A Distributed Information Fusion Method for Localization Based on Pareto Optimization

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***Abstract*—To overcome the limitations of specific positioning techniques for mobile wireless nodes and achieve a high accuracy, the fusion of heterogeneous sensor information is an appealing strategy. In this paper, the problem of optimal fusion of ranging information typically provided by Ultra-Wideband radio with speed and absolute orientation information is addressed. A new distributed recursive estimation method is proposed. The method does not assume any motion model of mobile nodes and is based on a Pareto optimization. The challenging part of the new estimator is the characterization of the statistical information needed to model the optimization problem. The proposed estimator is validated by Monte Carlo simulations, and the performance is compared to several Kalman-based filters commonly employed for localization and sensor fusion. Much better performance is achieved, but at the price of an increased computational complexity.**

I. Introduction

Localization is an essential service in many sensor network applications, where it is desirable to accurately estimate the position of mobile wireless nodes [1]. Localization applications range from autonomous mobile robots to vehicular networks, and from personnel localization to industrial and commercial asset tracking. In several cases, such as indoor or dense urban environments, the coverage or accuracy provided by commonly used global navigation satellite systems is not sufficient. Therefore, in recent years, research has been focused on alternative methods, particularly for wireless indoor localization [2].

Localization techniques based on Pulse-based Ultra-Wideband (UWB) are of particular interest. The main feature that makes pulse-based UWB suitable for localization is the fine time resolution, which allows for a theoretical ranging accuracy of the order of centimeters, when used in conjunction with time-of-arrival ranging methods [3]. Other potential benefits of the adoption of this technique are the resilience to multipath propagation effects and low-power device implementation. However, the fundamental limitations of UWB localization techniques can be overcome by a fusion with other source of information, such as speed and absolute orientation information that can be measured by on-board units.

In this paper, we propose a new fusion method based on a Pareto optimization that uses two heterogeneous sources of information. We consider the scenario in which a mobile node of unknown position (master) obtains measurements of its distance with respect to known-position anchors (slaves) and uses in addition its own speed and orientation information. On the one hand, the round-trip-time of UWB pulses give the ranging information that is then processed by a Weighted Least Squares (WLS) algorithm [4] to obtain an estimate of the position having low bias but a high variance of the error. On the other hand, measurements of the speed and absolute orientation (dead-reckoning) of the master give an estimate with a low variance but with a high bias. Then, a fusion stage consists in weighting the UWB and dead-reckoning estimates by a Pareto optimization problem where a tradeoff between the variance of the estimation error and its bias is exploited. Since the developed estimator requires only local processing at the master node, it is completely distributed and relies on the cooperation of the slaves. The method does not make any assumption about the model of the movement and is therefore applicable to a wide range of motion trajectories and dynamic scenarios.

The localization literature is vast. Concerning sensor fusion for localization, different methods have been studied for information sources such as GNSSs, inertial navigation systems, odometry, and local radio technologies [5], [6]. An extensive survey of the most common information sources and sensor fusion approaches, in the context of automotive positioning, is provided in [7]. However, to the best of our knowledge, no approach has been based on a Pareto optimization by trading-off between estimation error bias and variance. Compared to papers from the literature, we employ a new statistical characterization of the estimation errors which uses models from experimental tests on a UWB platform we presented in [8], [9]. By numerical simulations, we show that our new sensor fusion outperforms existing solutions, including several commonly employed techniques based on the Kalman filter. However, we pay the price of a slightly larger computational complexity.

The remainder of this paper is organized as follows: in Section II, the problem is formulated. In Section III, the statistical characterization of the estimation error is given. The positioning estimator is then given in Section IV. In Section V, simulation results are reported. Finally, we conclude the paper in Section VI.

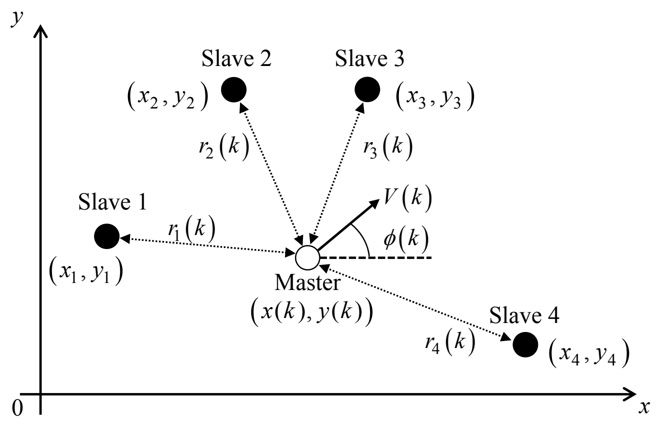

Fig. 1. Model of the localization system. The slave nodes are placed at fixed and known positions. At time k, the unknown-position master node measures its distance r_i with respect to each slave. Speed V and orientation ϕ of the master are measured using on-board sensors.

II. Problem formulation

The two-dimensional positions of $n \geq 4$ stationary UWB nodes, denoted as *slaves*, are assumed to be known and denoted as $[x_i \quad y_i]^T$, $i = 1 \dots n$. A mobile node, denoted as *master*, needs to estimate its own unknown position $\mathbf{z}(k) = [x(k) \quad y(k)]^T \in \mathbb{R}^2$ by measuring its distance $r_i(k)$ with respect to each one of the slaves, as shown in Fig. 1. The UWB range measurements are modeled later on based on experimental tests on the in-house developed platform described in [8], [9]. Furthermore, the master is able to measure its absolute speed and orientation, denoted as $V(k)$ and $\phi(k)$ respectively, by means of on-board sensors, to perform the so-called *dead-reckoning*. An approach for the modeling of such sensors is used in [10]. Coherently with experimental results discussed in the papers mentioned above, we assume that the ranging measurements and dead-reckoning measurements are affected by noises.

We propose a sensor fusion method that combines information from the ranging measurements and from the on-board sensors, without assuming any a-priori information on the motion model of the master. Furthermore, all processing is performed at the master node.

An estimate of the master's position at time $k+1$ is denoted by

$$\hat{\mathbf{z}}\,(k+1 \mid k+1) = [\hat{x}\,(k+1 \mid k+1) \quad \hat{y}\,(k+1 \mid k+1)]^T \,,$$

and is derived as follows:

$$\hat{x}\,(k+1 \mid k+1) = (1-\beta_{x,k})\hat{x}_r\,(k+1) + \beta_{x,k}\hat{x}_v\,(k+1)\ , \tag{1}$$

$$\hat{y}\,(k+1 \mid k+1) = (1-\beta_{y,k})\hat{y}_r\,(k+1) + \beta_{y,k}\hat{y}_v\,(k+1)\ , \tag{2}$$

where $\hat{\mathbf{z}}_r(k+1) = [\hat{x}_r(k+1) \quad \hat{y}_r(k+1)]^T$ is the position estimate based only on the ranging measurements, and $\hat{\mathbf{z}}_v(k+1) = [\hat{x}_v(k+1) \quad \hat{y}_v(k+1)]^T$ is the position estimate based on the dead-reckoning block, which gives the position estimate at the previous time step and the on-board speed and orientation sensors. The terms $\beta_{x,k}$ and $\beta_{y,k}$ are the sensor-fusion design parameters that must be optimally chosen based on a Pareto optimization, as we show later.

The estimation problem is separable on the x and y axes, in the sense that the estimates on the y axis does not affect the x axis, and vice versa. Therefore in the following we will provide derivations only for the x component, because the derivations for the y component are similar.

The dead-reckoning estimates have the following expressions:

$$\hat{x}_v(k+1) = \hat{x}\,(k \mid k) + \tilde{V}(k)T\cos(\tilde{\phi}(k)) \tag{3}$$

$$\hat{y}_v(k+1) = \hat{y}\,(k \mid k) + \tilde{V}(k)T\sin(\tilde{\phi}(k))\,, \tag{4}$$

where $\tilde{V}(k)$ is the measurement of the translational speed, $\tilde{\phi}(k)$ the measurement of the orientation and T is the sampling time interval. These measurements are expressed as

$$\tilde{V}(k) = V\,(k) + w_V\,(k)$$

$$\tilde{\phi}(k) = \phi\,(k) + w_\phi\,(k)$$

where the noise terms w_V and w_ϕ are zero-mean Gaussian random variables with known variances σ_V^2 and σ_ϕ^2, respectively. The estimates provided by the dead-reckoning block have a low variance in the estimation error but are biased. This is due to that the orientation measurement appears as the argument of a cosinus. It follows that the estimators of Eqs. (3) and (4) are biased and so are the estimates (1) and (2).

We can model our estimator (1) as

$$\hat{x}\,(k+1 \mid k+1) \triangleq x(k+1) + w_x(k+1)\,, \tag{5}$$

where $w_x(k+1)$ is the error in the position estimate. This error has a non zero average, namely the estimator is biased, since the dead-reckoning block gives a biased estimate. From (5) it follows that the error is

$$w_x(k+1) = \hat{x}\,(k+1 \mid k+1) - x(k+1)\,. \tag{6}$$

We denote the bias by $\mathbb{E}\,\{w_x(k+1)\}$, where $\mathbb{E}$ denotes statistical expectation.

In this paper we define a cost function that takes into account the estimator variance and bias simultaneously. We define the bias term of the estimation error as

$$P_1 \triangleq \mathbb{E}\,\{w_x(k+1)\}\ , \tag{7}$$

and the variance term of the estimation error as

$$P_2 \triangleq \mathbb{E}\,\left\{(\hat{x}\,(k+1 \mid k+1) - \mathbb{E}\,\{\hat{x}\,(k+1 \mid k+1)\})^2\right\}. \tag{8}$$

We are now ready to formulate a Pareto optimization problem [11] to select the fusion coefficients at each k:

$$\begin{aligned} \min_{\beta_{x,k}} \quad & (1-\rho_{x,k})\,P_2 + \rho_{x,k}P_1^2 \\ \text{s.t.} \quad & \beta_{x,k} \in \mathscr{B} = [-1, 1]\,. \end{aligned} \tag{9}$$

The minimization in this problem is motivated by that we would like to reduce as much as possible both the average of the estimation error, namely the bias, and the variance of the estimation error. The Pareto weighting factor $\rho_{x,k}$ must be chosen for each time k so to trade off the low bias and high variance of the error of the ranging estimate with the high bias and low variance of the error of the dead-reckoning estimate. Based on the statistical modelling of the bias, which we show later, the bias may grow unstable with time, which motivates the constraint $\beta_{x,k} \in \mathscr{B}$. The challenge for such an optimization is the analytical characterization of the cost function (9), which we study in the following section. Then, in Section IV we solve the optimization problem, thus achieving the optimal weighting coefficients $\beta_{x,k}$ and $\beta_{y,k}$ to use in Eqs. (1) and (2).

III. Characterization of the Estimation Error

A fundamental step for the development of the proposed sensor fusion technique is the characterization of the first and second order moments of the estimation error, which is provided in this section.

The expression for the position error at time instant k+1 is readily derived from (1) and (6):

$$w_x(k+1) = (1-\beta_{x,k})\, w_r(k+1) + \beta_{x,k} w_x(k) + \beta_{x,k} T \times \tilde{V}(k)\cos(\tilde{\phi}(k)) - \beta_{x,k} T V(k)\cos(\phi(k))\,, \tag{10}$$

where $w_r(k+1)$ is the error of the estimates obtained from the ranging. In the following subsections, we characterize the average and the correlation of this error, as well as of the error in the dead-reckoning estimates.

A. Ranging estimate

In this subsection we give a new statistical characterization of the estimation error of the ranging estimate. We consider the problem of finding the first and second order moments of the ranging estimates of the x and y component of the position, denoted as

$$\hat{\mathbf{z}}_r(k+1) = [\hat{x}_r(k+1) \quad \hat{y}_r(k+1)]^T \in \mathbb{R}^2\,.$$

This requires first a derivation of the ranging estimates. In the following, for the sake of simple notation, we drop the $k+1$ indices.

Since it must be that $n \geq 4$ for linear-based ranging estimates, the master's position estimate $\hat{\mathbf{z}}_r$ is obtained by

$$\tilde{\mathbf{s}} = \mathbf{H}\hat{\mathbf{z}}_r\,,$$

with

$$\mathbf{H} = \begin{bmatrix} 2\,(x_1 - x_n) & 2\,(y_1 - y_n) \\ 2\,(x_2 - x_n) & 2\,(y_2 - y_n) \\ \vdots & \vdots \\ 2\,(x_{n-1} - x_n) & 2\,(y_{n-1} - y_n) \end{bmatrix}, \tag{11}$$

$$\tilde{\mathbf{s}} = \begin{bmatrix} \tilde{r}_n^2 - \tilde{r}_1^2 \\ \vdots \\ \tilde{r}_n^2 - \tilde{r}_{n-1}^2 \end{bmatrix} + \mathbf{a}\,, \tag{12}$$

where the constant vector $\mathbf{a}$ is given by the following function of the known coordinates of the slaves:

$$\mathbf{a} = \begin{bmatrix} x_1^2 - x_n^2 + y_1^2 - y_n^2 \\ x_2^2 - x_n^2 + y_2^2 - y_n^2 \\ \vdots \\ x_{n-1}^2 - x_n^2 + y_{n-1}^2 - y_n^2 \end{bmatrix},$$

and $\tilde{r}_i$ is the measured value of the distance between the master and the i-th slave affected by additive zero-mean Gaussian noise w_{r_i}:

$$\tilde{r}_i = r_i + w_{r_i}\,. \tag{13}$$

We denote by $\sigma_{r_i}^2$ the variance of $\tilde{r}_i$. The variance of w_{r_i} depends on the range, according to the following exponential model that has been derived from real-world UWB measurement data in [12] and [13]:

$$\sigma_{w_{r_i}}^2 = \sigma_0^2 \exp(\kappa_\sigma r_i)\,. \tag{14}$$

By substituting (13) in (12), we can rewrite $\tilde{\mathrm{s}}$ as follows:

$$\begin{aligned} \tilde{\mathbf{s}} &= \begin{bmatrix} \tilde{r}_n^2 - \tilde{r}_1^2 \\ \vdots \\ \tilde{r}_n^2 - \tilde{r}_{n-1}^2 \end{bmatrix} + \mathbf{a} \\ &= \begin{bmatrix} r_n^2 - r_1^2 \\ \vdots \\ r_n^2 - r_{n-1}^2 \end{bmatrix} + \mathbf{a} \\ &+ \begin{bmatrix} w_{r_n}^2 + 2r_n w_{r_n} - w_{r_1}^2 - 2r_1 w_{r_1} \\ \vdots \\ w_{r_n}^2 + 2r_n w_{r_n} - w_{r_{n-1}}^2 - 2r_{n-1} w_{r_{n-1}} \end{bmatrix} \\ &= \mathbf{s} + \mathbf{w_s}\,, \end{aligned} \tag{15}$$

where

$$\mathbf{w_s} \triangleq \begin{bmatrix} w_{r_n}^2 + 2r_n w_{r_n} - w_{r_1}^2 - 2r_1 w_{r_1} \\ \vdots \\ w_{r_n}^2 + 2r_n w_{r_n} - w_{r_{n-1}}^2 - 2r_{n-1} w_{r_{n-1}} \end{bmatrix} \tag{16}$$

is the additive noise affecting the true value $\mathbf{s}$.

We derive the ranging estimate by the WLS technique, since ranging measurements are affected by unequal statistical errors across the various links. Then, the following classical WLS optimization problem gives the ranging estimates:

$$\min_{\hat{\mathbf{z}}_r} \left\| \mathbf{J}^{\frac{1}{2}} \left(\tilde{\mathbf{s}} - \mathbf{H}\hat{\mathbf{z}}_r \right) \right\|,$$

where $\mathbf{J}$ is a positive-definite weighting matrix introduced for the purpose of emphasizing the contribution of those range measurements that are deemed to be more reliable [4], and $\|\cdot\|$ is the Euclidean norm. A common choice for $\mathbf{J}$ is $\mathbf{R}^{-1}$, namely the inverse of the noise covariance matrix:

$$\mathbf{R} \triangleq \left(\mathbf{w}_s - \mathbb{E}\left\{\mathbf{w}_s\right\}\right)\left(\mathbf{w}_s - \mathbb{E}\left\{\mathbf{w}_s\right\}\right)^T\,. \tag{17}$$

Thus the well-known solution to the WLS problem is

$$\hat{\mathbf{z}}_r = \left(\mathbf{H}^T\mathbf{J}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{J}\tilde{\mathbf{s}}\,, \tag{18}$$

where $\mathbf{J} = \mathbf{R}^{-1}$. From (18) we can characterize the first and second order moments of the ranging estimate. First, however, we need the expression of the error covariance matrix and its inverse.

With such a goal in mind, we need to derive the average of Eq. (16):

$$\begin{aligned}\mathbb{E}\left\{\mathbf{w}_s\right\} &= \begin{bmatrix} \mathbb{E}\left\{w_{r_n}^2\right\} + 2r_n\,\mathbb{E}\left\{w_{r_n}\right\} \\ \vdots \\ \mathbb{E}\left\{w_{r_n}^2\right\} + 2r_n\,\mathbb{E}\left\{w_{r_n}\right\} \end{bmatrix} \\ &\quad - \begin{bmatrix} \mathbb{E}\left\{w_{r_1}^2\right\} + 2r_1\,\mathbb{E}\left\{w_{r_1}\right\} \\ \vdots \\ \mathbb{E}\left\{w_{r_{n-1}}^2\right\} + 2r_{n-1}\,\mathbb{E}\left\{w_{r_{n-1}}\right\} \end{bmatrix} \\ &= \begin{bmatrix} \sigma_{w_{r_n}}^2 - \sigma_{w_{r_1}}^2 \\ \vdots \\ \sigma_{w_{r_n}}^2 - \sigma_{w_{r_{n-1}}}^2 \end{bmatrix}. \end{aligned} \tag{19}$$

Here we used that the noises w_{r_i} affecting the range measurements have zero-mean. Finally, the expression of the covariance matrix $\mathbf{R}$ is obtained by substituting Eqs. (16) and (19) in (17). Tedious computations give that the diagonal elements $\mathbf{R}_{ll}$, $l = 1, \ldots, n-1$ are given by

$$\mathbf{R}_{ll} = 4r_n^2\sigma_{w_{r_n}}^2 + 2\sigma_{w_{r_n}}^4 + 4r_l^2\sigma_{w_{r_l}}^2 + 2\sigma_{w_{r_l}}^4$$

and that the off-diagonal elements $\mathbf{R}_{lj}$, with $l \neq j$, are identical and given by

$$\mathbf{R}_{lj} = 4r_n^2\sigma_{w_{r_n}}^2 + 2\sigma_{w_{r_n}}^4\,.$$

We are now in the position of deriving the expression of the inverse of $\mathbf{R}$:

Lemma 3.1: Consider the matrix $\mathbf{R}$ as defined in (17). The matrix has full rank, and the inverse is

$$\mathbf{R}^{-1} = \left(\mathbf{I} - \frac{1}{1+q}\mathbf{G}\mathbf{1}\mathbf{1}^T\right)\mathbf{D}^{-1}\,,$$

where $\mathbf{D}$ is a $(n-1)\times(n-1)$ diagonal matrix whose entries are $\mathbf{D}_{ll} = 4r_l^2\sigma_{w_{r_l}}^2 + 2\sigma_{w_{r_l}}^4$, $l = 1, \ldots, n-1$, $\mathbf{G} = p\mathbf{D}^{-1}$ is a $(n-1)\times(n-1)$ matrix with $p = 4r_n^2\sigma_{w_{r_n}}^2 + 2\sigma_{w_{r_n}}^4$, $q = \sum_{i=1}^{n-1} p/\mathbf{D}_{ii}$, and $\mathbf{1}$ is the all ones vector.

Proof: We start by observing that $\mathbf{R}$ can be written as $\mathbf{R} = \mathbf{D} + \mathbf{P}$ with $\mathbf{P} = p\mathbf{1}\mathbf{1}^T$. Then, $\mathbf{R}$ is always invertible: the eigenvalues are given by [14]

$$\lambda\left(\mathbf{R}\right) = \lambda\left(\mathbf{D} + p\mathbf{1}\mathbf{1}^T\right) = \lambda\left(\mathbf{D}\right) + \mathrm{tr}\left(p\mathbf{1}\mathbf{1}^T\right)\,,$$

where $\mathrm{tr}(\cdot)$ denotes the trace. The i-th eigenvalue is

$$\lambda_i = \mathbf{D}_{ii} + (n-1)p \neq 0,\ \forall\, i = 0 \ldots n-1\,,$$

whereby the we see that the eigenvalues of $\mathbf{R}$ are all non-zero. We can now turn our attention to derive the expression of the inverse.

We have that

$$\begin{aligned}\mathbf{R} &= \mathbf{D} + p\mathbf{1}\mathbf{1}^T \\ &= \mathbf{D}\left[\mathbf{I} + \mathbf{G}\mathbf{1}\mathbf{1}^T\right] = \mathbf{D}\mathbf{M}\,,\end{aligned}$$

where we have defined $\mathbf{G} \triangleq p\mathbf{D}^{-1}$ and $\mathbf{M} \triangleq \mathbf{I} + \mathbf{G}\mathbf{1}\mathbf{1}^T$. The inverse of $\mathbf{M}$ can be easily derived as [15]

$$\mathbf{M}^{-1} = \mathbf{I} - \frac{1}{1+q}\mathbf{G}\mathbf{1}\mathbf{1}^T\,,$$

whereby the lemma follows by observing that $\mathbf{R}^{-1} = \mathbf{M}^{-1}\mathbf{D}^{-1}$. ■

Based on Eq. (18) and the previous lemma, we are now in the position of deriving the expectation and the variance of the ranging estimation error $\mathbf{w}_r$. We define such an error as

$$\mathbf{w}_r \triangleq \hat{\mathbf{z}}_r - \mathbf{z}_r\,. \tag{20}$$

Lemma 3.2: The first component of the following vector gives the expectation of w_r, namely $\mathbb{E}\left\{w_r\right\}$:

$$\mathbb{E}\left\{\mathbf{w}_r\right\} = \left(\mathbf{H}^T\mathbf{J}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{J}\left(\mathbf{1}\sigma_{w_{r_n}}^2 - \left[\sigma_{w_{r_1}}^2, \sigma_{w_{r_2}}^2, \ldots, \sigma_{w_{r_{n-1}}}^2\right]^T\right).$$

Proof: By taking the mean of Eq. (18), we obtain:

$$\begin{aligned}\mathbb{E}\left\{\hat{\mathbf{z}}_r\right\} &= \left(\mathbf{H}^T\mathbf{J}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{J}\,\mathbb{E}\left\{\mathbf{r}'\right\} \\ &= \left(\mathbf{H}^T\mathbf{J}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{J}\left(\mathbf{1}\,\mathbb{E}\left\{\tilde{r}_n^2\right\}\right. \\ &\quad \left. - \mathbb{E}\left[\tilde{r}_1^2, \tilde{r}_2^2, \ldots, \tilde{r}_{n-1}^2\right]^T + \mathbf{a}\right) \\ &= \left(\mathbf{H}^T\mathbf{J}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{J}\left(\mathbf{1}\sigma_{r_n}^2\right. \\ &\quad \left. - \left[\sigma_{r_1}^2, \sigma_{r_2}^2, \ldots, \sigma_{r_{n-1}}^2\right]^T + \mathbf{a}\right) \\ &= \mathbf{z} + \left(\mathbf{H}^T\mathbf{J}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{J} \\ &\quad \times\left(\mathbf{1}\sigma_{w_{r_n}}^2 - \left[\sigma_{w_{r_1}}^2, \sigma_{w_{r_2}}^2, \ldots, \sigma_{w_{r_{n-1}}}^2\right]^T\right) \\ &= \mathbf{z} + \mathbb{E}\left\{\mathbf{w}_r\right\}\,,\end{aligned}$$

whereby the lemma follows. ■

In the following, we derive $\mathbb{E}\left\{w_r^2\right\}$. We start by taking the square of Eq. (13):

$$\tilde{r}_i^2 = r_i^2 + w_{r_i}^2 + 2r_iw_{r_i}\,, \tag{21}$$

then, using this expression in (16) and substituting in (18), we have the following expression for the ranging-based estimate:

$$\hat{\mathbf{z}}_r = \left(\mathbf{H}^T\mathbf{J}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{J}\left\{\begin{bmatrix} r_n^2 - r_1^2 \\ r_n^2 - r_2^2 \\ \vdots \\ r_n^2 - r_{n-1}^2 \end{bmatrix} + \mathbf{B}_k + \mathbf{a}\right\}, \tag{22}$$

where

$$\mathbf{B}_k = \begin{bmatrix} w_{r_n}^2 + 2r_nw_{r_n} - w_{r_1}^2 - 2r_1w_{r_1} \\ w_{r_n}^2 + 2r_nw_{r_n} - w_{r_2}^2 - 2r_2w_{r_2} \\ \vdots \\ w_{r_n}^2 + 2r_nw_{r_n} - w_{r_{n-1}}^2 - 2r_{n-1}w_{r_{n-1}} \end{bmatrix}. \tag{23}$$

Therefore the error can be expressed as

$$\mathbf{w}_r = \left(\mathbf{H}^T\mathbf{J}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{J}\mathbf{B}_k\,. \tag{24}$$

The second-order moment of the error is given by

$$\begin{aligned}\mathbb{E}\left\{\mathbf{w}_r^2\right\} =& \left(\mathbf{H}^T\mathbf{J}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{J}\,\mathbb{E}\left\{\mathbf{B}_k\mathbf{B}_k^T\right\}\\ &\cdot \mathbf{J}^T\mathbf{H}\left(\mathbf{H}^T\mathbf{J}\mathbf{H}\right)^{-1}\\ =& \left(\mathbf{H}^T\mathbf{J}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{J}\mathbf{C}\\ &\cdot \mathbf{J}^T\mathbf{H}\left(\mathbf{H}^T\mathbf{J}\mathbf{H}\right)^{-1}\,,\end{aligned} \tag{25}$$

where $\mathbf{C} = \mathbb{E}\left\{\mathbf{B}_k\mathbf{B}_k^T\right\}$. In this matrix, the diagonal elements are

$$\begin{aligned}\mathbf{C}_{ll} =& \mathbb{E}\left\{\left(w_{r_n}^2 + 2r_nw_{r_n} - w_{r_l}^2 - 2r_lw_{r_l}\right)^2\right\}\\ =& 3\sigma_{w_{r_n}}^4 + 4r_n^2\sigma_{w_{r_n}}^2 + 3\sigma_{w_{r_l}}^4 + 4r_l^2\sigma_{w_{r_l}}^2 - 2\sigma_{w_{r_n}}^2\sigma_{w_{r_l}}^2\,,\end{aligned}$$

whereas the off-diagonal elements are

$$\begin{aligned}\mathbf{C}_{lj} =& \mathbb{E}\left\{\left(w_{r_n}^2 + 2r_nw_{r_n} - w_{r_l}^2 - 2r_lw_{r_l}\right)\right.\\ &\times\left.\left(w_{r_n}^2 + 2r_nw_{r_n} - w_{r_j}^2 - 2r_jw_{r_j}\right)\right\}\\ =& 3\sigma_{w_{r_n}}^4 - \sigma_{w_{r_n}}^2\sigma_{w_{r_j}}^2 + 4r_n^2\sigma_{w_{r_n}}^2 - \sigma_{w_{r_l}}^2\sigma_{w_{r_n}}^2 + \sigma_{w_{r_l}}^2\sigma_{w_{r_j}}^2\,.\end{aligned}$$

This concludes the efforts to characterize the statistics of the ranging estimation error. In the following subsection, we focus on the statistics of the dead-reckoning estimation.

B. Dead-reckoning estimate

We have the following results for the first and second order moments of the dead-reckoning estimate:

Lemma 3.3: Let $\tilde{\phi}(k)$ be Gaussian, and $\tilde{V}(k)$ and $\tilde{\phi}(k)$ statistically independent, then

$$\mathbb{E}\left\{\tilde{V}(k)\cos(\tilde{\phi}(k))\right\} = V(k)\cos(\phi(k))e^{-\frac{\sigma_\phi^2}{2}}\,. \tag{26}$$

and

$$\mathbb{E}\left\{\tilde{V}^2(k)\cos^2(\tilde{\phi}(k))\right\} = \sigma_V^2\left(\frac{1}{2} + \frac{1}{2}\cos\left(2\phi(k)\right)e^{-2\sigma_\phi^2}\right)\,.$$

Proof: See [16]. ■

C. Estimation Bias

We can now put together the statistical characterization of the ranging and dead-reckoning estimations. We have the following results that give the terms $P1$ and $P2$ in (9):

Lemma 3.4: Consider the estimation error (6). The bias is

$$\begin{aligned}P_1 =& \mathbb{E}\left\{w_x(k+1)\right\} = (1-\beta_{x,k})\,\mathbb{E}\left\{w_r(k+1)\right\}\\ &\beta_{x,k}\,\mathbb{E}\left\{w_x(k)\right\} + \beta_{x,k}TV(k)\cos(\phi(k))\left(e^{-\frac{\sigma_\phi^2}{2}} - 1\right)\,.\end{aligned} \tag{27}$$

Proof: The expectation of (1) is

$$\begin{aligned}&\mathbb{E}\left\{\hat{x}\left(k+1 \mid k+1\right)\right\} =\\ &= (1-\beta_{x,k})\,x(k+1) + (1-\beta_{x,k})\,\mathbb{E}\left\{w_r(k+1)\right\}\\ &\quad +\beta_{x,k}\left(x(k) + \mathbb{E}\left\{w_x(k)\right\} + TV(k)\cos(\phi(k))e^{-\frac{\sigma_\phi^2}{2}}\right)\\ &= (1-\beta_{x,k})\,x(k+1) + \beta_{x,k}x(k+1)\\ &\quad + (1-\beta_{x,k})\,\mathbb{E}\left\{w_r(k+1)\right\} + \beta_{x,k}\,\mathbb{E}\left\{w_x(k)\right\}\\ &\quad +\beta_{x,k}TV(k)\cos(\phi(k))\left(e^{-\frac{\sigma_\phi^2}{2}} - 1\right)\\ &= x(k+1) + \mathbb{E}\left\{w_x(k+1)\right\}.\end{aligned}$$

whereby the lemma follows. ■

From the previous lemma, we see that the average of the estimation error at time $k+1$ depends on the bias of time k. To avoid a blow-up of the bias, we need to impose a condition on $\beta_{x,k}$. Thus, the absolute value of the average of the estimation error must be non-expansive, which can be easily achieved when $|\beta_{x,k}| \in [0,1]$, and thus $\mathscr{B} = [-1,1]$.

Proposition 3.5: Consider the estimation error (6). Then, the second order moment is

$$\mathbb{E}\left\{w_x^2(k+1)\right\} = \beta_{x,k}^2a_k + 2\beta_{x,k}b_k + \mathbb{E}\left\{w_r^2(k+1)\right\}$$

where

$$\begin{aligned}a_k =& \mathbb{E}\left\{w_r^2(k+1)\right\} + \mathbb{E}\left\{w_x^2(k)\right\}\\ &+ T^2\,\mathbb{E}\left\{\tilde{V}^2(k)\cos^2(\tilde{\phi}(k))\right\}\\ &+ T^2V^2(k)\cos^2(\phi(k))\left(1 - 2e^{-\frac{\sigma_\phi^2}{2}}\right)\\ &- 2\,\mathbb{E}\left\{w_x(k)\right\}\mathbb{E}\left\{w_r(k+1)\right\}\\ &- 2\,\mathbb{E}\left\{w_r(k+1)\right\}TV(k)\cos(\phi(k))\left(e^{-\frac{\sigma_\phi^2}{2}} - 1\right)\\ &+ 2\,\mathbb{E}\left\{w_x(k)\right\}TV(k)\cos(\phi(k))\left(e^{-\frac{\sigma_\phi^2}{2}} - 1\right)\,,\end{aligned} \tag{28}$$

and

$$\begin{aligned}b_k =& -\mathbb{E}\left\{w_r^2(k+1)\right\} + \mathbb{E}\left\{w_x(k)\right\}\mathbb{E}\left\{w_r(k+1)\right\}\\ &+ \mathbb{E}\left\{w_r(k+1)\right\}TV(k)\cos(\phi(k))\left(e^{-\frac{\sigma_\phi^2}{2}} - 1\right)\,.\end{aligned} \tag{29}$$

Proof: The result follows from Lemma 3.2, Eq. (25), and Proposition 3.3. See details in [16]. ■

Finally, to derive an expression for the variance of the estimation error P_2 that we need in optimization problem (9), we define the variables v_r, v_x and v_v as

$$\begin{aligned}v_r(k) \triangleq& w_r(k) - \mathbb{E}\left\{w_r(k)\right\}\,,\\ v_x(k) \triangleq& w_x(k) - \mathbb{E}\left\{w_x(k)\right\}\,,\\ v_v(k) \triangleq& T\tilde{V}(k)\cos\left(\tilde{\phi}(k)\right) - TV(k)\cos\left(\phi(k)\right)\left(1 - e^{-\frac{\sigma_\phi^2}{2}}\right)\,.\end{aligned}$$

Notice that v_r, v_x and v_v are all zero-mean. Furthermore we indicate their variance as $\sigma_{v_r}^2$, $\sigma_{v_x}^2$ and $\sigma_{v_v}^2$ respectively.

By substituting the expression for the error bias (27) in Eq. (8) we obtain

$$
\begin{aligned}
P_2 =& \mathbb{E}\left\{\left(w_x(k+1) - \mathbb{E}\left\{w_x(k+1)\right\}\right)^2\right\} \\
=& \mathbb{E}\left\{\left(\left(1-\beta_{x,k}\right) v_r(k+1) + \beta_{x,k} v_x(k) + \beta_{x,k} v_v(k)\right)^2\right\} \\
=& \mathbb{E}\left\{\left(1-\beta_{x,k}\right)^2 v_r^2(k+1) + \beta_{x,k}^2 v_x^2(k) + \beta_{x,k}^2 v_v^2(k)\right. \\
& +2\left(1-\beta_{x,k}\right)\beta_{x,k} v_r(k+1) v_x(k) \\
& \left. +2\left(1-\beta_{x,k}\right)\beta_{x,k} v_r(k+1) v_v(k) + 2\beta_{x,k}^2 v_x(k) v_v(k)\right\} .
\end{aligned}
$$

Since v_r, v_x and v_v are independent and $\mathbb{E}\left\{v_r\right\} = \mathbb{E}\left\{v_x\right\} = 0$, we can write

$$
\begin{aligned}
P_2 =& \mathbb{E}\left\{\left(1-\beta_{x,k}\right)^2 v_r^2(k+1) + \beta_{x,k}^2 v_x^2(k) + \beta_{x,k}^2 v_v^2(k)\right\} \\
=& \left(1-\beta_{x,k}\right)^2 \sigma_{v_r}^2(k+1) + \beta_{x,k}^2 \sigma_{v_x}^2(k) + \beta_{x,k}^2 \sigma_{v_v}^2(k).
\end{aligned}
$$

IV. Solution of the Pareto Optimization Problem

In the previous section we characterized the statistical moments of the estimation errors. Now, we have all the elements to provide the solution for problem (9).

It is easy to show that the optimization problem is convex. It follows that, after some tedious computations, the optimal value of $\beta_{x,k}$, for a fixed $\rho_{x,k}$, is

$$
\beta_{x,k}^*(\rho_{x,k}) = \max\left(-1, \min\left(\xi, 1\right)\right) , \tag{30}
$$

with

$$
\xi = \frac{2\left(1-\rho_{x,k}\right)\sigma_{v_r}^2(k+1) - 2\rho_{x,k}\gamma\,\mathbb{E}\left\{w_r(k+1)\right\}}{2\left(1-\rho_{x,k}\right)\eta + 2\rho_{x,k}\gamma^2} ,
$$

where

$$
\eta \triangleq \sigma_{v_r}^2(k+1) + \sigma_{v_x}^2(k) + \sigma_{v_v}^2(k) ,
$$

and

$$
\begin{aligned}
\gamma \triangleq& - \mathbb{E}\left\{w_r(k+1)\right\} + \mathbb{E}\left\{w_x(k)\right\} \\
& + TV(k)\cos(\phi(k))\left(e^{-\frac{\sigma_\phi^2}{2}} - 1\right) .
\end{aligned}
$$

Note that the optimal value of $\beta_{x,k}$ in (30) depends on $\rho_{x,k}$. The best value of $\rho_{x,k}$ is found by building the Pareto trade-off curve and selecting the “knee-point” on this curve [11]. Thus, we choose $\rho_{x,k}^*$ such that P_1 and $P2$ computed in $\beta_{x,k}^*(\rho_{x,k}^*)$ are $P_2 \simeq P_1^2$. This is given by the solution to the following optimization problem

$$
\min_{\rho_{x,k}} \left(P_2(\beta_{x,k}^*(\rho_{x,k})) - P_1^2\beta_{x,k}^*(\rho_{x,k})\right)^2 , \tag{31}
$$

where we have evidence that $P1$ and $P2$ are computed in $\beta_{x,k}^*(\rho_{x,k})$. This problem is highly non-linear. Simple numerical procedures, based on a discrimination of $\rho_{x,k}$ [11], can be employed to approximate the optimal Pareto coefficient $\rho_{x,k}^*$.

Finally, the values of the optimal $\beta_{y,k}$ and $\rho_{y,k}$ yielding the y component of the position estimate are obtained similarly to what done so far for the x component.

V. Simulation Results

In this section we present extensive simulation results of our new estimator (1) and (2) based on the optimal fusion coefficient of Eq. (30) and Pareto weighting factor of Eq. (31).

The parameter values used in the numerical simulations are the following:

- Speed measurement noise standard deviation $\sigma_v = 0.05$ m/s, to model the worst-case performance of an odometry sensor, [10].
- Orientation measurement noise standard deviation $\sigma_\phi = \pi/8$ rad, a relatively high value to model the worst-case performance of a magnetometer subject to disturbances due to the environment, see, e.g., [17], [18].
- Ranging noise model parameters (in eq. (14)), [8], [13]: $\sigma_0 = 0.25$, $\kappa_\sigma = 0.25$.
- Sampling time interval $T = 0.1$ s.

The following two scenarios have been considered:

(A) Linear trajectory with constant speed of 0.1 m/s, in Fig. 3.
(B) more complex and realistic piece-wise-linear (PWL)-acceleration trajectory with a maximum acceleration of 0.5 m/s^2, in Fig. 6.

The operation of the proposed method for selecting a point on the Pareto trade-off curve, by solving (31) numerically, is shown in an example in Fig. 2.

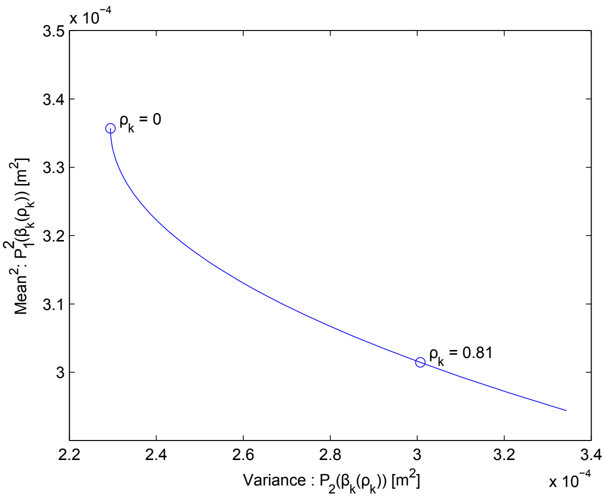

Fig. 2. Pareto tradeoff curve at an iteration k. Each point of this curve is calculated for a different value of ρ_k. In this specific example, $\rho_k^* = 0.81$ is chosen to make the estimator variance P_2 and the square of the mean P_1^2 approximately equal.

Numerical simulation results show that the proposed method yields a good trade-off between variance and bias of the estimated position.

The estimated trajectory, absolute error and empirical Cumulative Distribution Function (CDF) obtained by our new method for scenario A are shown in Fig. 3 – 5. In this scenario, a root-mean-squared error (RMSE) of 4.0 cm in the position estimate over the entire trajectory has been obtained by our method.

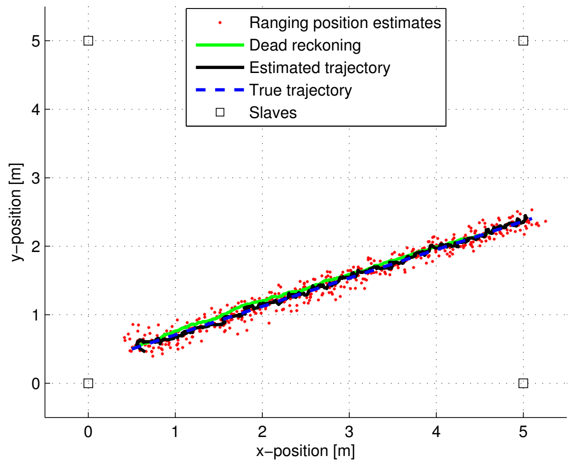

Fig. 3. Pareto optimization sensor fusion technique in the linear trajectory case, scenario A. The proposed method closely tracks the true trajectory.

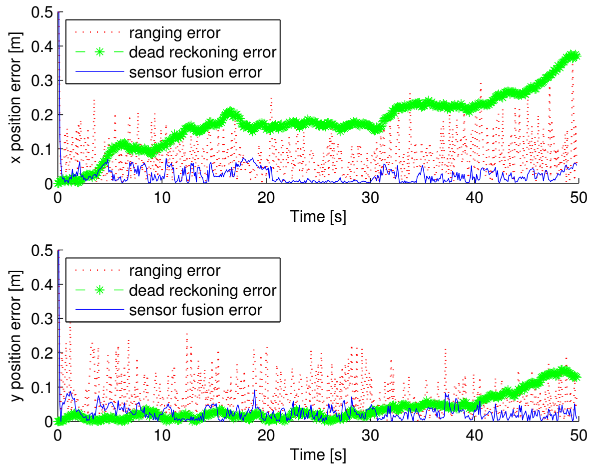

Fig. 4. Absolute errors in scenario A, for the x and y components. The dead-reckoning estimates, asterisks, have a high bias, while the WLS ranging estimates, dashed line, present a higher variance, but a reduced bias. The proposed sensor fusion technique is able to reach a good tradeoff between estimator bias and variance.

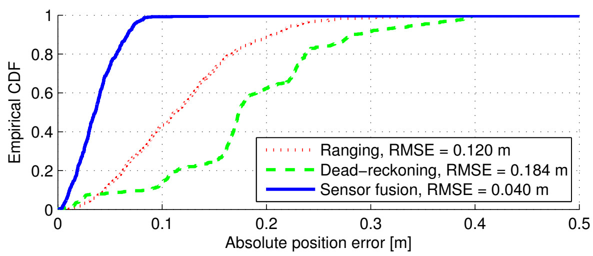

Fig. 5. Empirical CDF of the absolute positioning error in 2D for scenario A. Using the proposed method (blue line), about 95% of the position estimates has an error smaller than 7 cm, and the performance is improved with respect to both the ranging and the dead-reckoning stand-alone systems.

Furthermore, in Fig. 6 the more realistic PWL-acceleration case is shown, where an RMSE of 5.5 cm is obtained.

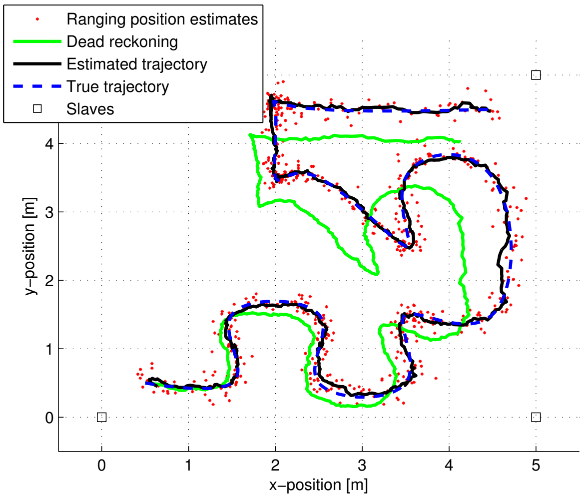

Fig. 6. PWL-acceleration trajectory case, scenario B. The proposed sensor fusion technique closely tracks the true trajectory with RMSE = 5.5 cm.

A. Performance Comparison

The method proposed in this paper has been compared to several other methods, in the two scenarios considered in the previous section. To provide an extensive comparison, the simulations were performed for different values of the sampling period T and of the speed V for scenario A, whereas for scenario B the simulations were performed for different values of T and of the maximum acceleration. Furthermore, for each configuration, the simulations were performed over 10 realizations of the random noises and the resulting RMSE values were averaged.

The sensor fusion methods available from the literature that we have considered are the following:

- the fusion method we proposed in [16], with the addition of the WLS in the ranging estimates, in order to provide a fair comparison with our new method proposed in the present paper.
- Extended Kalman Filter (EKF) [19], where the measured speed and orientation data are employed as inputs, and the ranges as measurements. No motion model is assumed.
- Unscented Kalman Filter (UKF) [20], with the same state-space model as the EKF above.
- Loosely coupled Kalman Filter (LC-KF), where the range measurements are preprocessed with the WLS algorithm of the present paper, to obtain preliminary position measurements, therefore using a linear measurement equation. The usual linearization is performed in the calculation of the input noise matrix.

Notice that the methods based on the Kalman filter theory have been implemented without assumptions about the motion model, to provide a fair comparison with the developed method of this paper. If prior information about motion models is

TABLE I
AVERAGE EXECUTION TIME PER ITERATION [S].

EKF	*[16] with WLS*	*LC-KF*	*UKF*	*Pareto Opt.*
1.3×10^{-4}	1.7×10^{-4}	1.9×10^{-4}	4.2×10^{-4}	4.5×10^{-4}

included in the state space formulation, the Kalman approaches might be able to provide better performance.

Comparisons for two selected simulation configurations are provided in Fig. 7 and 8, as obtained in the two scenarios with different values of T. The proposed Pareto method outperforms the other methods, while the KF-based methods have similar performance with respect to each other.

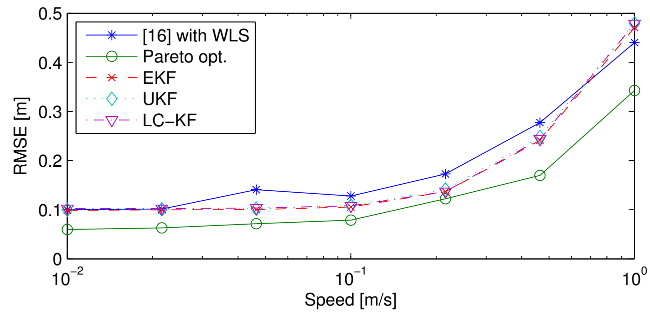

Fig. 7. RMSE vs speed of the different methods for Scenario A with T = 0.5 s.

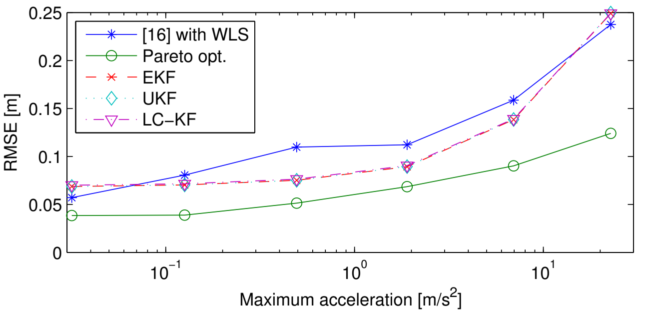

Fig. 8. RMSE vs maximum acceleration of several methods for Scenario B with T = 0.1 s.

However, the proposed Pareto method requires the highest computational load among the implemented techniques, as shown in the average simulation execution times listed in Table I. All the methods have been implemented in Matlab® on an Intel® Core2 Duo, 2.40 GHz PC with 3 GB of RAM.

VI. CONCLUSION

In this paper, we proposed a new sensor fusion method based on Pareto-optimization. We showed that an accurate tracking of a mobile node position was achieved while providing a good tradeoff between estimation error bias and variance. The method does not require assumptions on the motion model, therefore it is general and applicable to several motion scenarios. We showed that it outperformed classical approaches based on the Kalman filter theory.

Future work includes an extensive evaluation of the tradeoff between accuracy and computational complexity, and the characterization of the fundamental performance limitations (Cramer-Rao lower bounds) of the proposed fusion method.

VII. ACKNOWLEDGEMENTS

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