

STEVE SCHERRER

DEMYSTIFYING ANOVA

Linear Regression Review

Univariate Regression

$$y = mx + B$$

Univariate Regression

$$y = mx + B$$

slope

A diagram illustrating the components of the univariate regression equation $y = mx + B$. The equation is displayed in a large, bold, light gray font. Below the equation, the word "slope" is written in a bold, reddish-orange font. A thin, reddish-orange arrow points from the word "slope" to the coefficient 'm' in the equation. Similarly, the word "intercept" is written in a bold, reddish-orange font to the right of "slope". A thin, reddish-orange arrow points from the word "intercept" to the constant 'B' in the equation.

intercept

Univariate Regression

Predicted Values

x values

$$f(x) = \beta_1 X + \beta_0 + \varepsilon$$

slope

intercept

error

Univariate Regression

Predicted values

Values - Feature

$$f(x) - \varepsilon = \beta_1 X + \beta_0$$

error

slope

intercept

Univariate Regression

Values - Y (target) Values - Feature

The diagram illustrates the univariate regression equation $f(x) - \epsilon = \beta_1 X + \beta_0$. A red curly brace is positioned above the left side of the equation, $f(x) - \epsilon$, with the label "Values - Y (target)" above it. A red arrow points from the label "Values - Feature" to the term $\beta_1 X$. Another red arrow points from the label "slope" to the coefficient β_1 . A final red arrow points from the label "intercept" to the term β_0 .

$$f(x) - \epsilon = \beta_1 X + \beta_0$$

slope intercept

Multivariate Regression

The diagram illustrates the multivariate regression equation $f(x) - \epsilon = \beta_1 X_1 + \beta_2 X_2 + \beta_0$. A red curly brace is positioned above the left side of the equation, labeled "values - y (target)". Red arrows point from descriptive labels to the coefficients: "values - Feature 1" points to β_1 , "values - Feature 2" points to β_2 , "slope - Feature 1" points to β_1 , "slope - Feature 2" points to β_2 , and "intercept" points to β_0 .

values - y (target)

values - Feature 1

values - Feature 2

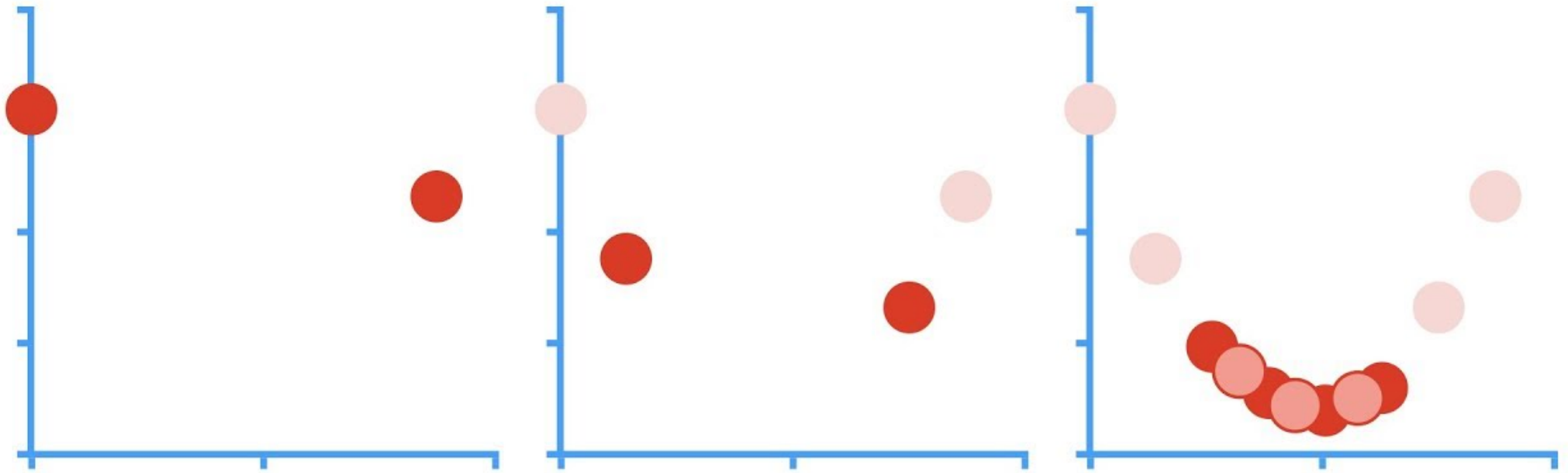
$$f(x) - \epsilon = \beta_1 X_1 + \beta_2 X_2 + \beta_0$$

slope - Feature 1

slope - Feature 2

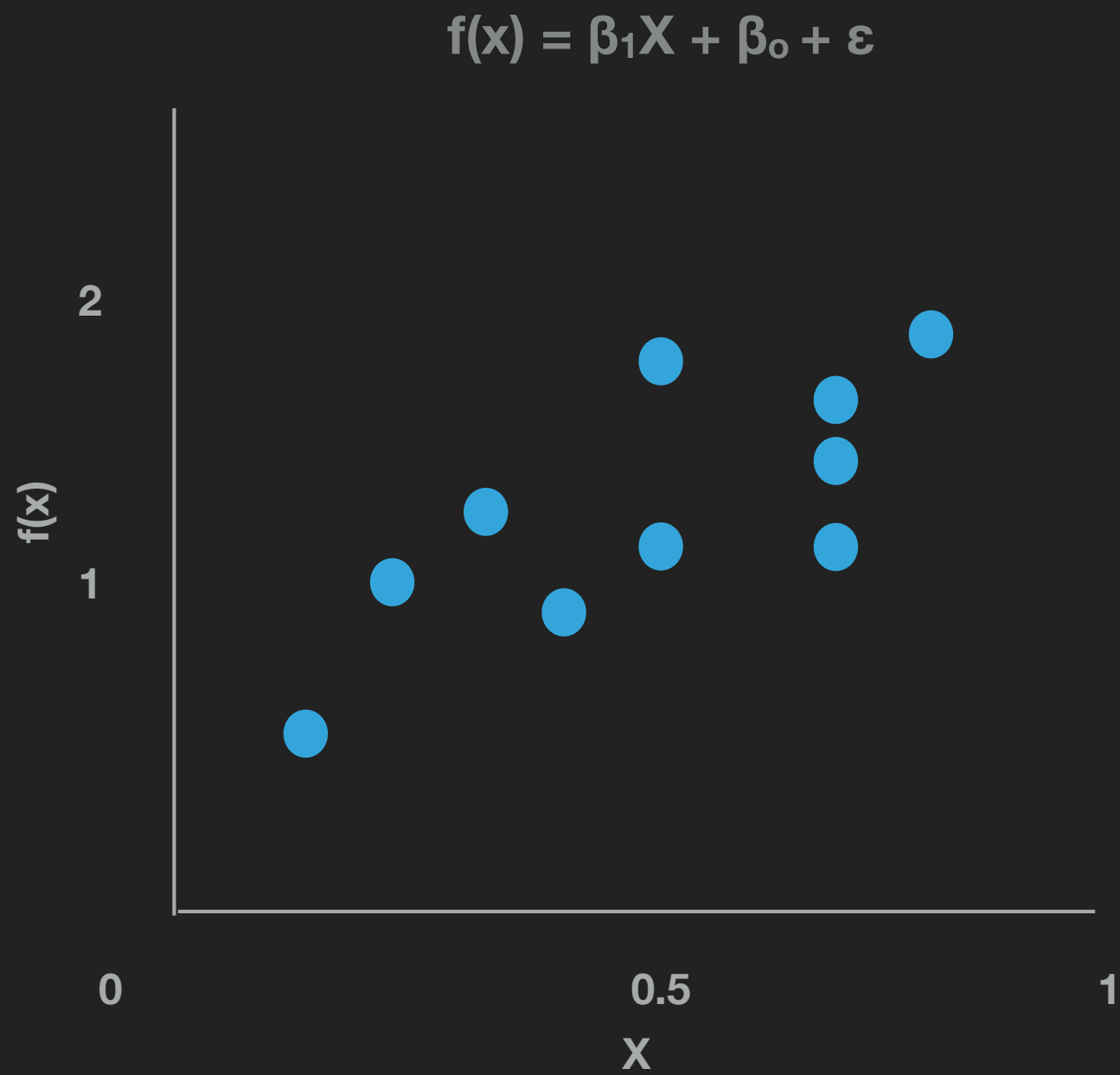
intercept

Gradient Descent....



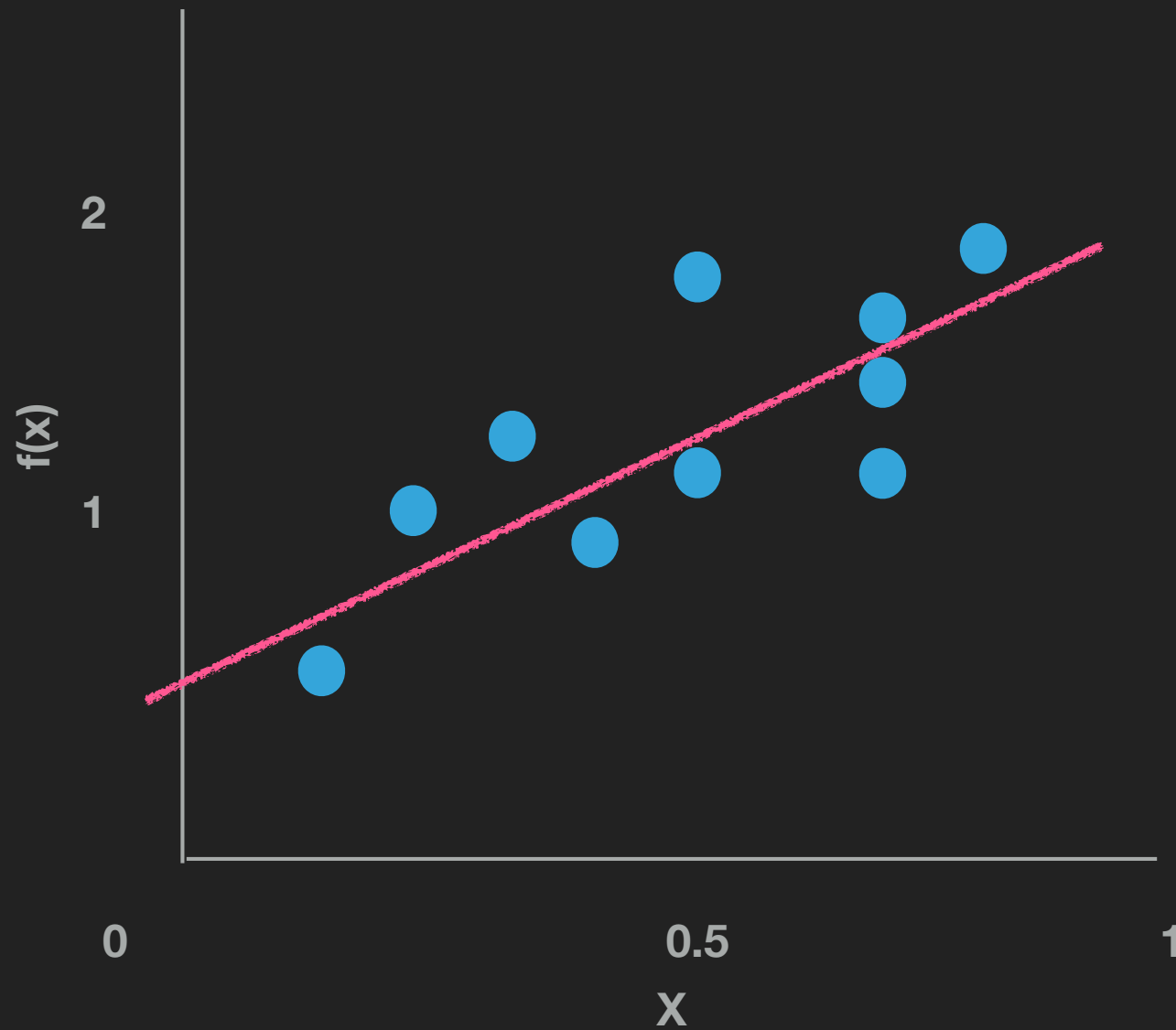
...Step-by-Step!!!

y	x
0.5	0.1
0.9	0.4
1.0	0.2
1.1	0.7
1.1	0.5
1.2	0.3
1.5	0.7
1.6	0.7
1.7	0.5
1.8	0.8



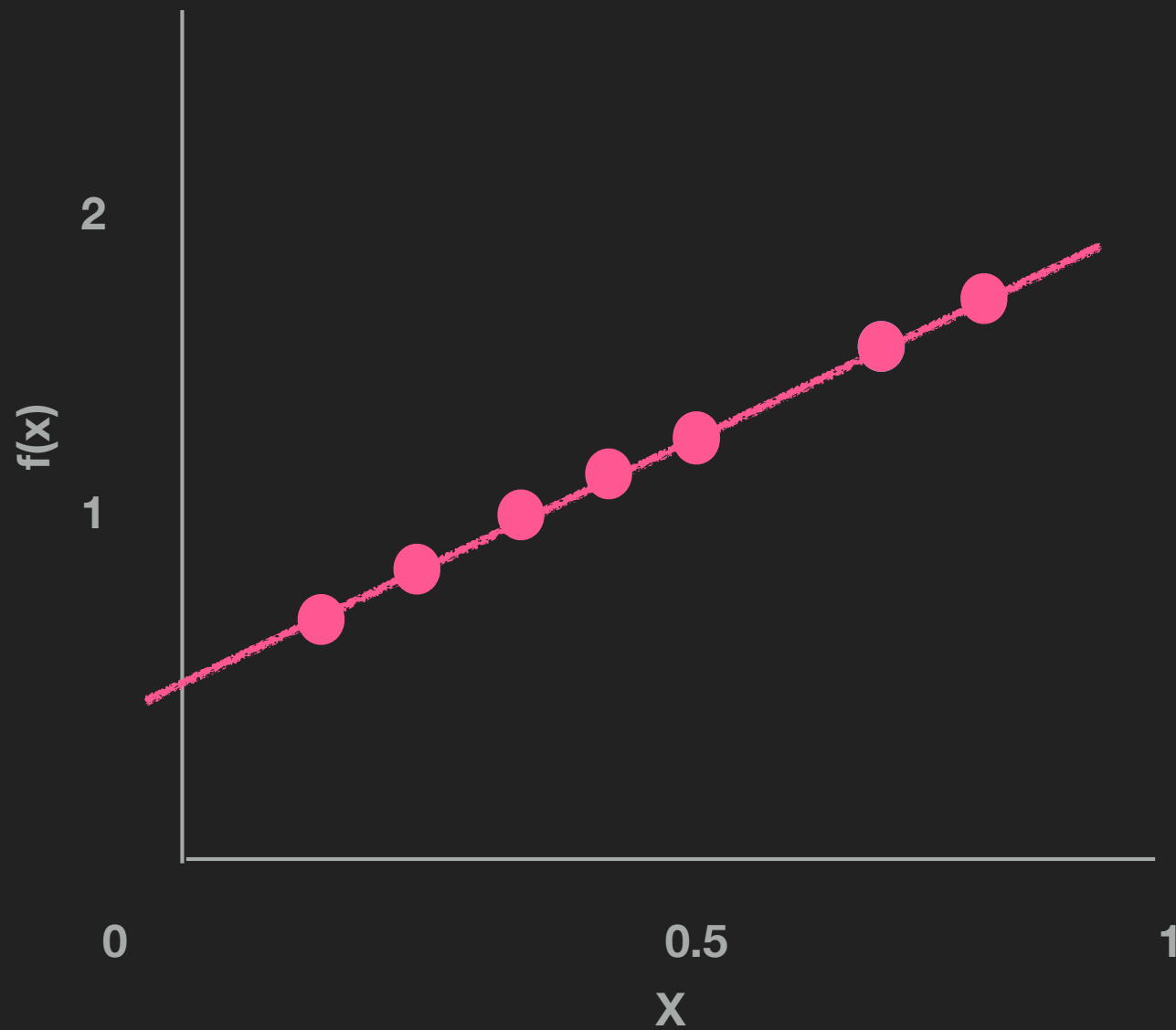
y	x
0.5	0.1
0.9	0.4
1.0	0.2
1.1	0.7
1.1	0.5
1.2	0.3
1.5	0.7
1.6	0.7
1.7	0.5
1.8	0.8

$$f(x) = \beta_1 X + \beta_0 + \varepsilon$$



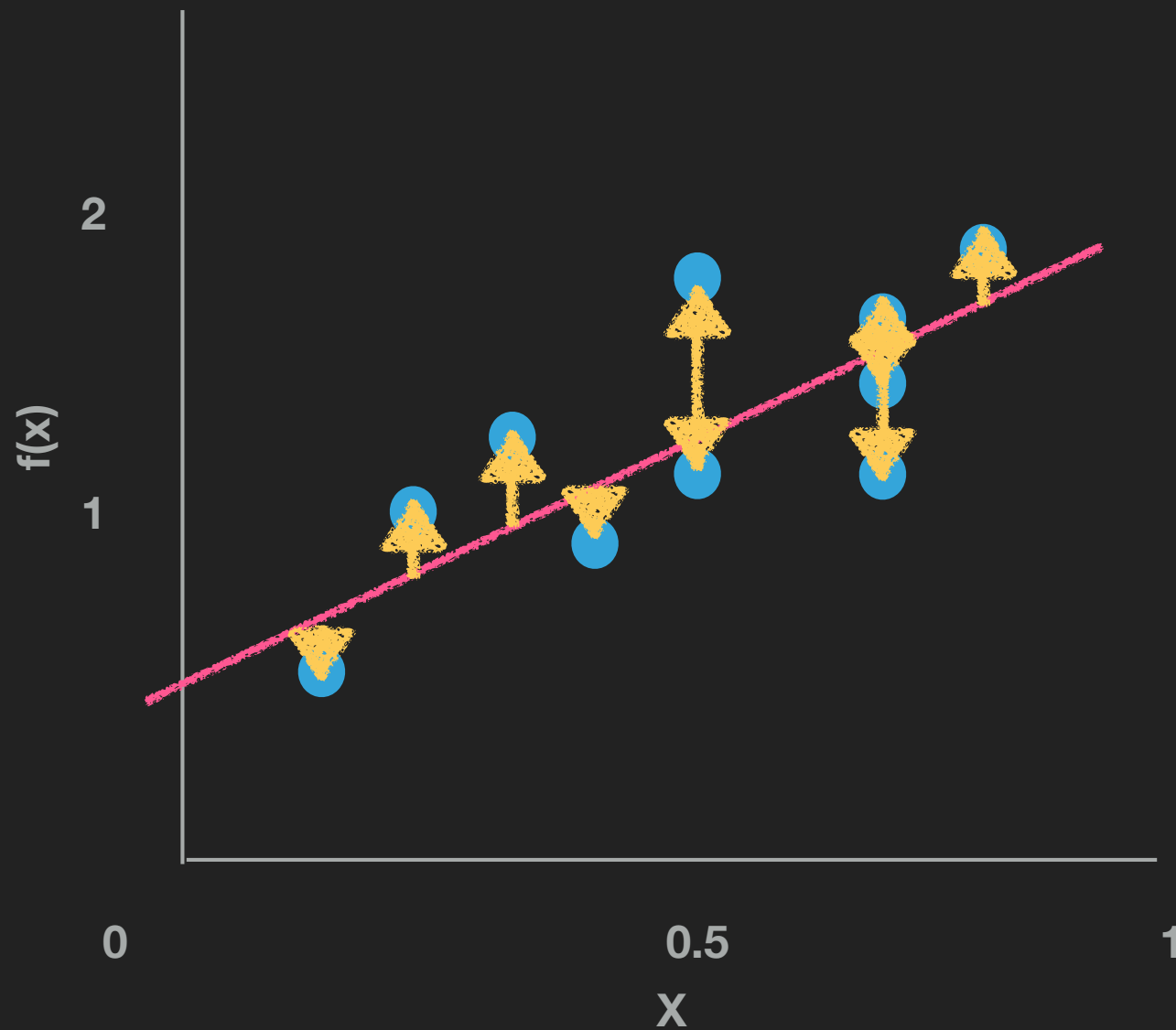
y	x
0.5	0.1
0.9	0.4
1.0	0.2
1.1	0.7
1.1	0.5
1.2	0.3
1.5	0.7
1.6	0.7
1.7	0.5
1.8	0.8

$$f(x) = \beta_1 X + \beta_0 + \varepsilon$$



y	x	f(x)
0.5	0.1	0.73
0.9	0.4	1.1
1.0	0.2	0.86
1.1	0.7	1.50
1.1	0.5	1.24
1.2	0.3	0.99
1.5	0.7	1.50
1.6	0.7	1.50
1.7	0.5	1.24
1.8	0.8	1.62

$$f(x) = \beta_1 X + \beta_0 + \varepsilon$$



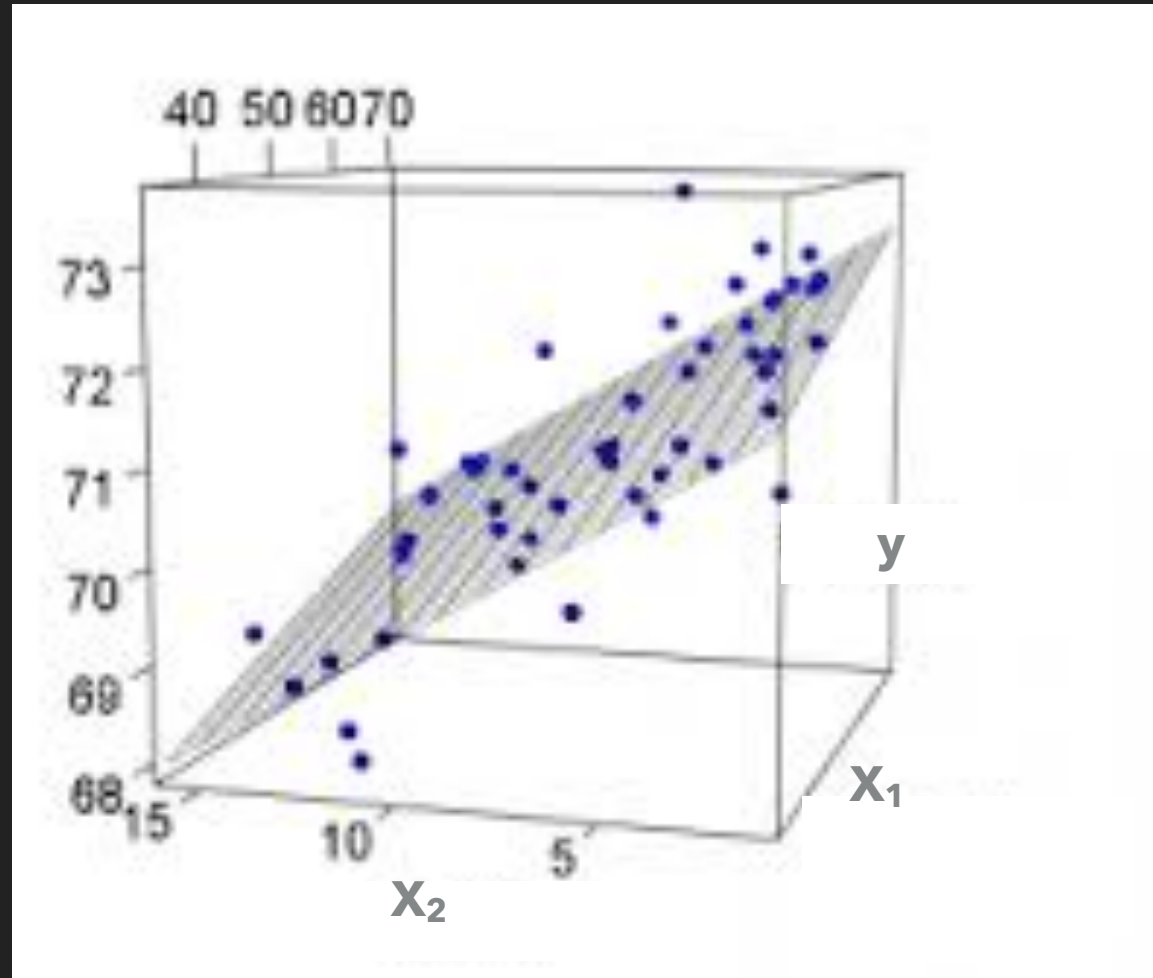
y	x	f(x)	ε
0.5	0.1	0.73	-0.23
0.9	0.4	1.1	-0.2
1.0	0.2	0.86	0.14
1.1	0.7	1.50	-0.4
1.1	0.5	1.24	-0.14
1.2	0.3	0.99	0.21
1.5	0.7	1.50	0
1.6	0.7	1.50	0.1
1.7	0.5	1.24	0.46
1.8	0.8	1.62	0.18

Assumptions of Linear Regression

1. Observations are independent
2. Residuals are normally distributed and centered around zero
 - ▶ Shapiro-Wilk's test
3. Residuals are homoskedastic (no underlying pattern)
 - ▶ - Breusch- Pagan test
4. If using multiple features (multivariate regression), features are not correlated

Multivariate Regression

$$f(\mathbf{x}) = \beta_1 X_1 + \beta_2 X_2 + \beta_0 + \varepsilon$$



ANOVA is a Special Case of Multivariate Regression

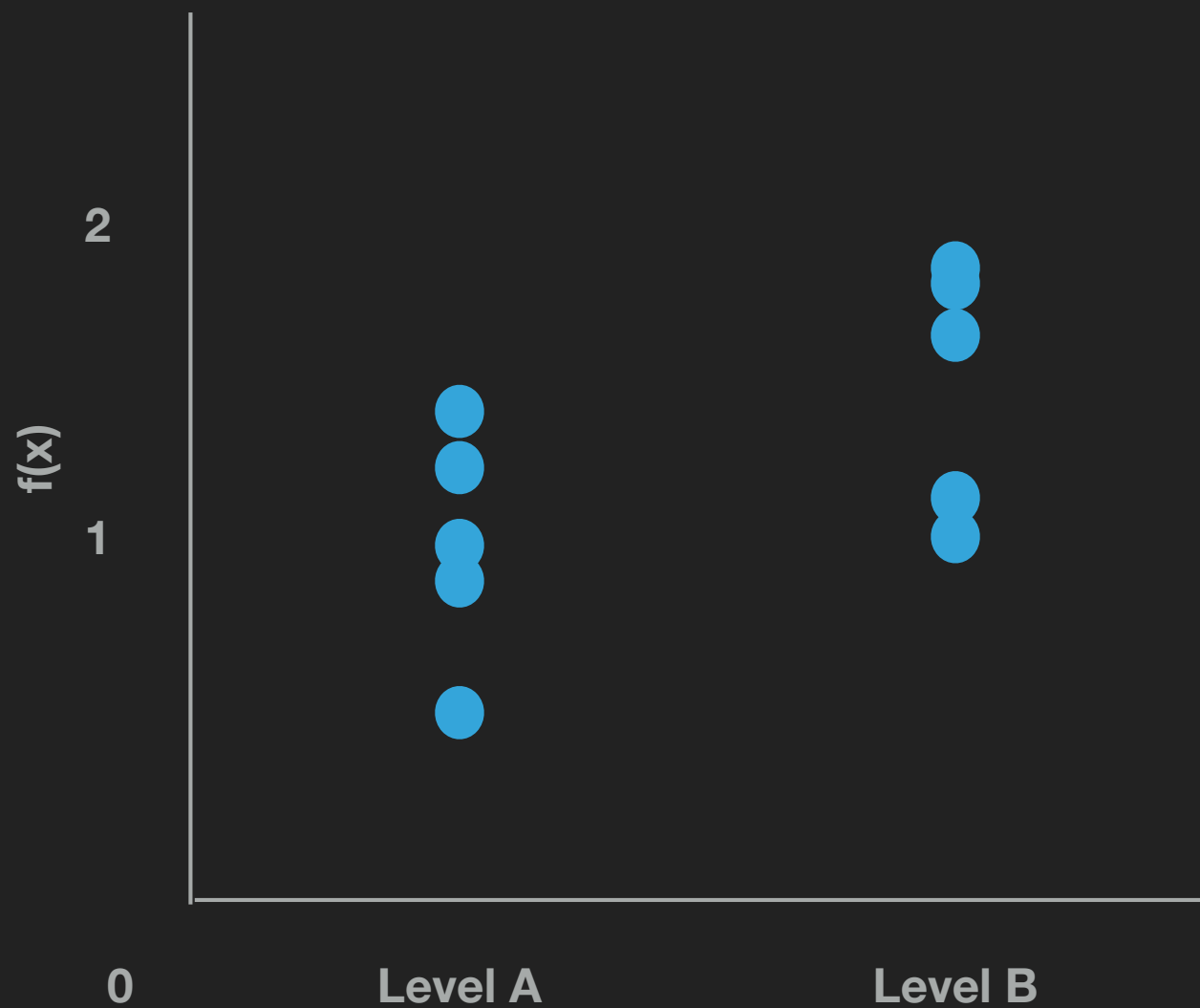
y	x
0.5	A
0.9	A
1.0	A
1.1	B
1.1	A
1.2	B
1.5	A
1.6	B
1.7	B
1.8	B

One Hot Encoded



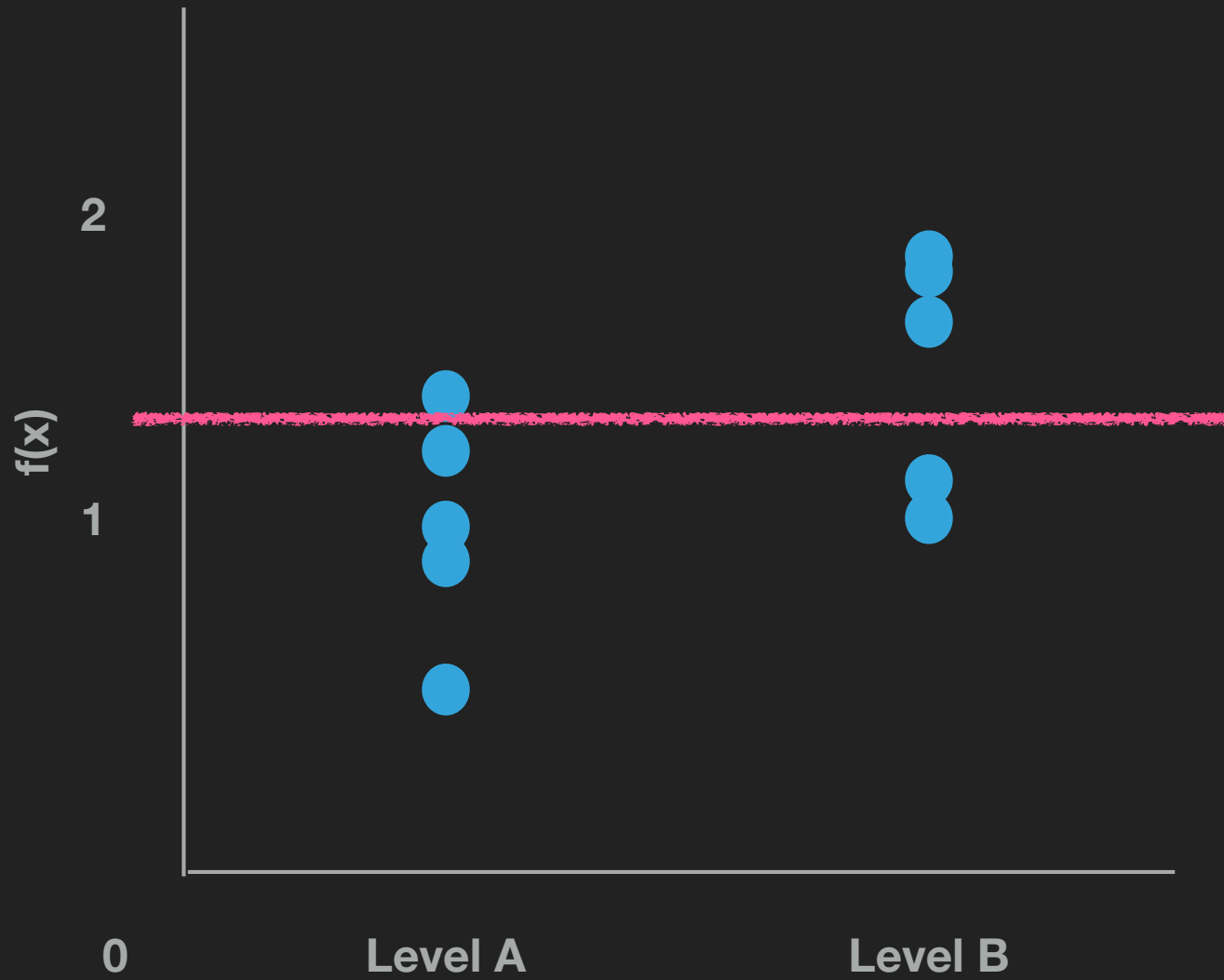
y	X _a	X _b
0.5	1	0
0.9	1	0
1.0	1	0
1.1	0	1
1.1	1	0
1.2	0	1
1.5	1	0
1.6	0	1
1.7	0	1
1.8	0	1

$$Y = \beta_a X_a + \beta_b X_b + \beta_o$$



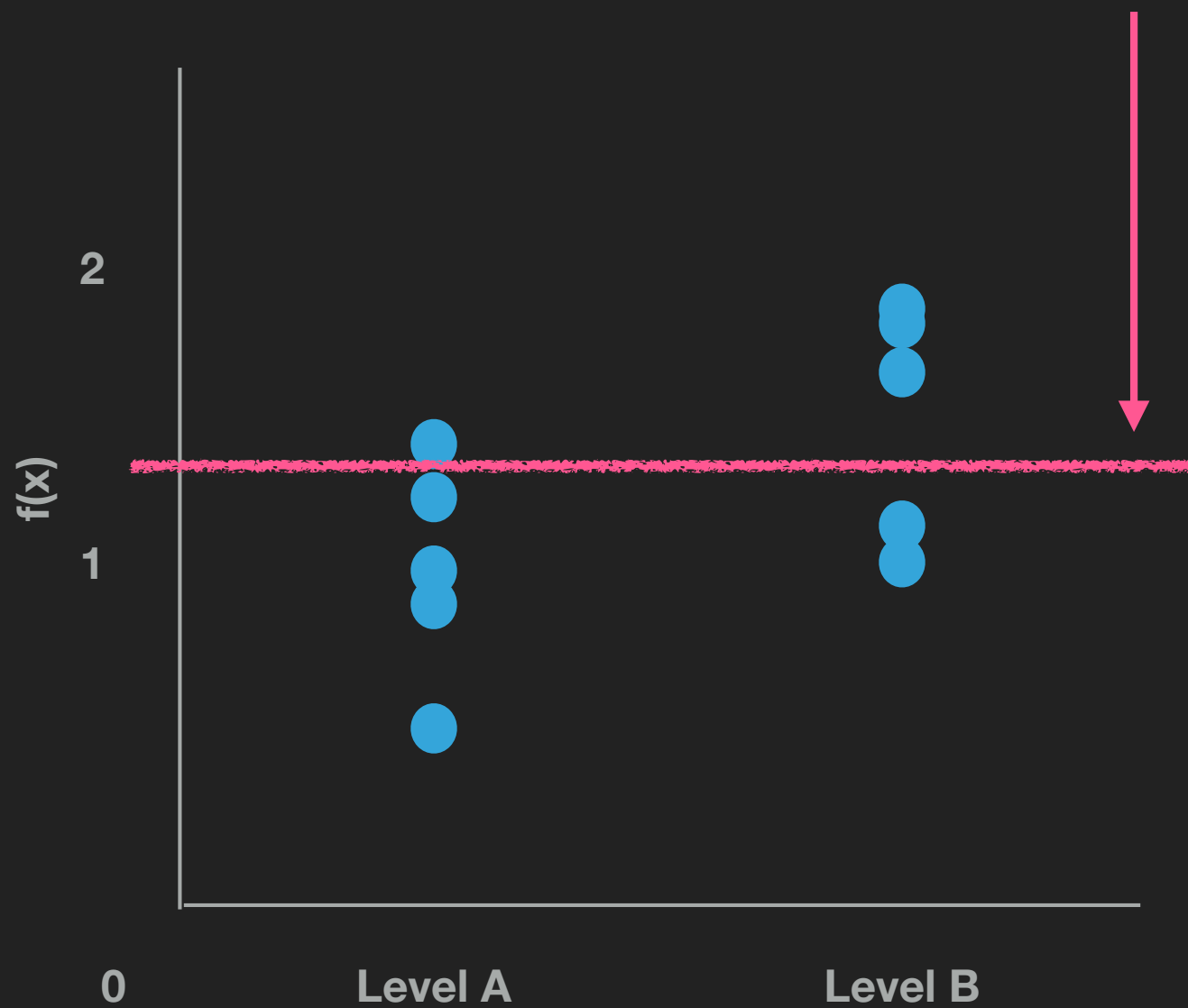
y	X_a	X_b
0.5	1	0
0.9	1	0
1.0	1	0
1.1	0	1
1.1	1	0
1.2	0	1
1.5	1	0
1.6	0	1
1.7	0	1
1.8	0	1

$$Y = \beta_a X_a + \beta_b X_b + \beta_o$$



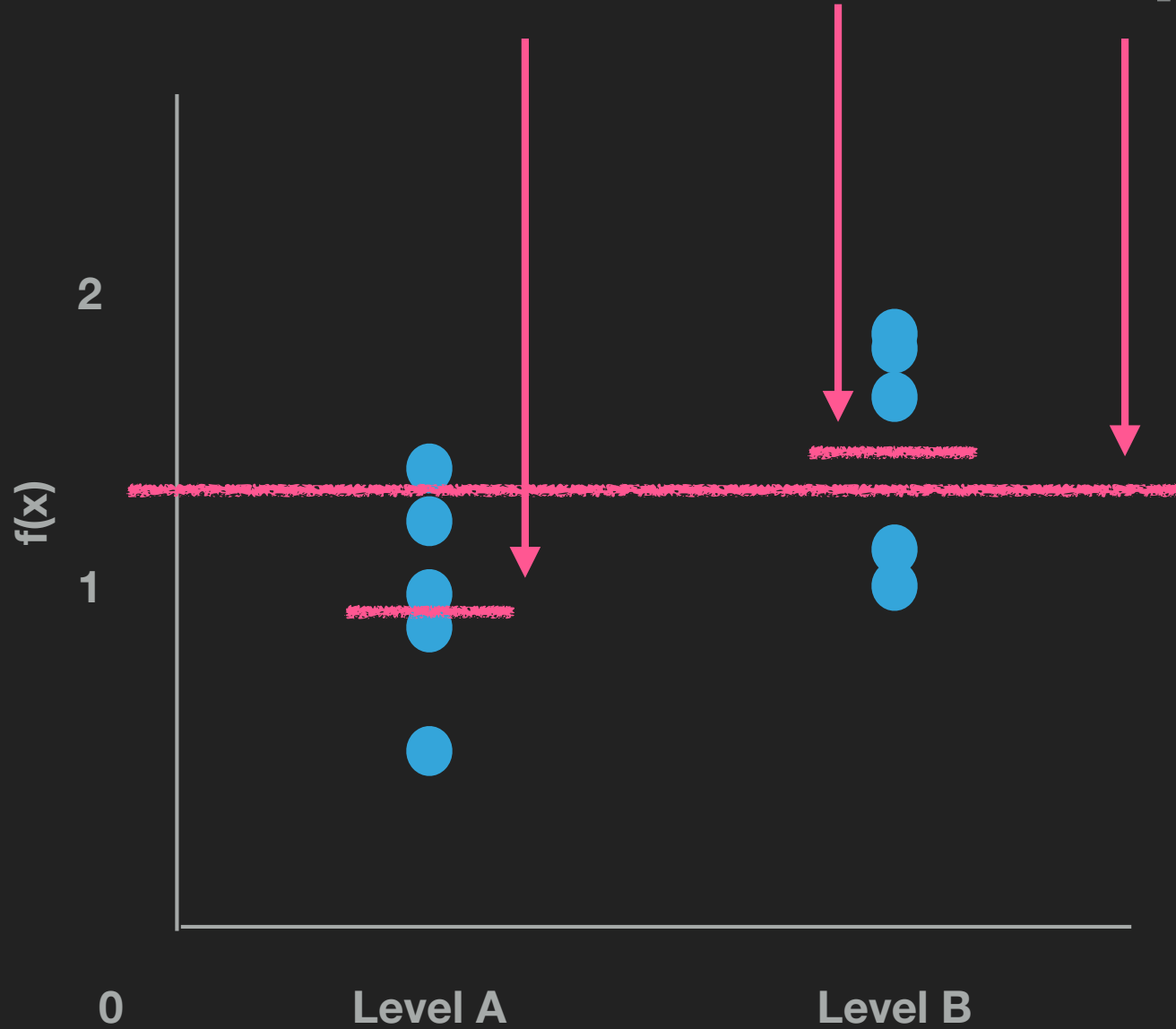
y	X _a	X _b
0.5	1	0
0.9	1	0
1.0	1	0
1.1	0	1
1.1	1	0
1.2	0	1
1.5	1	0
1.6	0	1
1.7	0	1
1.8	0	1

$$Y = \beta_a X_a + \beta_b X_b + \beta_o$$



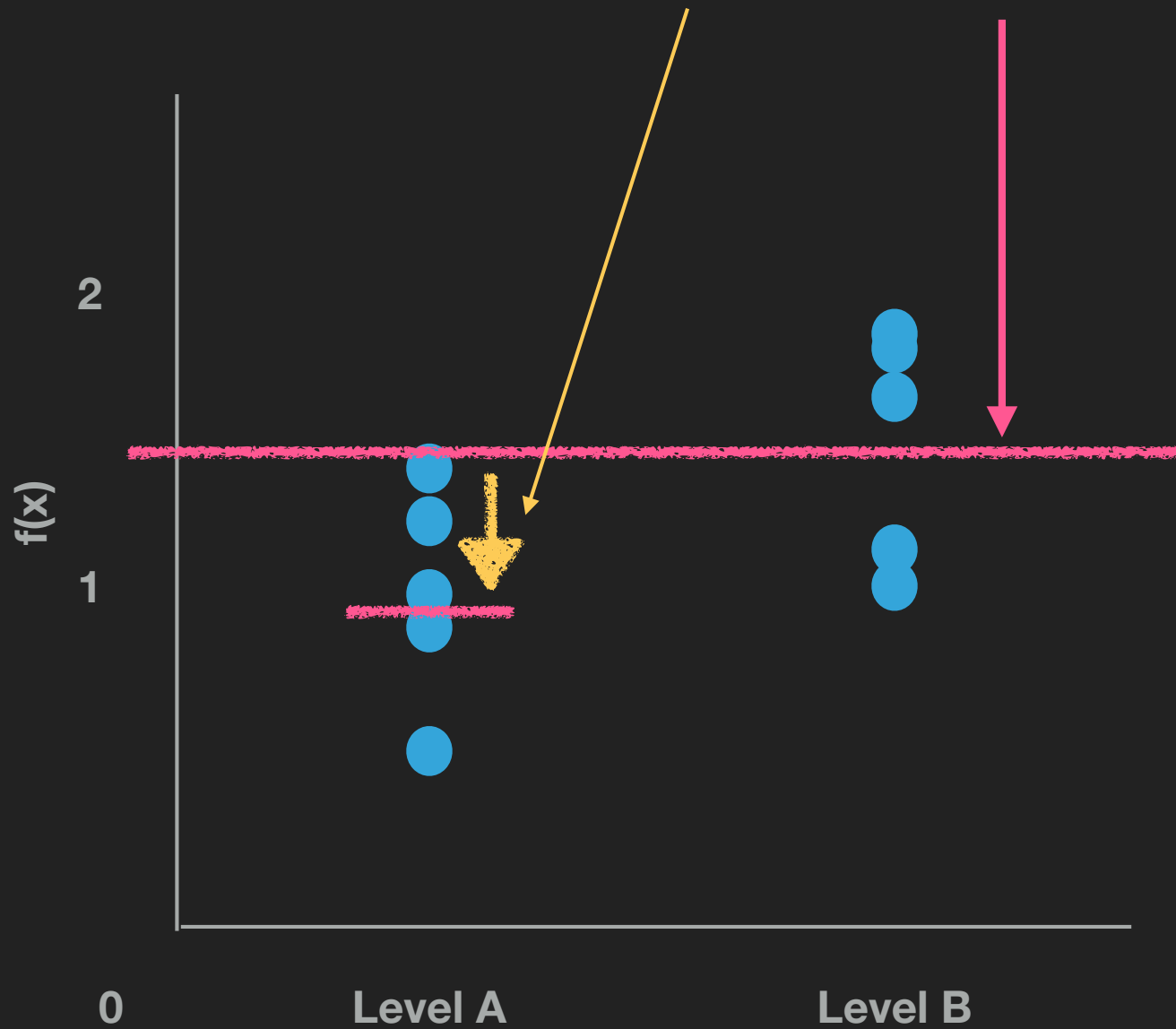
y	X_a	X_b
0.5	1	0
0.9	1	0
1.0	1	0
1.1	0	1
1.1	1	0
1.2	0	1
1.5	1	0
1.6	0	1
1.7	0	1
1.8	0	1

$$Y = 0.98X_a + 0.5X_b + \beta_o$$



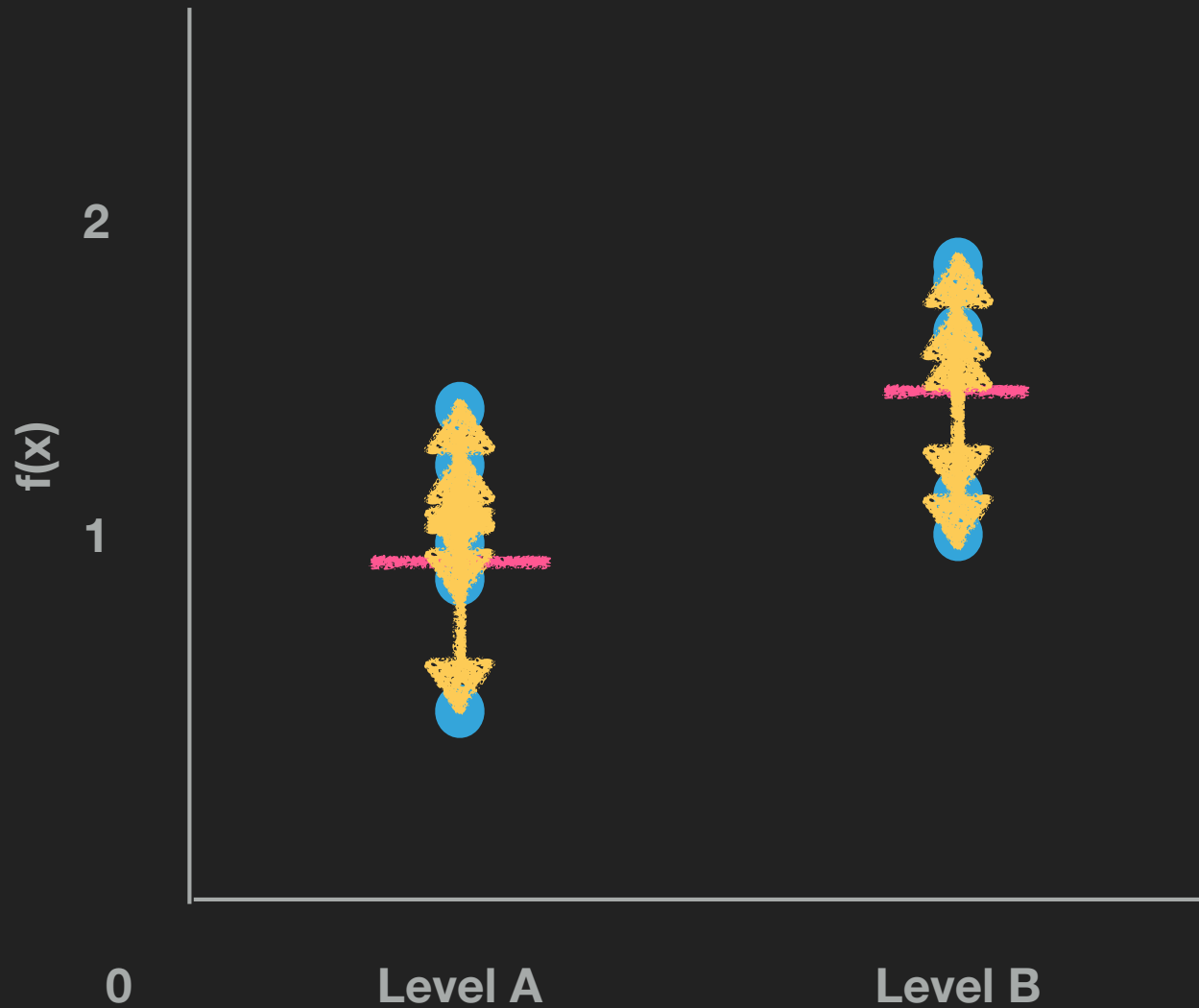
y	X_a	X_b
0.5	1	0
0.9	1	0
1.0	1	0
1.1	0	1
1.1	1	0
1.2	0	1
1.5	1	0
1.6	0	1
1.7	0	1
1.8	0	1

$$Y = 0.98 + 0.5 X_b$$



y	X_a	$f(x)$
0.5	1	0.98
0.9	1	1.1
1.0	1	0.98
1.1	0	1.48
1.1	1	0.98
1.2	0	1.48
1.5	1	0.97
1.6	0	1.48
1.7	0	1.48
1.8	0	1.48

$$Y = -0.5 X_a + 1.48$$



y	X_a	$f(x)$	ϵ
0.5	1	0.98	-0.48
0.9	1	1.1	-0.2
1.0	1	0.98	0.02
1.1	0	1.48	-0.38
1.1	1	0.98	0.12
1.2	0	1.48	-0.28
1.5	1	0.97	0.53
1.6	0	1.48	0.12
1.7	0	1.48	0.22
1.8	0	1.48	0.32

Assumptions of Linear Regression

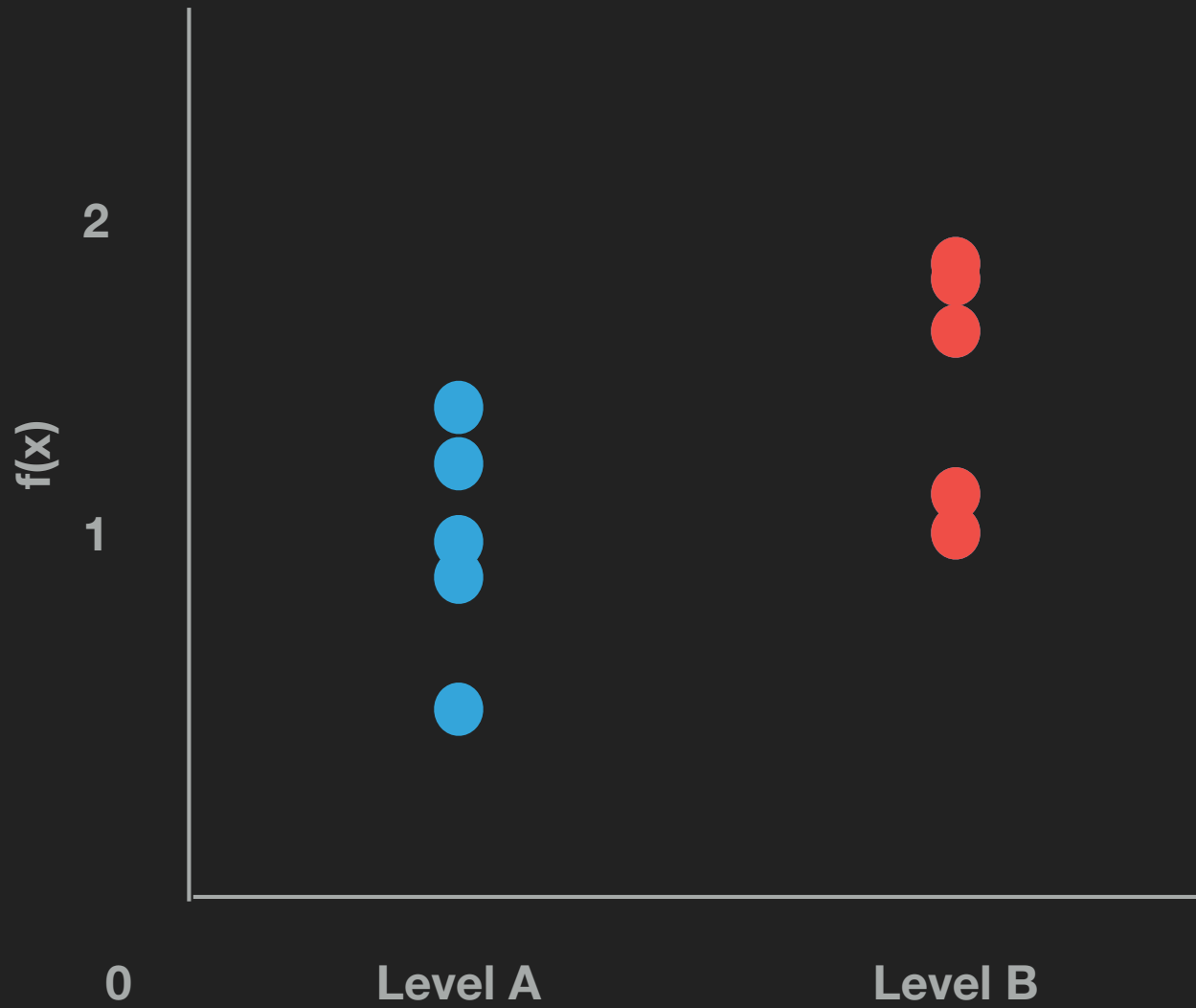
1. Observations are independent and normally distributed
2. Residuals are normally distributed and centered around zero
 - ▶ Shapiro-Wilk's test
3. Residuals are homoskedastic (no underlying pattern)
 - ▶ - Breusch- Pagan test
4. If using multiple features (multivariate regression), features are not correlated

Assumptions of ANOVA

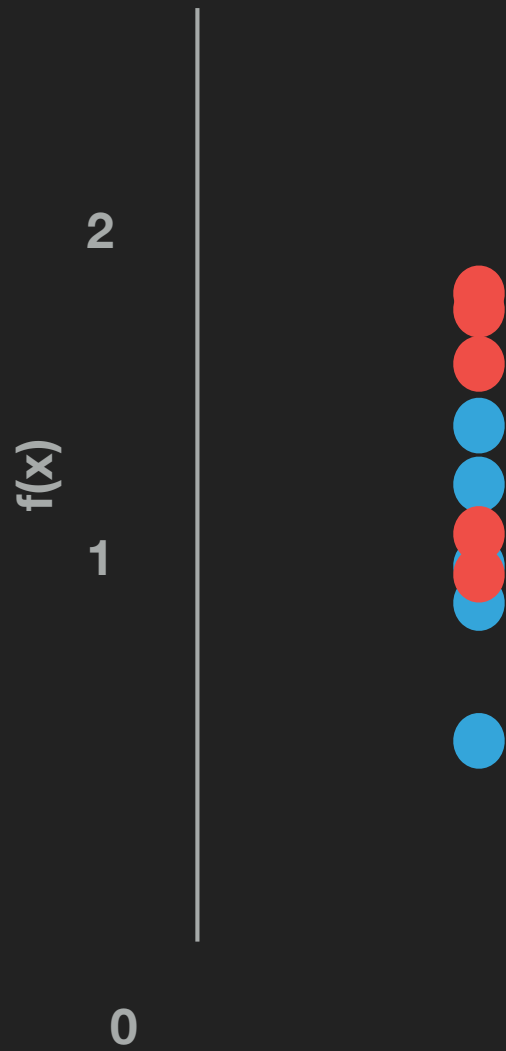
1. Observations are independent and normally distributed within groups
2. Residuals are normally distributed and centered around zero
 - ▶ Shapiro-Wilk's test
3. Variance between groups is similar ($\sim < 2x$)
 - ▶ $|\max([\sigma_a^2 - \sigma_b^2]) / \min([\sigma_a^2 - \sigma_b^2])| < 2$
4. If using multiple features (multivariate regression), features are not correlated

1. Observations are independent and normally distributed within groups

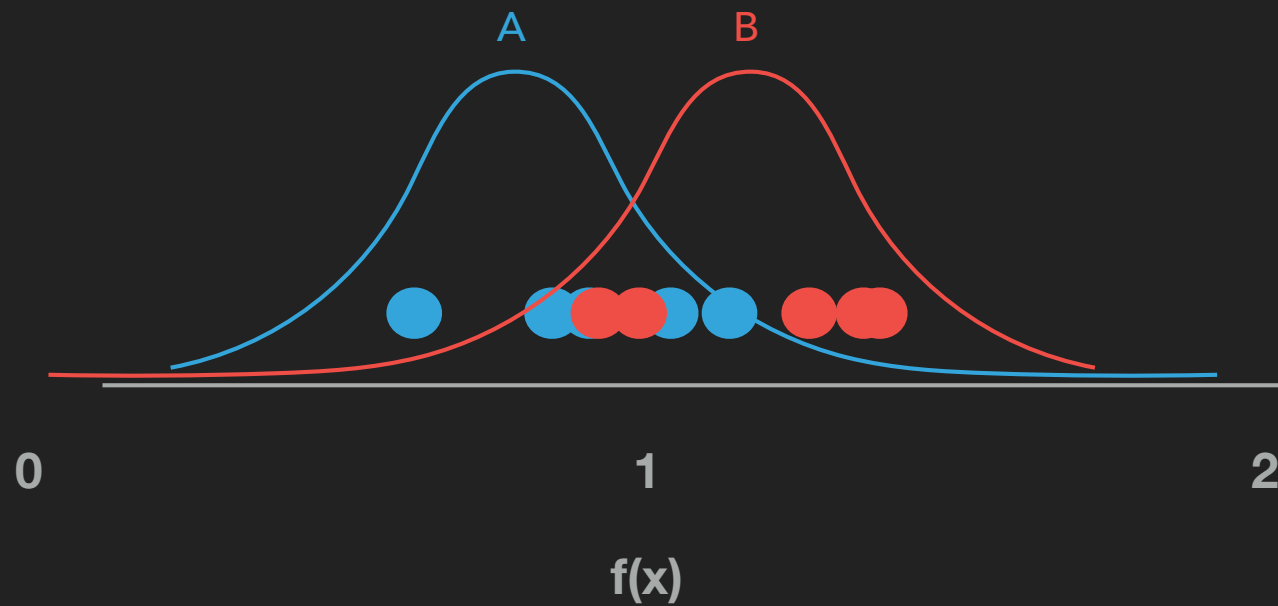
1. Observations are independent and normally distributed within groups



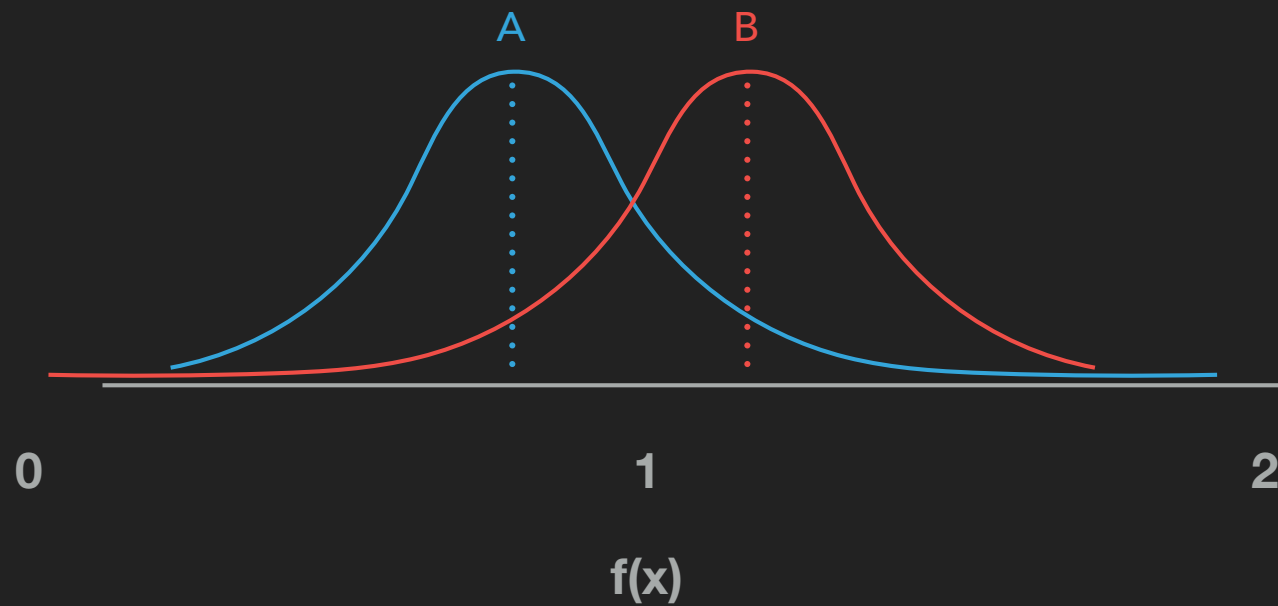
1. Observations are independent and normally distributed within groups



1. Observations are independent and normally distributed within groups



1. Observations are independent and normally distributed within groups



1. Observations are independent and normally distributed within groups

OLS Regression Results

```
=====
Dep. Variable:          y      R-squared:          0.434
Model:                  OLS    Adj. R-squared:       0.363
Method:                 Least Squares    F-statistic:      6.127
Date:                  Sun, 04 Oct 2020    Prob (F-statistic): 0.0384
Time:                  14:37:07    Log-Likelihood:    -1.6598
No. Observations:      10      AIC:              7.320
Df Residuals:          8      BIC:              7.925
Df Model:              1
Covariance Type:       nonrobust
=====
```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	0.9800	0.143	6.861	0.000	0.651	1.309
z[T.b]	0.5000	0.202	2.475	0.038	click to scroll output; double click to hi	

```
=====
Omnibus:                0.887    Durbin-Watson:      1.642
Prob(Omnibus):          0.642    Jarque-Bera (JB):    0.670
Skew:                   -0.287    Prob(JB):            0.715
Kurtosis:               1.870    Cond. No.            2.62
=====
```

```
1 scipy.stats.ttest_ind(a = aov_data.y.loc[aov_data.z == 'a'], b = aov_data.y.loc[aov_data.z == 'b'])
```

executed in 6ms, finished 13:58:17 2020-10-04

Ttest_indResult(statistic=-2.475368857441685, pvalue=0.03838779259171958)

1. Observations are independent and normally distributed within groups

OLS Regression Results						
Dep. Variable:	y	R-squared:	0.434			
Model:	OLS	Adj. R-squared:	0.363			
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	coef	std err	t	P> t	[0.025	0.975]
Intercept	0.9800	0.143	6.861	0.000	0.651	1.309
z[T.b]	0.5000	0.202	2.475	0.038	click to scroll output; double click to hide	
Omnibus:	0.887	Durbin-Watson:		1.642		
Prob(Omnibus):	0.642	Jarque-Bera (JB):		0.670		
Skew:	-0.287	Prob(JB):		0.715		
Kurtosis:	1.870	Cond. No.		2.62		

For more information, look into contrast coding

Setting your intercept as your "Control" will allow you to see how your different candidates (A, B, ...) compete