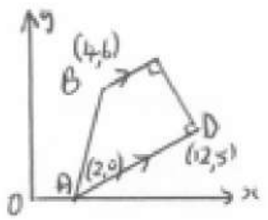
CAMBRIDGE
INTERNATIONAL EXAMINATIONS

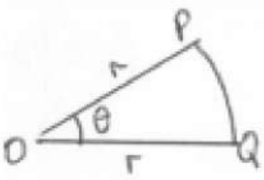
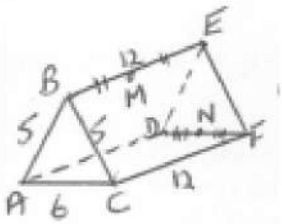
November 2003

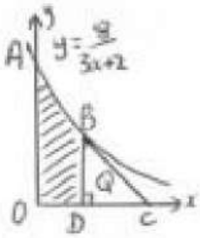
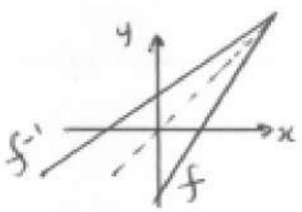
GCE A AND AS LEVEL

MARK SCHEME
MAXIMUM MARK: 75
SYLLABUS/COMPONENT: 9709/01 MATHEMATICS Pure Mathematics : Paper One

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1	$x(11-2x) = 12$ $2x^2-11x+12=0$ Solution of quadratic → $(1\frac{1}{2},8)$ and $(4,3)$	M1 A1 DM1 A1 [4]	Complete elimination of x, or of y. Correct quadratic. (or $y^2-11y+24=0$) Correct method of solution→2values All correct (guesswork or TI B1 for one pair of values, full marks for both)
2	(i) $4s^4+5=7(1-s^2) \rightarrow 4x^2+7x-2=0$ (ii) $4s^4+7s^2-2=0$ → $s^2 = \frac{1}{4}$ or $s^2 = -2$ → $\sin\theta = \pm\frac{1}{2}$ → $\theta = 30°$ and $150°$ and $\theta = 210°$ and $330°$	B1 [1] M1 A1A1√ A1√ [4]	Use of $s^2+c^2=1$. Answer given. Recognition of quadratic in s^2 Co. For 180° - "his value" For other 2 answers from "his value", providing no extra answers in the range or answers from $s^2=-1$
3	(a) $a=60$, $n=48$, $S_n=3726$ S_n formula used → $d = \$0.75$ 3rd term = $a+2d$ = \$61.50 (b) $a=6$ $ar=4$ $\therefore r=\frac{2}{3}$ $S_\infty = a/(1-r) = 18$	 M1 A1 A1√ [3] M1 M1A1 [3]	 Correct formula (M0 if nth term used) Co Use of a+2d with his d. 61.5 ok. a, ar correct, and r evaluated Correct formula used, but needs r <1 for M mark
4	(i) $y = x^3 - 2x^2 + x$ (+c) (1,5) used to give c= 5 (ii) $3x^2-4x+1>0$ → end values of 1 and $\frac{1}{3}$ → $x<\frac{1}{3}$ and $x>1$	B2,1,0 B1√ [3] M1 A1 A1 [3]	Co - unsimplified ok. Must have integrated + use of x=1and y=5 for c Set to 0 and attempt to solve. Co for end values – even if <,>,=,etc Co (allow ≤ and ≥). Allow $1<x<\frac{1}{3}$
5	 (i) m of BC = ½ Eqn BC $y-6=\frac{1}{2}(x-4)$ m of CD = -2 eqn CD $y-5=-2(x-12)$ (ii) Sim eqns $2y=x+8$ and $y+2x=29$ → C (10,9)	B1 M1A1√ M1 A1√ [5] M1 A1 [2]	Co Correct form of eqn. √ on m="½." Use of $m_1m_2=-1$ √ on his "½" but needs both M marks. Method for solving Co Diagram only for (ii), allow B1 for (10,9)

6  (i) $20 = 2r + r\theta$ $\rightarrow\ \theta = (20 / r) - 2$	M1 A1 [2]	Eqn formed + use of $r\theta$ + at least one r Answer given.
(ii) $A = \frac{1}{2}r^2\theta$ $\rightarrow A = 10r - r^2$	M1 A1 [2]	Appropriate use of $\frac{1}{2}r^2\theta$ Co – but ok unsimplified –eg $\frac{1}{2}r^2(20/r)-2)$
(iii) Cos rule $PQ^2 = 8^2+8^2-2.8.8\cos 0.5$ Or trig $PQ = 2\times 8\sin 0.25$ $\rightarrow PQ = 3.96$ (allow 3.95).	M1 A1 A1 [3]	Recognition of "chord" +any attempt at trigonometry in triangle. Correct expression for PQ or PQ^2. Co
7  (i) Height = 4	B1 [1]	Pythagoras or guess – anywhere, 4**k** ok.
(ii) **MC** = 3**i**-6**j**-4**k** **MN** = 6**j** – 4**k**	B2,1√ B1√ [3]	√ for "4". Special case B1 for –3**i**+6**j**+4**k** √ on "4". Accept column vectors. (nb if (ii) incorrect, but answers are correct in (iii) allow feedback).
(iii) **MC.MN** = -36+16 = -20 **MC.MN** = $\sqrt{61}\sqrt{52}\cos\theta$ $\rightarrow \theta = 111°$	M1A1√ M1 A1 [4]	Use of $x_1y_1+x_2y_2+x_3y_3$. √ on **MC** and **MN** Product of two moduli and cos θ. Co. Nb If both **MC** and **MN** "reversed", allow 111° for full marks.
8 (i) $y = 72\div(2x^2)$ or $36\div x^2$ $A = 4x^2+6xy$ $\rightarrow\ A = 4x^2 + 216\div x$	B1 M1 A1 [3]	Co from volume = lbh . Attempts most of the faces(4 or more) Co – answer was given.
(ii) $dA/dx = 8x - 216\div x^2$ $= 0$ when $8x^3=216$ $\rightarrow\ x = 3$	M1 DM1 A1 [3]	Reasonable attempt at differentiation. Sets his differential to 0 and uses. Co. (answer = ±3 loses last A mark)
(iii) Stationary value = 108 cm² $d^2A/dx^2=8+432\div x^3$ $\rightarrow$ Positive when x=3 Minimum.	A1√ M1 A1 [3]	For putting his x into his A. Allow in (ii). Correct method – could be signs of dA/dx A mark needs d^2A/dx^2 correct algebraically, + x=3 + minimum. It does not need "24".

9  (i) $dy/dx = -24/(3x+2)^2$	M1A1	Use of fn of fn. Needs ×3 for M mark. Co.
Eqn of tangent $y-1=-\frac{3}{8}(x-2)$ Cuts y=0 when $x=4\frac{2}{3}$	M1A1√	Use of line form with dy/dx. Must use calculus. √ on his dy/dx. Normal M0.
Area of Q = $\frac{1}{2}\times2\frac{2}{3}\times1 = \frac{4}{3}$	M1A1 [6]	Needs y=0 and ½bh for M mark. (beware fortuitous answers)
(ii) Vol $=\pi\int y^2dx = \pi\int 64(3x+2)^{-2}dx$ $=\pi\,[-64(3x+2)^{-1} \div 3]$ Limits from 0 to 2 $\rightarrow 8\pi$	M1 A1A1 DM1 A1 [5]	Uses $\int y^2$ + some integration → $(3x+2)^k$. A1 without the ÷3. A1 for ÷3 and π Correct use of 0 and 2. DMO if 0 ignored. Co. Beware fortuitous answers.
10 (i) fg(x) = g first, then f $= 8/(2-x) - 5 = 7$ $\rightarrow x = 1\frac{1}{3}$	M1 DM1 A1	Correct order - g first, then into f. Correct method of solution of fg=7. Co. (nb gf gets 0/3)
(or f(A)=7, A = 6, g(x) = 6, → $x = 1\frac{1}{3}$)	[3]	(M1 for 6. M1 for g(x)=6. A1)
(ii) $f^{-1} = \frac{1}{2}(x+5)$ Makes y the subject $y = 4\div(2-x)$ $\rightarrow g^{-1} = 2 - (4\div x)$	B1 M1 A1 [3]	Anywhere in the question. For changing the subject. Co – any correct answer. (A0 if f(y).)
(iii) $2-4/x = \frac{1}{2}(x+5)$ $\rightarrow x^2+x+8=0$ Use of b^2-4ac → Negative value → No roots.	M1 M1 A1 [3]	Algebra leading to a quadratic. Quadratic=0 + use of b^2-4ac. Correct deduction from correct quadratic.
(iv) 	B1 B1 B1 [3]	Sketch of f Sketch of f^{-1} Evidence of symmetry about y=x.