Homework #1

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Econ 236B, Spring 2022
Due: Feb 7, 2022

Reminder: Empirical and simulation exercises must be submitted with "stand-alone" code and (if appropriate) data in txt (tab delimited) format. *The homework is due next Monday (Feb 7) by 11:59 pm Pacific*.

1. A time series process is deterministic if its stochastic nature has no relation with time so that it can be predicted with certainty. The simplest example of a deterministic process is $x_t = x$ for all t. Consider $x_t = \alpha * \cos(t) + \beta * \sin(t)$ with $\alpha \sim N(0,1)$ and $\beta \sim N(0,1)$

   a) What is the mean and variance of x_t?
   b) Does $E(x_t x_{t-k})$ depend on time t? Prove.

2. Suppose $e_t = u_t\sqrt{b + \alpha e_{t-1}^2}$ where $u_t \sim iid(0,1)$ The process e_t is called an ARCH process.

   a) Is e_t serially correlated?
   b) What is variance of e_t conditional on e_{t-1}? Is the conditional variance serially correlated?
   c) Let $x_t = e_t^2 - \sigma^2$ where $\sigma^2 = E(e_t^2 | e_{t-1}, e_{t-2}, \dots)$. Prove that x_t is mds.
   d) Prove that x_t is white noise.

3. ARMA + ARMA arithmetic

   a) Suppose that $y_{1t} \sim ARMA(2,1)$ and $y_{2t} \sim ARMA(1,2)$ are two independent processes. Process $x_t = y_{1t} + y_{2t}$ is also an $ARMA(p,q)$ process. What p and q describe x? Prove.
   b) Suppose that $y_{1t} \sim ARMA(p_1, q_1)$ and $y_{2t} \sim ARMA(p_2, q_2)$ are two independent processes. Again, $x_t = y_{1t} + y_{2t}$ is also an $ARMA(p,q)$ process. What p and q describe x in this general case? Prove.

4. Working problem. Problem 8.3 in Romer's textbook (4th edition; also see problem 8.12 for derivations; scans for these are on p. 3). Contrast ACF/PACF for actual and measured consumption. Download quarterly US series for consumption of non-durables (FRED database). Plot ACF/PACF. Compare your results with results and predictions in Mankiw (1982).

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5. Download U.S. series for CPI inflation rate, real GDP growth rate, civilian unemployment rate, USD/UK pound exchange rate (You may want to use St. Louis Fed data base FRED). All series are at quarterly frequency (aggregate [average] to quarterly frequency if necessary). Exclude data after December 2019 (i.e., do not cover the COVID19 period).

a) Estimate and plot ACF and PACF. Describe common patterns.
b) Using Box-Jenkins approach, what ARMA(p,q) model may be appropriate for these series? Discuss.
c) Using BIC, choose a parsimonious parametric AR(p) model. Report the values of BIC for p=0,…,10.
d) Conduct the diagnostic checking of the estimated optimal AR model: serial correlation of residuals (e.g., Box-Ljung; you may get the code for it at http://ideas.repec.org/c/boc/bocode/t961403.html); ACF/PACF for estimated residuals; normality of the error term (Jarque-Bera test; "jbtest" in Matlab).
e) Estimate the model using $1, \ldots, T-10$ observations and compute point forecasts and forecast errors for $T-9, \ldots, T$ (for now, ignore sampling uncertainty in parameter estimates).
f) Estimate the same model using $T-50, \ldots, T-10$ observations and compute point forecasts and forecast errors for $T-9, \ldots, T$ (for now, ignore sampling uncertainty in parameter estimates). Compare with forecasts and forecast errors in e). Discuss.

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Romer 8.3:

8.3. The time-averaging problem. (Working, 1960.) Actual data give not consumption at a point in time, but average consumption over an extended period, such as a quarter. This problem asks you to examine the effects of this fact.

Suppose that consumption follows a random walk: $C_t = C_{t-1} + e_t$, where e is white noise. Suppose, however, that the data provide average consumption over two-period intervals; that is, one observes $(C_t + C_{t+1})/2$, $(C_{t+2} + C_{t+3})/2$, and so on.

(*a*) Find an expression for the change in measured consumption from one two-period interval to the next in terms of the e's.

(*b*) Is the change in measured consumption uncorrelated with the previous value of the change in measured consumption? In light of this, is measured consumption a random walk?

(*c*) Given your result in part (*a*), is the change in consumption from one two-period interval to the next necessarily uncorrelated with anything known as of the first of these two-period intervals? Is it necessarily uncorrelated with anything known as of the two-period interval immediately preceding the first of the two-period intervals?

(*d*) Suppose that measured consumption for a two-period interval is not the average over the interval, but consumption in the second of the two periods. That is, one observes C_{t+1}, C_{t+3}, and so on. In this case, is measured consumption a random walk?

Romer 8.12

8.12. Consumption of durable goods. (Mankiw, 1982.) Suppose that, as in Section 8.2, the instantaneous utility function is quadratic and the interest rate and the discount rate are zero. Suppose, however, that goods are durable; specifically, $C_t = (1 - \delta)C_{t-1} + X_t$, where X_t is purchases in period t and $0 \le \delta < 1$.

(*a*) Consider a marginal reduction in purchases in period t of dX_t. Find values of dX_{t+1} and dX_{t+2} such that the combined changes in X_t, X_{t+1}, and X_{t+2} leave the present value of spending unchanged (so $dX_t + dX_{t+1} + dX_{t+2} = 0$) and leave C_{t+2} unchanged (so $(1 - \delta)^2 dX_t + (1 - \delta)dX_{t+1} + dX_{t+2} = 0$).

(*b*) What is the effect of the change in part (*a*) on C_t and C_{t+1}? What is the effect on expected utility?

(*c*) What condition must C_t and $E_t[C_{t+1}]$ satisfy for the change in part (*a*) not to affect expected utility? Does C follow a random walk?

(*d*) Does X follow a random walk? (Hint: Write $X_t - X_{t-1}$ in terms of $C_t - C_{t-1}$ and $C_{t-1} - C_{t-2}$.) Explain intuitively. If $\delta = 0$, what is the behavior of X?