An Overview of MIMO Systems in Wireless Communications

Lecture in "Communication Theory for Wireless Channels"

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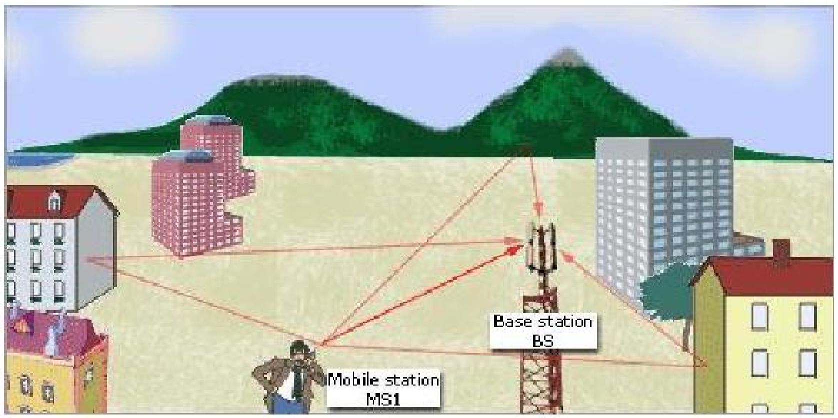
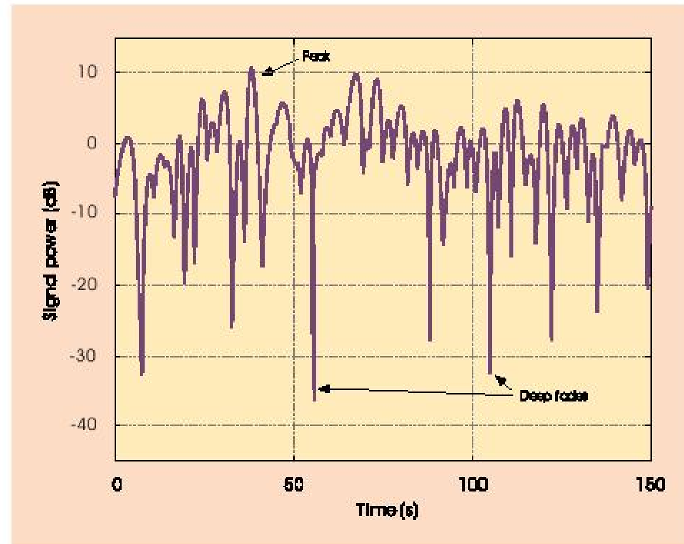
Future Broadband Wireless Systems

- **Desired attributes**
  - Significant increase in spectral efficiency and data rates
  - High Quality–of–Service (QoS) — bit error rate, outage, . . .
  - Wide coverage
  - Low deployment, maintenance and operation costs

- The **wireless channel** is very **hostile**
  - Severe fluctuations in signal level (fading)
  - Co–channel interference
  - Signal power falls off with distance (path loss)
  - Scarce available bandwidth
  - . . .

The Wireless Channel

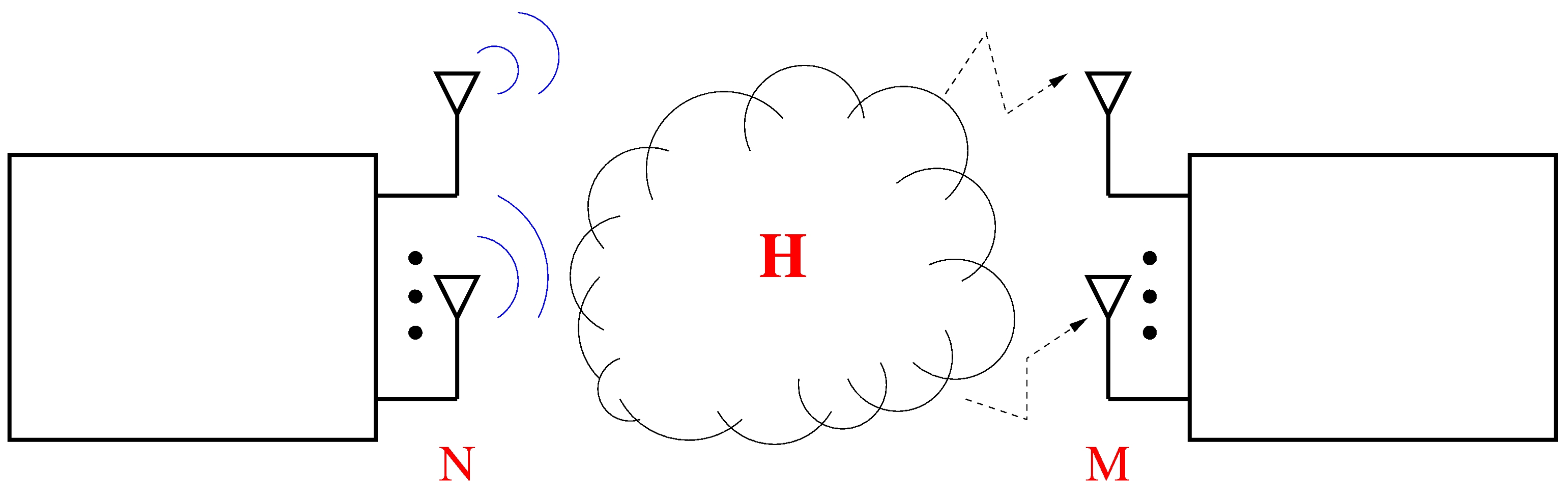




- **Multipath propagation** causes signal **fading**

[1]

MIMO System

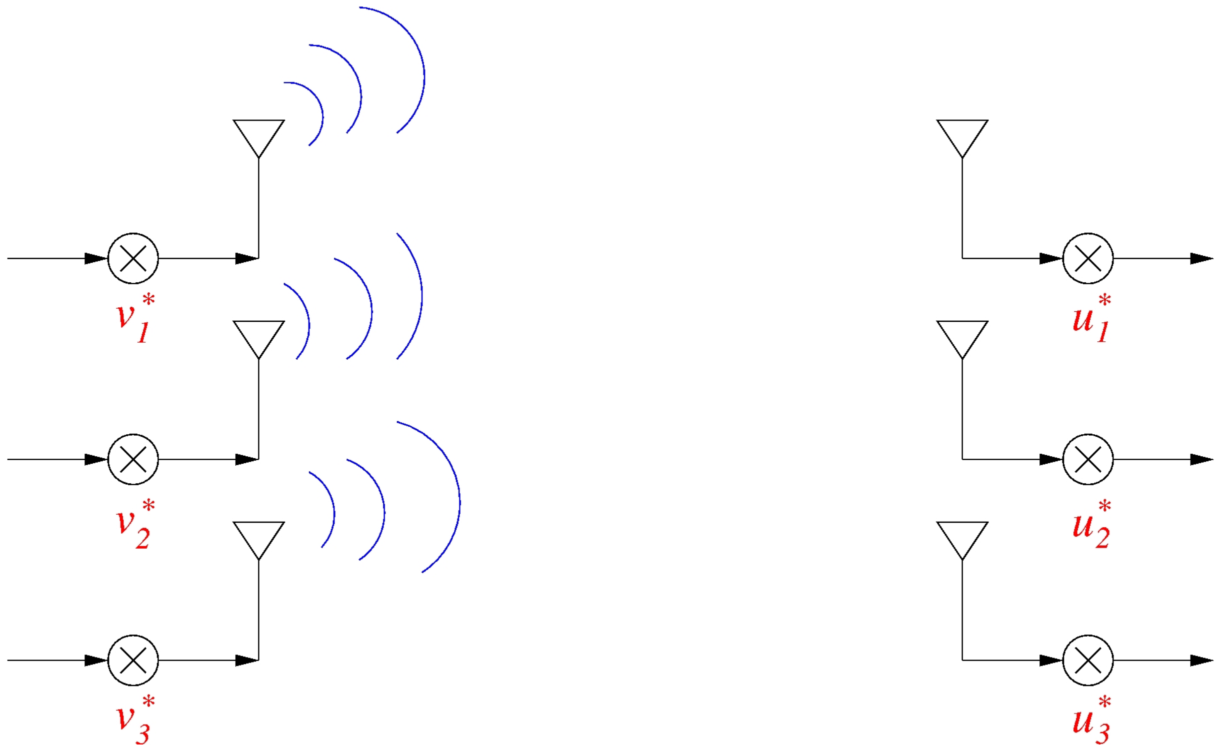


Performance Improvements Using MIMO Systems

- Array gain $\Longrightarrow$ increase coverage and QoS
- Diversity gain $\Longrightarrow$ increase coverage and QoS
- Multiplexing gain $\Longrightarrow$ increase spectral efficiency
- Co–channel interference reduction $\Longrightarrow$ increase cellular capacity

[1]

Array Gain



- Increase in **average received SNR** obtained by **coherently combining** the incoming / outgoing signals

- **Requires channel knowledge** at the transmitter / receiver

[2, 3]

Array Gain

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

- $\mathbf{H} \in \mathbb{C}^{M \times N}$ ($\mathcal{E}|\mathbf{H}_{ik}|^2 = 1$). $\mathbf{x} \in \mathbb{C}^N$, $\mathbf{y} \in \mathbb{C}^M$
- $\mathbf{n} \in \mathbb{C}^M$: zero–mean complex Gaussian noise
- **Principle:** To obtain the **full array gain**, one should transmit using the **maximum eigenmode** of the channel
- The **singular value decomposition (SVD)** $\mathbf{H} = \mathbf{U}\mathbf{D}\mathbf{V}^\dagger$, with $\mathbf{D} = \mathrm{diag}(\sqrt{\lambda_1}, \ldots, \sqrt{\lambda_m}, 0, \ldots, 0)$ and $m = \min\{N, M\}$, yields m **equivalent SISO channels**

$$\lambda_1, \ldots, \lambda_m = \begin{cases} \mathrm{eig}(\mathbf{H}\mathbf{H}^\dagger) & \text{if } M < N \\ \mathrm{eig}(\mathbf{H}^\dagger\mathbf{H}) & \text{if } M \geq N \end{cases}$$

$$\widetilde{\mathbf{y}} = \mathbf{D}\widetilde{\mathbf{x}} + \widetilde{\mathbf{n}},$$

where $\widetilde{\mathbf{y}} = \mathbf{U}^\dagger\mathbf{y}$, $\widetilde{\mathbf{x}} = \mathbf{V}^\dagger\mathbf{x}$ and $\widetilde{\mathbf{n}} = \mathbf{U}^\dagger\mathbf{n}$ ($\mathbf{U}, \mathbf{V}$ unitary)

[2, 3]

Array Gain

$$\widetilde{\mathbf{y}} = \mathbf{D}\widetilde{\mathbf{x}} + \widetilde{\mathbf{n}}$$

- If $\lambda_i = \lambda_{\max} = \max\{\lambda_1, \ldots, \lambda_m\}$, **(maximum eigenmode)**

$$\widetilde{y}_i = \sqrt{\lambda_{\max}}\, \widetilde{x}_i + \widetilde{n}_i$$

- **Known results**
  - For $N \times 1$ and $1 \times M$ arrays, the **array gain** (increase in average SNR) is respectively of $10 \log_{10} N$ and $10 \log_{10} M$ dB
  - In the **asymptotic limit**, with M large:

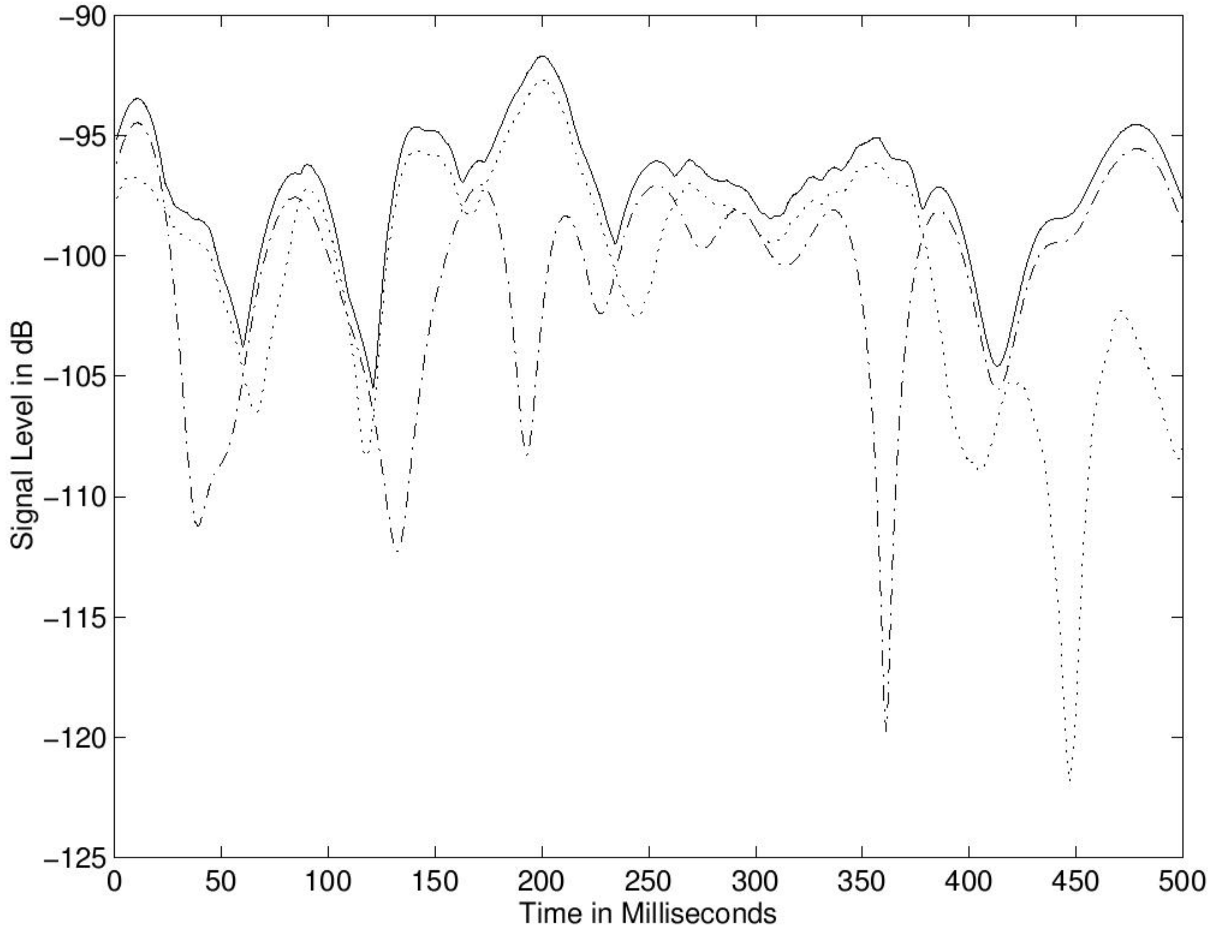
$$\lambda_{\max} < (\sqrt{c} + 1)^2 M \qquad c = \frac{N}{M} \geq 1$$
$$\lambda_{\min} > (\sqrt{c} - 1)^2 M \qquad c = \frac{N}{M} > 1$$

- For maximum
  - Capacity: **waterfilling** (later in this presentation)
  - Array gain: use only the **maximum eigenchannel**

[2, 3]

Diversity Gain

- **Principle:** provide the receiver with **multiple identical copies** of a given signal to **combat fading** $\Longrightarrow$ gain in **instantaneous SNR**



[4]

Diversity Gain

- Intuitively, the **more independently fading, identical copies** of a given signal the receiver is provided with, the **faster** the bit error rate (BER) decreases as a function of the per signal SNR. At **high SNR values**, it has been shown that

$$P_e \approx (G_c \cdot \mathrm{SNR})^{-d}$$

  where d represents the diversity gain and G_c the coding gain

- **Definition:** For a **given transmission rate** R, the **diversity gain** is

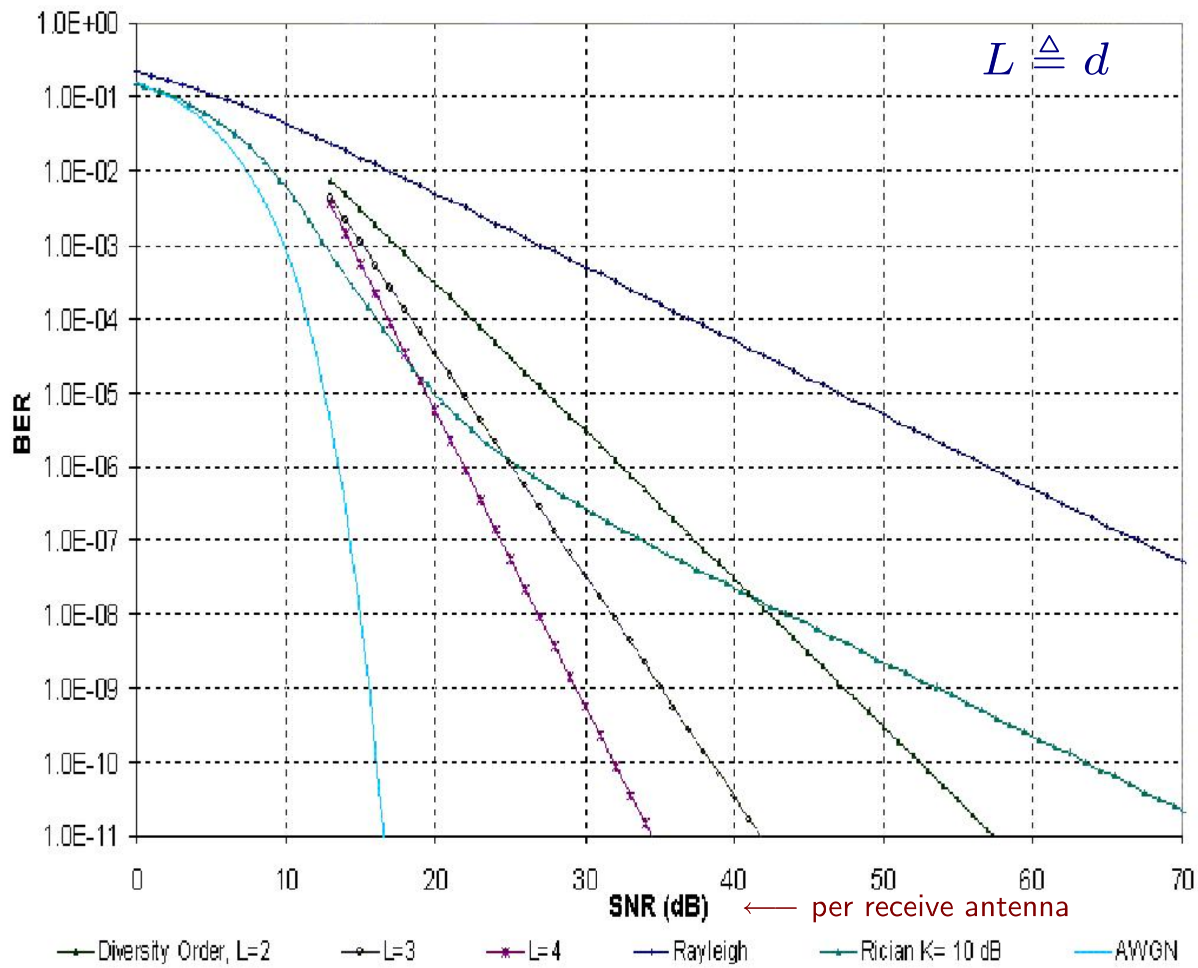
$$d(R) = - \lim_{SNR \to \infty} \frac{\log(P_e(R, \mathrm{SNR}))}{\log \mathrm{SNR}}, \tag{1}$$

  where $P_e(R, \mathrm{SNR})$ is the BER at the given rate and SNR

- **Independent** versus **correlated** fading
- **Diminishing return** for each extra signal copy

[3, 5, 6]

Diversity Gain

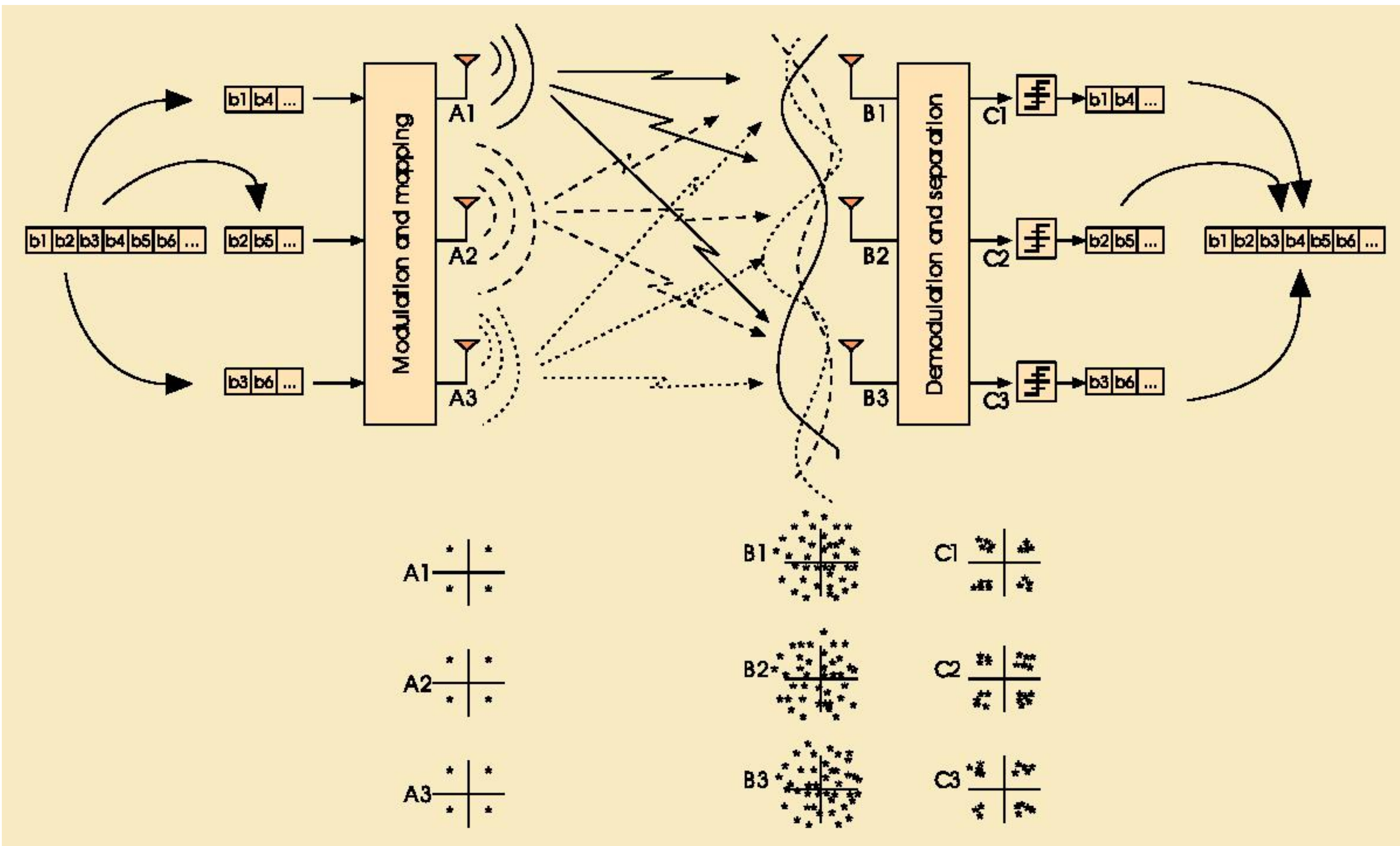


- The diversity gain is the **magnitude of the slope** of the BER $P_e(R, \mathrm{SNR})$ plotted as a function of SNR on a log–log scale

[4, 6]

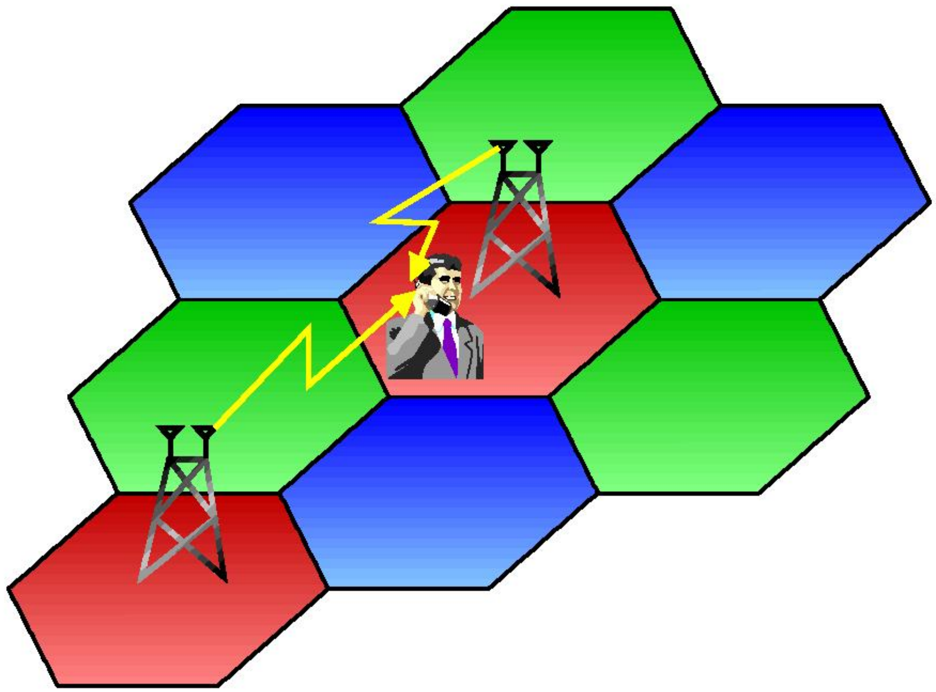
Multiplexing Gain

- **Principle:** Transmit **independent data signals** from **different antennas** to **increase the throughput**



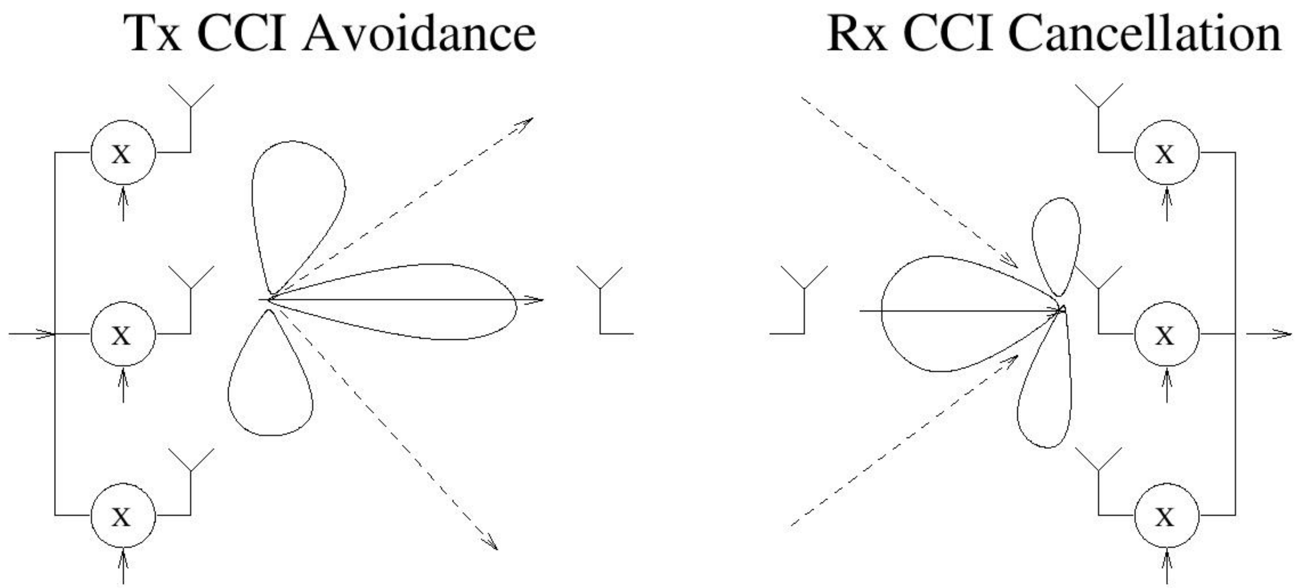
[1]

Co–Channel Interference



[1]

Co–Channel Interference Reduction



- $N-1$ interferees can be cancelled with N transmit antennas
- $M-1$ interferers can be cancelled with M receive antennas

[1]

Capacity of MIMO Systems — The Gaussian Channel

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n},$$

with:

- $\mathbf{H} \in \mathbb{C}^{M \times N}$ with **uniform phase** and **Rayleigh** magnitude (**Rayleigh fading environment**)—i.i.d. Gaussian, zero–mean, independent real and imaginary parts, variance $1/2$

- $\mathbf{x} \in \mathbb{C}^N$, $\mathbf{y} \in \mathbb{C}^M$

- $\mathbf{n}$: zero–mean complex Gaussian noise. **Independent and equal variance** real and imaginary parts. $\mathcal{E}[\mathbf{n}\mathbf{n}^\dagger] = I_M$

- Transmitter **power constraint**: $\mathcal{E}[\mathbf{x}^\dagger\mathbf{x}] = \mathrm{tr}\big(\mathcal{E}[\mathbf{x}\mathbf{x}^\dagger]\big) \leq P$

[7]

Circularly Symmetric Random Vectors

Definition: A complex Gaussian random vector $\mathbf{x} \in \mathbb{C}^n$ is said to be **circularly symmetric** if the corresponding vector

$$\hat{\mathbf{x}} \in \mathbb{R}^{2n} = \begin{bmatrix} \mathfrak{Re}(\mathbf{x}) \\ \mathfrak{Im}(\mathbf{x}) \end{bmatrix}$$

has the structure

$$\mathcal{E}\left[(\hat{\mathbf{x}} - \mathcal{E}[\hat{\mathbf{x}}])(\hat{\mathbf{x}} - \mathcal{E}[\hat{\mathbf{x}}])^{\dagger}\right] = \frac{1}{2}\begin{bmatrix} \mathfrak{Re}(\mathbf{Q}) & -\mathfrak{Im}(\mathbf{Q}) \\ \mathfrak{Im}(\mathbf{Q}) & \mathfrak{Re}(\mathbf{Q}) \end{bmatrix}$$

for some Hermitian non–negative definite $\mathbf{Q} \in \mathbb{C}^{n \times n}$

[7]

Circularly Symmetric Random Vectors

The **pdf** of a CSCG random vector $\mathbf{x}$ with mean μ and covariance matrix $\mathbf{Q}$ is given by

$$f_{\mu,\mathbf{Q}}(\mathbf{x}) = \frac{1}{\det \pi\mathbf{Q}} \exp\left[-(\mathbf{x}-\mu)^\dagger \mathbf{Q}^{-1} (\mathbf{x}-\mu) \right]$$

and has **differential entropy**

$$\begin{aligned} h(\mathbf{X}) &= -\int_{\mathbb{C}^n} f_{\mu,\mathbf{Q}}(\mathbf{x}) \, \log f_{\mu,\mathbf{Q}}(\mathbf{x}) \, d\mathbf{x} \\ &= \log\det \pi e \mathbf{Q} \end{aligned}$$

[7]

The Deterministic Gaussian Channel — Capacity

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \qquad \mathcal{E}[\mathbf{x}^\dagger \mathbf{x}] \leq P$$

Idea: Maximize the **mutual information** between $\mathbf{x}$ and $\mathbf{y}$

$$\begin{aligned} I(\mathbf{X};\mathbf{Y}) &= h(\mathbf{Y}) - h(\mathbf{Y}|\mathbf{X}) \\ &= h(\mathbf{Y}) - h(\mathbf{N}) \end{aligned}$$

$\Longrightarrow$ Maximize $h(\mathbf{Y})$

[7]

Maximizing $h(\mathbf{Y})$

It can be shown that:

- If $\mathbf{x}$ satisfies $\mathcal{E}[\mathbf{x}^\dagger \mathbf{x}] \leq P$, then so does $\mathbf{x} - \mathcal{E}[\mathbf{x}]$
- For all $\mathbf{y} \in \mathbb{C}^M$, $h(\mathbf{Y})$ is maximized if $\mathbf{y}$ is **Circularly Symmetric Complex Gaussian (CSCG)**
- If $\mathbf{x} \in \mathbb{C}^N$ is CSCG with covariance $\mathbf{Q}$, then $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \in \mathbb{C}^M$ is also CSCG

$$
\begin{aligned}
\Longrightarrow I(\mathbf{X};\mathbf{Y}) &= \log\det \pi e(I_M + \mathbf{H}\mathbf{Q}\mathbf{H}^\dagger) - \log\det \pi e \\
&= \log\det(I_M + \mathbf{H}\mathbf{Q}\mathbf{H}^\dagger)
\end{aligned}
$$

- A non–negative definite $\mathbf{Q}$ such that $I(\mathbf{X};\mathbf{Y})$ is maximum and $\mathrm{tr}(\mathbf{Q}) \leq P$ remains to be found

[7]

Deterministic Gaussian MIMO Channel

- $\mathbf{H}$ known at the transmitter ("waterfilling solution"): Choose $\mathbf{Q}$ diagonal, such that

$$\mathbf{Q}_{ii} = (\alpha - \lambda_i^{-1})^+, \qquad i = 1, \ldots, N$$

with $(\cdot)^+ \triangleq \max(\cdot, 0)$, $(\lambda_1, \ldots, \lambda_N)$ the eigenvalues of $\mathbf{H}^\dagger\mathbf{H}$ and α such that $\sum_i \mathbf{Q}_{ii} = P$. The capacity is given by:

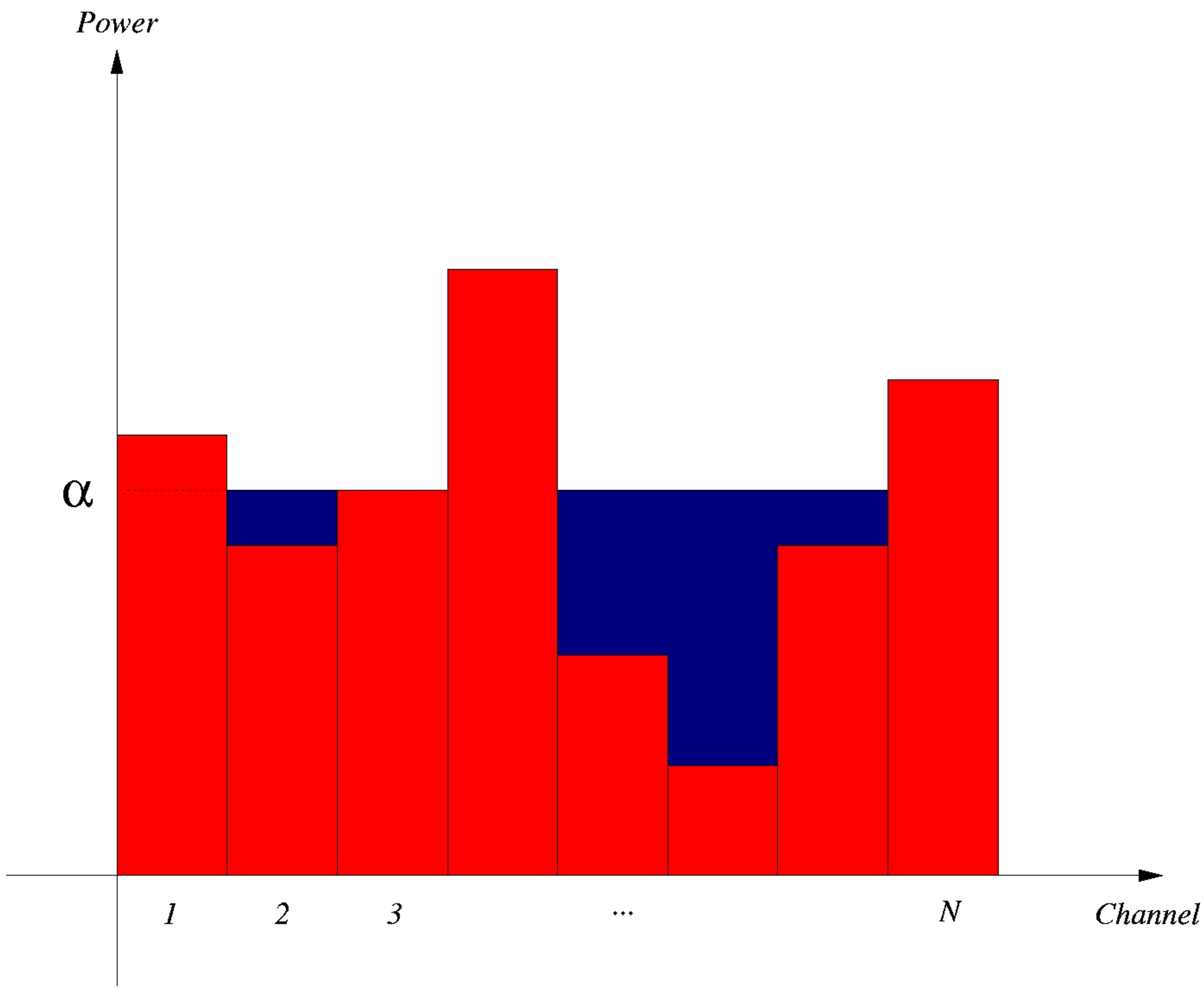
$$C_{\mathrm{WF}} = \sum_{i=1}^{N} \big(\log(\alpha\lambda_i)\big)^+ \qquad [\mathrm{bits/s/Hz}]$$

- $\mathbf{H}$ unknown at the transmitter: Choose $\mathbf{Q} = \frac{P}{N} I_N$ (**equal power**). Then,

$$C_{\mathrm{EP}} = \log\det(I_M + \tfrac{P}{N}\mathbf{H}\mathbf{H}^\dagger) \qquad [\mathrm{bits/s/Hz}]$$

[3, 7]

Waterfilling Solution



Rayleigh Fading MIMO Channel

- **Memoryless** Rayleigh fading Gaussian channel (**unknown** at the transmitter)

- Choose $\mathbf{x}$ CSCG and $\mathbf{Q} = \frac{P}{N} I_N$. The **ergodic capacity** is given by:

$$\begin{aligned} C_{\mathrm{EP}} &= \mathcal{E}_{\mathbf{H}}\Big[\log\det(I_M + \tfrac{P}{N}\mathbf{H}\mathbf{H}^\dagger)\Big] \qquad [\mathrm{bits/s/Hz}] \\ &= \mathcal{E}_{\mathbf{H}}\Big[\sum_{i=1}^{m}\log\left(1 + \tfrac{P}{N}\lambda_i\right)\Big], \end{aligned}$$

where $m = \min(N, M)$ and $\lambda_1, \ldots, \lambda_m$ are the eigenvalues of the **Wishart** matrix

$$\mathbf{W} = \begin{cases} \mathbf{H}\mathbf{H}^\dagger & M < N \\ \mathbf{H}^\dagger\mathbf{H} & M \geq N \end{cases}$$

- For large SNR, $C_{\mathrm{EP}} = \min(N, M)\log P + \mathcal{O}(1)$, i.e. the **capacity grows linearly** with $\min(N, M)$!

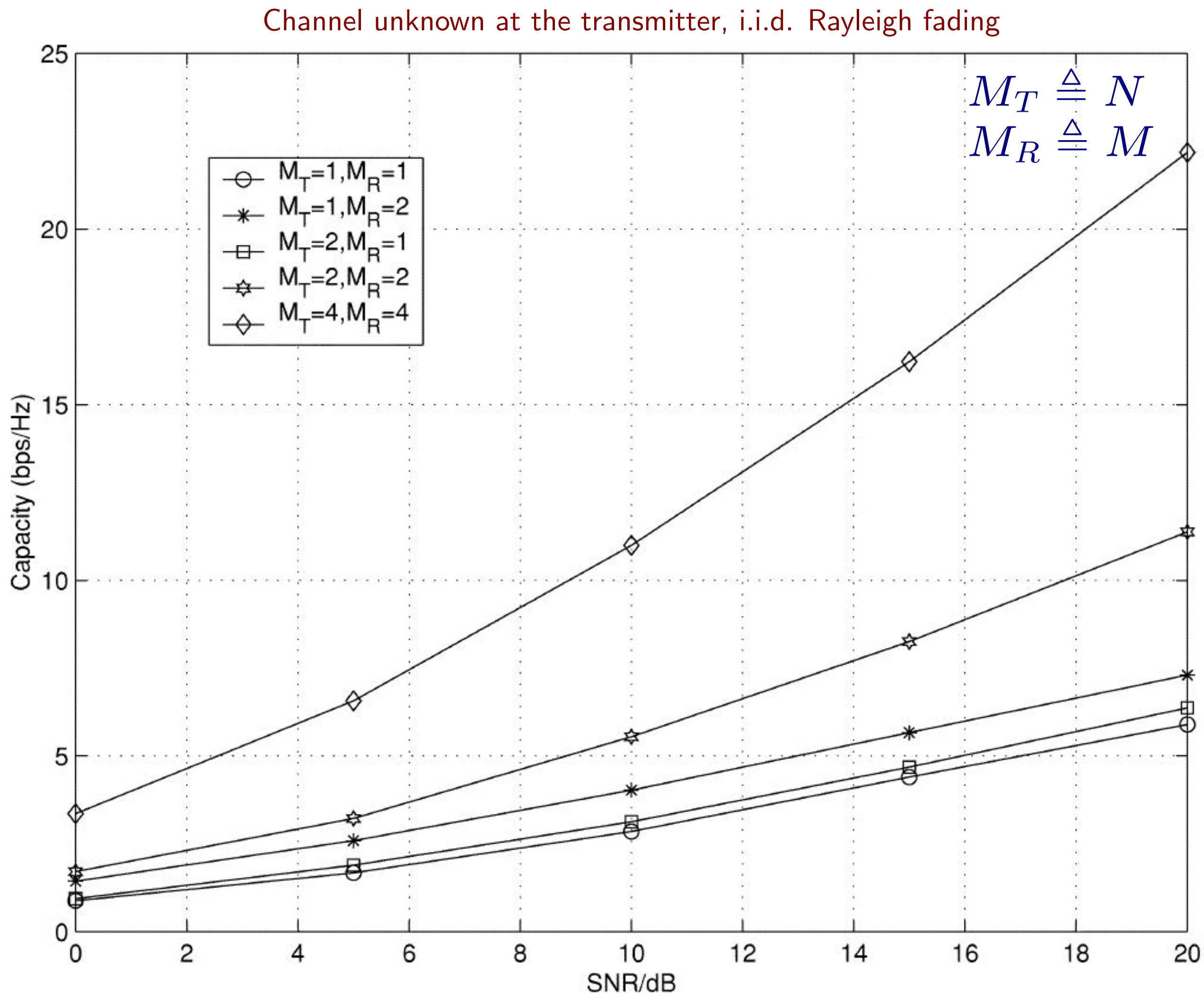
[3, 7]

Capacity of Fading Channels

- **Rayleigh fading**: the capacity grows linearly with $\min(N, M)$
- **Ricean channels**: Increasing the line–of–sight (LOS) strength at fixed SNR reduces the capacity
- If the gains in $\mathbf{H}$ become **highly correlated**, there is a capacity loss
- **Waterfilling (WF) capacity** gains over **Equal Power (EP) capacity** are significant at low SNR but converge to zero as the SNR increases

  $\Longrightarrow$ **Question:** Is it beneficial to feed the channel state back to the transmitter ?
- Many **exact** capacity results are known for **i.i.d. Rayleigh** channels. For other channels (**Rice, etc.**), we have many **limiting** results

Ergodic Capacity of Ideal MIMO Systems



[6]

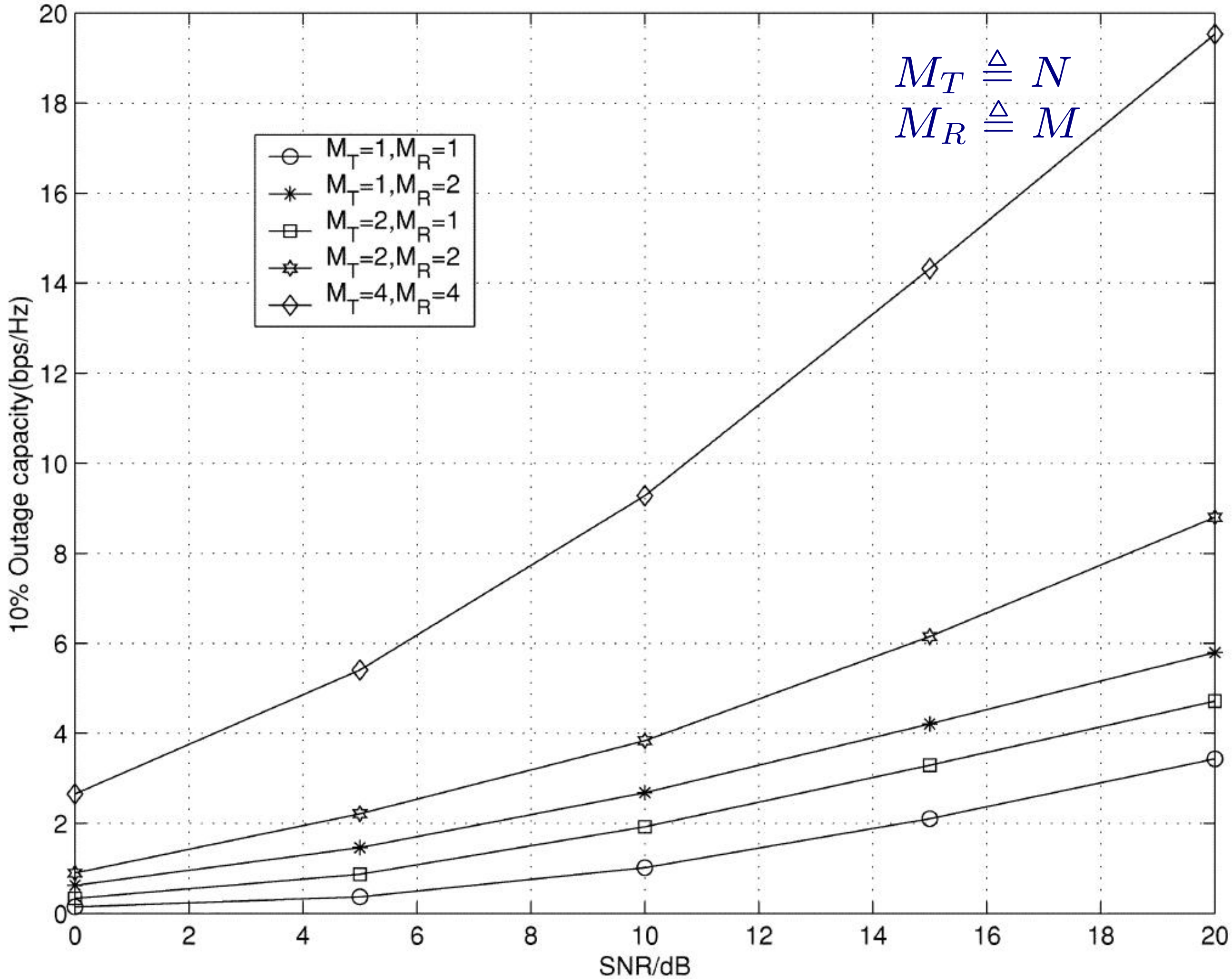
Outage Capacity

- The capacity of a fading channel is a **random variable**
- **Definition:** The $q\%$ **outage capacity** $C_{\mathrm{out,q}}$ of a fading channel is the information rate that is guaranteed for $(100 - q)\%$ of the channel realizations, i.e.
$$P(I(\mathbf{X};\mathbf{Y}) \leq C_{\mathrm{out,q}}) = q\%$$
- Since, for large SNR and i.i.d. Rayleigh fading,
$$C = \min(N, M) \log \mathrm{SNR} + \mathcal{O}(1),$$
we can define the **multiplexing gain** r as
$$r = \lim_{\mathrm{SNR}\to\infty} \frac{C(\mathrm{SNR})}{\log \mathrm{SNR}},$$
which comes at **no extra bandwidth or power**

[1, 3, 6]

Outage Capacity of Ideal MIMO Systems

Channel unknown at the transmitter, i.i.d. Rayleigh fading



[6]

Transmission over MIMO channels

We can use the advantages provided by MIMO channels to:

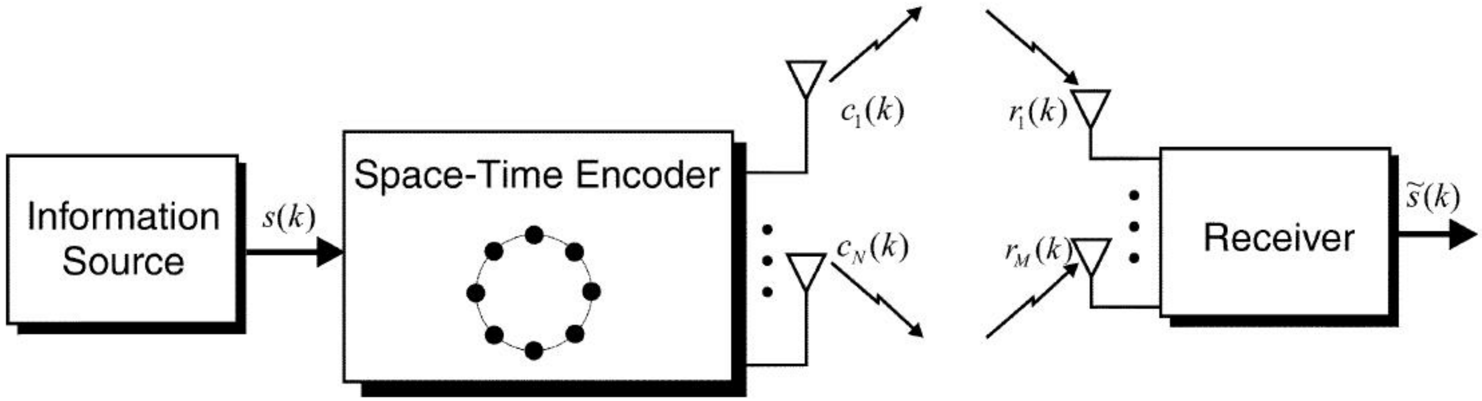
- Maximize **diversity** to combat channel fading and decrease the **bit error rate (BER)** $\Longrightarrow$ **space–time codes (STC)**

- Maximize the **throughput** $\Longrightarrow$ **spatial multiplexing, V–BLAST** (Bell laboratories layered space–time)

- Try to do **both** at the same time $\Longrightarrow$ **trade–off** between increasing the throughput and increasing diversity

[3, 6, 8]

Maximizing Diversity with Space–Time Codes

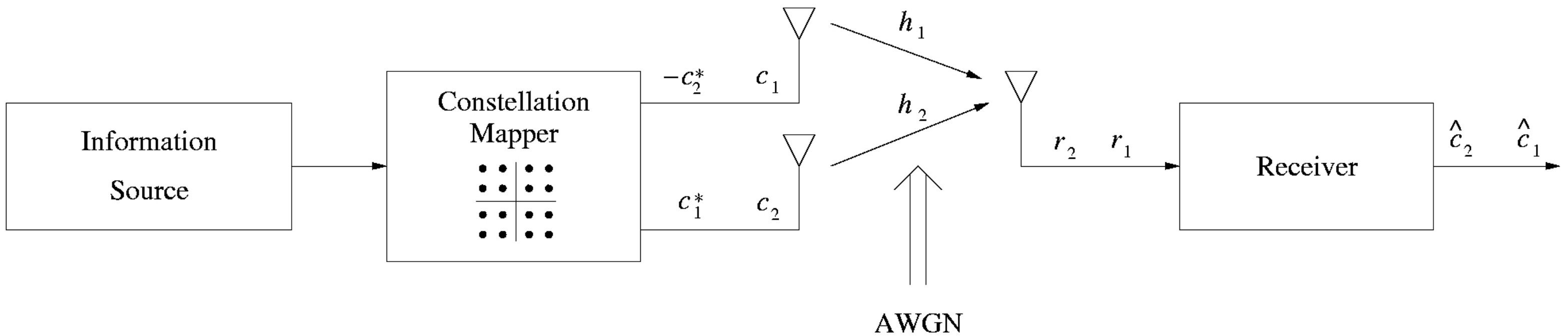


- **Space–Time Trellis Codes (STTC)** ⟵ often better performance at the cost of increased complexity
  - Complex decoding (vector version of the Viterbi algorithm) — increases exponentially with the transmission rate
  - Full diversity. Coding gain

- **Space–Time Block Codes (STBC)**
  - Simple maximum–likelihood (ML) decoding based on linear processing
  - Full diversity. Minimal or no coding gain

[3]

Alamouti Scheme for Transmit Diversity (STBC)



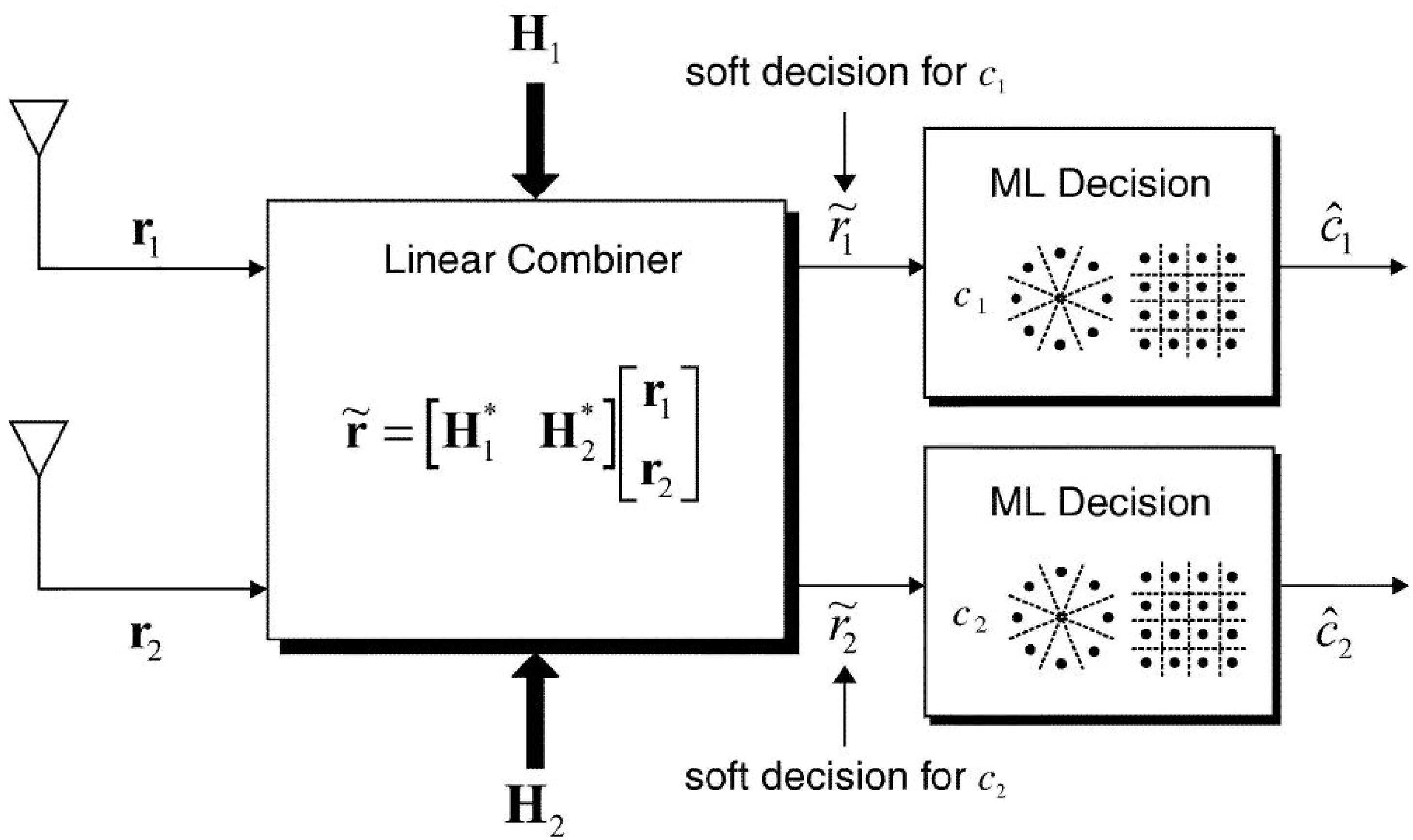
$$\begin{cases} r_1 = h_1 c_1 + h_2 c_2 + n_1 & \text{[time } t\text{]} \\ r_2 = -h_1 c_2^* + h_2 c_1^* + n_2 & \text{[time } t+T\text{]} \end{cases}$$

$$\Longrightarrow \begin{cases} \widetilde{r}_1 = h_1^* r_1 + h_2 r_2^* & = & (|h_1|^2 + |h_2|^2) c_1 + h_1^* n_1 + h_2 n_2^* & \longrightarrow & \widehat{c}_1 \\ \widetilde{r}_2 = h_2^* r_1 - h_1 r_2^* & = & (|h_1|^2 + |h_2|^2) c_2 - h_1 n_2^* + h_2^* n_1 & \longrightarrow & \widehat{c}_2 \end{cases}$$

- **Assumption:** the channel remains unchanged over two consecutive symbols
- **Rate = 1 — Diversity order = 2 — Simple decoding**

[9]

STBC Receiver Structure



[3]

STBCs from Complex Orthogonal Designs

- Alamouti's scheme works only when $N = 2 \Longrightarrow$ **Generalization**

- **Definition:** A *complex orthogonal design* $\mathcal{O}_c$ of size N is an **orthogonal matrix** with entries in the indeterminates $\pm x_1, \pm x_2, \ldots, \pm x_N$, their conjugates $\pm x_1^*, \pm x_2^*, \ldots, \pm x_N^*$ or multiples of these indeterminates by $\pm\sqrt{-1}$

- **Example** (2×2):
$$\mathcal{O}_c(x_1, x_2) = \begin{array}{cc} \text{space} \longrightarrow & \text{time} \\ \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix} & \downarrow \end{array}$$

- **Coding scheme** (using a constellation $\mathcal{A}$ with 2^b elements):

  1. At time slot t, Nb bits arrive at the encoder. Select constellation signals $c_1, \ldots, c_N$
  2. Set $x_i = c_i$ to obtain a matrix $\mathcal{C} = \mathcal{O}_c(c_1, \ldots, c_N)$
  3. At each time slot $t = 1, \ldots, N$, the entries $\mathcal{C}_{ti}, i = 1, \ldots, N$ are transmitted simultaneously from transmit antennas $1, 2, \ldots, N$

[10]

STBCs from Complex Orthogonal Designs

- The maximum–likelihood detection rule reduces to **simple linear processing** for STBCs

- One can obtain the **maximum possible diversity order** MN at transmission rate $R = 1$ using STBCs based on orthogonal designs

- **However:** complex orthogonal designs exist only if $n = 2 \ldots$!

[10]

Generalized Complex Orthogonal Designs (GCOD)

- **Definition:** Let $\mathcal{G}_c$ be a $p \times N$ matrix with entries in the indeterminates $\pm x_1, \pm x_2, \ldots, \pm x_k$, their conjugates $\pm x_1^*, \pm x_2^*, \ldots, \pm x_k^*$ or multiples of these indeterminates by $\pm\sqrt{-1}$ or 0. If $\mathcal{G}_c^\dagger \mathcal{G}_c = (|x_1|^2 + \cdots + |x_k|^2)I$, then $\mathcal{G}_c$ is referred to as a *generalized complex orthogonal design* of size N and rate $R = k/p$

- **Definition:** *Generalized complex linear processing orthogonal design* (GCLPOD) $\mathcal{L}_c$: exactly like above, but the entries can be linear combinations of $x_1, \ldots, x_k$ and their conjugates

- One can obtain a **diversity order of** MN at rate R using a STBC based on a GCOD or a GCLPOD of size N and rate R

[10]

Generalized Complex Orthogonal Designs

- Generalized complex linear processing orthogonal designs of rates:
  - $R = 1$ exist for $N = 2$
  - $R = 3/4$ exist for $N = 3$ and $N = 4$
  - $R = 1/2$ exist for $N \geq 5$

- For $N \geq 3$, **it is not known** whether GCLPODs with higher rates exist

- **Example** (GCLPOD, $R = \frac{3}{4}$, $N = 3$ and GCOD, $R = \frac{1}{2}$, $N = 3$):

$$\mathcal{L}_c^3 = \begin{pmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} \\ \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & \frac{-x_1 - x_1^* + x_2 - x_2^*}{2} \\ \frac{x_3^*}{\sqrt{2}} & -\frac{x_3^*}{\sqrt{2}} & \frac{x_2 + x_2^* + x_1 - x_1^*}{2} \end{pmatrix} \qquad \mathcal{G}_c^3 = \begin{pmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \\ x_1^* & x_2^* & x_3^* \\ -x_2^* & x_1^* & -x_4^* \\ -x_3^* & x_4^* & x_1^* \\ -x_4^* & -x_3^* & x_2^* \end{pmatrix}$$

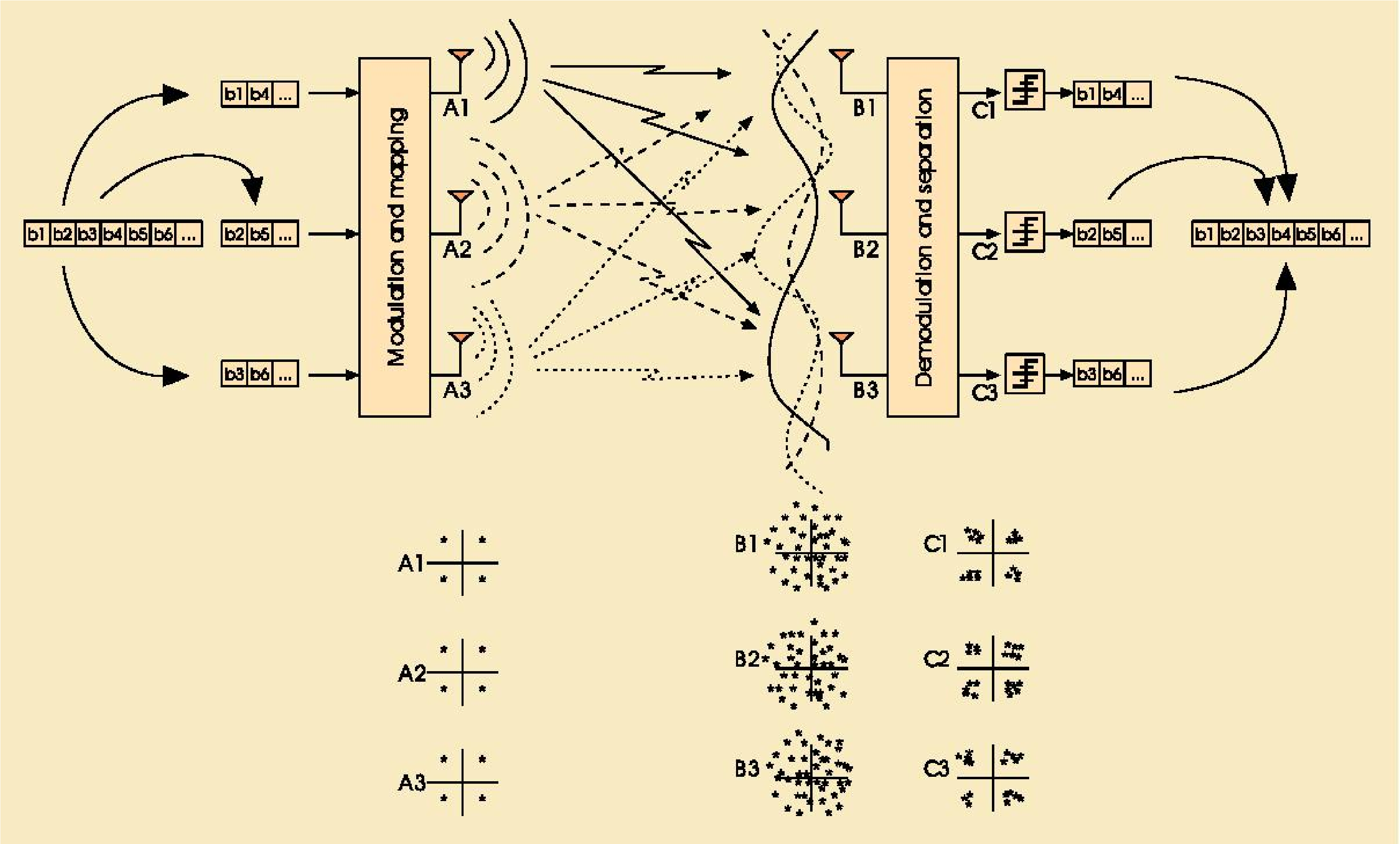
[10]

Capacity and Space–Time Block Codes

- Space–time block codes
  - have **extremely low** encoder/decoder complexity
  - provide **full diversity**
- However
  - For the i.i.d. Rayleigh channel, STBCs result in a **capacity loss** in the presence of multiple receive antennas
  - STBCs are **only optimal with respect to capacity** when they have rate $R = 1$ and there is **one receive antenna**

[11]

Maximizing the Throughput with V–BLAST



[1]

Maximizing the Throughput with V–BLAST

Description

- Transmitters operate **co–channel**, symbol synchronized
- Substreams are **exactly independent** (no coding across the transmit antennas — each substream can be individually coded)
- Individual **transmit powers scaled** by $\frac{1}{N}$ so the total power is kept constant
- Channel estimation burst by burst using a training sequence
- Requires near–independent channel coefficients

[4]

Receivers for Spatial Multiplexing

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \qquad i.e.$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N} \\ h_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ h_{M1} & \cdots & \cdots & h_{MN} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_M \end{bmatrix}$$

- If we transmit a block of $N \times T$ symbols, we have $\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N}$, with $\mathbf{Y}, \mathbf{N} \in \mathbb{C}^{M \times T}$ and $\mathbf{X} \in \mathbb{C}^{N \times T}$

- **Optimal (ML)** Receiver: $\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \left\| \mathbf{y} - \mathbf{H}\mathbf{x} \right\|$
  - Exhaustive search (often prohibitive complexity)
  - Diversity order for each data stream: $M \qquad (N \leq M)$

[3, 4, 6]

Receivers for Spatial Multiplexing

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

- **Zero–forcing (ZF)** Receiver:

$$\hat{\mathbf{x}} = \mathbf{H}^{\#}\mathbf{y}$$

with $\mathbf{H}^{\#} = (\mathbf{H}^{\dagger}\mathbf{H})^{-1}\mathbf{H}^{\dagger}$ **(pseudo–inverse)**

  - **Significantly reduced** receiver complexity
  - **Noise enhancement** problem
  - Diversity order for each data stream: $M - N + 1$ $(N \leq M)$

[3, 4, 6, 12]

Receivers for Spatial Multiplexing

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

- **Minimum mean–square error (MMSE)** Receiver:

$$\hat{\mathbf{x}} = \widetilde{\mathbf{W}} \cdot \mathbf{y}, \qquad \text{where } \widetilde{\mathbf{W}} = \arg\min_{\mathbf{W}} \mathcal{E}\left[\left\|\mathbf{W}\mathbf{y} - \mathbf{x}\right\|^2\right].$$

We obtain:

$$\hat{\mathbf{x}} = \mathbf{H}^{\dagger}\left(\mathbf{H}\mathbf{H}^{\dagger} + \mathcal{E}\left[\mathbf{n}\mathbf{n}^{\dagger}\right]\right)^{-1} \cdot \mathbf{y}$$

  - Minimizes the **overall error** due to noise and mutual interference
  - Equivalent to the zero–forcing receiver at high SNR
  - Diversity order for each data stream: approximately $M - N + 1$ $(N \leq M)$

[3, 4, 6, 12]

Receivers for Spatial Multiplexing

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \qquad\qquad \mathbf{H} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \cdots & \mathbf{h}_N \end{bmatrix}$$

- **V–BLAST** receiver — successive interference cancellation (SIC):

$$\begin{aligned} \widetilde{x}_1 &= \mathbf{w}_1^T \mathbf{y} \\ \hat{x}_1 &= Q(\widetilde{x}_1) && \text{(quantization)} \\ \mathbf{y}_2 &= \mathbf{y} - \hat{x}_1 \mathbf{h}_1 && \text{(interference cancellation)} \\ \widetilde{x}_2 &= \mathbf{w}_2^T \mathbf{y}_2, && \text{etc.} \end{aligned}$$

- The ith ZF–nulling vector $\mathbf{w}_i$ is defined as the **unique minimum–norm vector** satisfying

$$\mathbf{w}_i^T \mathbf{h}_j = \begin{cases} 0 & j > i \\ 1 & j = i, \end{cases}$$

  is **orthogonal** to the subspace spanned by the contributions to $\mathbf{y}_i$ due to the symbols not yet estimated and cancelled and is given by the ith row of $\mathbf{H}^{\#} = (\mathbf{H}^{\dagger}\mathbf{H})^{-1}\mathbf{H}^{\dagger} \qquad (N \leq M)$

[13]

Receivers for Spatial Multiplexing

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \qquad \mathbf{H} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \cdots & \mathbf{h}_N \end{bmatrix}$$

- **V–BLAST** receiver
  - The SNR of $\widetilde{x}_i$ is proportional to $1/\|\mathbf{w}_i\|^2$
  - **Idea:** detect the components x_i in **order of decreasing SNR** $\Longrightarrow$ ordered successive interference cancellation (OSIC)

$$\begin{array}{lrcl}
\text{initialization:} & \mathbf{G}_1 & = & \mathbf{H}^{\#} \qquad \mathbf{G}_i = \begin{bmatrix} \mathbf{g}_i^1 & \mathbf{g}_i^2 & \cdots & \mathbf{g}_i^N \end{bmatrix}^T \\
 & i & = & 1 \\
 & \mathbf{y}_1 & = & \mathbf{y} \\
\text{recursion:} & k_i & = & \arg\min_{j \notin \{k_1,\ldots,k_{i-1}\}} \left\|\mathbf{g}_i^j\right\|^2 \\
 & \mathbf{w}_{k_i} & = & \mathbf{g}_i^{k_i} \\
 & \widetilde{x}_{k_i} & = & \mathbf{w}_{k_i}^T \mathbf{y}_i \\
 & \hat{x}_{k_i} & = & Q(\widetilde{x}_{k_i}) \\
 & \mathbf{y}_{i+1} & = & \mathbf{y}_i - \hat{x}_{k_i}\mathbf{h}_{k_i} \\
 & \mathbf{G}_{i+1} & = & \mathbf{H}_{\overline{k_i}}^{\#} \qquad \mathbf{H}_{\overline{k_i}} \triangleq \mathbf{H} \text{ with columns } k_1, \cdots, k_i \text{ set to } 0 \\
 & i & = & i+1
\end{array}$$

[13]

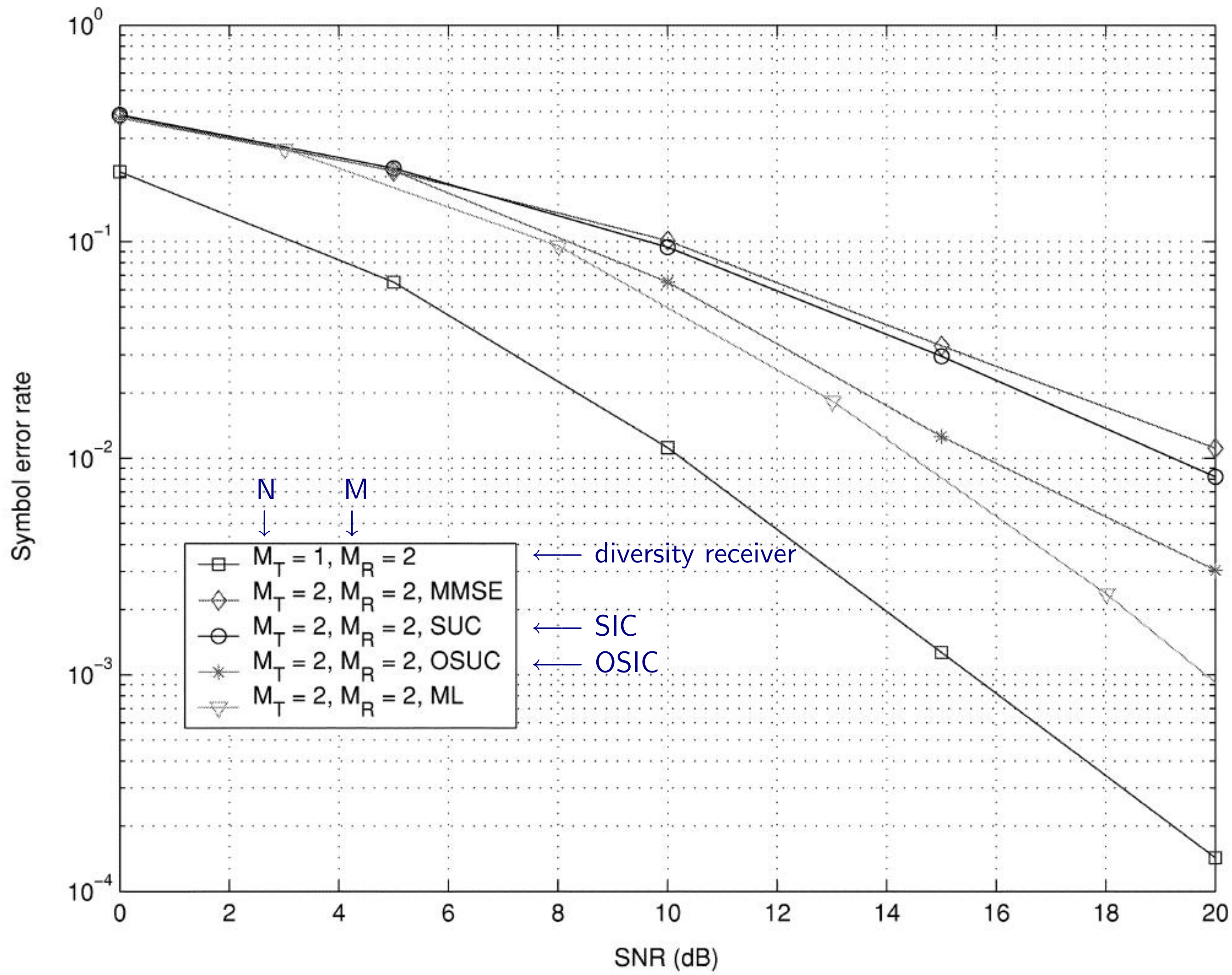
Receivers for Spatial Multiplexing

- The **V–BLAST SIC** receiver:
  - Provides a reasonable trade–off between complexity and performance (between MMSE and ML receivers)
  - Achieves a diversity order of approximately $M - N + 1$ per data stream $(N \leq M)$

- **The V–BLAST OSIC** receiver:
  - Provides a reasonable trade–off between complexity and performance (between MMSE and ML receivers)
  - Achieves a diversity order which lies between $M - N + 1$ and M for each data stream $(N \leq M)$

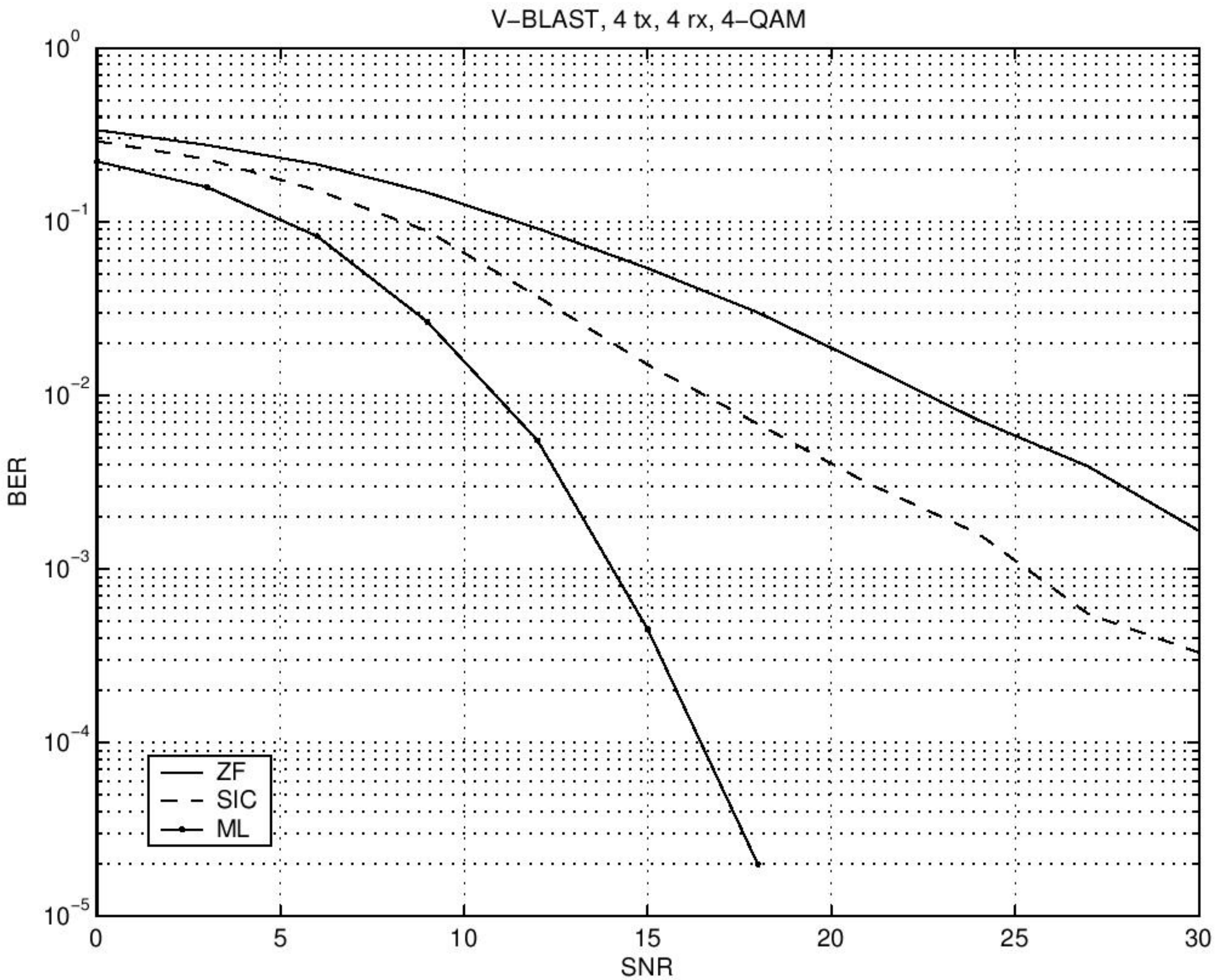
[3, 6]

Performance Comparison



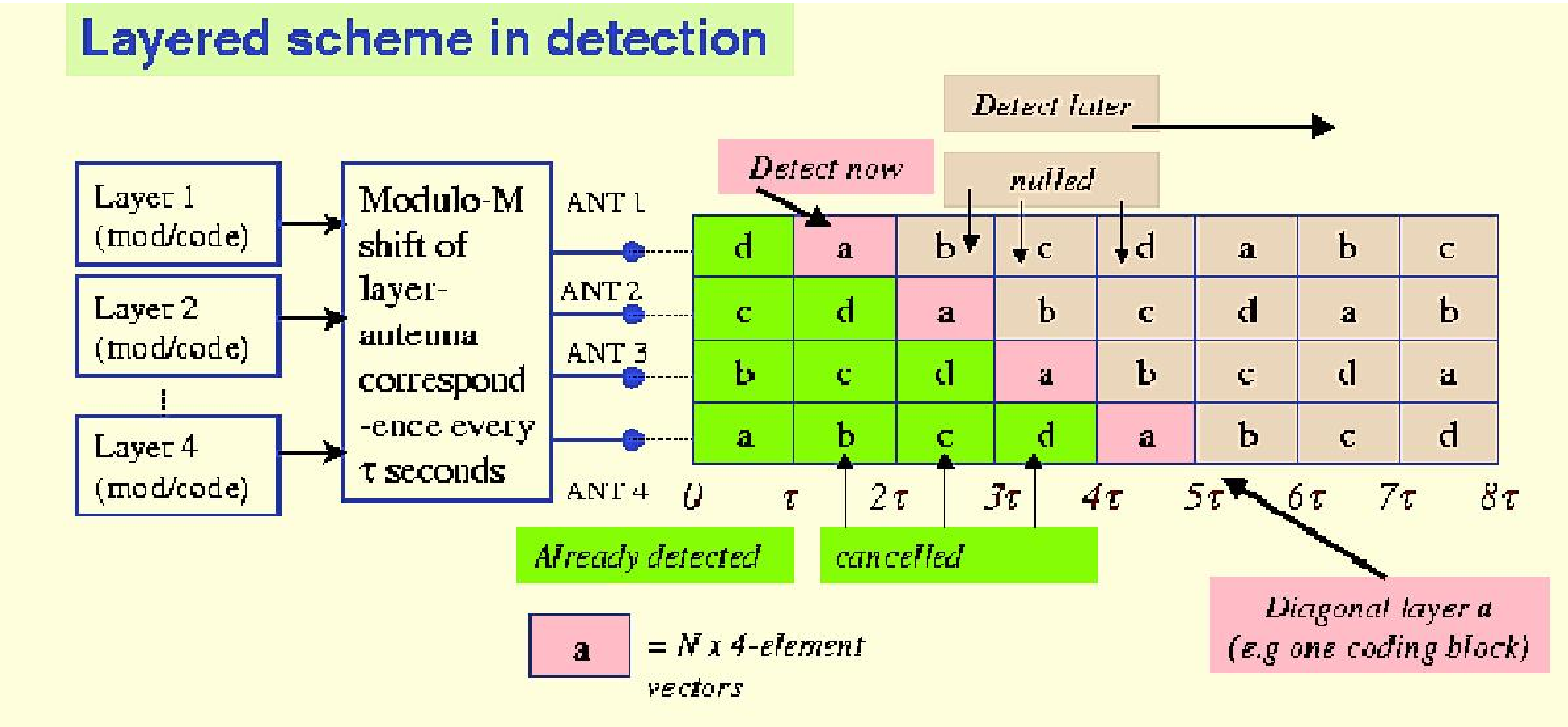
[6]

Performance Comparison



[4]

D–BLAST



Layered scheme in detection
Detect later
Detect now
nulled
Layer 1 (mod/code)
Layer 2 (mod/code)
Layer 4 (mod/code)
Modulo-M shift of layer-antenna correspond-ence every τ seconds
ANT 1
ANT 2
ANT 3
ANT 4
d a b c d a b c
c d a b c d a b
b c d a b c d a
a b c d a b c d
0 τ 2τ 3τ 4τ 5τ 6τ 7τ 8τ
Already detected
cancelled
Diagonal layer a (e.g one coding block)
a = N x 4-element vectors

[4, 14]

Linear Dispersion Codes

- **V–BLAST**
  - is unable to work with **fewer receive than transmit** antennas
  - doesn't have any built–in spatial coding

- **Space–time codes** do not perform well at high data rates

- **Linear dispersion codes**
  - include V–BLAST and the orthogonal design STBCs as special cases
  - can be used for any number of transmit and receive antennas
  - can be decoded with V–BLAST like algorithms
  - satisfy an information–theoretic optimality criterion

[4, 15]

Linear Dispersion Codes

- A **linear dispersion code** of rate $R = \frac{k}{p}b$ is one for which

$$\mathbf{X} = \sum_{i=1}^{k} (c_i \mathbf{C}_i + c_i^* \mathbf{D}_i), \qquad \mathbf{X} = \begin{bmatrix} \mathbf{x}^1 \\ \mathbf{x}^2 \\ \vdots \\ \mathbf{x}^p \end{bmatrix}$$

where $c_i, \ldots, c_k$ belong to a constellation $\mathcal{A}$ with 2^b symbols and $\mathbf{C}_i, \mathbf{D}_i \in \mathbb{C}^{p \times N}$

Number of transmit antennas: N
Number of receive antennas: M

[15]

Linear Dispersion Codes

- If $\mathbf{Y} = \mathbf{X}\mathbf{H}^T + \mathbf{N}$, it can be shown that: $(\mathbf{H} \in \mathbb{C}^{M \times N};\ \mathbf{Y}, \mathbf{N} \in \mathbb{C}^{p \times M})$

$$\underbrace{\begin{bmatrix} \hat{\mathbf{y}}_1 \\ \vdots \\ \hat{\mathbf{y}}_M \end{bmatrix}}_{\eta} = \mathcal{H} \underbrace{\begin{bmatrix} \hat{c}_1 \\ \vdots \\ \hat{c}_k \end{bmatrix}}_{\xi} + \begin{bmatrix} \hat{\mathbf{n}}_1 \\ \vdots \\ \hat{\mathbf{n}}_M \end{bmatrix}, \qquad \begin{aligned} \mathbf{Y} &= \begin{bmatrix} \mathbf{y}_1 & \cdots & \mathbf{y}_M \end{bmatrix} \\ \mathbf{N} &= \begin{bmatrix} \mathbf{n}_1 & \cdots & \mathbf{n}_M \end{bmatrix} \end{aligned}$$

where $\hat{\mathbf{y}}_i \triangleq \begin{bmatrix} \mathfrak{Re}(\mathbf{y}_i) \\ \mathfrak{Im}(\mathbf{y}_i) \end{bmatrix}$, $\hat{\mathbf{n}}_i \triangleq \begin{bmatrix} \mathfrak{Re}(\mathbf{n}_i) \\ \mathfrak{Im}(\mathbf{n}_i) \end{bmatrix}$, $\hat{c}_i \triangleq \begin{bmatrix} \mathfrak{Re}(c_i) \\ \mathfrak{Im}(c_i) \end{bmatrix}$ and

$$\mathcal{H} \in \mathbb{C}^{2Mp \times 2k} = f(\mathbf{H}, \mathbf{C}_1, \ldots \mathbf{C}_k, \mathbf{D}_1, \ldots \mathbf{D}_k)$$

- **V–BLAST like techniques** can thus be used to decode linear dispersion codes
- $\{\mathbf{C}_1, \ldots, \mathbf{C}_k, \mathbf{D}_1, \ldots, \mathbf{D}_k\}$ are dispersion matrices designed to optimize given criteria (e.g. **maximum mutual information** between η and ξ)

[15]

Diversity vs. Multiplexing Trade–off

$$C = \min\{N, M\} \log \mathrm{SNR} + \mathcal{O}(1)$$

- **Definition:** A **scheme** $\{\mathcal{C}(\mathrm{SNR})\}$ is a **family of codes** of block length l, one for each SNR level. $R(\mathrm{SNR})$ [b/symbol] denotes the **rate** of the code $\mathcal{C}(\mathrm{SNR})$

- **Definition:** A scheme $\{\mathcal{C}(\mathrm{SNR})\}$ is said to achieve **spatial multiplexing gain** r and **diversity gain** d if the data rate

$$\lim_{\mathrm{SNR}\to\infty} \frac{R(\mathrm{SNR})}{\log \mathrm{SNR}} = r$$

and the average error probability

$$\lim_{\mathrm{SNR}\to\infty} \frac{\log P_e(\mathrm{SNR})}{\log \mathrm{SNR}} = -d \qquad (2)$$

[8]

Diversity vs. Multiplexing Trade–off

- For each r, $d^*(r)$ is the **supremum of the diversity gains** achieved over all schemes

- We also define:
  - $d^*_{\max} \triangleq d^*(0)$, the **maximal diversity gain**
  - $r^*_{\max} \triangleq \sup\{r | d^*(r) > 0\}$, the **maximal spatial multiplexing gain**

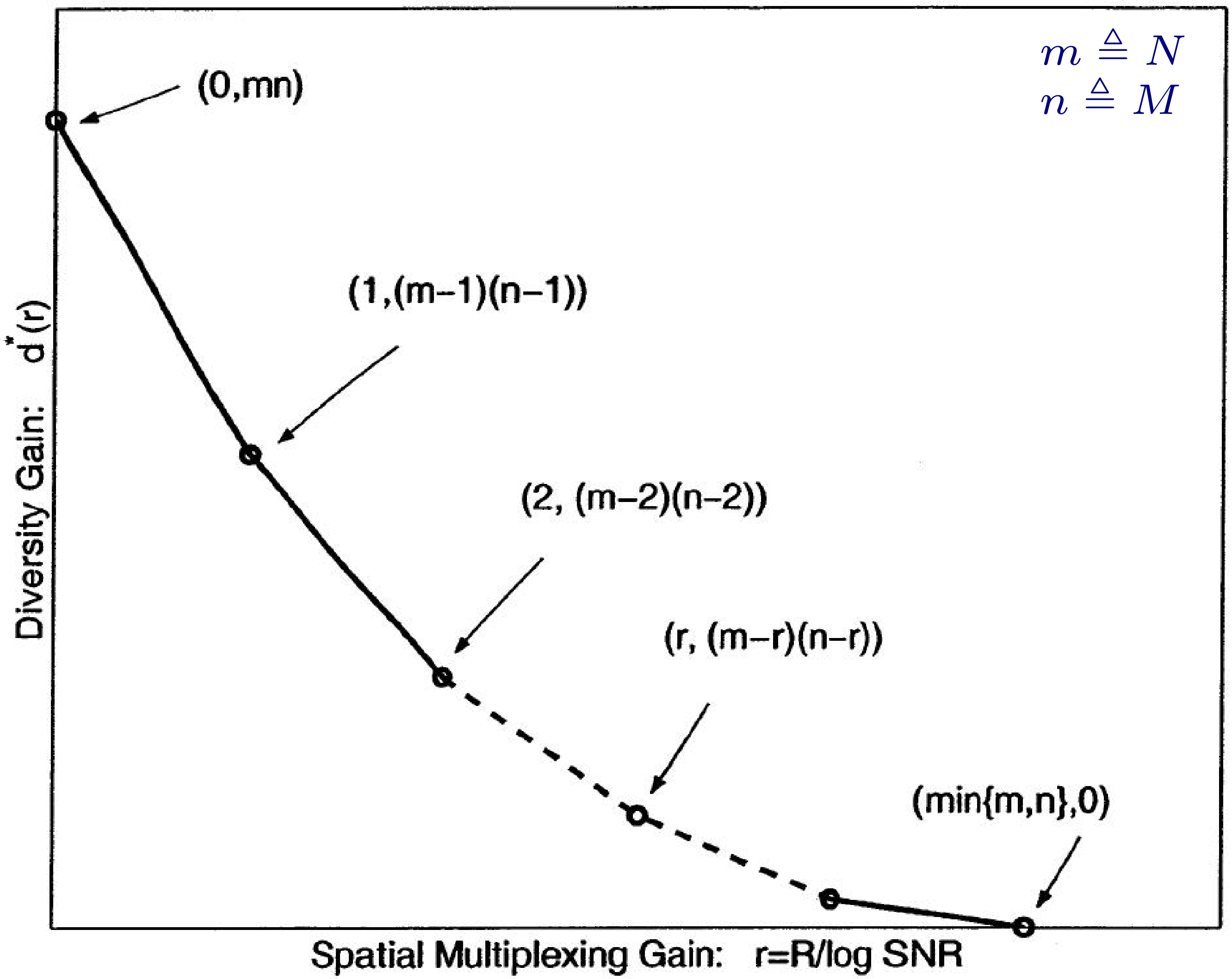
- **Theorem:** Assume $l \geq N + M - 1$. The **optimal trade–off curve** $d^*(r)$ is given by the **piecewise–linear function** connecting the points $(k, d^*(k))$, $k = 0, 1, \ldots, \min\{N, M\}$, where

$$d^*(k) = (N - k)(M - k).$$

In particular, $d^*_{\max} = NM$ and $r^*_{\max} = \min\{N, M\}$.

[8]

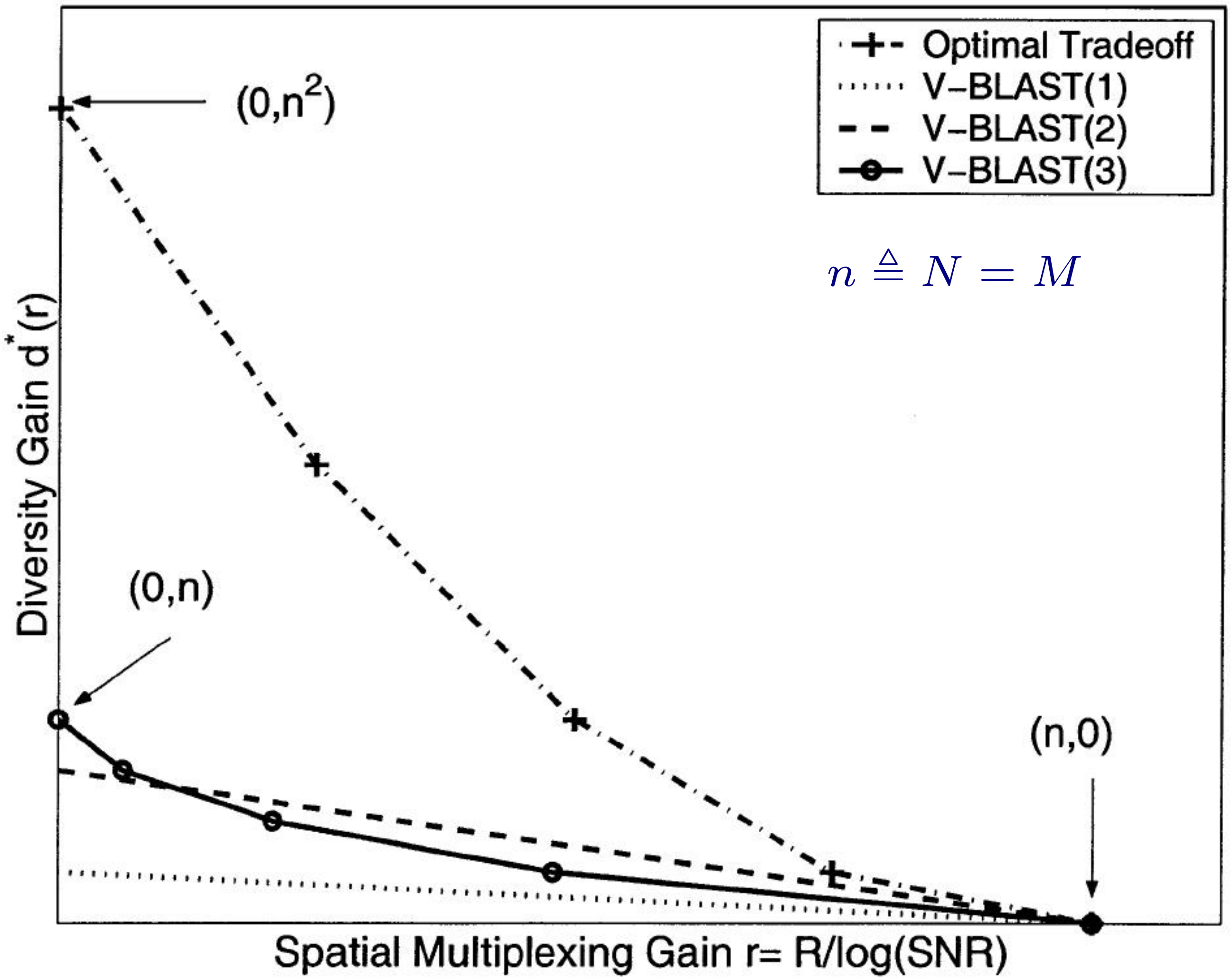
Diversity vs. Multiplexing: Optimal Trade–off



$m \triangleq N$
$n \triangleq M$

[8]

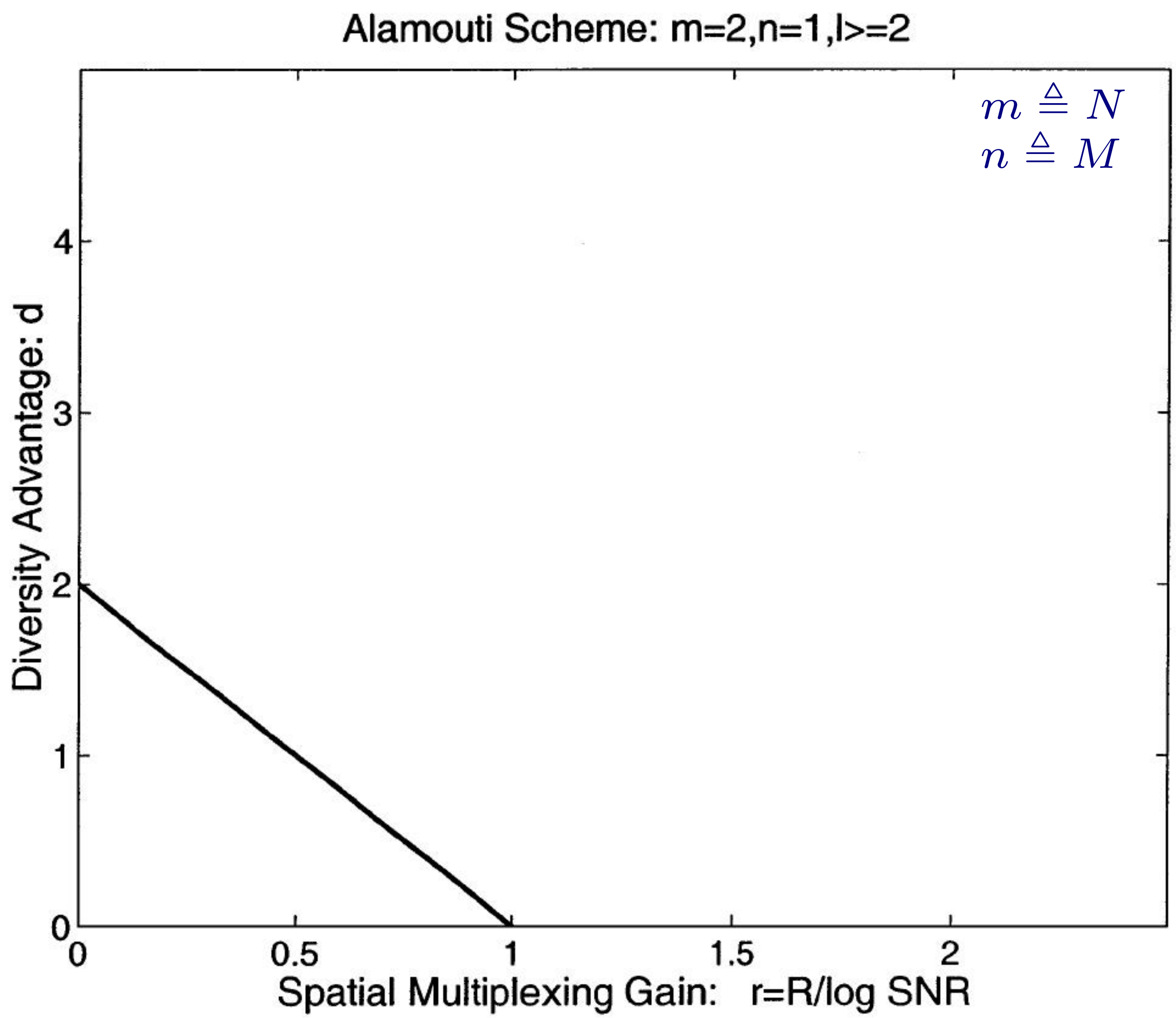
Diversity vs. Multiplexing Trade–off: V–BLAST



$n \triangleq N = M$

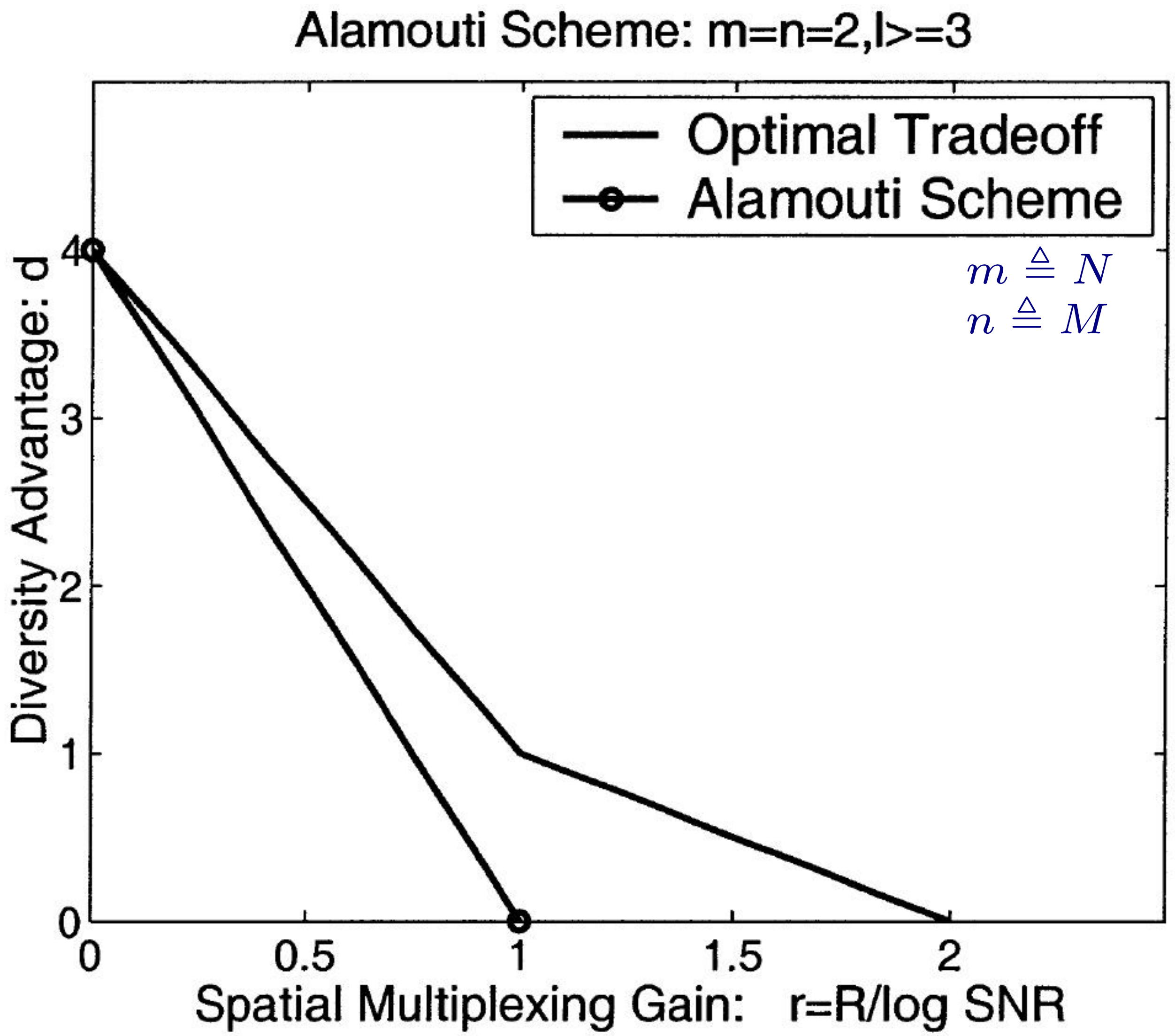
[8]

Diversity vs. Multiplexing Trade–off: Alamouti Scheme



[8]

Diversity vs. Multiplexing Trade–off: Alamouti Scheme



[8]

Diversity vs. Multiplexing Trade–off

- Definitions (1) and (2) for the **diversity gain** are **not equivalent**: in the former one, a **fixed data rate** is assumed for all SNRs, whereas in the latter one, the data rate is a **fraction of** $C(\mathrm{SNR})$, and hence increases with the SNR

- Definition (1) is the **most widely used** in the literature

- Definition (2) allows to **quantify** the diversity vs. multiplexing trade–off

[6, 8]

MIMO Channel Modeling

- **A good MIMO channel model must include**:
  - Path loss
  - Shadowing
  - Doppler and delay spread profiles
  - Ricean K factor distribution
  - Joint antenna correlation at transmit and receive ends
  - Channel matrix singular value distribution

[3]

Ricean K factor distribution

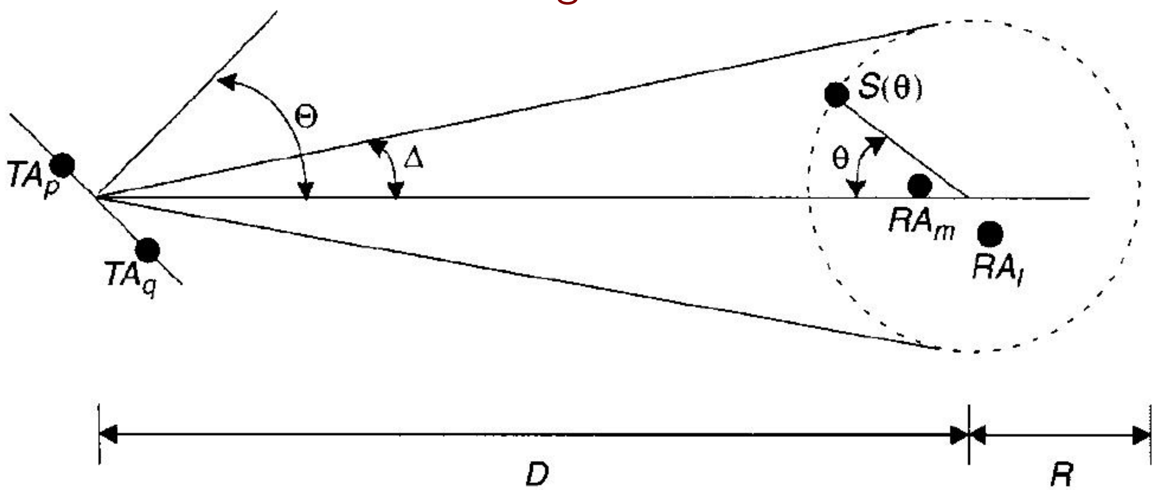
$$\mathbf{H} = \mathbf{H}_{\mathrm{LOS}} + \mathbf{H}_{\mathrm{NLOS}}$$

- The higher the Ricean K factor, the more dominant $\mathbf{H}_{\mathrm{LOS}}$ (line–of–sight)
- $\mathbf{H}_{\mathrm{LOS}}$ is a time–invariant, often **low rank** matrix $\Longrightarrow$ high K factor channels often exhibit a **low capacity**
- In a near–LOS link, the improvement in link budget often more than compensates for the loss of MIMO capacity $\Longrightarrow$ usually, the LOS component is **not intentionally reduced**
- **Experimental measurements** show that, in general:
  - K **increases** with antenna height
  - K **decreases** with transmitter–receiver distance $\Longrightarrow$ MIMO substantially increases throughput in areas far away from the base station

[3]

Correlation Model for $\mathbf{H}_{\mathrm{NLOS}}$

"One–ring" model



- Base Station (BS) usually elevated and unobstructed by local scatterers
- Subscriber Unit (SU) often **surrounded** by local scatterers — assumed here uniformly distributed in θ

TA_l : lth transmitting antenna element
RA_l : lth receiving antenna element
$S(\theta)$: scatterer located at angle θ

Θ : angle of arrival
Δ : angle spread

[16]

Correlation Model for $\mathbf{H}_{\mathrm{NLOS}}$

- Correlation from one BS antenna element to two SU antenna elements:

$$\mathcal{E}[\mathbf{H}_{l,p}\mathbf{H}^*_{m,p}] \approx J_0\left(\frac{2\pi}{\lambda}\, \underset{\substack{\uparrow \\ \text{distance between antennas } l \text{ and } m}}{d(l,m)}\right)$$

- Correlation from two BS antenna elements to one SU antenna element in the **broadside** direction ($\Theta = 0$):

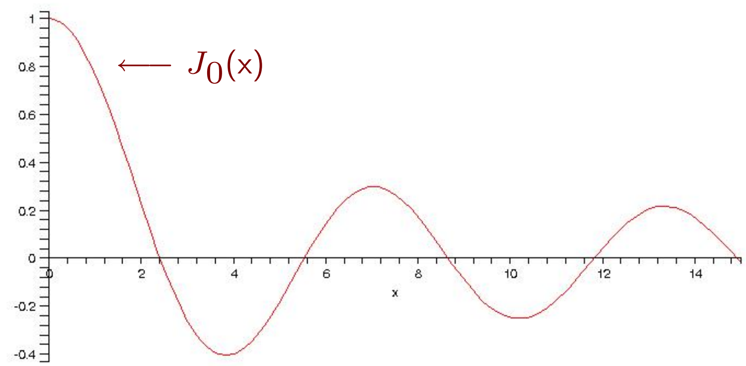
$$\mathcal{E}[\mathbf{H}_{m,p}\mathbf{H}^*_{m,q}] \approx J_0\left(\Delta\frac{2\pi}{\lambda}\, \underset{\substack{\uparrow \\ \text{distance between antennas } p \text{ and } q}}{d(p,q)}\right)$$

- Correlation from two BS antenna elements to one SU antenna element in the **inline** direction ($\Theta = \frac{\pi}{2}$):

$$\mathcal{E}[\mathbf{H}_{m,p}\mathbf{H}^*_{m,q}] \approx e^{-j\frac{2\pi}{\lambda}\, d(p,q)\, \left(1-\frac{\Delta^2}{4}\right)} \cdot J_0\left(\left(\frac{\Delta}{2}\right)^2 \frac{2\pi}{\lambda}\, d(p,q)\right)$$

[3, 16]

Correlation Model for $\mathbf{H}_{\text{NLOS}}$



- The mobiles have to be in the **broadside** direction to obtain the highest diversity

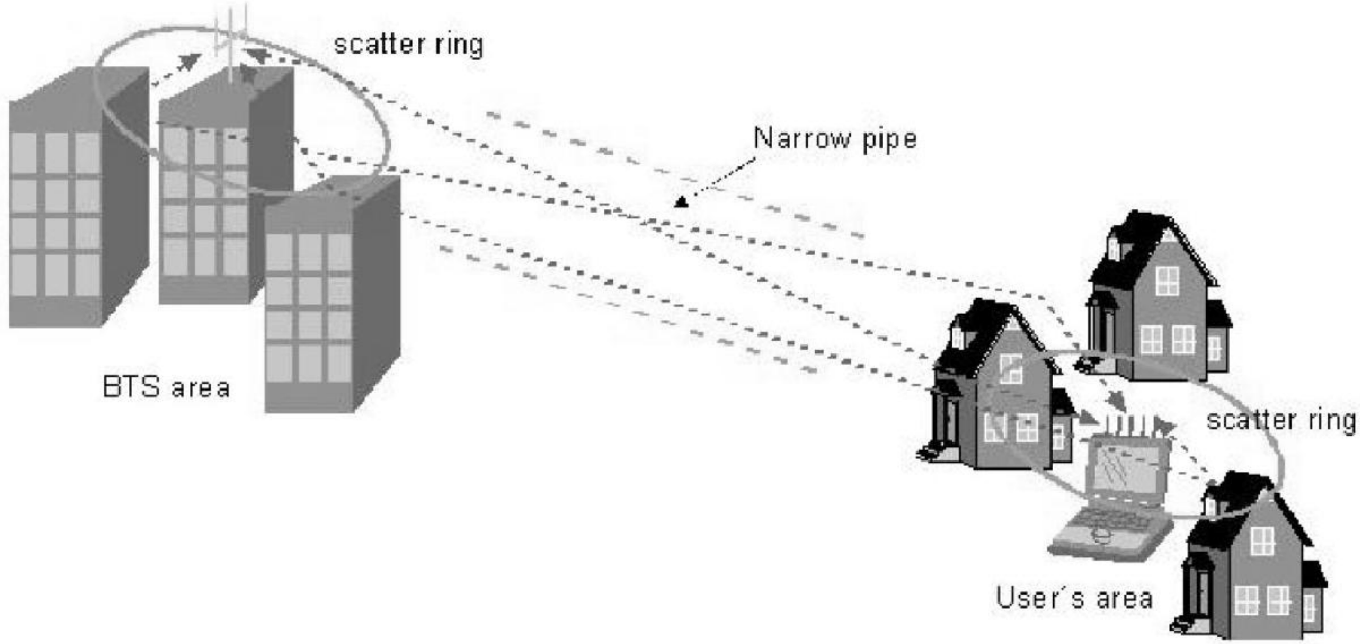
- **Interelement spacing** has to be high to have low correlation $\Longrightarrow$ beamforming and MIMO yield **conflicting criteria**

- Using the above results, one can obtain **upper bounds for the MIMO capacity**

[3, 16]

Decoupling Between Rank and Correlation

Pinhole channel



- Uncorrelated fading at both ends **doesn't necessarily** imply a high–rank channel

[3, 4]

MIMO Channel Modeling

- **Time–varying** wideband MIMO channel:

$$\mathbf{H}(\tau) = \sum_{i=1}^{L} \mathbf{H}_i \delta(\tau - \tau_i)$$

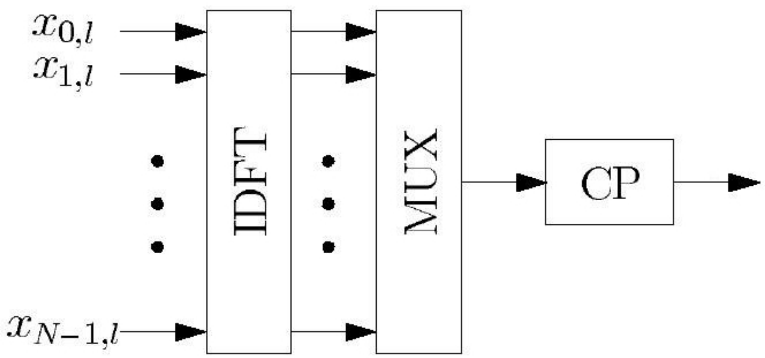
where $\mathbf{H}(\tau) \in \mathbb{C}^{M \times N}$ and only $\mathbf{H}_1$ contains a LOS component

- Typical **interelement spacing**:
  - Base station: 10λ (due to the absence of local scatterers)
  - Subscriber unit: $\frac{1}{2}\lambda$ (rich scattering)

[3]

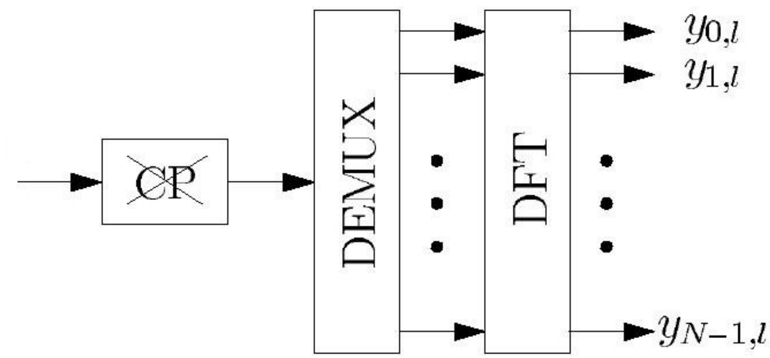
MIMO–OFDM Systems

SISO OFDM Transmitter

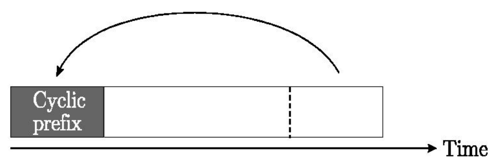


$N \triangleq K$, l = OFDM symbol number

SISO OFDM Receiver



$N \triangleq K$



- **Net result**: The **frequency selective** fading channel of bandwidth B is decomposed into K **parallel frequency-flat** fading channels, each having bandwidth $\frac{B}{K}$. (Condition: The **impulse response** of the channel is **shorter** than the **length of the cyclic prefix**)

[6, 17]

MIMO–OFDM Systems

- OFDM can be **extended to MIMO** systems by performing the IDFT/DFT and CP operations at **each of the transmit and receive antennas** (with the appropriate condition on the length of the cyclic prefix)

- **Diversity systems:** (Ex: Alamouti scheme)
  - Send c_1 and c_2 over OFDM tone i over antennas 1 and 2
  - Send $-c_2^*$ and c_1^* over OFDM tone $i+1$ over antennas 1 and 2 within the same OFDM symbol
  - **Alternative technique**: Code on a **per–tone** basis across OFDM symbols in time

[6]

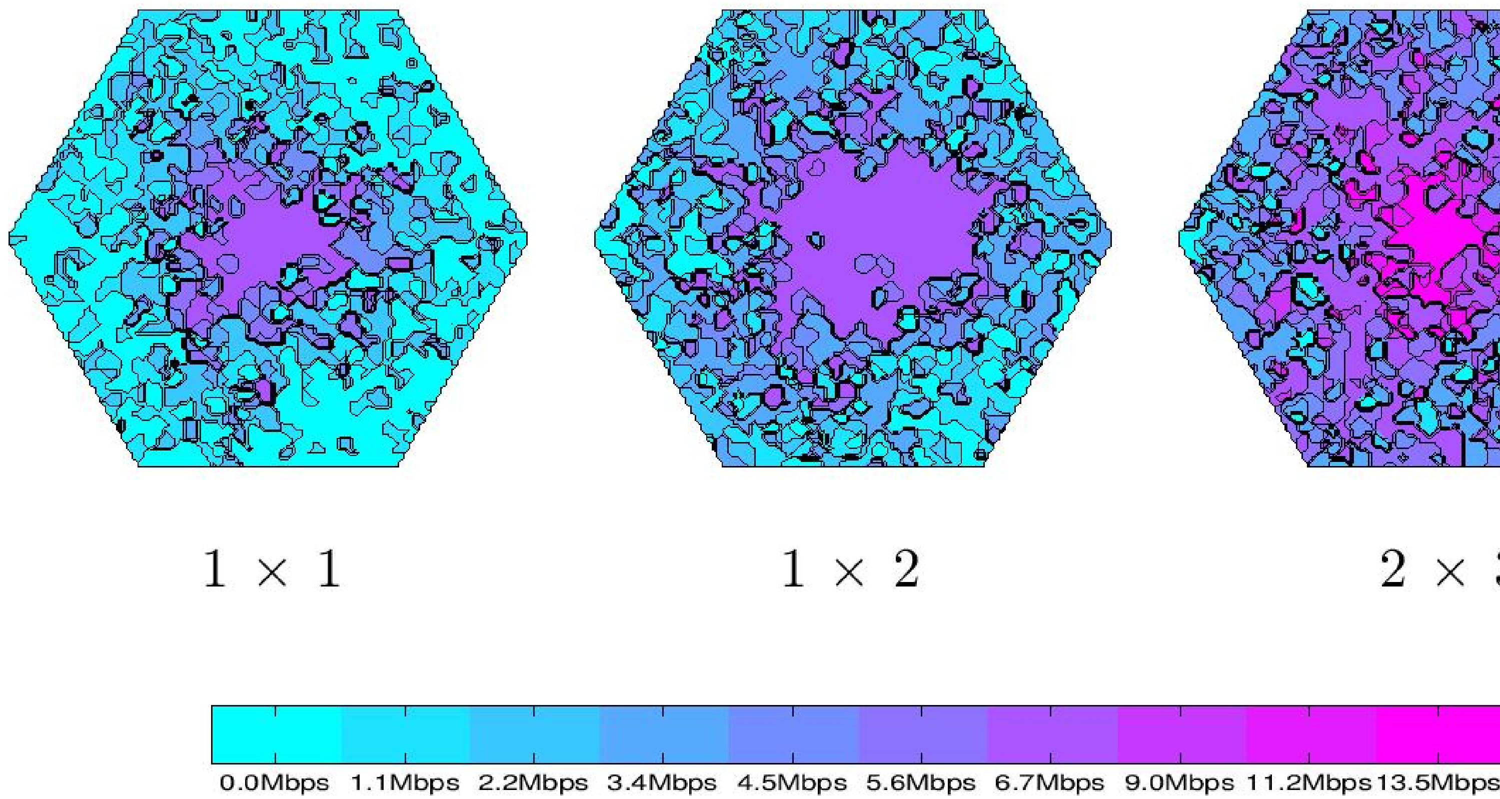
MIMO–OFDM Systems

- **Spatial multiplexing:** Maximize spatial rate $(r = \min\{N, M\})$ by transmitting independent data streams over different antennas $\Longrightarrow$ spatial multiplexing over each tone

- **Space–frequency** coded MIMO–OFDM
  - OFDM tones with spacing **larger than the coherence bandwidth** B_C experience **independent fading**
  - If $D_{\text{eff}} = \frac{B}{B_C}$, the **total diversity gain** that can be realized is of NMD_{eff}

[6]

Throughput in MIMO Cellular Systems



[1, 4]

Conclusions

- MIMO channels offer **multiplexing gain, diversity gain, array gain** and a **co–channel interference cancellation gain**

- **Careful balancing** between those gains is required

- MIMO systems offer a **promising solution** for future generation wireless networks

- Ongoing **research**
  - Space–time coding (orthogonal designs, etc.)
  - Receiver design (ML receiver is too complex)
  - Channel modeling
  - Capacity of non–ideal MIMO channels
  - . . .

[1, 4]

References

[1] H. Bölcskei, "MIMO: what shall we do with all these degrees of freedom?," presentation, 2003, available at `http://www.tele.ntnu.no/projects/beats/seminar.htm`.

[2] J. B. Andersen, "Array gain and capacity for known random channels with multiple element arrays at both ends," *IEEE J. Select. Areas Commun.*, vol. 18, no. 11, pp. 2172–2178, Nov. 2000.

[3] D. Shiu P. J. Smith D. Gesbert, M. Shafi and A. Nayguib, "From theory to practice: An overview of MIMO space–time coded wireless systems," *IEEE J. Select. Areas Commun.*, vol. 21, no. 3, pp. 281–302, Apr. 2003.

[4] D. Gesbert, "MIMO space–time coded wireless systems," presentation, Sept. 2003, available at `http://www.tele.ntnu.no/projects/beats/course.htm`.

[5] Z. Wang and G. B. Giannakis, "A simple and general parametrization quantifying performance in fading channels," *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1389–1398, Aug. 2003.

[6] R. U. Nabar A. J. Paulraj, D. A. Gore and H. Bölcskei, "An overview of MIMO communications—a key to gigabit wireless," *Proceedings of the IEEE*, vol. 92, no. 2, pp. 198–218, Feb. 2004.

[7] E. Teletar, "Capacity of multi-antenna Gaussian channels," Tech. Rep., AT&T Bell Laboratories, June 1995.

[8] D. N. C. Tse L. Zheng, "Diversity and multiplexing: a fundamental trade–off in multiple antenna channels," *IEEE Trans. Inform. Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.

[9] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.

[10] H. Jafarkhani V. Tarokh and A. R. Calderbank, "Space–time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456–1467, July 1999.

[11] S. Sandhu and A. Paulraj, "Space–time block codes: a capacity perspective," *IEEE Commun. Lett.*, vol. 4, no. 12, pp. 384–386, Dec. 2000.

[12] H. Bölcskei and A. Paulraj, "Multiple–input multiple–output (MIMO) wireless systems," unpublished.

[13] R. A. Valenzuela G. D. Golden, C. J. Foschini and P. W. Wolniansky, "Detection algorithm and initial laboratory results using V–BLAST space–time communication architecture," *Electronics Lett.*, vol. 35, no. 1, Jan. 1999.

[14] G. J. Foschini, "Layered space–time architecture for wireless communication in a fading environment using multi–element antennas," *Bell-Labs Techn. J.*, pp. 41–59, 1996.

[15] B. Hassibi and B. M. Hochwald, "High–rate codes that are linear in space and time," *IEEE Trans. Inform. Theory*, vol. 48, no. 7, pp. 1804–1824, July 2002.

[16] M. J. Gans D. Shiu, G. J. Foschini and J. M. Kahn, "Fading correlation and its effect on the capacity of multielement antenna systems," *IEEE Trans. Commun.*, vol. 48, no. 3, pp. 502–513, Mar. 2000.

[17] M. Sandell, *Design and analysis of estimators for multicarrier modulation and ultrasonic imaging*, Ph.D. thesis, Luleå University, Sweden, 1996.