Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2020**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Blank pages are indicated.

1 The lengths, X centimetres, of a random sample of 7 leaves from a certain variety of tree are as follows.

$$5.2 \quad 4.8 \quad 5.5 \quad 6.1 \quad 4.8 \quad 3.9 \quad 4.4$$

(a) Calculate unbiased estimates of the population mean and variance of X.   [3]

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It is now given that the true value of the population variance of X is 0.55, and that X has a normal distribution.

(b) Find a 95% confidence interval for the population mean of X.   [3]

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2 In the past the yield of a certain crop, in tonnes per hectare, had mean 0.56 and standard deviation 0.08. Following the introduction of a new fertilizer, the farmer intends to test at the 2.5% significance level whether the mean yield has increased. He finds that the mean yield over 10 years is 0.61 tonnes per hectare.

(a) State two assumptions that are necessary for the test. [2]

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(b) Carry out the test. [5]

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3 The masses, in kilograms, of large sacks of flour and small sacks of flour have the independent distributions N(40, 1.5^2) and N(12, 0.7^2) respectively.

(a) Find the probability that the total mass of 6 randomly chosen large sacks of flour is more than 245 kg. [4]

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(b) Find the probability that the mass of a randomly chosen large sack of flour is less than 4 times the mass of a randomly chosen small sack of flour. [6]

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4 A fair spinner has five sides numbered 1, 2, 3, 4, 5. The score on one spin is denoted by X.

(a) Show that $\text{Var}(X) = 2$. [1]

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Fiona has another spinner, also with five sides numbered 1, 2, 3, 4, 5. She suspects that it is biased so that the expected score is less than 3. In order to test her suspicion, she plans to spin her spinner 40 times. If the mean score is less than 2.6 she will conclude that her spinner is biased in this way.

(b) Find the probability of a Type I error. [4]

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(c) State what is meant by a Type II error in this context. [1]

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5 Each week a sports team plays one home match and one away match. In their home matches they score goals at a constant average rate of 2.1 goals per match. In their away matches they score goals at a constant average rate of 0.8 goals per match. You may assume that goals are scored at random times and independently of one another.

(a) A week is chosen at random.

    (i) Find the probability that the team scores a total of 4 goals in their two matches. [2]

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    (ii) Find the probability that the team scores a total of 4 goals, with more goals scored in the home match than in the away match. [3]

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(b) Use a suitable approximating distribution to find the probability that the team scores fewer than 25 goals in 10 randomly chosen weeks. [4]

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(c) Justify the use of the approximating distribution used in part **(b)**. [1]

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6 The length of time, T minutes, that a passenger has to wait for a bus at a certain bus stop is modelled by the probability density function given by

$$f(t) = \begin{cases} \frac{3}{4000}(20t - t^2) & 0 \leqslant t \leqslant 20, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Sketch the graph of $y = f(t)$. [1]

(b) Hence explain, without calculation, why $E(T) = 10$. [1]

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(c) Find $Var(T)$. [3]

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(d) It is given that $P(T < 10 + a) = p$, where $0 < a < 10$.

Find $P(10 - a < T < 10 + a)$ in terms of p. [2]

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(e) Find $P(8 < T < 12)$. [3]

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(f) Give one reason why this model may be unrealistic. [1]

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Additional Page

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