Advanced Location-Tracking Systems in Home, Automotive and Public Transportation Environments

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***Abstract*—In this paper, a novel distance based source-localization algorithm is proposed for location-tracking applications in home, automotive and public transportation environments. Based on the least squares (LS) formulation of the source-localization problem, we develop an optimization algorithm that proves mostly unaffected by the number of local minima of the LS-objective function. The algorithm, referred to as range global distance continuation (R-GDC), consists of an iterative procedure in which, at each k-th iteration, the original objective is smoothed by a Gaussian kernel (smoothing function) and minimized by a steepest-descent (SD) method. The results reveal that the R-GDC algorithm, not only outperforms the alternative methods, but also achieves a localization error that is close to theoretical position error bound (PEB).**

I. INTRODUCTION

Location-tracking (LT) is a new exploitable feature to use in the development of novel wireless applications. For instance, in a audio-system, the loudspeaker locations can be exploited to create immersive surround sound effects. In public transportation, heterogeneous networks and automotive environment, the location information of wireless devices can be used to develop enhanced location-based services such as identification, entertainments, etc. [1]. However, the utilization of these applications in real-life is still prevented by the lack of accurate and low-complexity localization techniques.

Typically, localization algorithms are based on a multivariate least-squares (LS) optimization, where the objective(cost)-function is designed to minimize the mean square error computed between the estimate and the measured distances for all pair of nodes in the network [2]. Although current technologies, such as the impulse radio ultra wideband (IR-UWB), can provide accurate distance measurements [3]–[5], the unreliability of the node location estimates is due to LS-objective. Indeed, such a function is known to be non convex and its local minima depend on both the amount of error in the measurements and the network geometry [6] [7].

In the literature, several algorithm have been proposed to solve this problem. Amongst all the alternatives, the most effective are those based on semidefinite programming (SDP) [8], [9], majorization (SMACOF) [6] and global distance continuation (GDC) [10] techniques. In some cases, however, such algorithms are still prohibitive from a computational complexity perspective and they do not provide sufficiently accurate performance.

In this paper, a novel approach based on GDC technique [11] is proposed and proved very robust to the local minima problem. The algorithm, referred to as range-GDC (R-GDC) exploits the low-complexity of a steepest-descent (SD) optimization method to iteratively minimize a smoothed LS-objective function. The simulation results show that the localization error achieved with the R-GDC algorithm closely approaches the position error bound (PEB) derived in [12].

The remainder of the article is as follows. In section II the localization problem is put into a mathematical context, in section III the recipe of the R-GDC algorithm is given. In section IV, numerical results show that the R-GDC algorithm is very effective for LT-applications in home, automotive and public transportation environment. In section V some final conclusions are drawn.

II. PROBLEM STATEMENT

Consider a wireless network of N_{A} anchors and a target deployed in the η-dimensional space. An anchor is a node whose position is known *a priori*, while a target is a node whose location is to be determined. Let $\mathbf{a}_i \in \mathbb{R}^\eta$ and $\mathbf{x} \in \mathbb{R}^\eta$ be row-vectors whose elements are the coordinates of the i-th anchor's and the target's location, respectively.

The Euclidean distance between the i-th anchor and the target, denoted by d_i, is given by $d_i = \|\mathbf{a}_i - \mathbf{x}\|_{\mathrm{F}}$, where $\| \, \|_{\mathrm{F}}$ is the Frobenius norm. A sample of the distance d_i, denoted by $\tilde{d}_i$, is affected by an error ϵ_i as in the following model

$$\tilde{d}_i = d_i + \epsilon_i. \tag{1}$$

The distance based least-square (LS) formulation of the source-localization problem is

$$\hat{\mathbf{x}} = \arg \min_{\hat{\mathbf{x}} \in \mathbb{R}^\eta} \sum_{i=1}^{N_{\mathrm{A}}} \left(\tilde{d}_i - \hat{d}_i \right)^2, \tag{2}$$

where $\hat{d}_i \triangleq \|\mathbf{a}_i - \hat{\mathbf{x}}\|_{\mathrm{F}}$ and $\hat{\mathbf{x}}$ is the target location estimate.

For the sake of convenience, $s(\hat{\mathbf{x}})$ will refer to the cost-function in equation (2).

As previously mentioned, the main challenge of the LS-based source-localization problem is the minimization of $s(\hat{\mathbf{x}})$, since such cost-function presents several local minima. In the sequel, we provide the recipe of a novel optimization algorithm based on GDC technique, and we refer to [11] for further details and mathematical derivations of the results.

III. Range-GDC Source Localization Algorithm

The GDC-based optimization technique consists of an iterative procedure in which, at each k-th iteration, the original objective is smoothed by a kernel (smoothing function) prior to its minimization. The kernel is parameterized by a smoothing parameter λ, adjusted at each iteration, such that the sequence of successive $\lambda^{(k)}$ is strictly monotonically decreasing to zero ($\lambda^{(K)} = 0$) and, at $\lambda^{(1)}$, the smoothed objective is convex.

The smoothing function is given by the Gaussian kernel [10]

$$g(u, \lambda) \triangleq e^{-u^2/\lambda^2}. \tag{3}$$

The smoothed objective, denoted by $\langle s\rangle_\lambda(\hat{\mathbf{x}})$, is

$$\langle s\rangle_\lambda(\hat{\mathbf{x}}) \triangleq \frac{1}{\pi^{\eta/2}\lambda^\eta}\int_{\mathbb{R}^\eta} s(\mathbf{u}) e^{-\dfrac{\|\mathbf{u}-\hat{\mathbf{x}}\|_2^2}{\lambda^2}} d\mathbf{u}, \tag{4}$$

where $s(\mathbf{u})$ is the objective function in equation (2), $\mathbf{u} \in \mathbb{R}^\eta$ and $\lambda \in \mathbb{R}^+$ is a parameter that controls the degree of smoothing ($\lambda \gg 0$ strong smoothing).

In [11] it is shown that equation (4) becomes

$$\langle s\rangle_\lambda(\hat{\mathbf{x}}) = \frac{1}{\pi}\int_{\mathbb{R}^\eta} \sum_{i=1}^{N_A} w_i \left(\tilde{d}_i - \|\mathbf{a}_i - \hat{\mathbf{x}} + \lambda\mathbf{u}\|_2\right)^2 e^{-\|\mathbf{u}\|_2^2} d\mathbf{u}. \tag{5}$$

The benefits of the R-GDC algorithm are manifolds. First, it provides an estimate of the global minimum of $s(\hat{\mathbf{x}})$ with statistical consistency. Second, it is a low-complexity algorithm since $\langle s\rangle_\lambda(\hat{\mathbf{x}})$ is minimized with any gradient-based method such as the SD technique. Third, the number of iterations can be kept relatively small without compromising the effectiveness of the method.

The R-GDC algorithm can be efficiently implemented using the following results.

A. Evaluation of the smoothed objective

With $\eta = 2$, equation (5) reduces to

$$\langle s\rangle_\lambda(\hat{\mathbf{x}}) = \sum_{i=1}^{N_A} w_i \left(\lambda^2 - 2\lambda\tilde{d}_i\Gamma\left(\frac{3}{2}\right) {}_1F_1\left(\frac{3}{2};1;\frac{\hat{d}_i^2}{\lambda^2}\right) e^{\frac{-\hat{d}_i^2}{\lambda^2}} + \tilde{d}_i^2 + \hat{d}_i^2\right), \tag{6}$$

where $\Gamma(a)$ is the gamma function and ${}_1F_1(a; b; c)$ is the confluent hypergeometric function [13].

The numeric evaluation of equation (6) requires care, especially when λ is very small. Numerically stable computations can be achieved using known equivalences for the confluent hypergeometric function [13]. In particular,

$${}_1F_1\left(\frac{3}{2}; 1; z\right) = 1 + \sum_{m=1}^{+\infty}\left(z^m \prod_{k=1}^{m} \frac{(1/2 + k)}{k^2}\right),\ z < 10, \tag{7}$$

$${}_1F_1\left(\frac{3}{2}; 1; z\right) \approx \frac{z^{-3/2}}{\Gamma(-\frac{1}{2})}\left(\sum_{m=0}^{M-1} \frac{(-z)^{-m}}{m!} \prod_{k=0}^{m-1}\left(\frac{3}{2} + k\right)^2\right) + \frac{e^z z^{1/2}}{\Gamma(\frac{3}{2})}\left(\sum_{q=0}^{Q-1} \frac{z^{-q}}{q!} \prod_{k=0}^{q-1}\left(k - \frac{1}{2}\right)^2\right),\ z \geq 10, \tag{8}$$

where $z \triangleq \frac{\hat{d}^2}{\lambda^2}$ and M and Q are sufficiently large numbers to ensure an accurate approximation (typically $(M, Q) \geq 5$).

B. Evaluation of the gradient of $\langle s\rangle_\lambda(\hat{\mathbf{x}})$

A simple SD algorithm requires the evaluation of the gradient $\nabla_{\hat{\mathbf{x}}}\langle s\rangle_\lambda(\hat{\mathbf{x}})$, which is given by

$$\nabla_{\hat{\mathbf{x}}}\langle s\rangle_\lambda(\hat{\mathbf{x}}) = \sum_{i=1}^{N_A} s_i'(r_i; \lambda)\nabla_{\hat{\mathbf{x}}} r_i, \tag{9}$$

where the i-th $\eta \times \eta$ block $\nabla_{\hat{\mathbf{x}}} r_i$ is

$$[\nabla_{\hat{\mathbf{x}}} r_i]_i = \frac{\hat{\mathbf{a}}_i - \hat{\mathbf{x}}}{\|\hat{\mathbf{a}}_i - \hat{\mathbf{x}}\|_2}, \tag{10}$$

and s_i' is

$$s_i'(r_i; \lambda) = 2r_i + \frac{r_i\sqrt{\pi}\tilde{d}_i}{\lambda} S_1(r_i; \lambda), \tag{11}$$

and

$$S_1(r_i; \lambda) \triangleq e^{-\frac{r_i^2}{\lambda^2}}\left(2\,{}_1F_1\left(\frac{3}{2};1;\frac{r_i^2}{\lambda^2}\right) - 3\,{}_1F_1\left(\frac{5}{2};2;\frac{r_i^2}{\lambda^2}\right)\right), \tag{12}$$

where for notational simplicity we have replaced $r_i \triangleq \hat{d}_i$.

C. Computation of $\lambda^{(1)}$

The last critical step required to implement the above-described GDC algorithm is to determine the initial smoothing parameter $\lambda^{(1)}$, such that $\langle s\rangle_\lambda$ is convex.

In [11], it was shown that is sufficient to compute

$$\lambda^{(1)} \geq \frac{\sqrt{\pi}}{2}\max_i\{\tilde{d}_i\}. \tag{13}$$

IV. LT-Application for Home, Automotive and Public Transportation Environment

In this section, the proposed localization algorithm is applied to the development of advanced LT-applications for home, automotive and public transportation environments.

A. LT-Application for Home Environment

For the home environment, the LT-application is an advanced home-theatre system, where the sound is tuned accordingly to the loudspeakers' and remote-control's locations. A sketch of the audio-system is illustrated in figure 1.

Each device is equipped with a low-complexity UWB transceiver, that enables distance (ranging) measurements. This information is used by a localization engine to estimate the position of each node. First, loudspeakers' positions are obtained with an anchor-free based localization algorithm and then, using this information, a source-localization algorithm computes the remote control's location.

One of the problems in this LT-application is the fact that anchors are randomly placed in the environment. Indeed, as mentioned in section I, the network geometry affects the number of local minima of $s(\hat{\mathbf{x}})$, compromising therefore the accuracy of the localization algorithm.

In this work, we will focus on the localization problem of the remote control (anchor locations are known). The results will show that the R-GDC algorithm can constantly find the global minimum of $s(\hat{\mathbf{x}})$ regardless the random location of the anchors.

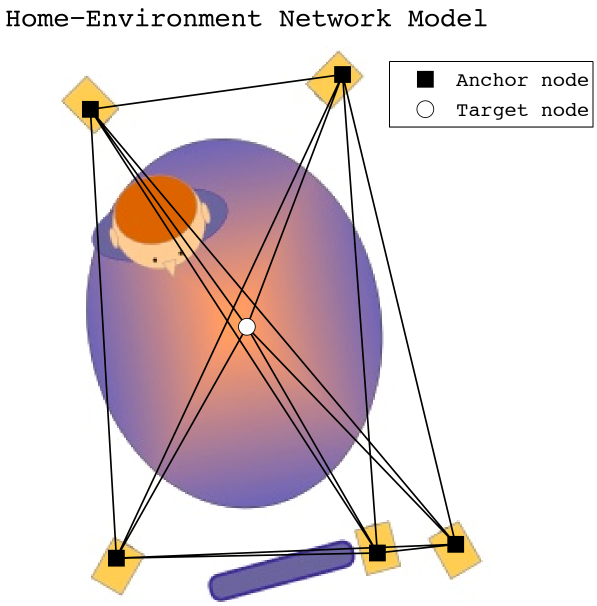

Fig. 1. Localization system for UWB-LT audio home-environment application

The first step in this study is to quantify the impact of the anchor locations onto the localization error ζ

$$\zeta = \|\mathbf{x} - \hat{\mathbf{x}}\|_{\mathrm{F}}. \tag{14}$$

To this end, we recall that in geo-localization the geometric dilution of precision $(GDOP)$ is a measure of such impact. A lower $GDOP$ indicates a good location of the anchors and viceversa. Mathematically,

$$GDOP = \mathrm{trace}(\mathbf{Q}), \tag{15}$$

where $\mathbf{Q} \triangleq (\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}$, $\mathbf{A} \triangleq [\frac{\mathbf{a}_1 - \mathbf{x}}{d_1}, c; \ldots; \frac{\mathbf{a}_{N_{\mathrm{A}}} - \mathbf{x}}{d_{N_{\mathrm{A}}}}, c]$ and c is the speed of light.

For the sake of completion, the proposed R-GDC algorithm is compared with 2 alternative methods, namely, SMACOF [6] and squared R-GDC (SR-GDC) [10]. The network is randomly deployed within an area of 10×10 meters. The remote control is at the location $\mathbf{x} = (5, 5)$, 3 loudspeakers are placed in the front and 2 on the rear of the remote control, as illustrated in figure 1. Ranging measurements are affected by an error with a zero-mean Gaussian distribution with variance σ^2.

The result reveals that the R-GDC algorithm is less sensitive to the variations of the $GDOP$, especially with high noise level. Indeed, consider the results obtained for $\sigma = 0.5$. Varying the GDOP, the localization error achieved with the SMACOF and SR-GDC algorithms increases faster than that achieved with the R-GDC.

Next, we consider the localization error ζ as a function of noise standard deviation σ. The performance of the algorithms are also compared to PEB [12] used as theoretical benchmark.

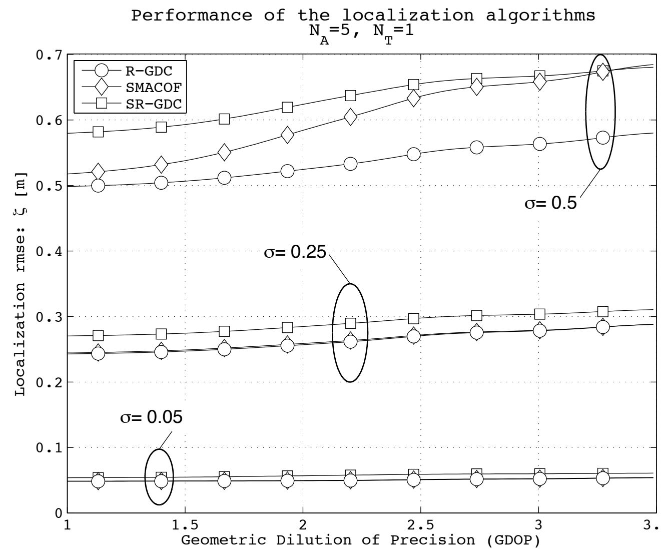

Fig. 2. Performance comparison of the localization error as a function of the $GDOP$.

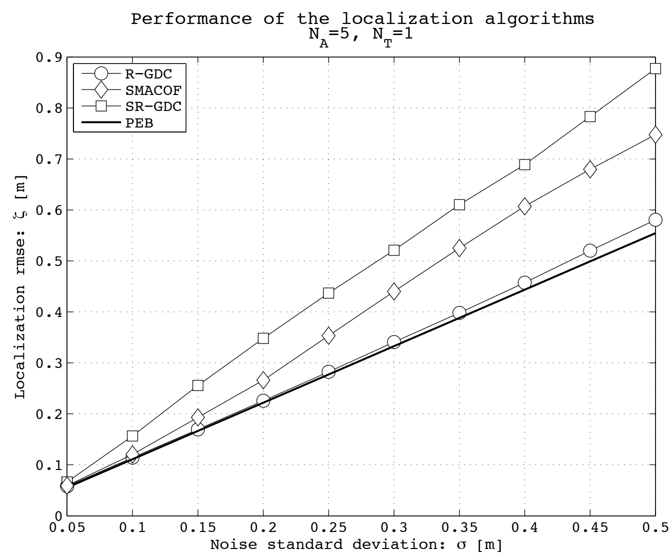

Fig. 3. Performance comparison of the localization error as a function of the noise standard deviation σ.

In this study, it is shown that the R-GDC algorithm performs optimally for almost the entire range of noise level. For large values of σ, however, we observe that the R-GDC's performance slightly deviates from the PEB. This effect should not be interpret as a degradation of the algorithm's performance, but rather as a deficiency of the PEB. Indeed, it is typical that under large a noise the PEB, or generally the Cramér-Rao Lower Bound (CRLB), becomes a loose bound. An example, for instance, is given in [14], where the mean-square-estimate error bound is studied for the bearing estimation problem.

B. Automotive Environment

For the automotive environment, the LT-application is to find the location of the car-key. The UWB wireless network is formed with a set of devices mounted in the car-frame, and a UWB transmitter in the car-key as shown in figure 4.

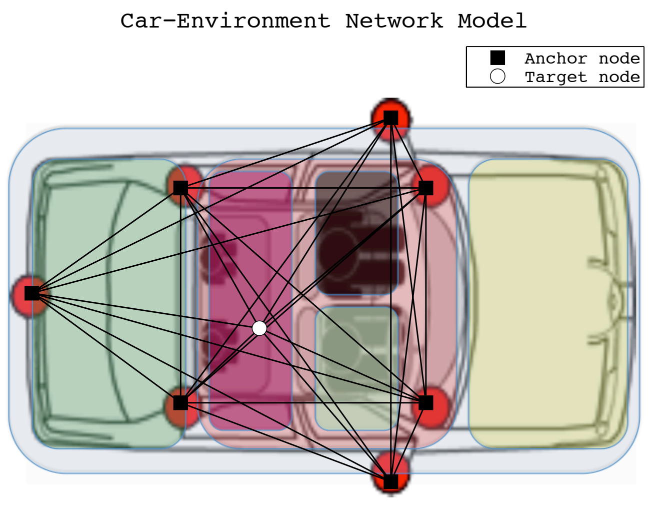

Fig. 4. Deployment of the UWB wireless network in the car. Colors indicate different area of the car.

The UWB transmitter will send encoded signals to the UWB devices embedded in the car, which will perform ranging. Using such distance information, a localization algorithm will estimate the location of the car-key and it will recognize, with a given error, for instance ± 0.1m, if the key is outside or inside the car. It is also desirable to identify the location inside the car, $i.e.$ trunk, passenger, engine department.

In figure 4, a possible deployment of the anchor nodes is illustrated. The monitored area is separated in different zones, uniquely identified with a color. The first separation is between the area outside and inside the car, denoted by zone B1 and B2, respectively. Zone B2 is divided in 3 sub-zones, namely, the trunk (B3), the passenger department (B4) and the engine department (B5). Finally, zone B4 is further divided in 3 sub-sub-zones, namely, back sits (B6), front-left sit (B7) and front-right sit (B8).

In figure 5, we show with an histogram the probability of incorrect zone detection obtained via simulations. We assume that ranging errors are zero-mean Gaussian variables with $\sigma = 0.3$m, and the car size is approximately 4×2m.

The result shows that the proposed R-GDC algorithm outperforms the alternatives and provides, in general, a probability of incorrect zone detection smaller than 0.2. The worse performance, however, corresponds to a zone with small area such as B7 and B8. This is an expected result, since the PEB proportionally increases with the noise.

In figure 6, we show the performance of the localization algorithms (measured with respect to the localization error ζ) as a function of the noise standard deviation σ. As in the previous study, we use the PEB to give a theoretical performance bound. Also in this case, it is shown that the R-GDC outperforms the alternative techniques and provides an error close to the bound. The gap between R-GDC and PEB is again a consequence of the fact that the PEB is no longer the tightest bound.

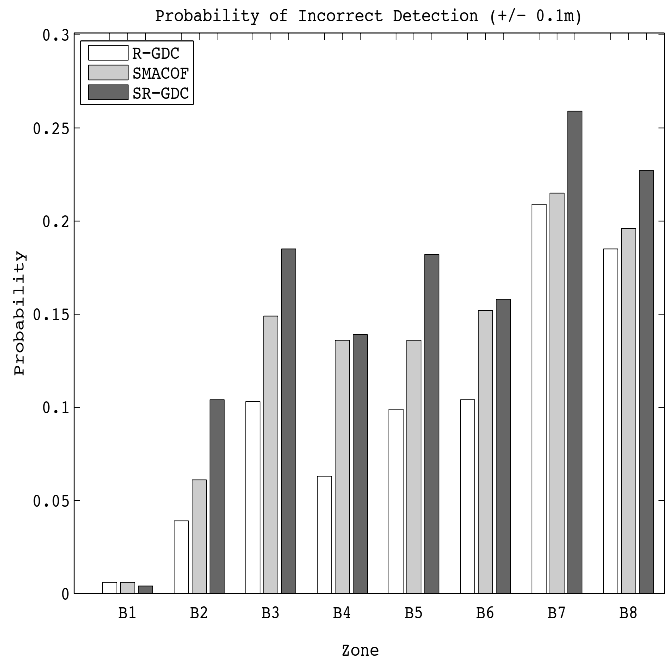

Fig. 5. Comparison of the localization algorithms in terms of the probability of incorrect zone detection.

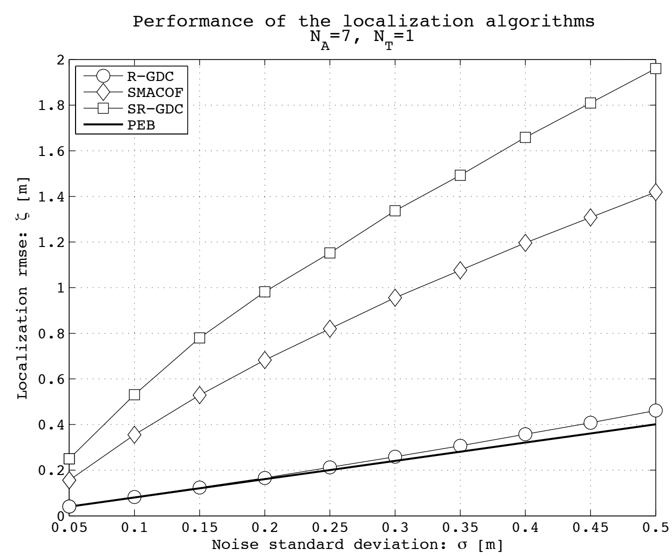

Fig. 6. Comparison of the localization algorithms in terms of the localization error

C. Public Transportation Environment

For the public transportation environment, the LT-application scenario is relatively simpler than the others. In general, we can consider a scenario where few UWB nodes are mounted in the structure of the public transportation (airplane, train, bus, etc.) and a mobile UWB terminal needs to be localized. For instance, in figure 7 we illustrate the concept of this LT-application used in the cabin of an aircraft.

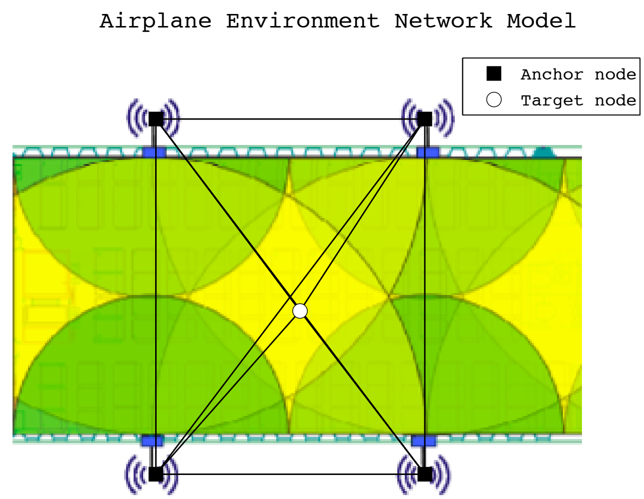

Fig. 7. Network model for an UWB-based localization system in the aircraft.

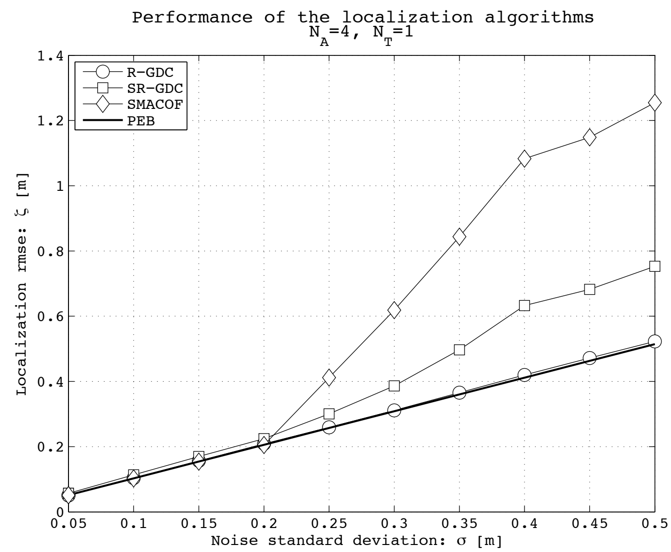

Fig. 8. Comparison of the localization algorithms with respect to ζ.

The challenge in this scenario is the high noise level due to the multiple systems coexisting in the airplane.

In this simulation, we assume that the aircraft cell is $10 \times$ 10m size and the target device is randomly located within this area. Access points are positioned at the corner of this cell and each anchor measures the distance to the target.

In figure 8, the localization error performance of the considered algorithms are shown as a function of σ. The results are also compared with the PEB. Once again, the proposed R-GDC algorithm outperforms all the alternatives and, it reaches the PEB within the entire scale of noise level.

In this scenario, the performance of the SR-GDC and SMACOF algorithms are accurate and optimal under low noise level conditions. Indeed, the regular deployment of the anchors, the fact that the target is always within the convex-hull formed by the anchors and the relative large size of the cell favorite the convexity of the cost-function and therefore, the improvement of the SR-GDC's and SMACOF's performance.

V. Conclusions

We proposed a localization algorithm based on global distance continuation technique to accurately compute the location of a target node, given the position of few anchor nodes and a set of imperfect distance measurements. We evaluated the performance of the R-GDC algorithm under different scenario conditions, which characterize LT-applications for home, automotive and public transportation environments. Throughout simulations, we have shown that the performance of such algorithm is very close to the theoretical PEB.

VI. Acknowledgment

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References

[1] [Online]. Available: http://www.euwb.eu/

[2] G. Mao, B. Fidan, and B. D. O. Anderson, "Wireless sensor network localization techniques," *Computer Networks: The Intern. J. of Comp. and Telecomm. Networking*, vol. 51, no. 10, pp. 2529–2553, July 2007.

[3] B. Alavi and K. Pahlavan, "Modeling of the TOA-based distance measurement error using UWB indoor radio measurements," *IEEE Commun. Lett.*, vol. 10, no. 4, pp. 275–277, 2006.

[4] I. Guvenc, Z. Sahinoglu, and P. Orlik, "TOA estimation for IR-UWB systems with different transceiver types," *IEEE Transactions on Microwave Theory and Techniques*, vol. 54, pp. 1876–1886, 2006.

[5] L. Joon-Yong and R. Scholtz, "Ranging in a dense multipath environment using an UWB radio link." *IEEE J. Sel. Areas Commun.*, vol. 20, pp. 1667–1683, December 2002.

[6] T. F. Cox and M. A. A. Cox, *Multidimensional Scaling*, 2nd ed. Chapman & Hall/CRC, 2000.

[7] J. Dattorro, *Convex Optimization and Euclidean Distance Geometry*. Meboo Publishing, 2005.

[8] P. Biswas, T.-C. Liang, K.-C. Toh, and T.-C. Wang, "Semidefinite programming based algorithms for sensor network localization with noisy distance measurements," *ACM Trans. on Sensor Netw. (TOSN)*, vol. 2, no. 2, pp. 188–220, May 2006.

[9] A. Y. Alfakih, H. Wolkowicz, and A. Khandani, "Solving euclidean distance matrix completion problems via semidefinite programming," *Journ. on Comp. Opt. and App.*, vol. 12, no. 1, pp. 13 – 30, 1999.

[10] J. More and Z. Wu, "Global continuation for distance geometry problems," *SIAM J. Optim.*, vol. 7, pp. 814–836, 1997.

[11] G. Destino and G. Abreu, "Solving the source localization problem via global distance continuation," in *Proc. IEEE International Conference on Communications (ICC)*, 2009, p. (accepted).

[12] D. Jourdan, D. Dardari, and M. Win, "Position error bound for UWB localization in dense cluttered environments," in *Proc. IEEE International Conference on Communcations*, vol. 8, June 2006, pp. 3705–3710.

[13] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 10th ed. Dover Publications, 1965.

[14] J. Abel, "A bound on mean-square-estimate error," *IEEE Transactions on Information Theory*, vol. 39, no. 5, pp. 1675–1680, Sep 1993.