Economics 202A
Macroeconomics
Fall 2021

Problem Set 3
Due on November 16, 2020
Please upload solutions to Gradescope

You are allowed to work in groups. But please write up your own solutions. Please note that copying from old solutions constitutes plagiarism. Please write up your solutions as clearly as possible. Your grade will be reduced if your solution is unreasonably difficult to follow.

This problem set was originally meant to solve a relatively simple version of the Zeldes-Deaton-Carroll model using the methods described in Alisdair McKay's numerical methods notes which are available here:

https://alisdairmckay.com/Notes/NumericalCrashCourse/index.html#.

After much work, Ethan and I have realized that these methods do not lend themselves to an accurate solution of that problem. The problem is that a polynomial approximation of the value function sometimes results in slight non-monotonicity of the value function, which leads the golden-search maximization method to find a local maximum rather than the global maximum. This can result in large errors in the solution. As a consequence, we have slightly altered the problem to add additional "curvature" to the value function so that this problem doesn't arise. The resulting problem is not as interesting from an economic point of view, unfortunately. (But it does nest the Zeldes-Deaton-Carroll problem.) We now simply view this problem set as a second "warm-up" problem set. You will get practice writing certain pieces of code that we will then modify in problem set 4 where we will use related but somewhat different methods to really solve the Zeldes-Deaton-Carroll model. Those that are interested can also vary the added "curvature" parameters and explore how the solution method runs into trouble. (See optional part O.)

Consider the following household consumption-savings problem with uninsurable idiosyncratic income risk. The household's utility function is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma},$$

where C_t denotes consumption, β is the household's subjective discount factor, and γ is the coefficient of relative risk aversion and also the reciprocal of the elasticity of intertemporal substitution. The household starts off with assets A_0 and each period receives labor income Y_t. Labor income is risky. The logarithm of labor income follows an AR(1) process:

$$\log Y_t = (1-\rho)\log \mu + \rho \log Y_{t-1} + \epsilon_t$$

and $\epsilon_t \sim \mathrm{N}(0, \sigma^2)$. (Here $\log$ denotes the natural logarithm.) The parameter μ denotes the unconditional mean of labor income, ρ is the persistence of fluctuations in labor income, and σ^2 is the variance of innovations to labor income.

The household has access to a risk-free technology for borrowing and lending. We assume for simplicity that the interest rate is the same for borrowing and landing and use r denote that interest rate. Each period, the household faces a choice of how much to consume and how much to borrow or save. Its budget constraint is therefore

$$C_t + \frac{A_{t+1}}{1+r} = Y_t + A_t.$$

We assume for simplicity that A_t is only constrained by the natural borrowing limit which does not bind and can therefore be ignored when solving the problem.

The problem as stated above is the problem we would like to solve. However, due to the numerical issues discussed above, we now modify this problem to add "curvature." Notice that the budget constraint can be written:

$$A_{t+1} = (1+r)(Y_t - C_t + A_t).$$

We now replace this budget constraint with the following:

$$A_{t+1} = Z(Y_t - C_t + A_t)^{\alpha}.$$

In other words, savings is invested in a home production technology of the form ZS_t^{α} where $S_t = Y_t - C_t + A_t$. Notice that this budget constraint nests the original one in the case where $\alpha = 1$ and $Z = (1+r)$.

The rest of the problem set asks you to solve this problem using value function iteration. As in problem set 1, we will follow closely the steps laid out in Alisdair McKay's numerical analysis notes. I have written the rest of the problem as a set of step-by-step instructions assuming that you are basing your work on McKay's notes and code. However, you are welcome to solve the problem without reference to these steps and this code. If you do, please describe the steps you take and report the relevant code.

When solving the problem, please consider the following set of parameter values to be the baseline parameter values and use these unless otherwise instructed: $\gamma = 2$, $\beta = 0.94$, $\mu = 1$, $b = 0.4$, $\rho = 0.9$, $\sigma^2 = 0.01$, $r = 0.05$, $\alpha = 1/3$, and $Z = 20$.

A. Write the household's problem recursively. Be sure to state what variables are chosen and all the constraints.

B. Write a version of McKay's PolyBasis function for this problem. Use a 2nd order polynomial basis (which will have 6 terms).

C. Write a version of McKay's PolyGetCoeff function for this problem.

D. Start your main Matlab program by reading in the parameter values into a structure.

E. Create a grid on A and $\log Y$. Feel free to use McKay's tauchen function as needed. Use 7 grid points for $\log Y$ and 100 grid points for A. (Please create the grid for $\log Y$, not Y. Uniformity will make grading easier.) Calculate the steady-state of A using the first-order conditions of the sequence problem and the budget constraint. Then create an equally spaced grid from $0.05\bar{A}$ to $1.95\bar{A}$, where $\bar{A}$ is the steady state.

F. Write a version of McKay's Bellman Matlab function for this problem. To do this, you will need to solve for consumption C_t as a function of the variables A_t, A_{t+1}, Y_t.

G. Write a version of McKay's MaxBellman function for this problem. Similarly to McKay's program make sure to bound the upper constraint such that consumption is positive. Additionally, make sure that the choices of next period's assets are bounded to be within the upper and lower bounds of the grid.

H. Write the value function iteration for-loop for this problem. Once the iterations converge, plot the value function, policy function, and the consumption on the meshgrid of (A, Y) using the *surf* function in Matlab.

I. (Optional) Implement the Howard acceleration for this problem. Report the speed improvement that you are able to achieve. (You will need to experiment with the number of iterations to figure out what works well in terms of giving a speed improvement.)

J. Adapt McKay's Simulate function for this problem.

Now you can view some results:

K. Using your new Simulate function, produce a 10,000 period simulation of the evolution of A, Y, and C. Report a histogram of A, Y, and C. (Note: Y_t not $\log Y_t$.) Report the mean and standard deviation of each variable. Plot the evolution of A, Y, and C over a 100 period stretch starting from period 1000. How do these mean values compare to the steady-state values calculated earlier?

L. Plot consumption as a function of A for several values of Y. Do this for the entire range of A on your grid.

M. Plot change in assets $Y - C$ as a function A for several values of Y. Do this for the entire range of A on your grid.

N. Plot the marginal propensity to consume as a function of A for several values of Y. Do this for the entire range of A on your grid. You can approximate the marginal propensity to consume as the extra consumption in the period that results from a windfall gain of 1 unit of A. Does this plot make economic sense? (Hint: It might not due to the limitations of the polynomial approximation methods we are using in this problem set.)

O. (Optional) Explore how the solution method runs into trouble if you try to increase α towards 1. As you do this, you may want to vary the range of assets on the grid and also the polynomial basis. If you consider higher order polynomials than 2nd order, it may be interesting for you to plot the value function for a particular value of Y_t as a function of A_t during intermediate steps in the value function iteration. You may start seeing cases where the value function becomes slightly non-monotonic. You can think about how this will lead the golden search algorithm to run into problems. (This is the problem that we couldn't get around easily in writing the problem.)