CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/03, 8719/03

MATHEMATICS
Mathematics and Higher Mathematics : Paper 3

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 *EITHER*: State or imply non-modular inequality $-5 < 2^x - 8 < 5$, or $(2^x - 8)^2 < 5^2$ or corresponding pair of linear equations or quadratic equation — B1

Use correct method for solving an equation of the form $2^x = a$ — M1

Obtain critical values 1.58 and 3.70, or exact equivalents — A1

State correct answer $1.58 < x < 3.70$ — A1

OR: Use correct method for solving an equation of the form $2^x = a$ — M1

Obtain one critical value (probably 3.70), or exact equivalent — A1

Obtain the other critical value, or exact equivalent — A1

State correct answer $1.58 < x < 3.70$ — A1

[4]

[Allow 1.59 and 3.7. Condone ≤ for <. Allow final answers given separately. Exact equivalents must be in terms of ln or logarithms to base 10.]
[SR: Solutions given as logarithms to base 2 can only earn M1 and B1 of the first scheme.]

2 *EITHER*: Obtain correct unsimplified version of the x^2 or x^4 term of the expansion of $(1+\frac{1}{2}x^2)^{-2}$ or $(2+x^2)^{-2}$ — M1

State correct first term $\frac{1}{4}$ — B1

Obtain next two terms $-\frac{1}{4}x^2 + \frac{3}{16}x^4$ — A1+A1

[The M mark is not earned by versions with unexpanded binomial coefficients such as $\begin{pmatrix}-2\\1\end{pmatrix}$.]

[SR: Answers given as $\frac{1}{4}(1 - x^2 + \frac{3}{4}x^4)$ earn M1B1A1.]

[SR: Solutions involving $k(1+\frac{1}{2}x^2)^{-2}$, where $k = 2$, 4 or $\frac{1}{2}$ can earn M1 and A1 for a correct simplified term in x^2 or x^4.]

OR: Differentiate expression and evaluate f(0) and f ′(0), where $f'(x) = kx(2+x^2)^{-3}$ — M1

State correct first term $\frac{1}{4}$ — B1

Obtain next two terms $-\frac{1}{4}x^2 + \frac{3}{16}x^4$ — A1+A1

[Allow exact decimal equivalents as coefficients.]

[4]

3 Use correct cos $2A$ formula, or equivalent pair of correct formulas, to obtain an equation in cos θ — M1

Obtain 3-term quadratic $6\cos^2\theta + \cos\theta - 5 = 0$, or equivalent — A1

Attempt to solve quadratic and reach $\theta = \cos^{-1}(a)$ — M1

Obtain answer 33.6° (or 33.5°) or 0.586 (or 0.585) radians — A1

Obtain answer 180° or π (or 3.14) radians and no others in range — A1

[The answer θ = 180° found by inspection can earn B1.]
[Ignore answers outside the given range.]

[5]

4(i)	*EITHER*	Obtain terms $\frac{1}{2\sqrt{x}}$ and $\frac{1}{2\sqrt{y}}\frac{dy}{dx}$, or equivalent	B1+B1
		Obtain answer in any correct form, e.g. $\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$	B1
	OR:	Using chain or product rule, differentiate $(\sqrt{a}-\sqrt{x})^2$	M1
		Obtain derivative in any correct form	A1
		Express $\frac{dy}{dx}$ in terms of x and y only in any correct form	A1
	OR:	Expand $(\sqrt{a}-\sqrt{x})^2$, differentiate and obtain term $-2.\frac{\sqrt{a}}{2\sqrt{x}}$, or equivalent	B1
		Obtain term 1 by differentiating an expansion of the form $a + x \pm 2\sqrt{a}\sqrt{x}$	B1
		Express $\frac{dy}{dx}$ in terms of x and y only in any correct form	B1
			[3]
(ii)		State or imply coordinates of P are $(\frac{1}{4}a, \frac{1}{4}a)$	B1
		Form equation of the tangent at P	M1
		Obtain 3 term answer $x + y = \frac{1}{2}a$ correctly, or equivalent	A1
			[3]
5 (i)		Make recognizable sketch of $y = \sec x$ or $y = 3 - x^2$, for $0 < x < \frac{1}{2}\pi$	B1
		Sketch the other graph correctly and justify the given statement	B1
			[2]

[Award B1 for a sketch with positive y-intercept and correct concavity. A correct sketch of $y = \cos x$ can only earn B1 in the presence of $1/(3-x^2)$. Allow a correct single graph and its intersection with $y = 0$ to earn full marks.]

(ii)	State or imply equation $\alpha = \cos^{-1}(1/(3-\alpha^2))$ or $\cos\alpha = 1/(3-\alpha^2)$	B1
	Rearrange this in the form given in part **(i)** i.e. sec $\alpha = 3 - \alpha^2$	B1
		[2]

[Or work *vice versa*.]

(iii)	Use the iterative formula with $0 \le x_1 \le \sqrt{2}$	M1
	Obtain final answer 1.03	A1
	Show sufficient iterations to justify its accuracy to 2d.p. or show there is a sign change in the interval (1.025, 1.035)	A1
		[3]

6 (i)	Use product or quotient rule to find derivative	M1
	Obtain derivative in any correct form	A1
	Equate derivative to zero and solve a linear equation in x	M1
	Obtain answer $3\frac{1}{2}$ only	A1
		[4]
(ii)	State first step of the form $\pm\frac{1}{2}(3-x)e^{-2x} \pm \frac{1}{2}\int e^{-2x}dx$, with or without 3	M1
	State correct first step e.g. $-\frac{1}{2}(3-x)e^{-2x} - \frac{1}{2}\int e^{-2x}dx$, or equivalent, with or without 3	A1
	Complete the integration correctly obtaining $-\frac{1}{2}(3-x)e^{-2x} + \frac{1}{4}e^{-2x}$, or equivalent	A1
	Substitute limits $x = 0$ and $x = 3$ correctly in the complete integral	M1
	Obtain answer $\frac{1}{4}(5 + e^{-6})$, or exact equivalent (allow e^0 in place of 1)	A1
		[5]
7 (i) *EITHER*:	Attempt multiplication of numerator and denominator by 3 + 2i, or equivalent	M1
	Simplify denominator to 13 or numerator to 13 + 26i	A1
	Obtain answer $u = 1 + 2i$	A1
OR:	Using correct processes, find the modulus and argument of u	M1
	Obtain modulus $\sqrt{5}$ (or 2.24) or argument $\tan^{-1}2$ (or 63.4° or 1.11 radians)	A1
	Obtain answer $u = 1 + 2i$	A1
		[3]
(ii)	Show the point U on an Argand diagram in a relatively correct position	B1√
	Show a circle with centre U	B1√
	Show a circle with radius consistent with 2	B1√
		[3]

[f.t. on the value of u.]

(iii)	State or imply relevance of the appropriate tangent from O to the circle	B1√
	Carry out a complete strategy for finding max arg z	M1
	Obtain final answer 126.9° (2.21 radians)	A1
		[3]

[Drawing the appropriate tangent is sufficient for B1√.]
[A final answer obtained by measurement earns M1 only.]

8 (**i**)	*EITHER*:	Divide by denominator and obtain a quadratic remainder	M1
		Obtain $A = 1$	A1
		Use any relevant method to obtain B, C or D	M1
		Obtain one correct answer	A1
		Obtain $B = -1$, $C = 2$, $D = 0$	A1
	OR:	Reduce *RHS* to a single fraction and identify numerator with that of $f(x)$	M1
		Obtain $A = 1$	A1
		Use any relevant method to obtain B, C or D	M1
		Obtain one correct answer	A1
		Obtain $B = -1$, $C = 2$, $D = 0$	A1
			[5]
(**ii**)		Integrate and obtain terms $x - \ln(x-1)$, or equivalent	B1√
		Obtain third term $\ln(x^2+1)$, or equivalent	B1√
		Substitute correct limits correctly in the complete integral	M1
		Obtain given answer following full and exact working	A1
			[4]

[If $B = 0$ the first B1√ is not available.]
[SR: If A is omitted in part (**i**), treat as if $A = 0$. Thus only M1M1 and B1√B1√M1 are available.]

9 (**i**)	Separate variables and attempt to integrate $\dfrac{1}{\sqrt{(P-A)}}$	M1
	Obtain term $2\sqrt{(P-A)}$	A1
	Obtain term $-kt$	A1
		[3]
(**ii**)	Use limits $P = 5A$, $t = 0$ and attempt to find constant c	M1
	Obtain $c = 4\sqrt{A}$, or equivalent	A1
	Use limits $P = 2A$, $t = 2$ and attempt to find k	M1
	Obtain given answer $k = \sqrt{A}$ correctly	A1
		[4]
(**iii**)	Substitute $P = A$ and attempt to calculate t	M1
	Obtain answer $t = 4$	A1
		[2]
(**iv**)	Using answers to part (**ii**), attempt to rearrange solution to give P in terms of A and t	M1
	Obtain $P = \frac{1}{4}A(4 + (4-t)^2)$, or equivalent, having squared $\sqrt{A}$	A1
		[2]

[For the M1, $\sqrt{(P-A)}$ must be treated correctly.]

10 **(i)**	Express general point of l or m in component form e.g. $(1 + 2s, s, -2 + 3s)$ or $(6 + t, -5 - 2t, 4 + t)$	B1
	Equate at least two corresponding pairs of components and attempt to solve for s or t	M1
	Obtain $s = 1$ or $t = -3$	A1
	Verify that all three component equations are satisfied	A1
	Obtain position vector $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ of intersection point, or equivalent	A1
		[5]
(ii) *EITHER:*	Use scalar product to obtain $2a + b + 3c = 0$ and $a - 2b + c = 0$	B1
	Solve and find one ratio e.g. $a : b$	M1
	State one correct ratio	A1
	Obtain answer $a : b : c = 7 : 1 : -5$, or equivalent	A1
	Substitute coordinates of a relevant point and values of a, b and c in general equation of plane and calculate d	M1
	Obtain answer $7x + y - 5z = 17$, or equivalent	A1
OR:	Using two points on l and one on m (or *vice versa*) state three simultaneous equations in a, b, c and d e.g. $3a + b + c = d$, $a - 2c = d$ and $6a - 5b + 4c = d$	B1√
	Solve and find one ratio e.g. $a : b$	M1
	State one correct ratio	A1
	Obtain a ratio of three unknowns e.g. $a : b : c = 7 : 1 : -5$, or equivalent	A1
	Use coordinates of a relevant point and found ratio to find fourth unknown e.g. d	M1
	Obtain answer $7x + y - 5z = 17$, or equivalent	A1
OR:	Form a correct 2-parameter equation for the plane, e.g. $\mathbf{r} = \mathbf{i} - 2\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + \mu(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	B1√
	State 3 equations in x, y, z, λ and μ	M1
	State 3 correct equations	A1√
	Eliminate λ and μ	M1
	Obtain equation in any correct unsimplified form	A1
	Obtain $7x + y - 5z = 17$, or equivalent	A1
OR:	Attempt to calculate vector product of vectors parallel to l and m	M1
	Obtain two correct components of the product	A1
	Obtain correct product, e.g. $7\mathbf{i} + \mathbf{j} - 5\mathbf{z}$	A1
	State that the plane has equation of the form $7x + y - 5z = d$	A1√
	Substitute coordinates of a relevant point and calculate d	M1
	Obtain answer $7x + y - 5z = 17$, or equivalent	A1
		[6]

[The follow through is on $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ only.]