Weighted Combination Method of Conflict Evidences Based On Basic Point

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***Abstract*—In the circumstances that evidences are highly or full conflict, D-S evidence combination method may lose effectiveness, for this reason some improved methods of evidence combination are given in succession. Some scholars think, different evidence source is under different circumstance, so the importance of each evidence source is different. In the conditions that evidences are highly or full conflict, it is necessary to give weight to every evidence source, then conflict probability can only just be assigned.**

In this paper, we construct an evaluating standard with basic proposition and its rough combined probability value, to calculate the weight coefficient of every evidence source, based on that, conflict probability is assigned, combination of evidences is built. In this method, subjective information from decision-makers is used, and objective information from evidences is also used, only little artificial job is needed. This method can be achieved on computer easily, and satisfying consequence can be obtained.

I. Introduction

EVIDENCE combination method is the core of evidence theory. In the circumstances that evidences are highly or full conflict, D-S evidence combination method may lose effectiveness, so some scholars give some improved methods of evidence combination in succession [1-10]. In the improved methods of evidence combination, some methods use weight coefficients. The methods using weight coefficients can be divided into two kinds, one kind assigns conflict probability into related focal elements according to certain weights, the other kind gives weight to every evidence source beforehand, then assigns conflict probability. The approach in this paper falls into the later.

In fact, how to combine conflict evidences, how to determine weight coefficients, those depend on people's subjective judging, depend on decision-makers' acceptance, to a great extent. In this paper, we construct an evaluating reference with basic proposition and its rough combined probability value, to calculate the weight coefficient of every evidence source. In this method, subjective information from decision-makers is used, and objective information from evidences is also used, only little artificial job is needed. This method can be achieved on computer easily, and satisfying consequence can be obtained. Thereby, this method provides a way to combine subjective information and objective information for weighted combination of conflict evidences.

This work was supported in part by Basic Reserch Project of Shaanxi Province Under No.SJ08A02

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II. D-S Evidence Theory

The set Θ made of limited mutual exclusive elements is called identification framework. The map m is defined to any proposition A : $2^{\Theta} \to [0,1]$. The map m is called basic probability distribution function, m is satisfied with

(1) $m(\Phi)=0$,

(2) $0 \le m(A) \le 1$,

(3) $\sum_{A \subseteq \Theta} m(A) = 1$.

If $A \subseteq \Theta$ and $m(A)$>0, then A is called focal element.

Follow combining formulas are given in D-S evidence theory to combine evidences from different evidence sources.

$$m(\Theta) = 0 \tag{1}$$

$$m(A) = \frac{\sum_{\bigcap A_j = A} \prod_{i=1}^{n} m_i(A_j)}{1-k} \tag{2}$$

Where $k = \sum_{\bigcap A_j = \Phi} \prod_{i=1}^{n} m_i(A_j)$, it shows total conflict degree between all evidences.

III. Weighted Combination Method of Conflict Evidences

In the process of combining evidences, it is improper to treat every evidence source at equal importance. In fact, different evidence source is under different circumstance, so the importance of evidence from different evidence source is different. Therefore, weight coefficients of evidences ought to be used in the process of assigning conflict probability, to improve rationality of combining evidences.

Assume there are n evidence sources, the evidence set is $E = \{E_1, E_2, ..., E_n\}$. Let weight coefficient of evidence E_i be w_i, $w_i \in [0,1]$, w_i will be fixed later. All weight coefficients form weight vector $\mathbf{W} = (w_1, w_2, ..., w_n)^{\mathrm{T}}$, $\sum_{i=1}^{n} w_i = 1$.

Because $w_i \le 1$ for every w_i, and, combined probability consists of product of different weight coefficients, as a result, combined probability will be very small. To avoid that, let

$$\mathbf{W}^* = (w_1^*, w_2^*, ..., w_n^*)^{\mathrm{T}}$$
$$= (\frac{w_1}{w_{\max}}, \frac{w_2}{w_{\max}}, ..., \frac{w_n}{w_{\max}})^{\mathrm{T}}$$

where $w_{\max} = \max\{w_1, w_2, ..., w_n\}$. In this way, $\mathbf{W} = (w_1, w_2, ..., w_n)^{\mathrm{T}}$ is enlarged, $\mathbf{W}^* = (w_1^*, w_2^*, ..., w_n^*)^{\mathrm{T}}$ is called relative weight vector.

Suppose there are s exclusive propositions $A_1, A_2, ..., A_s$. For proposition A_j , let its weighed probability distribution be

$$m_i^*(A_j) = w_i^* m_i(A_j) \quad i = 1,2,...,n; j = 1,2,...,s \qquad (3)$$

Because $\sum_{j=1}^{s} m_i^*(A_j) < 1$, a definition is added as below.

$$m_i^*(\Theta) = 1 - \sum_{j=1}^{s} m_i^*(A_j) \qquad (4)$$

As a result, the third condition of basic probability distribution function definition gets satisfied. The function defined by formula (3) and (4) can be called a basic probability distribution function[10].

For proposition A_j , $\frac{1}{n}\sum_{i=1}^{n} m_i^*(A)$ is the mean of its new probabilities. Larger the mean is, more tenable proposition A is. Assigning conflict probability according to the mean[4], a set of combining formulas are given as below.

$$m(\Phi) = 0$$
$$m(A_j) = \prod_{i=1}^{n} m_i^*(A_j) + k \bullet \frac{1}{n}\sum_{i=1}^{n} m_i^*(A_j) \qquad A_j \neq \Phi, \Theta$$
$$m(\Theta) = 1 - \sum_{j=1}^{s} m(A_j)$$

where $k = \sum_{\bigcap A_j = \Phi} \prod_{i=1}^{n} m_i^*(A_j)$.

IV. Using Basic Point to Calculate the Weight Coefficient of Every Evidence

Here is an example of conflict evidence[10]. Let $\Theta = \{A, B, C\}$, 3 evidences are as follows:

E_1 : $m_1(A) = 0.98$, $m_1(B) = 0.01$, $m_1(C) = 0.01$,

E_2 : $m_2(A) = 0$, $m_2(B) = 0.01$, $m_2(C) = 0.99$,

E_3 : $m_3(A) = 0.9$, $m_3(B) = 0$, $m_3(C) = 0.1$.

In the 3 evidences, 2 evidences support proposition A with probability of 0.9 and upwards, only one does not support proposition A . That means most of evidences support proposition A with large probability. So if evidences are combined, the combined probability of proposition A should be the maximum in the 3 propositions. In addition, the mean value of 3 probabilities of proposition A is (0.98+0+0.9)/3 = 0.6267, so if unknown proposition is not considered, we think intuitively the combined probability of proposition A ought to be between the mean 0.6267 and the maximum 0.98 . Similarly, we think the combined probability of proposition B ought to be the minimum. The mean value of 3 probabilities of proposition B is (0.01+0.01+0)/3 = 0.0067, so we think the combined probability of proposition B ought to be between 0 and the mean 0.0067 .

In the ordinary circumstances, by observing probabilities of propositions, we can get some tentative knowledge about combined probability of each proposition, especially about the propositions which are given maximum or minimum probability by most evidence sources. Based on those thought, we establish following algorithm to calculate weight coefficient of every evidence source.

Suppose there are n evidence sources and s exclusive propositions, all given probabilities form a matrix

$$\begin{pmatrix} m_1(A_1) & m_1(A_2) & \ldots & m_1(A_s) \\ m_2(A_1) & m_2(A_2) & \ldots & m_2(A_s) \\ \ldots\ldots & & & \\ m_n(A_1) & m_n(A_2) & \ldots & m_n(A_s) \end{pmatrix}$$

So n probabilities of proposition A_j form a n -dimensional probability vector $M(A_j)$

$$M(A_j) = \begin{pmatrix} m_1(A_j) \\ m_2(A_j) \\ \vdots \\ m_n(A_j) \end{pmatrix} \quad (j = 1,2,...,s)$$

In this matrix, we can find out a proposition which is given maximum probability by most evidence sources, such as proposition A in the above example, we call it maximum probability proposition. We can also find out a proposition which is given minimum probability by most evidence sources, such as proposition B in the above example, we call it minimum probability proposition. According to above-mentioned thought, we can get rough ranges of combined probability of maximum probability proposition and minimum probability proposition.

We set maximum probability proposition as basic point, suppose the basic point proposition is $A_{j^\cdot}$. Because we can get rough range of combined probability of maximum probability proposition, so we can give the proposition a rough combined probability value, suppose the rough combined probability value is $P_{j^\cdot}$. In this way, the basic point proposition and its rough combined probability value can be regarded as a reference which is one of rules to calculate weight coefficients as follows.

In the people's direct-vision consciousness, to combine many evidences means people have to mediate their probabilities to some extent. How to mediate after all? What is form of the mediating? That is difficult to conclude. In order to calculate conveniently, we suppose the form is linear. In this way, probabilities of the basic point proposition are combined in linear form, the combined probability value is equal to the rough combined probability value P_{j^*}, so we get

$\sum_{i=1}^{n} w_i\, m_i(A_{j^*}) = P_{j^*}$, where $w_i (i = 1, 2, ..., n)$ is weight coefficient of evidence i.

To calculate weight vector, an absolutely credible proposition G is invented. Because G is absolutely credible, so we suppose $m_1(G) = m_2(G) = ... = m_n(G) = 1$. An absolutely incredible proposition H is also invented, similarly we suppose $m_1(H) = m_2(H) = ... = m_n(H) = 0$.

Then, the n-dimensional probability vectors of proposition G and H are as below.

$$M(G) = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \quad M(H) = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

In the ordinary circumstances, for any proposition A_j, A_j cannot be absolutely credible, and cannot be absolutely incredible, it should be between absolutely credible and absolutely incredible. So we can get the distance between probability vector $M(A_j)$ and $M(G)$, and the distance between $M(A_j)$ and $M(H)$. We construct the sum of square of the weighted distances

$$\begin{aligned} l_j(\mathbf{W}) &= l_j(w_1, w_2, ...w_n) \\ &= \sum_{i=1}^{n} [w_i^2 (m_i(A_j) - 1)^2 + w_i^2 (m_i(A_j) - 0)^2] \end{aligned}$$

where weight coefficient $w_i (i = 1, 2, ..., n)$ is not fixed yet. As a distance, $l_j(\mathbf{W})$ should be minimum, for any $j = 1, 2, ..., s$. Thus we build a multiple objective programming

$$\min \quad \mathbf{L}(\mathbf{W}) = (l_1(\mathbf{W}), l_2(\mathbf{W}), ..., l_S(\mathbf{W}))^{\mathrm{T}}$$

$$\text{s.t.} \begin{cases} \sum_{i=1}^{n} w_i = 1 \\ \sum_{i=1}^{n} w_i\, m_i(A_{j^*}) = P_{j^*} \\ w_i \geq 0 \ \ (i = 1, 2, ..., n) \end{cases} \tag{5}$$

Because $l_j(\mathbf{W}) \geq 0$ for any j $(j = 1, 2, ..., s)$, therefore above multiple objective programming (5) can be turned to a single objective programming

$$\min \quad \sum_{j=1}^{s} l_j(\mathbf{W}) = \sum_{j=1}^{s}\sum_{i=1}^{n} [w_i^2 (m_i(A_j) - 1)^2 + w_i^2 (m_i(A_j) - 0)^2]$$

$$\text{s.t.} \begin{cases} \sum_{i=1}^{n} w_i = 1 \\ \sum_{i=1}^{n} w_i\, m_i(A_{j^*}) = P_{j^*} \\ w_i \geq 0 \ \ (i = 1, 2, ..., n) \end{cases} \tag{6}$$

For the present we wouldn't consider the condition $w_i \geq 0$ ($i = 1, 2, ..., n$), construct lagrange function

$$\begin{aligned} F(\mathbf{W}, \lambda) &= \sum_{j=1}^{s}\sum_{i=1}^{n} [w_i^2 (m_i(A_j) - 1)^2 + w_i^2 (m_i(A_j) - 0)^2] \\ &\quad - \lambda_1 (\sum_{i=1}^{n} w_i - 1) - \lambda_2 (\sum_{i=1}^{n} w_i\, m_i(A_{j^*}) - P_{j^*}) \end{aligned}$$

let

$$\begin{cases} \dfrac{\partial F}{\partial w_i} = 2 w_i \sum_{j=1}^{s} [(m_i(A_j) - 1)^2 + m_i(A_j)^2] \\ \qquad - \lambda_1 - \lambda_2 m_i(A_{j^*}) = 0 \quad i = 1, 2, ..., n \\ \dfrac{\partial F}{\partial \lambda_1} = \sum_{i=1}^{n} w_i - 1 = 0 \\ \dfrac{\partial F}{\partial \lambda_2} = \sum_{i=1}^{n} w_i\, m_i(A_{j^*}) - P_{j^*} = 0 \end{cases} \tag{7}$$

Solving system of linear equations (7), we get[11]

$$w_i = \frac{\sum_{v=1}^{n} [\frac{1}{u_v}(m_v(A_{j^*}) - m_i(A_{j^*}))(m_v(A_{j^*}) - P_{j^*})]}{u_i [(\sum_{v=1}^{n} \frac{1}{u_v})(\sum_{v=1}^{n} \frac{m_v^2(A_{j^*})}{u_v}) - (\sum_{v=1}^{n} \frac{m_v(A_{j^*})}{u_v})^2]} \tag{8}$$

where $u_i = \sum_{j=1}^{s} [(m_i(A_j) - 1)^2 + (m_i(A_j))^2]$.

In this way, we get $\mathbf{W} = (w_1, w_2, \cdots, w_n)^{\mathrm{T}}$. When $\mathbf{W} = (w_1, w_2, \cdots, w_n)^{\mathrm{T}} \geq (0, 0, \cdots, 0)^{\mathrm{T}}$, restraint condition $w_i \geq 0$ ($i = 1, 2, ..., n$) get satisfied, so $\mathbf{W} = (w_1, w_2, \cdots, w_n)^{\mathrm{T}}$ is the weight vector we want to seek. If the weight vector $\mathbf{W} = (w_1, w_2, \cdots, w_n)^{\mathrm{T}}$ is not satisfied with the demand $\mathbf{W} = (w_1, w_2, \cdots, w_n)^{\mathrm{T}} \geq (0, 0, \cdots, 0)^{\mathrm{T}}$, that means the rough combined probability value is not suitable, we should give a new rough combined probability value and calculate again.

Minimum probability proposition can also be set as basic point, but its probability value is very small, errors produced in calculation will accumulate, thus weight coefficients got in calculation may not be very satisfying, but sort of the weight coefficients will be justified, so the sort can be used to confirm the weight coefficients obtained on maximum probability proposition.

Besides, combining formula of probabilities of basic point proposition can be not in linear form $\sum_{i=1}^{n} w_i m_i(A_{j^*}) = P_{j^*}$, but thereby formula (7) will not be a system of linear equations yet, calculation will be much more complex.

V. Calculation and Analysis on the Example

First, let us give above example direct-vision analysis.

Probability distribution of evidence E_1 is similar to probability distribution of evidence E_3, thus the difference of weight coefficients between evidence E_1 and E_3 should be little. Evidence E_1 and E_3 account for $\frac{2}{3}$ of 3 evidences, play a leading role, so weight coefficients of evidence E_1 and E_3 should be large. In addition, evidence E_1 is a bit extreme, but evidence E_3 is a bit moderate, so weight coefficient of E_3 should be a bit larger than weight coefficient of E_1. Evidence E_2 conflicts strongly with the others, and is in the minority, so weight coefficients of evidence E_2 should be small.

The mean value of probabilities of proposition A, B, C is respectively 0.6267, 0.00667, 0.3667. There are large differences in 3 means, so we think, combined probability of proposition A > combined probability of proposition C > combined probability of proposition B.

The example is calculated as below with the method in this paper.

Let proposition A be the basic point, let the rough combined probability value of A be respectively 0.9800, 0.9000, 0.8000, 0.7000, 0.6267, calculating according to formula (8), we get table 1.

From table 1, we can make out, when the rough combined probability value of A takes respectively 0.9000, 0.8000, 0.7000, the weight vectors got in calculating are a bit different, but sort of 3 components of each vector is identical. In addition, the difference of weight coefficients between evidence E_1 and E_3 is little, and, weight coefficients of E_1 and E_3 are larger than weight coefficient of E_2. These are in keeping with above analysis conclusions. The example show, if rough combined probability value of basic point proposition is justified, this method will produce satisfying weight vector.

With above justified weight vectors, combined probability of each proposition is calculated according to the combination method in this paper. The combined probabilities are compared with D-S combined probabilities and results in [4], see table 2.

Table 1 weight vectors got in calculating for different rough combined probability value of A

rough combined probability value	weight vectors got in calculating	analysis
$P_{j^*}=0.9800$	$\begin{pmatrix} 0.5191 \\ -0.0427 \\ 0.5236 \end{pmatrix}$	Rough combined probability value is too large, so condition $\mathbf{w} \ge \mathbf{0}$ can not get satisfied. This weight vector should be abandoned.
$P_{j^*}=0.9000$	$\begin{pmatrix} 0.4732 \\ 0.0421 \\ 0.4847 \end{pmatrix}$	
$P_{j^*}=0.8000$	$\begin{pmatrix} 0.4158 \\ 0.1481 \\ 0.4361 \end{pmatrix}$	
$P_{j^*}=0.7000$	$\begin{pmatrix} 0.3584 \\ 0.2541 \\ 0.3875 \end{pmatrix}$	
$P_{j^*}=0.6267$	$\begin{pmatrix} 0.3164 \\ 0.3318 \\ 0.3519 \end{pmatrix}$	Rough combined probability value is too small, as a results, weight coefficient of E_2 is too large. This weight vector should be abandoned.

Table 2 3 types of calculation results

combining method		$m^*(A)$	$m^*(B)$	$m^*(C)$	$m^*(\Theta)$
D-S formula		0	0	1	0
method in [4]		0.6260	0.0067	0.3673	0
method in this paper	$P_{j^*}=0.9$	0.61886	0.00354	0.06220	0.31540
	$P_{j^*}=0.8$	0.61127	0.00431	0.14884	0.23557
	$P_{j^*}=0.7$	0.60177	0.00527	0.25266	0.14030

From table 2, we can get, because part of probability is given to unknown proposition in this paper, as a result, combined probability of proposition A is not as large as the desired, but the sort and proportions of combined probabilities of all propositions are satisfying. When the rough combined probability value of proposition changes from large to small, in responses the combined probability of proposition turns to a bit small, but account for maximum proportion throughout. Besides, when the rough combined probability value of proposition changes from large to small, the combined probability of proposition become large, that indicates, when trust in proposition is cut down, part of trust is transferred to proposition , that is reasonable.

Results in this paper are similar to results in [4], the difficult problem in D-S combining formula is overcome. Some difference is, probability of unknown proposition is 0 in [4], but the probability is larger than 0 in this paper, that indicates there is some indeterminacy in evidence combining, that corresponds to reality much more.

VI. Conclusions

How to combine conflict evidences, how to determine weight coefficients, those depend on people's subjective judging, depend on decision-makers' acceptance, to a great extent. In this paper, weighted combination method of conflict evidences based on basic point is proposed. In this method, subjective information from decision-makers is used, and objective information from evidences is also used, only little artificial job is needed. This method can be achieved on computer easily, and satisfying consequences can be obtained.

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