CAMBRIDGE
INTERNATIONAL EXAMINATIONS

June 2003

GCE A AND AS LEVEL

MARK SCHEME

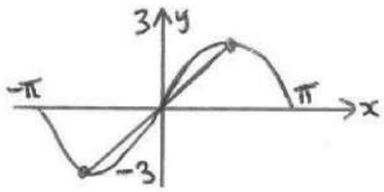
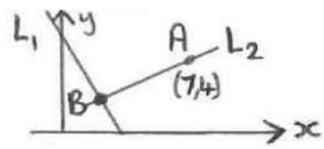
MAXIMUM MARK: 75

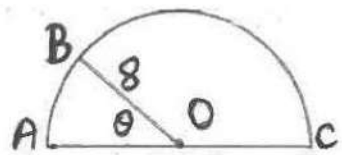
SYLLABUS/COMPONENT: 9709/01

MATHEMATICS
Paper 1 (Pure 1)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1. $(2x - 1/x)^5$. 4^{th} term needed. $\rightarrow {}_5C_3 = 5.4/2$ $\rightarrow x\ 2^2\ x\ (-1)^3$ $\rightarrow -40$	 M1 DM1 A1 [3]	 Must be 4^{th} term – needs $(2x)^2\ (1/x)^3$ Includes and converts ${}_5C_2$ or ${}_5C_3$ Co Whole series given and correct term not quoted, allow 2/3
2. sin3x + 2cos3x = 0 tan3x = -2 x = 38.9 (8) and x = 98.9 (8) and x = 158.9 (8) NB. $\sin^2 3x + \cos^2 3x = 0$ etc. M0 But $\sin^2 3x = (-2\cos 3x)^2$ plus use of $s^2 + c^2 = 1$ is OK Alt. $\sqrt{5}\sin(3x + \alpha)$ or $\sqrt{5}\cos(3x - \alpha)$ both OK	 M1 A1 A1√ A1√ [4]	 Use of tan = sin ÷ cos with 3x Co For 60 + "his" For 120 + "his" and no others in range (ignore excess ans. outside range) Loses last A mark if excess answers in the range
3. (a) $dy/dx = 4 - 12x^{-3}$ (b) $\int = 2x^2 - 6x^{-1} + c$ (a) (quotient OK M1 correct formula, A1 co)	B2, 1 [2] 3 x B1 [3]	One off for each error (4, -, 12, -3) One for each term – only give +c if obvious attempt at integration
4. a = -10 a + 14d = 11 d = $\frac{3}{2}$ a + (n – 1)d = 41 n = 35 Either $S_n = n/2(2a + (n-1)d)$ or $n/2(a + l)$ = 542.5	M1 M1 A1 M1 A1 [5]	Using a = (n – 1)d Correct method – not for a + nd Co Either of these used correctly For his d and any n
5. (i) 2a + b = 1 and 5a + b = 7 $\rightarrow$ a = 2 and b = -3 (ii) f(x) = 2x - 3 ff(x) = 2(2x – 3)-3 $\rightarrow$ 4x – 9 = 0 when x = 2.25	M1 A1 [2] M1 DM1 A1 [3]	Realising how one of these is formed Co Replacing "x" by "his ax + b" and "+b" For his a and b and solved = 0 Co

6. (i) 	B2, 1 [2]	For complete cycle, shape including curves, not lines, -3 to +3 shown or implied, for $-\pi$ to π. Degrees ok
(ii) $x = \pi/2$, $y = 3$ (allow if 90°) $\rightarrow k = 6/\pi$ co.	M1 A1 [2]	Realising maximum is $(\pi/2, 3)$ + sub Co (even if no graph)
(iii) $(-\pi/2, -3)$ – must be radians	B1 [1]	Co (could come from incorrect graph)
7. (i) 		
Gradient of $L_1 = -2$ Gradient of $L_2 = ½$ Eqn of L_2 $y - 4 = ½(x - 7)$	B1 M1 M1A1√ [4]	Co – anywhere Use of $m_1m_2 = -1$ Use of line eqn – or $y = mx + c$. Line must be through (7, 4) and non-parallel
(ii) Sim Eqns $\rightarrow x = 3, y = 2$	M1 A1	Solution of 2 linear eqns Co
$AB = \sqrt{(2^2 + 4^2)} = \sqrt{20}$ or 4.47	M1A1 [4]	Correct use of distance formula. Co
8. (i) $\overrightarrow{BA} = \mathbf{a} - \mathbf{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ $\overrightarrow{BC} = \mathbf{c} - \mathbf{b} = -2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ Dot product = -2 + 8 – 6 = 0 $\rightarrow$ Perpendicular	M1 M1A1 A1 [4]	Knowing how to use position vector for $\overrightarrow{BA}$ or $\overrightarrow{BC}$ – not for $\overrightarrow{AB}$ or $\overrightarrow{CB}$ Knowing how to use $x_1y_1 + x_2y_2 + x_3y_3$. Co Correct deduction. Beware fortuitous (uses $\overrightarrow{AB}$ or $\overrightarrow{CB}$ – can get 3 out of 4)
(ii) $\overrightarrow{BC} = \mathbf{c} - \mathbf{b} = -2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ $\overrightarrow{AD} = \mathbf{d} - \mathbf{a} = -5\mathbf{i} + 10\mathbf{j} + 5\mathbf{k}$ These are in the same ratio \ parallel	M1 M1	Knowing how to get one of these Both correct + conclusion. Could be dot product = 60 $\rightarrow$ angle = 0°
Ratio = 2:5 (or $\sqrt{24}$: $\sqrt{150}$)	M1A1 [4]	Knowing what to do. Co. Allow 5:2

9.		
		
(i) $\theta = 1$ angle BOC = $\pi-\theta$ Area = $\frac{1}{2}r^2\theta$ = 68.5 or $32(\pi-1)$ (or ½circle-sector)	B1 M1 A1 [3]	For $\pi-\theta$ or for $\frac{1}{2}\pi r^2$ – sector Use of $\frac{1}{2}r^2\theta$ Co NB. 32 gets M1 only
(ii) $8 + 8 + 8\theta = \frac{1}{2}(8 + 8 + 8(\pi-\theta))$ Solution of this eqn $\rightarrow$ 0.381 or $^1/_3(\pi-2)$	M1 M1 A1 [3]	Relevant use of $s = r\theta$ twice Needs θ – collected – needs perimeters Co.
(iii) $\theta = \pi/3$ AB = 8cm BC = 2 x $8\sin\pi/3 = 8\sqrt{3}$ Perimeter = $24 + 8\sqrt{3}$	B1 M1 A1 [3]	Co. Valid method for BC – cos rule, Pyth allow decimals here Everything OK. Answer given NB. Decimal check loses this mark
10. $y = \sqrt{(5x + 4)}$		
(i) $dy/dx = \frac{1}{2}(5x + 4)^{-\frac{1}{2}} \times 5$ $x = 1$, $dy/dx = 5/6$	B1B1 B1 [3]	$\frac{1}{2}(5x + 4)^{-\frac{1}{2}}$ x 5 B1 for each part Co
(ii) $dy/dt = dy/dx \times dx/dt$ $= 5/6 \times 0.03$ $\rightarrow$ 0.025	M1 A1√ [2]	Chain rule correctly used For (i) x 0.03
(iii) realises that area $\rightarrow$ integration $\int = (5x + 4)^{3/2} \div {}^3/_2 \div 5$ Use of limits $\rightarrow$ 54/15 - 16/15 = 38/15 = 2.53	M1 A1A1 DM1 A1 [5]	Realisation + attempt – must be $(5x + 4)^k$ For $(5x + 4)^{3/2} \div {}^3/_2$. For $\div$ 5 Must use "0" to "1" Co

11. (i) $8x - x^2 = a - x^2 - b^2 - 2bx$ + equating	M1	Knows what to do – some equating
$\rightarrow b = -4$	B1	Anywhere – may be independent
$a = b^2 = 16$ (i.e. $16 - (x - 4)^2$)	A1 [3]	For 16- $(\ )^2$
(ii) $dy/dx = 8 - 2x = o$ when	M1	Any valid complete method
$\rightarrow (4, 16)$ (or from $-b$ and a)	A1 [2]	Needs both values
(iii) $8x - x^2 \geq -20$		
$x^2 - 8x - 20 = (x - 10)(x + 2)$	M1	Sets to 0 + correct method of solution
End values -2 and 10	A1	Co – independent of < or > or =
Interval $-2 \leq x \leq 10$	A1 [3]	Co – including $\leq$ (< gets A0)
$g: x \rightarrow 8x - x^2$ for $x \geq 4$		
(iv) domain of g^{-1} is $x \leq 16$	B1√	From answer to (i) or (ii). Accept <16
range of g^{-1} is $g^{-1} \geq 4$	B1 [2]	Not f.t since domain of g given
(v) $y = 8x - x^2 \rightarrow x^2 - 8x + y = 0$	M1	Use of quadratic or completed square expression to make x subject
$x = 8 \pm \sqrt{(64 - 4y)} \div 2$	DM1	Replaces y by x
$g^{-1}(x) = 4 + \sqrt{(16 - x)}$	A1 [3]	Co (inc. omission of -)
or $(x - 4)^2 = 16 - y \rightarrow x = 4 + \sqrt{(16 - y)}$ $\rightarrow y = 4 + \sqrt{(16 - x)}$		