This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE INFOCOM 2009 proceedings.

Reliable Broadcast of Safety Messages in Vehicular Ad Hoc Networks

Farzad Farnoud (Hassanzadeh) and Shahrokh Valaee

***Abstract*—Broadcast communications is critically important in vehicular networks. Many safety applications need safety warning messages to be broadcast to all vehicles present in an area. Design of a Medium Access Control (MAC) protocol for vehicular networks is an interesting problem because of challenges posed by broadcast traffic, high mobility, high reliability and low delay requirements of these networks. In this article, we propose a topology-transparent broadcast protocol and present a detailed mathematical analysis for obtaining the probability of success and the average delay. We show, by analysis and simulations, that the proposed protocol outperforms two existing protocols for vehicular networks with topology-transparent properties and provides reliable broadcast communications for delivering safety messages under load conditions deemed to be common in vehicular environments.**

I. Introduction

Each year, road accidents cause approximately 1.2 million deaths worldwide [1]. Despite the large number of these fatalities, they are, in principle, avoidable. Of 43,000 annual road accident deaths in the US, 21,000 are caused by roadway departures and intersection related incidents [2]. This number can be significantly lowered by deploying active/cooperative safety systems enabled by vehicular communications.

Unlike conventional safety systems, which try to minimize the casualties of collisions by using devices such as air bags and shock absorbers, active/cooperative safety systems are capable of preventing accidents. These systems are part of a broad range of emerging communications, electronics, and informatics technologies, unified under Intelligent Transportation Systems (ITS), being developed to fundamentally enhance safety and productivity in surface transportation.

ITS development relies, at its core, on a communication platform enabling fast and reliable communication in vehicular environments. Dedicated Short Range Communication (DSRC) standard, adopted by IEEE and ASTM International (ASTM E 2213-03 [3]), provides such communication platform for ITS [4].

Based on DSRC standard, 75MHz bandwidth at 5.9GHz is allocated to public and private vehicular communication applications [5]. The 75MHz bandwidth is divided into seven 10MHz channels. Among the seven designated channels, one channel is the control channel (ch 178) used mainly for broadcast traffic.

This works has been done with partial support from AUTO21 Network of Centers of Excellence and Mark IV Industries, IVHS Division, Canada.

F. Farnoud was with the University of Toronto, Toronto, Canada. He is now with the University of Illinois at Urbana-Champaign. (email: farnoud@ifp.uiuc.edu)

S. Valaee is with the University of Toronto, Toronto, Canada. (email: s.valaee@utoronto.ca)

Our goal in this work is to provide a Medium Access Control (MAC) protocol in ad hoc mode for broadcast communication. Such a MAC protocol must be able to reliably deliver safety-critical messages. Due to stringent delay requirements of safety traffic, transmission delay of a protocol designed for vehicular communication must be very low. Furthermore, a vehicular MAC must be capable of supporting mobility and effectively coordinating tens of sources of broadcast traffic.

The ad hoc MAC protocol presented here is a topology-transparent protocol that uses positive orthogonal codes as its transmission patterns as explained in Section IV. This protocol was introduced and discussed for vehicular communications in [6] and [7] wherein elementary analysis and simulation results were presented and code generation and improvements on the basic protocol were described. In this work, we provide a detailed analysis for probability of success and average delay that can be generalized to other topology-transparent protocols with a constant number of transmissions in each frame. We present simulation results showing how the proposed topology-transparent protocol performs in vehicular environments. Furthermore, we show that simulation results and analysis are in agreement.

The rest of this article is organized as follows. In the next section, we discuss the characteristics of safety traffic, produced by active/cooperative safety systems, that determine the capabilities required of a vehicular communication network. We review the related works in Section III. The proposed broadcast protocol is explained in Section IV. Analytical performance study and simulation results are presented in Sections V and VI, respectively. Finally, we conclude this paper in Section VII.

II. Characteristics of Safety Traffic in Vehicular Networks

We assume that safety systems installed on each vehicle require a map of relative positions of neighboring vehicles. Neighbors of a vehicle are vehicles geographically “close” to it. Here, two vehicles are neighbors if they are within each other’s communication range.

If positions of neighboring vehicles are known to the safety system, many collisions can be avoided. If the velocities of the neighbors is also known, each vehicle can predict future positions and avoid possible collision-prone situations.

Building a local map in each vehicle requires that: 1) each vehicle be able to discover its own absolute or relative position, and 2) vehicles be able to broadcast position information to their neighbors. Discovering the position of a vehicle can be done via GPS [8], radio ranging techniques [9], and/or, radar.

Our focus in this work is on designing a MAC protocol that is capable of delivering position information messages, as well as other data.

At 100km/hr, a vehicle moves 6m (approximately the accuracy of GPS) in 216ms. Therefore, update frequency of approximately 5 messages/second guarantees accurate and up-to-date maps.

Since vehicular update messages need to deliver limited information such as vehicle ID, message ID, position, velocity, road condition, warning, etc, we argue that the size of these messages is well under a few hundred bytes.

Location information, on Earth's surface with 1cm resolution, can be delivered with $\log_2(2\pi 6.4 \times 10^6\,\mathrm{m}/10^{-2}\,\mathrm{m}) + \log_2(\pi 6.4 \times 10^6\,\mathrm{m}/10^{-2}\,\mathrm{m}) = 62.81 \leq 63$ bits where $6.4 \times 10^6 m$ is the Earth's radius. Relative location information within 100m (in a 200m×200m square centered at the reference point) in a Cartesian system with 1cm resolution can be delivered with $2\log_2(200\,\mathrm{m}/10^{-2}\,\mathrm{m}) = 28.6 \leq 29$ bits.

Assuming each vehicle transmits its position in absolute form, its velocity, and the relative positions and velocities of vehicles immediately in front, behind, left, and right, $63 + 29 + 4(29 + 29) = 324$ bits or 41 bytes need to be transmitted. Adding 2 bytes for the ID of each vehicle, in total 51 bytes are needed. If 49 bytes are allocated for other uses, such as detection of an obstacle and its position, emergency car and its position, emergency braking, etc, the length of the safety message is about 100 bytes. Therefore, in vehicular communications, safety messages are short compared to data or multimedia messages.

An automatic safety system is successful if it can recognize a dangerous situation before the driver of a vehicle does. The driver reaction time, i.e., the time from the moment an event occurs until the moment a decision is made, is between 500ms to 1.2s, depending on how unexpected the event is [10]. Noting that a warning message alerting a driver, itself needs to be processed, communication delay must not exceed 100-200ms.

III. Related Work

A major difference between an ad hoc network for safety messages in vehicular environments and a conventional ad hoc network is that in vehicular networks, as discussed earlier, traffic is of broadcast type; routine safety messages are issued from all vehicles several times per second and are intended for all their neighbors. Transmission of safety messages must be reliable and with very low delay. Conventional random access MAC protocols for ad hoc networks are not designed to handle broadcast traffic from many nodes in the network. For example, in IEEE 802.11 no mechanism exists to reduce the probability of collision for broadcast traffic.

In IEEE 802.11, Request To Send (RTS) and Clear To Send (CTS) packets can be transmitted before unicast communications to avoid collisions. It may seem straightforward to add RTS/CTS handshake to broadcast communications as well. However, in vehicular communication, the length of broadcast messages is short and comparable to that of RTS. Therefore, the probability of collision is not significantly lower for RTS packets. The short length of messages also contributes to inefficiency since the payload (safety message) is not significantly larger than the overhead (RTS+CTS). Furthermore, RTS/CTS handshake needs to be performed with more than one receiver to obtain the same reliability as that of unicast communication. Hence protocols such as Broadcast Medium Window (BMW) [11] and Batch Mode Multicast MAC (BMMM) [12], which rely on RTS/CTS handshake with multiple nodes, are not effective methods for the delivery of short broadcast messages in a vehicular environment. Even unicasting such short messages with 802.11 approach is very inefficient. According to a model devised by Bianchi [13], the maximum bandwidth utilization of 802.11a with RTS/CTS handshake, at 54 Mb/s with payload size of 100 bytes, is less than 7% [5]. Multiple RTS/CTS handshakes, as proposed by the above protocols, will further decrease efficiency.

While random access protocols have shortcomings in the context of vehicular communications, topology-transparent broadcast protocols appear to be a suitable option because they support broadcast communications without significant penalty due to mobility.

In topology-dependent protocols, scheduling is a function of the detailed topology of the network. Therefore, changes of topology require recalculation of scheduling. In highly dynamic mobile ad hoc networks, transmissions needed for rescheduling cause significant inefficiency and delay. To solve this problem, Chlamtac and Fargò [14] proposed a topology-transparent algorithm in which scheduling is a function of only high-level network parameters, namely, the maximum number of nodes in the network, M, and the maximum number of neighbors of a node, N, i.e., the nodes within its communication range. The fundamental idea behind topology-transparent broadcast is transmitting a packet several times in a frame in a manner that the number of possible collisions is smaller than the number of transmissions.

In the protocol proposed in [14], time is divided into frames, each containing w subframes and each subframe is divided into w timeslots. Each node is assigned w timeslots, here called a *transmission pattern*, one in each subframe, in which they are allowed to transmit. Transmission patterns are constructed from polynomials of order λ over the Galois Field $\mathrm{GF}(w)$ so that each two nodes have at most λ timeslots in common in their transmission patterns, and therefore, each two users may collide at most in λ timeslots. If $w > \lambda N$, each node is guaranteed collision-free transmission in at least one timeslot. Since there are $w^{\lambda+1}$ polynomials of degree λ, we have $M \leq w^{\lambda+1}$.

In Chlamtac and Fargó [14] the goal is the minimization of frame length while satisfying the above conditions. Ju and Li [15] attempt to maximize the throughput by guaranteeing more collision-free timeslots in each frame. Youn and Bose [16] claim that the use of constant-weight codes as transmission patterns that satisfy the condition $w > \lambda N$ improves the performance of topology-independent protocols. Other design methods, using Galois field theory and Latin squares theory, and an extension to multi-channel networks are proposed by Cai et al [17].

The above protocols eliminate the dependence of scheduling on topology and perform efficiently when all nodes have data to send but, in practical networks with random loads, they may have inferior performance compared to random access protocols in terms of throughput and delay. The attempt to guarantee free timeslots for all users in every frame results in long frames, and therefore, low throughput and high delay. By combining random access protocols and topology-transparent algorithm, we introduce a protocol based on constant-weight codes that is capable of ensuring reliable broadcast in highly mobile networks while maintaining low delay.

We compare the performance of the proposed protocol with two random repetition protocols, namely, Synchronous p-Persistent Retransmission (SPR) and Synchronous Fixed Retransmission (SFR), proposed [18] [19] [20] to solve said broadcast problems in vehicular environments. In a random repetition protocol, time is divided into frames with L timeslots. The timeslots in which a node is allowed to transmit, i.e., its transmission pattern, are chosen randomly. In SPR, a node with a packet to send transmits the packet in each timeslot in a frame with probability p and remains idle with probability $1-p$. In SFR, the packet is transmitted w times in a frame, i.e., w timeslots are randomly chosen out of the L available timeslots for repeated transmissions of the packet.

It is shown in [20], via simulation, that SFR decreases the probability of failure by one order of magnitude, compared to IEEE 802.11a. The performance of SPR is worse than SFR but better than IEEE 802.11a. Although each message is repeated several times, the overhead may still be less than many of the protocols that rely on multiple RTS/CTS handshakes to transmit a broadcast packet because of the short length of safety messages. SFR and SPR are robust against mobility because they also exhibit topology-transparent properties. However, these protocols, unlike topology-transparent protocols mentioned above, do not attempt to combat collision, resulting in relatively high probability of collision.

In the following, as it is the case with other topology-transparent protocols, we only consider synchronous protocols. In a synchronous protocol, timeslots and frames are synchronized among different users. Synchronization can be achieved using a variety of methods. One that is particularly appealing to vehicular communication applications is Global Positioning System (GPS) [8] because many vehicles are already equipped with GPS devices and more will be equipped in the future.

IV. BROADCAST USING POSITIVE ORTHOGONAL CODES

Youn and Bose [16] propose the use of constant-weight codes as transmission patterns in topology-transparent networks. A transmission pattern with w transmissions in a frame with L timeslots can be obtained from a binary codeword of length L and weight w; each 1 corresponds to a transmission and each 0 corresponds to idle/receive mode. Consider two codewords $\mathbf{x}$ and $\mathbf{y}$, from a constant-weight code with weight w, length L, and minimum distance 2δ, used as transmission patterns. Since the Hamming distance between $\mathbf{x}$ and $\mathbf{y}$ is at least 2δ, the maximum number of collisions, λ, between two users transmitting with these patterns is $w-\delta$. Therefore, a topology-transparent scheduling can be obtained from a code with minimum distance $2\delta = 2(w-\lambda)$ where $\lambda N < w$.

In general however, as long as the weights of all codewords are larger than $N\lambda$, it is not required for the code to have a constant weight. The critical characteristic is that the correlation between two codewords be bounded. Hence we define,

Definition 1. *A* ***Positive Orthogonal Code (POC)****, C, is a code in which each two codewords $\mathbf{x}$ and $\mathbf{y}$ satisfy $\sum_{i=1}^{L} x_i y_i \leq \lambda$ where L is the length of the code and λ is the maximum cross correlation.*

Codes with variable weight can be used to establish different levels of Quality of Service (QoS) in a topology-transparent network [7] and to increase the number of available codewords in the network. Moreover, the above definition can be modified to obtain asynchronous positive orthogonal codes which can be used in asynchronous networks. In this work however, we only consider constant-weight POCs with $\lambda = 1$, which is equivalent to constant-weight codes with minimum distance $2(w-1)$.

To accommodate all users in the network, the total number of users, M, must be less than or equal to the cardinality of the code. The size of the largest constant-weight code with given parameters is unknown in the general case [21]. Johnson [22] provides an upper bound for the number of codewords in such code

$$\|C\| \leq \left\lfloor \frac{L}{w} \left\lfloor \frac{L-1}{w-1} \cdots \left\lfloor \frac{L-\lambda}{w-\lambda} \right\rfloor \cdot \right\rfloor \right\rfloor \tag{1}$$

where $\lfloor x \rfloor$ is the largest integer less than or equal to x. For example, for $L = 64$ and $w = 6$, code cardinality is bounded by 128 and for $L = 128$ and $w = 6$, code cardinality is bounded by 525. Note that strict orthogonality, i.e., $\lambda = 0$, leads to a very low code cardinality, namely, at most L/w.

Previous topology-transparent protocols, as mentioned earlier, require that $1+N\lambda \leq w$. For constant-weight codes, since $M \leq \|C\|$,

$$M \leq \frac{L(L-1)\cdots(L-\lambda)}{(1+N\lambda)N\lambda\cdots(1+(N-1)\lambda)} \approx \left(\frac{L}{N\lambda}\right)^{\lambda+1}, \tag{2}$$

and hence, $L \geq N\lambda M^{\frac{1}{\lambda+1}}$. If $\lambda = 1$, this condition implies that $L \geq N\sqrt{M}$. Hence if $M < N^2$, conventional TDMA, which has frame length equal to M, is more efficient than topology-transparent scheduling. For example, for a network with $N = 30$, the number of users must be at least 900 to justify the use of topology-transparent protocols based on constant-weight codes with $\lambda = 1$. However, the condition $1+N\lambda \leq w$ assumes that all nodes have a packet for transmission in every frame, which in many practical networks does not hold. By removing this constraint, in networks with random load, higher efficiency can be obtained while maintaining reliability. In the following, we find the probability of failure and delay for POC-based topology-transparent networks that do not guarantee a free timeslot in each frame for each user and show that it is possible to maintain high probability of success without requiring $1+N\lambda \leq w$.

A. Distributed Code Assignment

The number of nodes, M, in a vehicular network is extremely large. Therefore, it is impossible to assign distinct timeslots (TDMA) or transmission patterns (topology-transparent) to all nodes. However, the limited coverage of wireless communication enables us to spatially reuse codewords. When codewords are reused, with the changes of topology, code assignment needs to be updated to ensure that nodes within each other's radio coverage do not use the same codeword. Since the number of available codewords can be significantly larger than the number of timeslots in conventional TDMA, reassignment of codewords in a topology-transparent network needs to be performed with much less frequency than the reassignment of timeslots in conventional TDMA, resulting in less overhead.

To design a distributed code assignment protocol, we assume a subset C_a of code C is reserved only for network association. Once a vehicle enters a road, it randomly selects a tentative codeword from C_a. In the network association phase, the vehicle joining the network can start transmitting its packets as usual using the chosen codeword. However, it must also acquire a permanent codeword that is unique within its two-hop communication range.

To obtain information about codewords used in the two-hop neighborhood, the joining node issues Code Information Requests (CIQ). Every node i receiving a CIQ transmits a Code Information Response (CIR) which contains the index of its codeword and its ID, the codewords of the node's one-hop neighbors, and the ID of those neighbors. The codewords indicated in the CIR received from node i, denoted by C_i, are used by other nodes and hence unusable by the joining node. After receiving several of these packets, the node with the tentative codeword chooses a permanent codeword from the set $C_p = C \backslash C_a \backslash \cup_i C_i$.

While network association is performed only once when the vehicle enters the road, each node with a permanent codeword also periodically transmits a CIR with frequency of once every few seconds. This enables the network to adapt to topology changes. If a node with a permanent codeword discovers that its codeword is being used by one of its two-hop neighbors, it releases that codeword and chooses another one from C_p.

Code Information Response Window: When a joining node issues a CIQ, if all neighbors transmit CIR packets in the next frame, the additional load caused by several immediate CIR packets results in performance degradation. To resolve this issue, we introduce the Code Information Response Window (CIRW). Each node that receives a CIQ, sets a counter to a random number uniformly chosen between 1 and CIRW. At the end of each frame, the counter is decreased by one. When the counter reaches zero, CIR is transmitted in the next frame. The joining node determines its permanent codeword after CIRW frames have passed.

The number of permanent codewords required depends on the desired communication range, R_c. The cardinality of C must be large enough to support all vehicles in length $4R_c$ of a road. For example, assuming $R_c = 100m$ and adjacent cars in the same lane are 30m apart, in a four-lane road at least $4 \times 4 \times 100/30 \approx 53$ permanent codewords are required. If more codewords are available, the frequency of CIR transmissions needed to keep the code assignment up-to-date, and hence the overhead due to CIR transmissions, decreases.

If code cardinality is not large enough to allocate sufficient number of codes for C_a, more codewords can be added to the code with higher cross-correlation for use in code assignment phase. Since these codes are in use only for a short time, performance degradation caused by their higher cross-correlation is minimal.

B. Adaptive Elimination

If two users have transmission patterns that include transmission in a common timeslot, a collision is likely to happen. However, if one user has a transmission before the common timeslot in its transmission pattern and includes some information in the transmitted packet in this timeslot with which the second user can identify the codeword used by the first user, provided that the second user successfully receives this transmission, it can prevent the collision simply by not transmitting in the common timeslot. We call this method *Adaptive Elimination*.

Adaptive elimination increases reliability by eliminating transmissions in timeslots that may potentially result in collision. To enable this, the codebook must be stored in all nodes and nodes must transmit the index of their codeword and indicate which timeslots are disabled. Each node adds a *codeword indicator* field with format (index of codeword, enabled/disabled timeslots) to the header of its data packets to inform other nodes of its codeword. Each part of this field has a predetermined length. It can be argued that adaptive elimination adds insignificant overhead.

V. Performance Study

In this section, we analytically study the performance of SPR, SFR, and the POC-based topology-transparent protocol. Methods used in this section are general and can also be applied to other topology-transparent protocols. Here, we do not consider adaptive elimination and also neglect the overhead due to code assignment. Furthermore, we assume a network that is interference limited and an ideal wireless channel, which carries signals with no attenuation and no noise. Hence the effect of noise on performance is neglected. As the traffic model, we use the binomial distribution. However, other traffic models can also be used.

A. Probability of Success

Transmission in a *timeslot* is successful if only one node transmits in that timeslot. If two nodes transmit in the same timeslot, a collision occurs and both transmissions fail. Since the channel is assumed ideal, all nodes are able to receive a transmission when there is no collision. In each frame, each node transmits in several timeslots. The message is successfully transmitted if at least one of the transmissions in the *frame* is successful. Probability of success is defined as the number of messages successfully transmitted by a node divided by the number of messages that the node has attempted to transmit. Probability of success depends on the number of *interfering users*, i.e., the number of users transmitting in the same frame as the desired user.

In order to obtain the probability of success, we introduce the following events. $\mathcal{S}$ is the event that at least one transmission is successful in a frame. When discussing protocols with exactly w transmissions in a frame, such as POC and SFR, $\mathcal{S}_i$ denotes the event that the ith transmission among w transmissions is successful and $\hat{\mathcal{S}}_i$ denotes the event that the ith transmission is the first successful transmission. When discussing SPR, $\mathcal{S}_i$ is the event that the transmission in the ith timeslot is successful and $\hat{\mathcal{S}}_i$ is the event that the first successful transmission occurs in the ith timeslot.

Assume that the desired user is transmitting a message. The probability of success, P_s, can be written as

$$P_s = \sum_{n=0}^{N-1} P_n(\mathcal{S}) P(\boldsymbol{n} = n) \tag{3}$$

where, $\boldsymbol{n}$ is the random variable denoting the number of interfering users, $P_n(\mathcal{S})$ is the probability of the event $\mathcal{S}$ given that there are n interfering users, and N is the number of the neighbors of the desired user.

The probability mass function of $\boldsymbol{n}$ depends on the traffic model.

1) Probability of Success for SPR: For SPR, the desired transmitter is successful in the ith timeslot if it transmits in that timeslot and all other users are silent. Assuming each user transmits with probability p in each timeslot, the probability of success in a timeslot, s, is

$$s \triangleq P_n(\mathcal{S}_i) = p\,(1-p)^n \qquad 1 \le i \le L. \tag{4}$$

The desired user fails to transmit its packet successfully if it fails in all L timeslots. The probability of failure in all timeslots is $(1-s)^L$. Therefore, the probability of success, $P_n(\mathcal{S})$, is

$$P_n^{(SPR)}(\mathcal{S}) = 1 - (1-s)^L = 1 - \left(1 - p\,(1-p)^n\right)^L . \tag{5}$$

2) Probability of Success for SFR and POC: In SFR and topology-transparent protocols with exactly w transmissions such as POC, since there are exactly w repetitions, different timeslots are not independent. Therefore, the probability of success cannot be obtained as easily as that of SPR.

The probability that at least one transmission is successful among w transmissions, $P_n(\mathcal{S}_1 \cup \cdots \cup \mathcal{S}_w)$, can be written as

$$\sum_{k=1}^{w} (-1)^{k+1} \sum_{\{a_1,\ldots,a_k\} \in \binom{[w]}{k}} P_n(\mathcal{S}_{a_1} \cap \cdots \cap \mathcal{S}_{a_k}) \tag{6}$$

where $\binom{[w]}{k}$ is the set of all k-subsets of $\{1, \cdots, w\}$. As we will see, $P_n(\mathcal{S}_{a_1} \cap \cdots \cap \mathcal{S}_{a_k})$ does not depend on $a_1, \ldots, a_k$ but rather only on k. Therefore, by defining

$$\gamma_k = P_n(\mathcal{S}_{a_k} \cap \cdots \cap \mathcal{S}_{a_1}), \quad \{a_1, \ldots, a_k\} \in \binom{[w]}{k} \tag{7}$$

we have

$$P_n(\mathcal{S}) = P_n(\mathcal{S}_1 \cup \cdots \cup \mathcal{S}_w) = \sum_{k=1}^{w} (-1)^{k+1} \binom{w}{k} \gamma_k. \tag{8}$$

Next, we find γ_k for SFR and POC and substitute it in (8) to obtain success probability for SFR and POC.

a) SFR: The probability that a certain interfering user, i.e., a user that transmits in the same frame, does not transmit in timeslots $a_k, a_{k-1}, \ldots, a_1$ with the desired user is equal to $\binom{L-k}{w} / \binom{L}{w}$ where the transmission pattern of the interfering user can be any of the $\binom{L}{w}$ patterns with equal probability. Among the possible patterns, $\binom{L-k}{w}$ patterns do not include transmission in the prohibited timeslots $a_k, a_{k-1}, \ldots, a_1$.

Since the n interfering users are independent,

$$\gamma_k = P_n(\mathcal{S}_{a_k} \cap \mathcal{S}_{a_{k-1}} \cap \cdots \cap \mathcal{S}_{a_1}) = \left(\frac{\binom{L-k}{w}}{\binom{L}{w}} \right)^n \tag{9}$$

Therefore, as claimed earlier, $P_n(\mathcal{S}_{a_k} \cap \mathcal{S}_{a_{k-1}} \cap \cdots \cap \mathcal{S}_{a_1})$ does not depend on $a_1, \ldots, a_k$ but only on k. Hence,

$$P_n^{(SFR)}(\mathcal{S}) = \sum_{k=1}^{w} (-1)^{k+1} \binom{w}{k} \left(\frac{\binom{L-k}{w}}{\binom{L}{w}} \right)^n . \tag{10}$$

b) POC: In POC with $\lambda = 1$, by definition, an interfering user may transmit in only one timeslot in which the desired user also transmits. Consider the a_jth transmission of the desired user. Let p_1 be the probability that a certain interfering user transmits in the same timeslot as the a_jth transmission of the desired user. The probability that the interfering user does not transmit at the same time as any of the transmission timeslots $a_k, a_{k-1}, \ldots, a_1$ of the desired user is $1 - kp_1$. Considering n independent interfering users, we have

$$\gamma_k = P_n(\mathcal{S}_{a_k} \cap \mathcal{S}_{a_{k-1}} \cap \cdots \cap \mathcal{S}_{a_1}) = (1 - kp_1)^n. \tag{11}$$

Note that $P_n(\mathcal{S}_{a_k} \cap \mathcal{S}_{a_{k-1}} \cap \cdots \cap \mathcal{S}_{a_1})$ only depends on k. Substitution in (8) yields

$$P_n^{(POC)}(\mathcal{S}) = \sum_{k=1}^{w} (-1)^{k+1} \binom{w}{k} (1 - kp_1)^n . \tag{12}$$

we should now find p_1. Assume that by reordering, the codeword of the desired user is written as $\overset{\xleftrightarrow{w}}{1111\cdots11}\overset{\xleftrightarrow{L-w}}{000\cdots0}$. Then, p_1 is equal to the number of codewords with form $\overset{\xleftrightarrow{w}}{1000\cdots00}\overset{\xleftrightarrow{L-w}}{\mathsf{xxx}\cdots\mathsf{x}}$ after reordering, divided by the total number of possible codewords. Codewords with this form are common in the first timeslot. Therefore, they cannot have any other common timeslot among the $L-w$ timeslots denoted by x. Since $w-1$ ones must be placed in the $L-w$ timeslots denoted by x with no overlap, there are at most $\frac{L-w}{w-1}$ codewords with this form. From (1), the total number of codewords is at most $\left\lfloor \frac{L}{w} \left\lfloor \frac{L-1}{w-1} \right\rfloor \right\rfloor$. Therefore, p_1 may be approximated by

$$p_1 \approx \frac{w(L-w)}{L(L-1)}. \tag{13}$$

p_1 can also be empirically obtained from a sample generated code using

$$p_1 \approx \frac{\sum_{i=1}^{\|C\|} \sum_{j=i+1}^{\|C\|} \langle \mathbf{c}_i, \mathbf{c}_j \rangle}{w\binom{\|C\|}{2}} \tag{14}$$

where C is the generated code and vectors $\mathbf{c}_i$ are codewords and $\|C\|$ is the size of the code (number of codewords). The

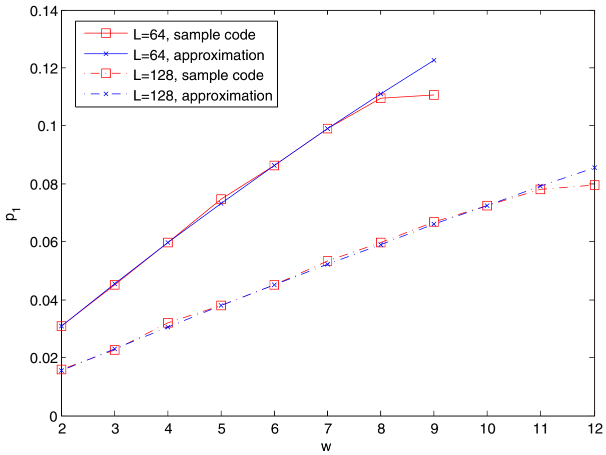

Fig. 1. Approximate and sample values of p_1 for POC

division by w is because p_1 corresponds only to one transmission among w transmissions. Comparison of approximate values and sample values of p_1 is presented in Fig. 1. It is observed that the approximation (13) provide values close to empirical results, especially when w is not very large.

B. Average Delay

When a packet is transmitted several times in a frame, delay $\boldsymbol{D}$, is defined as the first timeslot in which the packet is successfully received. The average delay, D_s, is defined as

$$D_s \triangleq E[\boldsymbol{D}|\mathcal{S}] = \sum_{n=0}^{N-1} D_s(n) P(\boldsymbol{n} = n) \tag{15}$$

where $D_s(n)$ is the average delay of a successful transmission when there are n interfering users. Note that $\boldsymbol{D}$ is not defined when all transmissions in a frame are unsuccessful.

1) SPR: The average delay for SPR, conditioned on successful transmission when there are n interfering users, can be obtained as

$$\begin{aligned} D_s(n) &\triangleq E_n[\boldsymbol{D}|\mathcal{S}] = \sum_{i=1}^{L} i P_n(\hat{\mathcal{S}}_i|\mathcal{S}) \\ &= \sum_{i=1}^{L} \frac{i P_n(\hat{\mathcal{S}}_i \cap \mathcal{S})}{P_n(\mathcal{S})} = \frac{\sum_{i=1}^{L} i P_n(\hat{\mathcal{S}}_i)}{P_n(\mathcal{S})} . \end{aligned} \tag{16}$$

Timeslot i is the first successful timeslot with probability

$$\begin{aligned} P_n(\hat{\mathcal{S}}_i) &= P_n(\mathcal{S}_i \cap \bar{\mathcal{S}}_{i-1} \cap \cdots \cap \bar{\mathcal{S}}_1) \\ &= P_n(\mathcal{S}_i) P_n(\bar{\mathcal{S}}_{i-1}) \cdots P_n(\bar{\mathcal{S}}_1) \\ &= s(1-s)^{i-1} \end{aligned} \tag{17}$$

where we have used (4). By substituting (17) in (16) we obtain

$$D_s^{(SPR)}(n) = \frac{1}{s} - \frac{L(1-s)^L}{1-(1-s)^L} . \tag{18}$$

Note that the right-hand-side of (18) implicitly depends on n through s.

2) SFR and POC: In SFR and topology-transparent protocols such as POC with exactly w transmissions we have

$$\begin{aligned} D_s(n) &\triangleq E_n[\boldsymbol{D}|\mathcal{S}] = \sum_{i=1}^{w} P_n(\hat{\mathcal{S}}_i|\mathcal{S}) E_n[\boldsymbol{D}|\hat{\mathcal{S}}_i] \\ &= \sum_{i=1}^{w} \frac{P_n(\hat{\mathcal{S}}_i)}{P_n(\mathcal{S})} \sum_{j=i}^{L-(w-i)} j P_n(\boldsymbol{D} = j|\hat{\mathcal{S}}_i) \end{aligned} \tag{19}$$

where $P_n(\boldsymbol{D} = j|\hat{\mathcal{S}}_i)$ is the probability that the delay is equal to j when the ith transmission is the first successful transmission. In other words, this is the probability that the ith transmission takes place in the jth timeslot given that the ith transmission is the first successful transmission. To calculate (19) we need to find $P_n(\hat{\mathcal{S}}_i)$ and $P_n(\boldsymbol{D} = j|\hat{\mathcal{S}}_i)$.

To obtain $P_n(\hat{\mathcal{S}}_i)$, we first find the probability that "transmission i is not successful but at least one previous transmission is successful", $P_n(\bar{\mathcal{S}}_i \cap (\mathcal{S}_{i-1} \cup \cdots \cup \mathcal{S}_1))$. We have

$$\begin{aligned} P_n(\bar{\mathcal{S}}_i \cap (\mathcal{S}_{i-1} \cup \cdots \cup \mathcal{S}_1)) &= P_n(\bigcup_{l=1}^{i-1} (\bar{\mathcal{S}}_i \cap \mathcal{S}_l)) \\ = \sum_{k=1}^{i-1} (-1)^{k+1} \sum_{\substack{\{a_1,\ldots,a_k\} \in \\ \binom{[w]}{k}}} & P_n(\bar{\mathcal{S}}_i \cap \mathcal{S}_{a_1} \cap \cdots \cap \mathcal{S}_{a_k}). \end{aligned} \tag{20}$$

Let

$$\begin{aligned} \gamma_k &\triangleq P_n(\mathcal{S}_{a_k} \cap \mathcal{S}_{a_{k-1}} \cap \cdots \cap \mathcal{S}_{a_1}) \\ \eta_k &\triangleq P_n(\bar{\mathcal{S}}_{a_k} \cap \mathcal{S}_{a_{k-1}} \cap \cdots \cap \mathcal{S}_{a_1}) . \end{aligned} \tag{21}$$

Therefore,

$$\begin{aligned} P_n(\bar{\mathcal{S}}_i \cap (\mathcal{S}_{i-1} \cup \cdots \cup \mathcal{S}_1)) &= \sum_{k=1}^{i-1} (-1)^{k+1} \binom{i-1}{k} \eta_{k+1} \\ &= \sum_{k=1}^{i-1} (-1)^{k} \binom{i-1}{k} (\gamma_{k+1} - \gamma_k) \end{aligned} \tag{22}$$

As discussed earlier, because the probability of an interference pattern only depends on the number of timeslots in which two users transmit simultaneously and not the position of those timeslots, γ_k and η_k only depend on k.

Using (22) we can write

$$\begin{aligned} P_n(\hat{\mathcal{S}}_i) &= P_n(\mathcal{S}_i \cap \bar{\mathcal{S}}_{i-1} \cap \cdots \cap \bar{\mathcal{S}}_1) \\ &= 1 - P_n(\bar{\mathcal{S}}_i) - P_n(\mathcal{S}_{i-1} \cup \cdots \cup \mathcal{S}_1) \\ &\quad + P_n(\bar{\mathcal{S}}_i \cap (\mathcal{S}_{i-1} \cup \cdots \cup \mathcal{S}_1)) \\ &= \sum_{k=1}^{i} (-1)^{k+1} \binom{i-1}{k-1} \gamma_k . \end{aligned} \tag{23}$$

Note that (23) can be used to obtain $P_n(\mathcal{S})$. As seen below, the result is the same as in (8).

$$\begin{aligned} P_n(\mathcal{S}) &= \sum_{i=1}^{w} P_n(\hat{\mathcal{S}}_i) = \sum_{i=1}^{w} \sum_{k=1}^{i} (-1)^{k+1} \binom{i-1}{k-1} \gamma_k \\ &= \sum_{k=1}^{w} (-1)^{k+1} \binom{w}{k} \gamma_k \end{aligned} \tag{24}$$

Next, we obtain $P_n(\boldsymbol{D} = j|\hat{\mathcal{S}}_i)$ for SFR and POC. Let $\boldsymbol{T}_i$ be the timeslot in which the ith transmission takes place.

$$P_n(\boldsymbol{D} = j|\hat{\mathcal{S}}_i) = P_n(\boldsymbol{T}_i = j|\hat{\mathcal{S}}_i) = P_n(\boldsymbol{T}_i = j) \qquad (25)$$

The last equality holds because the position of the ith transmission is independent of it being the first successful transmission.

For SFR, since the position of transmissions in the frame is strictly random, we have

$$P_n(\boldsymbol{D} = j|\hat{\mathcal{S}}_i) = P_n(\boldsymbol{T}_i = j) = \frac{\binom{j-1}{i-1}\binom{L-j}{w-i}}{\binom{L}{w}}. \qquad (26)$$

For POC, $P_n(\boldsymbol{T}_i = j)$ may depend on the code. Assuming '1's are distributed evenly in each codeword, we can use an expression identical to that of SFR as an approximation. Finally, substituting (23) and (26) in (19), for POC and SFR, yields $D_s(n)$. On average, messages wait $L/2$ timeslots in a buffer from their arrival at the network interface until the beginning of the next frame. If delay is defined from the moment that a packet arrives at the network interface, $L/2$ must be added to $D_s(n)$.

C. Numerical Results

In this section, we present numerical results for Sections V-A and V-B. Let L and N be 128 and 31 respectively. The value of p_1 for POC is calculated from (13). We assume each vehicle independently makes a local decision, whether or not to transmit its location to neighbor vehicles. Furthermore, we assume these periodical updates are generated according to a Bernoulli model in each frame with probability μ_p. Since the decisions for data transmission are independent, the number of nodes with an active packet in each frame is a Binomial random variable with parameters N and μ_p, where N is the number of cars whose signal can be received by the desired user.

The optimum probabilities of failure for SPR, SFR, and POC, when w changes, are plotted in Fig. 2. The optimum values of SPR and SFR is found when w ranges from 1 to 40. For POC, the optimum value is found when w changes from 2 to 12. POC for $w = 1$ is a trivial case and the code cardinality for $w > 12$ is not big enough to accommodate 31 users. It is observed that, for probability of user activity below 0.4, POC can offer a performance advantage of multiple orders of magnitude.

Fig. 3 shows the average delay of successful transmissions calculated using (15). It is observed that the delay is more or less the same for different protocols. This fact will also be observed in simulation results.

VI. Simulation Results

In Section V, we discussed the theoretical performance of the proposed topology-transparent broadcast protocol as well as similar protocols. As mentioned earlier, for obtaining analytical results, we assumed that nodes communicate in an ideal channel in which every node receives a signal from every other transmitting node. Furthermore, the capture effect and adaptive elimination are neglected in the analytical study.

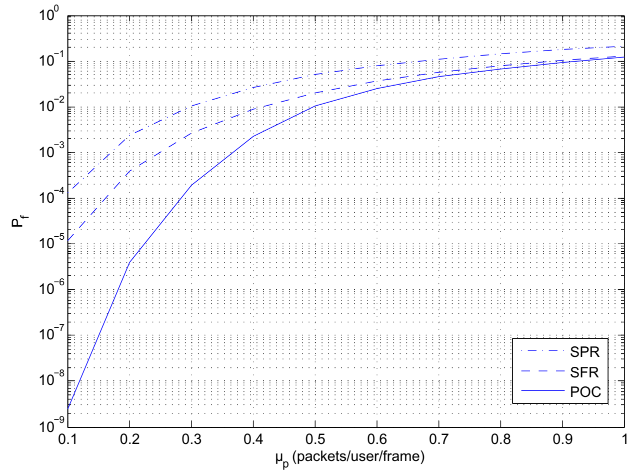

Fig. 2. Optimum probability of failure for POC, SFR, and, SPR, versus load, for $N = 31$, $L = 128$.

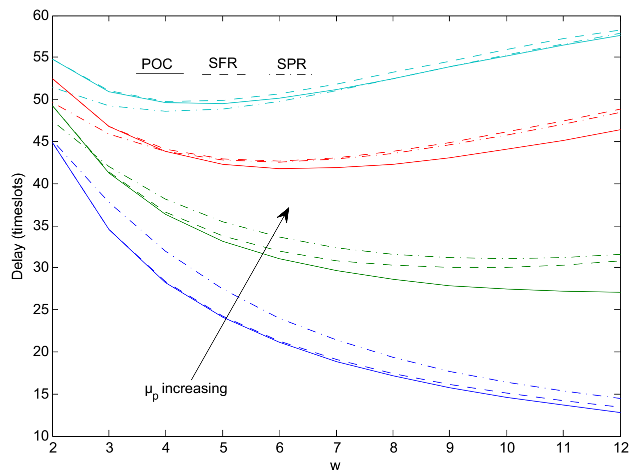

Fig. 3. Delay of successful transmissions for POC, SFR, and, SPR, for $N = 31$, $L = 128$, w ranging from 2 to 12 and $\mu_p = 0.1$, 0.4, 0.7, and 1 (packets/user/frame).

In this section, assuming a Rician channel with capture effect, we present simulation results for different protocols. We also consider the effect of adaptive elimination in the performance of the protocol in the simulation.

In an ideal channel, all simultaneous transmissions result in collision. In a non-ideal channel with capture, however, one of the many simultaneous transmissions may be successful. Note that since we are studying a multiple access system, we neglect the effect of noise in the system. Therefore, collision is the only contributor to packet loss.

A. Channel Model

In a Rician fading channel with Rice factor K, the pdf of the received power, $\boldsymbol{P}$, is

$$f_{\boldsymbol{P}}(P) = \frac{2K}{A^2}\exp\left(-K(1+\frac{2P}{A^2})\right)I_0\left(\sqrt{\frac{8K^2P}{A^2}}\right) \qquad (27)$$

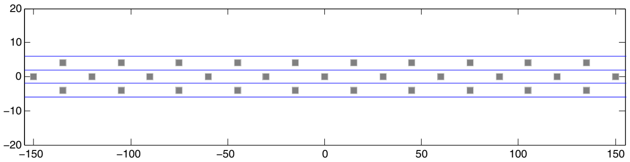

Fig. 4. Map of roadway and cars.

where A is the amplitude of the line-of-sight component, which is inversely proportional to the nth power of distance from transmitter where n is a constant called the path loss exponent.

In timeslot m, the desired receiver, denoted by u_0, receives the power $\boldsymbol{P}_i^{(m)}$ from user u_i. In an interference limited network, the desired transmitter, u_j, is successful in sending its packet to u_0 in the mth timeslot if

$$\begin{cases} u_j \in \boldsymbol{T}^{(m)}, u_0 \notin \boldsymbol{T}^{(m)} \\ \boldsymbol{P}_j^{(m)} > \frac{1}{\beta} \sum_{i:u_i \in \boldsymbol{T}^{(m)} \setminus \{u_j\}} \boldsymbol{P}_i^{(m)} \end{cases} \quad (28)$$

where $\boldsymbol{T}^{(m)}$ is the set of transmitting users in the mth timeslot and β is the capture ratio. A message transmitted by u_j is successfully delivered if (28) is satisfied at least for one timeslot in that frame.

B. Protocol Performance

1) Simulation Setup: In the simulation setup, cars are placed on a three-lane road with 4m lane separation and the distance between two adjacent cars in the same lane is 30m, as illustrated in Fig. 4. The received power by a vehicle from any other vehicle is randomly driven according to the Rician distribution with $K = 3$ and $n = 2$. The capture ratio, β is 0.2 unless otherwise stated. A neighborhood with $N = 31$ cars is considered which occupy 300m of road and the effects of mobility and other vehicles are neglected. Frame length, L, is 64. Data rate is 5Mbps and safety message size, after adding the overheads of different layers, is 200 bytes. When 200B is transmitted in each timeslot, the length of each timeslot is 320μs and each frame is 20.48ms. In an actual implementation, timeslots must be longer to compensate for non-ideal synchronization. The traffic model is binomial; in each frame, a message arrives at each node with probability μ_p.

C. Probability of Success

An important metric in the simulation results in this work is the probability that more than 90% of the nodes successfully receive the transmitted message, denoted by $P_s^{(0.9)}$. Fig. 5 shows $P_f^{(0.1)} = 1 - P_s^{(0.9)}$, i.e., the probability that more than 10% of the nodes in the network fail to receive a transmitted message successfully, for w's from 2 to 8. Quadratic curves are fitted to the simulation results. Average load, μ_p, is 0.2 (messages/user/frame); on average each car produces a 200B message every 20.48ms/0.2=102.4ms. This figure indicates that by choosing a good value for w, all protocols are capable of delivering messages reliably while POC performs better than the other protocols.

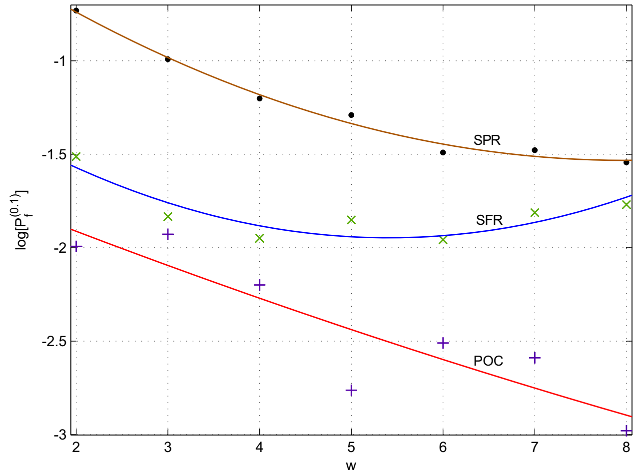

Fig. 5. Probability of failure versus w, for $\mu_p = 0.2$.

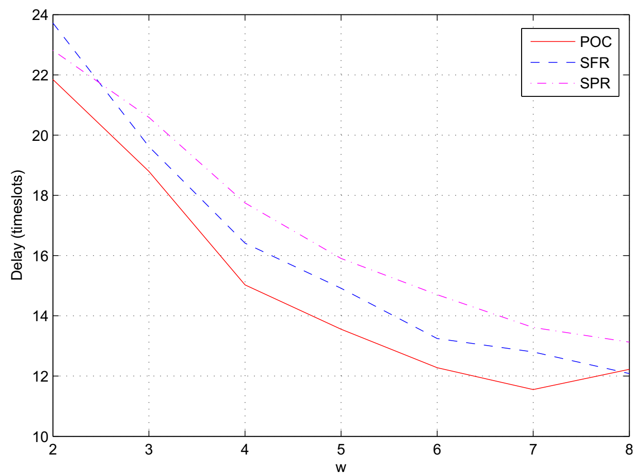

Fig. 6. Average delay versus w, for $\mu_p = 0.2$.

Fig. 6 shows the average delay versus w. The average delay of all protocols is more or less the same, as previously seen in Section V-C. For all protocols, the average delay is less than 24 timeslots or approximately 8ms. One may also consider the time that a message is buffered until the beginning of the frame by adding 20.48/2=10.24ms to the above values.

In the simulation results we have considered Rician channel with capture and adaptive elimination while the analytical results correspond to a case in which the wireless channel is perfect and capture and adaptive elimination are disabled. If capture and adaptive elimination are disabled, the analytical results and the simulation results agree. This is shown in Fig. 7, for the probability of success while w changes and in Fig. 8, for the delay while the average load changes. Irregularities can be observed more often in the plots for POC because POC has less intrinsic randomness compared to the other two methods.

VII. Conclusion

In most parts of the simulations, messages with length 200B are issued from each vehicle approximately 5 times

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE INFOCOM 2009 proceedings.

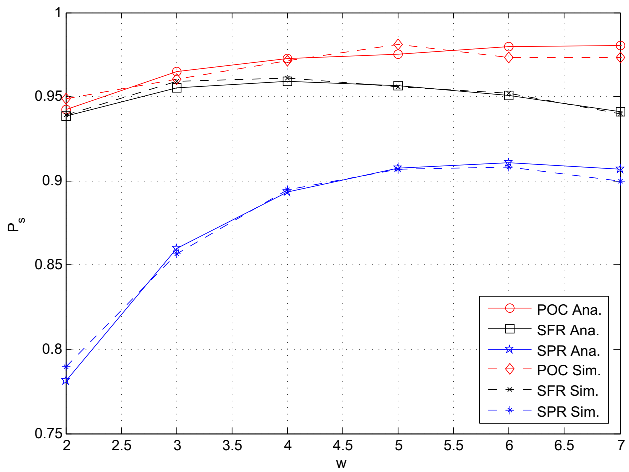

Fig. 7. Comparison of analytical and simulation results: P_s vs w, for $\mu_p = 0.3$.

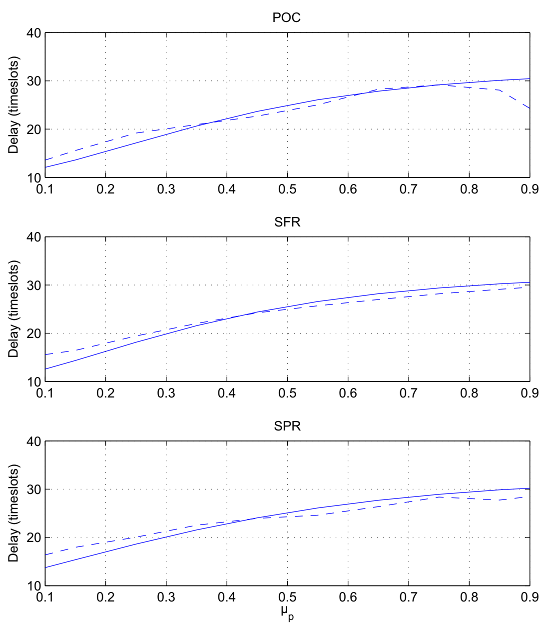

Fig. 8. Comparison of analytical and simulation results: Delay vs Average load, for $w = 6$ and $\beta = 0$.

per second. As explained in Section I, message frequency of approximately 5 messages with length 100B (per second per user) is sufficient for communicating position and other useful information. After adding different overheads, the message length will not exceed 200B assumed in the simulations. With the described load characteristics, we have shown that POC-based broadcast can reliably deliver safety messages with low delay. Furthermore, POC-based broadcast performs noticeably better than random repetition broadcast protocols, namely, SFR and SPR. We conclude that POC-based broadcast provides good performance in vehicular environments.

References

[1] The World Health Organization, "The World Health Report 2002 - Reducing Risks, Promoting Healthy Life." http://www.who.int/whr/2002/chapter4/en/index7.html, 2002.

[2] U.S. Department of Transportation, Research and Innovative Technology Administration (RITA), "Vehicular Infrastructure Integration (VII)." http://www.its.dot.gov/vii/.

[3] ASTM E2213-03, "Standard Specification for Telecommunication and Information Exchange Between Roadside and Vehicle Systems – 5 GHz Band Dedicated Short Range Communications (DSRC) Medium Access Control (MAC) and Physical Layer (PHY) Specificifications," *ASTM Int'l*, vol. July, 2003.

[4] S. Biswas, R. Tatchikou, and F. Dion, "Vehicle-to-vehicle Wireless Communication Protocols for Enhancing Highway Traffic Safety," *Communications Magazine, IEEE*, vol. 44, no. 1, pp. 74–82, 2006.

[5] J. Zhu and S. Roy, "MAC for Dedicated Short Range Communications in Intelligent Transport System," *Communications Magazine, IEEE*, vol. 41, no. 12, pp. 60–67, 2003.

[6] F. Farnoud, B. Hassanabadi, and S. Valaee, "Message Broadcast Using Optical Orthogonal Codes in Vehicular Communication Systems," *Wireless Networking for Intelligent Transportation Systems, ICST QSHINE Workshop*, Aug. 2007.

[7] F. Farnoud and S. Valaee, "Repetition-based Broadcast in Vehicular Ad Hoc Networks in Rician Channel with Capture," *IEEE INFOCOM MOVE Wrkshp*, Apr. 2008.

[8] B. Hofmann-Wellenhof, H. Lichtenegger, and J. Collins, *GPS Theory and Practice*. New York, NY, 2001.

[9] R. Parker, "Cooperative vehicle position estimation," Master's thesis, University of Toronto, 2007.

[10] M. Green, "" How Long Does It Take to Stop?" Methodological Analysis of Driver Perception-Brake Times," *Transportation Human Factors*, vol. 2, no. 3, pp. 195–216, 2000.

[11] K. Tang and M. Gerla, "MAC Reliable Broadcast in Ad hoc Networks," *Military Communications Conference, 2001. MILCOM 2001. Communications for Network-Centric Operations: Creating the Information Force. IEEE*, vol. 2, 2001.

[12] L. Min-Te Sun, A. Arora, and T. Lai, "Reliable MAC Layer Multicast in IEEE 802.11 Wireless Networks," *Proceedings of the International Conference on Parallel Processing*, pp. 527–536, 2002.

[13] G. Bianchi, "Performance Analysis Of The Ieee 802. 11 Distributed Coordination Function," *IEEE Journal on Selected Areas in Communications*, vol. 18, no. 3, pp. 535–547, 2000.

[14] I. Chlamtac and A. Farago, "Making transmission schedules immune to topology changes in multi-hop packet radio networks," *Networking, IEEE/ACM Transactions on*, vol. 2, no. 1, pp. 23–29, 1994.

[15] J. Ju and V. O. K. Li, "An optimal topology-transparent scheduling method in multihop packet radio networks," *IEEE/ACM Trans. Netw.*, vol. 6, no. 3, pp. 298–306, 1998.

[16] J.-H. Youn and B. Bose, "A topology-independent transmission scheduling in multihop packet radio networks," *Global Telecommunications Conference, 2001. GLOBECOM '01. IEEE*, vol. 3, pp. 1918–1922 vol.3, 2001.

[17] Z. Cai, M. Lu, and C. Georghiades, "Topology-transparent time division multiple access broadcast scheduling in multihop packet radio networks," *Vehicular Technology, IEEE Transactions on*, vol. 52, no. 4, pp. 970–984, 2003.

[18] Q. Xu, T. Mak, J. Ko, and R. Sengupta, "Layer-2 Protocol Design for Vehicle Safety Communications in Dedicated Short Range Communications Spectrum," *Intelligent Transportation Systems, 2004. Proceedings. The 7th International IEEE Conference on*, pp. 1092–1097, 2004.

[19] Q. Xu, T. Mak, and R. Sengupta, "Medium Access Control Protocol Design for Vehicle-vehicle Safety Messages," *Vehicular Technology, IEEE Transactions*, vol. 56, no. 2, pp. 499–518, 2007.

[20] Q. Xu, T. Mak, J. Ko, and R. Sengupta, "Vehicle-to-vehicle Safety Messaging in DSRC," *Proceedings of the 1st ACM international workshop on Vehicular ad hoc networks*, pp. 19–28, 2004.

[21] E. Agrell, A. Vardy, and K. Zeger, "Upper Bounds For Constant-weight Codes," *Information Theory, IEEE Transactions*, vol. 46, no. 7, pp. 2373–2395, 2000.

[22] S. Johnson, "A New Upper Bound for Error-correcting Codes," *Information Theory, IEEE Transactions*, vol. 8, no. 3, pp. 203–207, 1962.