time vector that starts

with the event: "the beacon is received by $\ell_i$". In the special case of $i = 0$, the event is the origin of the journey.

To compute the probability distribution of the segments, we notice that $s_i$ corresponds to a space-time motion trajectory of mobile node $\ell_i$ (the trajectory can be possibly zero if the node immediately retransmits the beacon), and a space-time vector of the beacon transmission via radio (where the time component of a transmission is zero). Therefore, in order to decompose a journey, we define two kinds of segments modeling the described situations:

- *emission segments* $S_e(\mathbf{u}, \mathbf{v})$: the node transmits immediately after receiving the beacon; $\mathbf{v}$ is the speed of the node that just received the beacon, and $\mathbf{u}$ is the emission space vector and is such that $|\mathbf{u}| \le 1$;
- *move-and-emit segments* $S_m(\mathbf{u}, \mathbf{v}, \mathbf{w}) = M(\mathbf{v}, \mathbf{w}) + \mathbf{u}$: $M(\mathbf{v}, \mathbf{w})$ is the space-time vector corresponding to the motion of the node carrying the beacon, where $\mathbf{v}$ is the initial vector speed of the node when it receives the beacon and $\mathbf{w}$ is the final speed of the node just before transmitting the beacon; the vector $\mathbf{u}$ is the emission space vector which ends the segment.

We remark that the vector $\mathbf{u}$ which ends the move-and-emit segments can be restricted to unitary segments.

*Lemma 2:* In a "fastest" journey decomposition (*i.e.,* with respect to an upper bound on the information propagation speed), move-end-emit segments $S(\mathbf{u}, \mathbf{v}, \mathbf{w})$ can be restricted to unitary emission vectors: $|\mathbf{u}| = 1$.

*Proof:* First, assume that $\ell_i$ and $\ell_{i+1}$ are not neighbors when $\ell_i$ receives the beacon. The earliest time at which $\ell_{i+1}$ will receive the beacon from $\ell_i$ is when both become neighbors, *i.e.,* when their distance is just equal to 1; therefore, the emit vector is unitary. Conversely, if $\ell_i$ and $\ell_{i+1}$ are already neighbors when $\ell_i$ receives the beacon, then $\ell_{i+1}$ can receive the packet immediately after $\ell_i$ and the segment would be an emission segment instead. ∎

Since we want to check when a beacon can be emitted at distance less than one from the destination, we do not include the last emission in our journey definition; therefore, the last segment $s_k$ corresponds only to the space-time motion trajectory of node $\ell_k$ (or simply, a motion equal to zero).

### C. Decomposition into Independent Segments

In this section, our aim is to decompose journeys into independent *emission* and *move-and-emit* segments. However, there is a dependence in successive node emissions and movements; for example, a node moving faster meets more nodes than a slower mobile node; similarly, the probability of a meeting between two nodes is in fact proportional to the relative speed between the nodes, hence two nodes that meet are more likely to move in (almost) opposite directions; therefore, the direction of an emission depends on the direction of the node movement. To overcome these difficulties, we will in fact work with an upper bound on the journey probability densities, and we show that this upper bound can be decomposed into independent segments.

Thus, our objective is to compute an upper bound on $P(\mathcal{C})$, the probability density that a journey $\mathcal{C}$ exists. For a fixed journey relay sequence $N(\mathcal{C})$ of size $k$, the probability density is a

vector in $R^{3k}$. Based on the journey decomposition, we have the expression $P(\mathcal{C}) = p(s_0|s_1)p(s_1|s_2)\cdots p(s_{k-1}|s_k)p(s_k)$, where $p(s_i|s_{i+1})$ is the conditional probability density of segment $s_i$, given the next segment $s_{i+1}$. We have the following expressions for the conditional probabilities, for all possible combinations of emission and move-and-emit segments:

- $p(S_e(\mathbf{u}_1, \mathbf{v}_1)|S_e(\mathbf{u}_2, \mathbf{v}_2)) = P(\mathbf{u}_1)P(\mathbf{v}_1)\frac{1}{\mathcal{A}}$;

this is the probability of emission segment $S_e(\mathbf{u}_1, \mathbf{v}_1)$, when we know the next segment (here, an emission segment): $P(\mathbf{u}_1)$ is the probability density of $\mathbf{u}_1$ inside the unit disk (emissions are equiprobable in the unit disk, hence $P(\mathbf{u}_1) = \frac{\partial}{\partial |\mathbf{u}_1|}\frac{\pi|\mathbf{u}_1|^2}{\pi} = 2|\mathbf{u}_1|$), $P(\mathbf{v}_1)$ is the probability that the node moves at speed $\mathbf{v}_1$, and $\frac{1}{\mathcal{A}}$ is the density of presence of a node on the second segment (to make the emission possible); there is no dependence on the parameters $\mathbf{u}_2, \mathbf{v}_2$ of the second segment, since the node receiving the packet re-emits it immediately to one of its neighbors (there is no new meeting);

- $p(S_e(\mathbf{u}_1, \mathbf{v}_1)|S_m(\mathbf{u}_2, \mathbf{v}_2, \mathbf{w}_2)) = P(\mathbf{u}_1)P(\mathbf{v}_1)\frac{1}{\mathcal{A}}$, for the same reason;
- $p(S_m(\mathbf{u}_1, \mathbf{v}_1, \mathbf{w}_1)|S_e(\mathbf{u}_2, \mathbf{v}_2)) = P(M(\mathbf{v}_1, \mathbf{w}_1))P(\mathbf{u}_1)P(\mathbf{u}_2)P(\mathbf{v}_2)\max\{0, \mathbf{u}_1 \cdot (\mathbf{w}_1 - \mathbf{v}_2)\}\frac{1}{\mathcal{A}}$;

this is the probability of the move-and-emit segment $S_m(\mathbf{u}_1, \mathbf{v}_1, \mathbf{w}_1)$, when we know the next segment (here, an emission segment); quantity $\max\{0, \mathbf{u}_1 \cdot (\mathbf{w}_1 - \mathbf{v}_2)\}\frac{1}{\mathcal{A}}$ is the average rate at which a node carrying the beacon on the second segment enters the neighborhood range of the previous node on the radius $\mathbf{u}_1$ with relative speed $\mathbf{v}_2 - \mathbf{w}_1$ (see the estimation of the rate at which two nodes meet in [4]); quantities $P(M(\mathbf{v}_1, \mathbf{w}_1)), P(\mathbf{u}_1), P(\mathbf{u}_2)$ and $P(\mathbf{v}_2)$ correspond to the probabilities of the respective space and speed vectors (we note the dependence on the parameters $\mathbf{u}_2, \mathbf{v}_2$ expressing the probability of the second segment, since the first segment includes a node motion, during which the packet is carried before being transmitted to a new neighbor);

- $p(S_m(\mathbf{u}_1, \mathbf{v}_1, \mathbf{w}_1)|S_m(\mathbf{u}_2, \mathbf{v}_2, \mathbf{w}_2)) = P(M(\mathbf{v}_1, \mathbf{w}_1))P(\mathbf{u}_1)P(\mathbf{u}_2)P(M(\mathbf{v}_2, \mathbf{w}_2))\max\{0, \mathbf{u}_1 \cdot (\mathbf{w}_1 - \mathbf{v}_2)\}\frac{1}{\mathcal{A}}$, for the same reason.

From the above, we notice that a journey cannot be directly decomposed into independent segments, because of the conditional probabilities. However, recall that in order to derive an upper bound on the information propagation speed, it suffices to compute when the probability of a journey becomes zero. Therefore, we can instead use an upper bound on the journey probabilities, and instead check when this upper bound becomes zero. Based on the previous expressions, we can upper bound the conditional probabilities. Hence, an upper bound of the density of $\mathcal{C}$ is $\tilde{P}(\mathcal{C})$, with $\tilde{P}(\mathcal{C}) = \tilde{P}(s_1)\tilde{P}(s_2)\cdots\tilde{P}(s_{k-1})\tilde{P}(s_k)$, and:

- $\tilde{P}(S_e(\mathbf{u}, \mathbf{v})) = P(\mathbf{u})P(\mathbf{v})\frac{1}{\mathcal{A}}$,
- $\tilde{P}(S_m(\mathbf{u}, \mathbf{v}, \mathbf{w})) = P(\mathbf{u})P(M(\mathbf{v}, \mathbf{w}))2\max\{v\}\frac{1}{\mathcal{A}}$,

where $\max\{v\}$ denotes the maximum node speed.

Looking at all the above equations, we observe that $\tilde{P}(s_i) \geq P(s_i|s_{i+1})$ for all $i$ and any combination of segments. Using the new segments probabilities, we have an upper bound journey model that can be decomposed into independent emission and move-and-emit segments.

## D. Journey Laplace Transform

Let $\sigma = (\zeta, \theta)$ be a is an inverse space-time vector: $\zeta$ is a space vector with components expressed in inverse distance units, and $\theta$ is a scalar in inverse time units. We define $\tilde{g}_k(\sigma)$ as the Laplace transform of the upper bound density of a journey $\mathcal{C}$ given that $N(\mathcal{C})$ is fixed of size $k$. In other words, we have by the Laplace transform definition: $\tilde{g}_k(\sigma) = E(\exp(-\sigma \cdot (s_0 + s_1 + \cdots + s_k)))$, under the probability weight $\tilde{P}$. Notice that the exponent is the dot product of two vectors, and that this product is a pure scalar without dimension, since $(\zeta, \theta)$ is an inverse space-time vector.

*Lemma 3:* The Laplace transform of the upper bound journey density given that the relay sequence $N(\mathcal{C})$ is fixed and is of length $k$, satisfies:

$$\tilde{g}_k(\sigma) = \left(2\max\{v\}E(e^{-\sigma \cdot (M(\mathbf{v}, \mathbf{w}) + \mathbf{u})}) + E(e^{-\zeta \cdot \mathbf{u}})\right)^k \left(\frac{1}{\mathcal{A}}\right)^k \times E(e^{-\sigma \cdot M(\mathbf{v}, \mathbf{w})}),$$

where $\max\{v\}$ denotes the maximum node speed.

*Proof:* This is a direct consequence of the independence of segments in the upper bound journey density model, which implies that the journey Laplace transform can be expressed as a product of the individual segment Laplace transforms. The first line expresses the Laplace transform of a sequence of $k$ emission *or* move-and-emit segments, while the last term corresponds to the last segment which corresponds only to a space-time motion trajectory. ∎

Let $\tilde{G}_n(\sigma)$ be the Laplace transform of the upper bound density of all journeys in a network of size $n$ in a square map of area size $\mathcal{A}$. Now, the remaining difficulty comes from the fact that $N(\mathcal{C})$ is not known or fixed. To tackle this problem, we define the Poisson generating function:

$$\tilde{G}(Z, \sigma) = \sum_{n \geq 0} \tilde{G}_n(\sigma)\frac{Z^n}{n!}e^{-Z}.$$

*Lemma 4:* The following identity holds:

$$\tilde{G}(Z, \sigma) = \sum_{k \geq 0} \tilde{g}_k(\sigma)Z^k .$$

*Proof:* This is a formal identity. Quantity $\tilde{g}_k(\sigma)$ depends only on the actual length of the relay sequence and not on the nodes that are actually in the relay sequence (from Lemma 1), thus the Laplace transform of the journeys that are made of $k$ segments is $\frac{n!}{(n-k)!}\tilde{g}_k(\sigma)$, since $\frac{n!}{(n-k)!}$ is the number of distinct relay sequences of size $k$.

This means that $[Z^n]\, \tilde{G}(Z, \sigma) = \sum_{k \geq n} \frac{n!}{(n-k)!}\tilde{g}_k(\sigma)$ , and

$$\tilde{G}(Z, \sigma)e^Z = \sum_{n \geq 0, k \geq n} \frac{n!}{(n-k)!}\tilde{g}_k(\sigma)\frac{Z^n}{n!}$$
$$= \sum_{k \geq 0} \tilde{g}_k(\sigma)e^Z.$$

∎

*Corollary 3:* We have (from Lemmas 3 and 4):

$$\tilde{G}(Z, \sigma) = \frac{E(e^{-\sigma \cdot M(\mathbf{v}, \mathbf{w})})}{1 - (2\max\{v\}E(e^{-\sigma \cdot (M(\mathbf{v}, \mathbf{w}) + \mathbf{u})}) + E(e^{-\zeta \cdot \mathbf{u}}))\frac{Z}{\mathcal{A}}}.$$

Therefore, we can now evaluate the Poisson generating function $\tilde{G}(Z, \sigma)$ of the journey Laplace transform by combining the segments Laplace transforms. In the following lemma, we evaluate the expressions for the Laplace transforms.

*Lemma 5:* We have:

- $E(e^{-\zeta \cdot \mathbf{u}}) = 2\pi \frac{2}{|\zeta|} I_1(|\zeta|)$ when $\mathbf{u}$ is unitary and uniform on the unit circle;
- $E(e^{-\zeta \cdot \mathbf{u}}) = \pi I_0(|\zeta|)$ when $\mathbf{u}$ is uniform in the unit disk;
- When all speeds are of modulus equal to $v$, we have $E(e^{-\sigma \cdot M(\mathbf{v},\mathbf{w})}) = \frac{1}{\sqrt{(\theta+\tau)^2 - |\zeta|^2 v^2} - \tau}$;

where $I_1()$ and $I_0()$ are modified Bessel functions.

*Proof:* See appendix. ∎

### E. Information Propagation Speed Analysis

Our aim is to obtain an estimate of $p_n(\mathbf{z}_0, \mathbf{z}_1, t)$, *i.e.,* the upper bound on the density of journeys that start at $\mathbf{z}_0$ at time 0 and end at $\mathbf{z}_1$ at time $t$. Let $p(Z, \mathbf{z}_0, \mathbf{z}_1, t)$ be the Poisson generating function of $p_n(\mathbf{z}_0, \mathbf{z}_1, t)$, that is: $p(Z, \mathbf{z}_0, \mathbf{z}_1, t, z) = \sum_n p_n(\mathbf{z}_0, \mathbf{z}_1, t) \frac{Z^n}{n!} e^{-Z}$.

*Lemma 6:* The generating function $p(Z, \mathbf{z}_0, \mathbf{z}_1, t)$ has positive coefficients.

*Proof:* From Lemma 4:

$$p(Z, \mathbf{z}_0, \mathbf{z}_1, t) = \sum_{k \geq 0} p_k(\mathbf{z}_0, \mathbf{z}_1, t) Z^k.$$

Hence, we can use the following depoissonization Lemma.

*Lemma 7:* When $n \to \infty$:

$$p_n(\mathbf{z}_0, \mathbf{z}_1, t) \leq p(n, \mathbf{z}_0, \mathbf{z}_1, t)(1 + o(1)).$$

*Proof:* See appendix. ∎

*1) Space-time Asymptotic Analysis:* We now evaluate the asymptotic behavior of the journey density $p_\nu(\mathbf{z}_0, \mathbf{z}_1, t)$. With a slight change of notation, we have substituted the node density instead of the number of nodes in the network, since in fact we are interested in the limit where $n$ tends to infinity, while $\nu$ remains constant. From Lemma 7, we see that we can equivalently evaluate the asymptotic behavior of the Poisson generating function coefficient $p(\nu, \mathbf{z}_0, \mathbf{z}_1, t)$ (where the number of nodes $n$ tends to infinity).

As we can observe by substituting the expressions of Lemma 5 in Corollary 3, the asymptotic coefficient of the Poisson generating function $\tilde{G}(\nu, \sigma)$, with $\sigma = (\zeta, \theta)$ a space-time vector, has a denominator $D(|\zeta|, \theta)$, such that (with $\rho = |\zeta|$):

$$D(\rho, \theta) = (1 - \frac{n}{\mathcal{A}} \pi \frac{2}{\rho} I_1(\rho)) \left( \sqrt{(\tau + \theta)^2 - \rho^2 v^2} - \tau \right) - \frac{n}{\mathcal{A}} 4\pi v I_0(\rho).$$

The key of the analysis is the set $\mathcal{K}$ of pairs $(\rho, \theta)$ such that $D(\rho, \theta) = 0$, called the *Kernel*. In fact any element of the Kernel (*i.e.,* a singularity of the Laplace transorm) can be used to obtain an asymptotic estimate of the journey probability density. We denote $(\rho_0, \theta_0)$ the element of the Kernel that attains the minimum value $\frac{\theta}{\rho}$. Notice that $(\rho_0, \theta_0)$ is a function of $\nu = \frac{n}{L^2}$. We prove the following lemma.

*Lemma 8:* Let $\nu$ be fixed and $\theta_1 > \theta_0$. There exists an $A_1$ such that, when $|\mathbf{z}| = |\mathbf{z}_1 - \mathbf{z}_0|$ and $t$ both tend to infinity:

$$p(\nu, \mathbf{z}_0, \mathbf{z}_1, t) \leq A_1 \exp(-\rho_0 |\mathbf{z}| + \theta_1 t) .$$

*Proof:* See appendix. ∎

*2) Information Propagation Speed:* Let $\mathbf{z}_0$ and $\mathbf{z}_1$ be fixed. Let $q_\nu(\mathbf{z}_0, \mathbf{z}_1, t)$ be the probability that there exists a journey that arrives at distance less than 1 to a destination node at $\mathbf{z}_1$ before time $t$.

*Lemma 9:* We have the upper bound:

$$q_\nu(\mathbf{z}_0, \mathbf{z}_1, t) \leq \int_{|\mathbf{z}_1 - \mathbf{z}'| < 1} p(\nu, \mathbf{z}', \mathbf{z}_1, t) d\mathbf{z}' .$$

*Proof:* By the definition of $q_\nu(\mathbf{z}_0, \mathbf{z}_1, t)$. ∎

Therefore, from Lemmas 7, 8 and 9, when $L \to \infty$ with $\mathbf{z}_0$, $\mathbf{z}_1$ fixed and $\mathbf{z} = \mathbf{z}_1 - \mathbf{z}_0$, we have the estimate $q_\nu(\mathbf{z}_0, \mathbf{z}_1, t) = O(\exp(\theta_1 t - |\mathbf{z}| \rho_0))$ for all $\theta_1 > \theta_0$.

Clearly, $q_\nu(\mathbf{z}_0, \mathbf{z}_1, t)$ vanishes very quickly when $t$ is smaller than the value such that $-\rho_0 |\mathbf{z}| + \theta_0 t = 0$, *i.e.,* when $\frac{\theta_0}{\rho_0} = \frac{|\mathbf{z}|}{t}$. This ratio gives the upper bound for the propagation speed. In other words, point $(\rho_0, \theta_0)$ achieves the lowest ratio $\frac{\theta}{\rho}$ in the kernel set $\mathcal{K}$. By expressing the kernel set $\mathcal{K}$ using the function $D(\rho, \theta)$ from the previous section, we obtain Theorem 1.

*Remark:* We note that this result corresponds to the situation where all nodes speeds are of modulus $v$ (as assumed in Lemma 5). Even if the speeds follow a different distribution, our analysis still applies, with the only change occurring in the Laplace transform of the motion vectors (but then the final form of Theorem 1 would be different). However, for an upper bound on the propagation speed, it suffices to consider $v$ as the maximum node speed.

To formally complete the proof, we need to address two remaining details: the contribution of the mirror images of the nodes (*i.e.,* to account for the nodes bouncing on the borders) and the destination's motion. We note here that all node mirror images induce a contribution factor of order $\exp(\theta_1 t - |\mathbf{z}| \rho_0 - x)$, where $x$ is the distance of the node from the border of the square network domain (see appendix); for almost all nodes, $x$ is of the order of $L$, *i.e.,* the edge length of the square, which tends to infinity; therefore the contribution of the mirror images is negligible, since it decays exponentially in $\exp(-L)$. For the destination's motion, it suffices to multiply the journey Laplace transform with the Laplace transform of the destination node excursion from its original position, to compute an upper bound on the propagation speed. Similarly, the destination's motion also induces a negligible factor (we omit the details of the proof due to space constraints; the proof follows Section IV-D in [6]).

## IV. SPARSE NETWORKS

### A. The Random Walk Model

*Corollary 1:* When nodes move at speed $v > 0$ in a random walk model (with node direction change rate $\tau > 0$), and when the square length $L \to \infty$ but such that the node density $\frac{n}{\mathcal{A}} \to 0$, the propagation speed upper bound is asymptotically equivalent to $O(\sqrt{\frac{nv}{\mathcal{A}\tau}} v)$.

*Proof:* Let $(\rho, \theta)$ be an element of the set $\mathcal{K}$. We have $\theta = \sqrt{(\tau + \nu H(\rho))^2 + \rho^2 v^2} - \tau$, with $H(\rho) = \frac{4\pi v I_0(\rho)}{1 - \frac{n}{\mathcal{A}} \pi \frac{2}{\rho} I_1(\rho)}$. We have: $H(\rho) = \frac{4\pi v}{1 - \pi \nu} + O(\rho^2)$, where $\nu = \frac{n}{\mathcal{A}}$. Therefore,

$$\theta = \sqrt{\tau^2 + \rho^2 v^2} - \tau + \frac{\tau}{\sqrt{\tau^2 + \rho^2 v^2}} H(\rho)\nu + O(\nu^2).$$

We obtain the ratio:

$$
\begin{aligned}
\frac{\theta}{\rho} &= \frac{H(\rho)\nu}{\rho}\frac{\tau}{\tau^2+\rho^2 v^2} + \frac{\sqrt{\tau^2+\rho^2 v^2}-\tau}{\rho} + O(\frac{\nu^2}{\rho}) \\
&= \frac{H(0)\nu}{\rho} + \frac{\rho v^2}{2\tau} + O(\frac{\nu^2}{\rho}+\nu\rho^2).
\end{aligned}
$$

Quantity $\frac{H(0)\nu}{\rho} + \frac{\rho v^2}{2\tau}$ is minimized with value $v\sqrt{\frac{2\nu H(0)}{\tau}}$ attained at $\rho = \frac{\sqrt{2\nu\tau H(0)}}{v}$. Therefore $\frac{\theta}{\rho}$ is minimized at value $v\sqrt{\frac{2\nu H(0)}{\tau}} + O(\nu^{3/2})$. ∎

### B. The Billiard Random Way-point Limit

The billiard limit is equivalent to setting $\tau = 0$.

*Corollary 2:* When nodes move at speed $v > 0$ with $\tau = 0$, and when $L \to \infty$ but with node density $\nu = \frac{n}{\mathcal{A}} \to 0$, the propagation speed upper bound is $(1 + O(\nu^2))v$.

*Proof:* Now, the kernel set $\mathcal{K}$ consists of the points $(\rho, \theta(\rho))$ where $\theta(\rho) = v\sqrt{\rho^2 + H_1(\rho)^2}$ with $H_1(\rho) = \frac{\frac{n}{\mathcal{A}}4\pi I_0(\rho)}{1-\frac{n}{\mathcal{A}}\pi\frac{2}{\rho}I_1(\rho)}$. In this case, the upper bound speed is proportional to $v$ with a factor of proportionality equal to $\sqrt{1 + \left(\frac{H_1(\rho_0)}{\rho_0}\right)^2}$ where $\rho_0$ minimizes $\frac{H_1(\rho)}{\rho}$. Since $\frac{H_1(\rho_0)}{\rho_0} = \frac{n}{\mathcal{A}}4\pi\min\{\frac{I_0(\rho)}{\rho}\} + O(\nu^2)$, we get the estimate $(1 + O(\nu^2))v$, proving Corollary 2. ∎

These corollaries are useful in order to see more intuitively the behavior of the upper bound of Theorem 1, when the node density is small, and consequently to better understand the fundamental performance limits of DTNs. Indeed, the case of sparse networks deserves special attention because of the potential applications and the necessity to use a delay tolerant architecture. For instance, in the random walk model, it is important to notice that the information propagation speed diminishes with the square root of the node density $\nu$. Furthermore, it is inversely proportional to the square root of the change of direction rate of the nodes (changing direction more frequently implies a smaller information propagation speed). In fact, the term in the square root in Corollary 1 is proportional to the expected number of neighbors that a node meets during a random step. Conversely, in the random way-point model, we notice that, surprisingly, the information propagation speed does not tend to 0 with the node density. In this case, the upper bound corresponds to the actual maximum speed of the mobile nodes (for instance, halving the node speed implies halving the information propagation speed).

### V. Slowness of Information Propagation Plots

To illustrate the behavior of the upper bound for the information propagation speed when the mobile density $\nu$ varies, we define the *slowness*, i.e., the inverse of the information propagation speed, for which our theoretical study now provides lower bounds. Plotting results (obtained by numerical resolution of Theorem 1) of our lower bounds are presented in Figure 1, where we consider a unit node speed: $v = 1ms^{-1}$.

We remark that the slowness drops to 0 at $\nu = 1/\pi$, which corresponds to the limit of our model. Recall, that this is a lower bound of the slowness (equivalent to the upper bound for the propagation speed). The actual slowness should continue
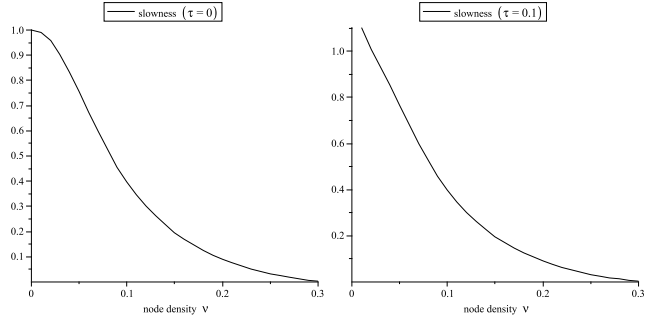


Fig. 1. Theoretical lower bound of slowness versus mobile node density $\nu$ when $\tau = 0$ (left) and $\tau = 0.1$ (right).

to be non-zero beyond $\nu = 1/\pi$. For the billiard - random way point limit (i.e., $\tau = 0$), we notice that the slowness is in $1 - O(\nu^2)$, confirming Corollary 2. For the random walk, we notice that the slowness is unbounded when $\nu \to 0$, confirming the $O(\frac{1}{\sqrt{\nu}})$ theoretical behavior proved in Corollary 1.

### VI. Simulations

In this section, we evaluate the accuracy of our theoretical upper bound in different scenarios by comparing it to the average information propagation time obtained by simulating a full epidemic broadcast in a two-dimensional network (as described in Section I). For all the simulations, we use a unit-disk graph model (i.e., a radio range of $1m$), and the mobile node speed is $1ms^{-1}$. Two commonly used mobility models are simulated: the random way-point model (which corresponds to our setting $\tau = 0$ as described in Subsection IV-B) and the random walk model (which corresponds to our setting $\tau = 0.1$). We study the two mobility models (Figure 2 for billiard random way-point mobility, and Figure 3 for random walk mobility) for different node density $\nu$ and area values ($\nu = 0.025$ on a $80 \times 80$ square, $\nu = 0.05$ on a $60 \times 60$ square, $\nu = 0.1$ on a $40 \times 40$ square, respectively).

In Figures 2 and 3, we depict the simulated propagation time versus the distance (plots), and we compare it to the theoretical bound, i.e., a line of fixed slope (in green - bottom). The slope is obtained from the analysis in Section III; it represents the slowness illustrated for corresponding density values in Figure 1. In the figures, time is measured in seconds, and distance in meters, therefore, the inverse slope of the plots provides us with the information propagation speed in $ms^{-1}$. What is important is the comparison of the two slopes at infinity. We notice that the measurements very quickly converge to a straight line of fixed slope, which implies a fixed information propagation speed.

Simulations show that the theoretical slope is clearly a lower bound on the slowness, as proved in Theorem 1. We also compare the simulation measurements with a second line of fixed slope (red - top). This line is provided only for comparison and corresponds to the heuristic situation where we assume that node movements and emissions are completely independent (according to the framework of [6] in an infinite network). Interestingly enough, the simulations show that the heuristic bound provides an accurate slope (the theoretical
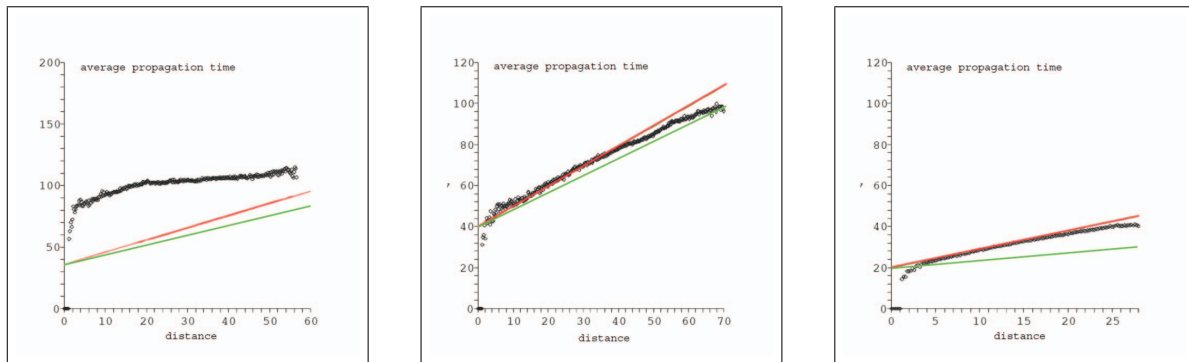
250

Fig. 2. Average propagation time (in seconds) versus distance to source (in meters), compared with theoretical (green - bottom) and heuristic (red - top) slope, for $\tau = 0$ and: $\nu = 0.025$ simulated in a $80 \times 80$ square (left); $\nu = 0.05$ in a $60 \times 60$ square (middle); $\nu = 0.1$ in a $40 \times 40$ square (right).
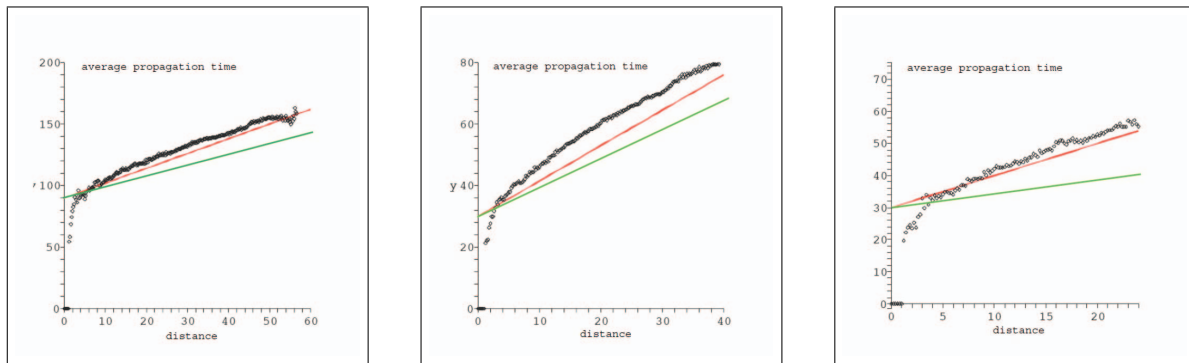


Fig. 3. Average propagation time (in seconds) versus distance to source (in meters), compared with theoretical (green - bottom) and heuristic (red - top) slope, for $\tau = 0.1$ and: $\nu = 0.025$ simulated in a $80 \times 80$ square (left); $\nu = 0.05$ in a $60 \times 60$ square (middle); $\nu = 0.1$ in a $40 \times 40$ square (right).

slope we provide is smaller, since in order to prove a rigorous bound on the information propagation speed, we work with an upper bound on the journey probability density for the journey decomposition in Section III-C).

## VII. CONCLUSION

In this paper, we have initiated a characterization of the information propagation speed of Delay Tolerant mobile Networks (DTNs) by providing a theoretical upper bound for large scale but finite two-dimensional networks (Theorem 1). Such theoretical bounds are useful in order to increase our understanding of the fundamental properties and performance limits of DTNs, as well as to evaluate and/or optimize the performance of specific routing algorithms. The model used in our analytical study is sufficiently general to encapsulate many popular mobility models (random way-point, random walk, Brownian motion). We also performed simulations for several scenarios to show the validity of our bounds.

Our methodology and space-time journey analysis provide a general framework for the derivation of analytical bounds on the information propagation speed in DTNs. Therefore, future investigations should consider extending the analysis to other neighboring models different from unit disk graphs (e.g., quasi-disk graphs, probabilistic models), proving tighter bounds (e.g., similar to the heuristic bound we discussed in the simulations section), and generalizing to other mobility

models or comparing the results with real traces. Another interesting direction for further research would be to compare the implications of our analysis on the delay of common routing schemes, such as epidemic routing, with the results presented in previous work on DTN modeling [4], [18], under the frequently used assumption that the inter-meeting time between pairs of nodes follows an exponential distribution.

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## APPENDIX

*Proof of Lemma 5 (Laplace Transform Expressions)*

The expression $E(e^{-\zeta \cdot \mathbf{u}})$, when $\mathbf{u}$ is uniform on the unit circle, is equal to $\frac{1}{2\pi} \int_0^{2\pi} e^{|\zeta| \cos \phi} d\phi = I_0(|\zeta|)$.

The expression $E(e^{-\zeta \cdot \mathbf{u}})$ when $\mathbf{u}$ is uniform on the unit disk with density $\nu$ is equal to $\nu \int_0^{2\pi} d\phi \int_0^1 e^{-r|\zeta| \cos \phi} 2\pi r dr$. In Taylor series we have $\nu \pi \sum_k (\frac{|\zeta|}{2})^{2k} \frac{1}{(k+1)!k!}$, which in turn is equal to $\nu \pi \frac{2}{|\zeta|} I_1(|\zeta|)$.

The expression $E(e^{-\sigma \cdot M(\mathbf{v}, \mathbf{w})})$ equals $\frac{1}{1 - \tau f(\zeta, \theta)}$, where $f(\zeta, \theta) = \int_0^\infty e^{|\zeta| v t - \theta t} e^{-\tau t} dt$ (see [6]).

*Proof of Lemma 7 (Depoissonization)*

By Cauchy integration (*cf.* [8]):

$$p_n(\mathbf{z}_0, \mathbf{z}_1, t) = \frac{n!}{2i\pi} \oint p(Z, \mathbf{z}_0, \mathbf{z}_1, t) \frac{e^Z}{Z^{n+1}} dZ,$$

where the integration loop encircles the origin on the complex plane. We take the circle of center 0 and radius $n$:

$$p_n(\mathbf{z}_0, \mathbf{z}_1, t) = \frac{n!}{2\pi} \int_0^{2\pi} p(ne^{i\phi}, \mathbf{z}_0, \mathbf{z}_1, t) \frac{\exp(ne^{i\phi})}{e^{in\phi}} d\phi.$$

Therefore,

$$
\begin{aligned}
p_n(\mathbf{z}_0, \mathbf{z}_1, t) &\leq p(n, \mathbf{z}_0, \mathbf{z}_1, t) \frac{n!}{2\pi n^n} \int_0^{2\pi} e^{n \cos \phi} d\phi \\
&= p(n, \mathbf{z}_0, \mathbf{z}_1, t) \frac{n!}{n^n} I_0(n).
\end{aligned}
$$

Using Stirling and Bessel asymptotics, $n! = \sqrt{2\pi n} n^n e^{-n}(1 + o(1))$ and $I(n) = \frac{1}{\sqrt{2\pi n}} e^n (1 + o(1))$, we complete the proof.

*Proof of Lemma 8 (Asymptotic Analysis)*

We prove this lemma for a random mobility model with speed of constant modulus, but generalization is straightforward. In this case, with $\sigma = (\zeta, \theta)$ a space-time vector: $\tilde{G}(\nu, \sigma) = \frac{1}{D(|\zeta|, \theta)}$, with $D(|\zeta|, \theta) = (1 - \nu \pi \frac{2}{|\zeta|} I_1(|\zeta|)) \left( \sqrt{(\theta + \tau)^2 - |\zeta|^2 v^2} - \tau - 2\pi v \nu I_0(|\zeta|) \right)$.

Notice that $\frac{1}{D(|\zeta|, \theta)}$ is in fact an analytic function of $|\zeta|^2$ and of $\theta$ with non negative coefficients. One must be aware that the quantity $|\zeta|^2$ refers to the sum of the square of the coefficients of $\zeta$ and not to the sum of the square of their modulus, and therefore induces an analytical function. Thus,

the definition domain of $\tilde{G}(\nu, \sigma)$ contains all tuples $\sigma = (\zeta, \theta)$ such that $(|\zeta|, \theta)$ belongs to the definition domain of $\frac{1}{D(|\zeta|, \theta)}$.

In particular, if there exists a real tuple $(\rho_1, \theta_1)$ in the Kernel $\mathcal{K}$, then the tuples $(\rho, \theta)$ such that $\Re(\theta) > \theta_1$ and $\Re(\rho) \leq \rho_1$ belong to the definition domain of $\frac{1}{D(\rho, \theta)}$.

Due to the asymptotic [1] on modified Bessel functions: $I_\alpha(\rho) \sim \sqrt{\frac{2i}{\pi \rho}} \cos \left( \frac{\rho}{i} - \frac{\alpha \pi}{2} - \frac{\pi}{4} \right)$, we can enlarge the definition domain of $\frac{1}{D(\rho, \theta)}$ to $Re(\rho) \leq \rho_1 + \frac{1}{2} \log |\Im(\rho)|$, when $\Im(\rho)$ is large.

Since the Laplace transform of space-time density $p(\nu, \mathbf{z}, t)$ is $\frac{\tilde{G}(\nu, \sigma)}{\theta}$ (the denominator $\theta$ comes from the fact that we cumulate all journeys arriving at $\mathbf{z}$ within $t$), $p(\nu, \mathbf{z}, t)$ can be expressed via the inverse Laplace transform:

$$p(\nu, \mathbf{z}, t) = (\frac{1}{2i\pi})^3 \int_{\Re(\theta) = \theta_1} \int_{\Re(\zeta) = \zeta_1} \tilde{G}(\nu, \sigma) e^{\zeta \cdot \mathbf{z} + \theta t} d\zeta \frac{d\theta}{\theta},$$

where $(\zeta_1, \theta_1)$ is any real element of the definition domain.

For our purpose, we will directly take $\theta_1 > \theta_0$ and $\zeta_1 = -\frac{\rho_0}{|\mathbf{z}|} \mathbf{z}$. Notice that the integration domain in $\zeta$ is an imaginary plane and in $\theta$ an imaginary axis (*i.e.,* integration is 3-dimensional); this explains the cubic factor $\frac{1}{(2i\pi)^3}$.

Without loss of generality, we assume that $\mathbf{z}$ is co-linear with the first axis: *i.e.,* $\mathbf{z} = (x, y)$ and $y = 0$. We denote $\zeta = (\zeta_x, \zeta_y)$. The integral in $\theta$ absolutely converges because it is in $\frac{1}{\theta^2}$, but we cannot conclude the same for the absolute convergence of $\tilde{G}(\zeta, \rho)$ which converges only in $\frac{1}{|\zeta|}$ when $\Im(\zeta)$ tends to infinity. To this end, we move the integration surface of $\zeta$ in a suitable way that will lead to an absolute convergence. This move is made possible by the multi-dimensional analytical nature of the functions.

We first define a function $\rho(a) = \rho_0 + ia$ for $|a| < B$ for some $B > 0$ and $\rho(a) = \rho_0 + ia + \frac{1}{2} \log(|a|)$ for $|a| > B$, such that $(\rho(a), \theta_1)$ always belong to the definition domain of $\frac{1}{D(\rho, \theta)}$. Second, we define the new surface of integration for $\zeta$ as the the union of Minkowski hyperbolic sections defined by $M(a) = \{\zeta, |\zeta| = \rho(a), \exists b > 0 : \Re(\zeta) = b\zeta_1\}$. In other words $\Re(\zeta_y) = 0$. From the identity $\zeta_x^2 = \rho(a)^2 + (\Im(\zeta_y))^2$, we get $|\Re(\zeta)| \geq \max\{\Re(\rho(a)), |\Im(\zeta)|\}$. Therefore,

$$
\begin{aligned}
| \int_{M(a)} \exp(-\mathbf{z} \cdot \zeta) d\zeta | &\leq \int_{M(a)} \exp(-|\mathbf{z}| \Re(\zeta)) |d\zeta| \\
&\leq \frac{\exp(-\Re(\rho(a)) |\mathbf{z}|)}{|\mathbf{z}|}.
\end{aligned}
$$

Since $\exp(-\Re(\rho(a)) |\mathbf{z}|) = e^{-\rho_0 |\mathbf{z}|} (a^{-|\mathbf{z}|})$, for $|a| > B$, the integral converges absolutely in $a$ when we add the contribution of all Minkowski sections, as soon as $|\mathbf{z}| > 2$.

*Contribution of Mirror Images*

Let us denote $p(\nu, \mathbf{z}, t)$ as the density distribution of the journeys that connect two points $\mathbf{z}_0$ and $\mathbf{z}_1$ in the original square which delimits the network domain, such that $\mathbf{z}_1 - \mathbf{z}_0 = \mathbf{z}$, with the journey starting at time 0 and arriving at the destination before $t$. Notice that the journey does not connect to the mirror images of $\mathbf{z}_1$. To account for the mirror images, when $\mathbf{z}_1 = (x_1, y_1)$, we need to add the three closest images at: $\mathbf{z}_1 + 2(L - x_1)e_x$, $\mathbf{z}_1 + 2(L - y_1)e_y$ and $\mathbf{z}_1 + 2(L - x_1)e_x + 2(L - y_1)e_y$ with $e_x = (1, 0)$ and $e_y = (0, 1)$. Adding all possible periodic mirror images, we get the identity:

$$
\begin{aligned}
p(\nu, \mathbf{z}_0, \mathbf{z}_1, t) &= p(\nu, \mathbf{z}, t) \\
&+ \sum_{j,k} p(\nu, \mathbf{z} + 2jLe_x + 2kLe_y, t) \\
&+ p(\nu, \mathbf{z} + 2(L - x_1)e_x + 2jLe_x + 2kLe_y, t) \\
&+ p(\nu, \mathbf{z} + 2(L - y_1)e_y + 2jLe_x + 2kLe_y, t) \\
&+ p(\nu, \mathbf{z} + 2(L - x_1)e_x + 2(L - y_1)e_y + 2jLe_x + 2kLe_y, t).
\end{aligned}
$$