CAMBRIDGE
INTERNATIONAL EXAMINATIONS

June 2003

GCE A AND AS LEVEL

MARK SCHEME
MAXIMUM MARK: 75
SYLLABUS/COMPONENT: 9709/03, 8719/03 **MATHEMATICS AND HIGHER MATHEMATICS** **Paper 3 (Pure 3)**

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1	**(i)**	Use trig formulae to express *LHS* in terms of sin x and cos x	M1
		Use cos 60° = sin 30° to reduce equation to given form cos $x = k$	M1
			[2]
	(ii)	State or imply that $k = -\dfrac{1}{\sqrt{3}}$ (accept -0.577 or -0.58)	A1
		Obtain answer x = 125.3° only	A1
		[Answer must be in degrees; ignore answers outside the given range.]	
		[SR: if $k = \dfrac{1}{\sqrt{3}}$ is followed by x = 54.7°, give A0A1√.]	
			[2]
2		State first step of the form $kxe^{2x} \pm \int ke^{2x}\,dx$	M1
		Complete the first step correctly	A1
		Substitute limits correctly having attempted the further integration of ke^{2x}	M1
		Obtain answer ¼ $(e^2 + 1)$ or exact equivalent of the form $ae^2 + b$, having used $e^0 = 1$ throughout	A1
			[4]
3	*EITHER*	State or imply non-modular inequality $(x-2)^2 < (3-2x)^2$, or corresponding equation	B1
		Expand and make a reasonable solution attempt at a 2- or 3-term quadratic, or equivalent	M1
		Obtain critical value $x = 1$	A1
		State answer $x < 1$ only	A1
	OR	State the relevant linear equation for a critical value, i.e. $2 - x = 3 - 2x$, or equivalent	B1
		Obtain critical value $x = 1$	B1
		State answer $x < 1$	B1
		State or imply by omission that no other answer exists	B1
	OR	Obtain the critical value $x = 1$ from a graphical method, or by inspection, or by solving a linear inequality	B2
		State answer $x < 1$	B1
		State or imply by omission that no other answer exists	B1
			[4]

4 (i)	*EITHER*	State or imply that $x - 2$ is a factor of $f(x)$	B1
		Substitute 2 for x and equate to zero	M1
		Obtain answer $a = 8$	A1
		[The statement $(x - 2)^2 = x^2 - 4x + 4$ earns B1.]	
	OR	Commence division by $x^2 - 4x + 4$ and obtain partial quotient $x^2 + 2x$	B1
		Complete the division and equate the remainder to zero	M1
		Obtain answer $a = 8$	A1
	OR	Commence inspection and obtain unknown factor $x^2 + 2x + c$	B1
		Obtain $4c = a$ and an equation in c	M1
		Obtain answer $a = 8$	A1
			[3]
(ii)	*EITHER*	Substitute $a = 8$ and find other factor $x^2 + 2x + 2$ by inspection or division	B1
		State that $x^2 - 4x + 4 \geq 0$ for all x (condone $>$ for $\geq$)	B1
		Attempt to establish sign of the other factor	M1
		Show that $x^2 + 2x + 2 > 0$ for all x and complete the proof	A1
		[An attempt to find the zeros of the other factor earns M1.]	
	OR	Equate derivative to zero and attempt to solve for x	M1
		Obtain $x = -\frac{1}{2}$ and 2	A1
		Show correctly that $f(x)$ has a minimum at each of these values	A1
		Having also obtained and considered $x = 0$, complete the proof	A1
			[4]
5 (i)		State or imply $w = \cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi$ (allow decimals)	B1
		Obtain answer $uw = -\sqrt{3} - i$ (allow decimals)	B1√
		Multiply numerator and denominator of $\frac{u}{w}$ by $-1 - i\sqrt{3}$, or equivalent	M1
		Obtain answer $\frac{u}{w} = \sqrt{3} - i$ (allow decimals)	A1
			[4]
(ii)		Show U on an Argand diagram correctly	B1
		Show A and B in relatively correct positions	B1√
			[2]
(iii)		Prove that $AB = UA$ (or UB), or prove that angle AUB = angle ABU (or angle BAU) or prove, for example, that $AO = OB$ and angle $AOB = 120°$, or prove that one angle of triangle UAB equals 60°	B1
		Complete a proof that triangle UAB is equilateral	B1
			[2]

6 (i) *EITHER*	State or imply $f(x) \equiv \dfrac{A}{2x+1} + \dfrac{B}{x-2} + \dfrac{C}{(x-2)^2}$	B1
	State or obtain $A = 1$	B1
	State or obtain $C = 8$	B1
	Use any relevant method to find B	M1
	Obtain value $B = 4$	A1
OR	State or imply $f(x) \equiv \dfrac{A}{2x+1} + \dfrac{Dx+E}{(x-2)^2}$	B1
	State or obtain $A = 1$	B1
	Use any relevant method to find D or E	M1
	Obtain value $D = 4$	A1
	Obtain value $E = 0$	A1
		[5]
(ii) *EITHER*	Use correct method to obtain the first two terms of the expansion of $(1+2x)^{-1}$ or $(x-2)^{-1}$ or $(x-2)^{-2}$ or $(1-\frac{1}{2}x)^{-1}$ or $(1-\frac{1}{2}x)^{-2}$	M1
	Obtain any correct sum of unsimplified expansions up to the terms in x^2 (deduct A1 for each incorrect expansion)	A2√
	Obtain the given answer correctly	A1

[Unexpanded binomial coefficients involving -1 or -2, e.g. $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ are not sufficient for the M1.]

[f.t. is on *A*, *B*, *C*, *D*, *E*.]

[Apply this scheme to attempts to expand $(9x^2+4)(1+2x)^{-1}(x-2)^{-2}$, giving M1A2 for a correct product of expansions and A1 for multiplying out and reaching the given answer correctly.]

[Allow attempts to multiply out $(1+2x)(x-2)^2(1-x+5x^2)$, giving B1 for reduction to a product of two expressions correct up to their terms in x^2, M1 for attempting to multiply out as far as terms in x^2, A1 for a correct expansion, and A1 for obtaining $9x^2+4$ correctly.]

[SR: *B* or *C* omitted from the form of partial fractions. In part **(i)** give the first B1, and M1 for the use of a relevant method to obtain *A*, *B*, or *C*, but no further marks. In part **(ii)** only the M1 and A1√ for an unsimplified sum are available.]

[SR: *E* omitted from the form of partial fractions. In part **(i)** give the first B1, and M1 for the use of a relevant method to obtain *A* or *D*, but no further marks. In part **(ii)** award M1A2√A1 as in the scheme.]

OR	Differentiate and evaluate $f(0)$ and $f'(0)$	M1
	Obtain $f(0) = 1$ and $f'(0) = -1$	A1
	Differentiate and obtain $f''(0) = 10$	A1
	Form the Maclaurin expansion and obtain the given answer correctly	A1
		[4]

7	**(i)**	State or imply that $\frac{dx}{dt} = k\,(100 - x)$	B1
		Justify $k = 0.02$	B1
			[2]
	(ii)	Separate variables and attempt to integrate $\frac{1}{100-x}$	M1
		Obtain term – ln (100 - x), or equivalent	A1
		Obtain term 0.02t, or equivalent	A1
		Use x = 5, t = 0 to evaluate a constant, or as limits	M1
		Obtain correct answer in any form, e.g. -ln(100 - x) = 0.02t - ln 95	A1
		Rearrange to give x in terms of t in any correct form, e.g. x = 100 - 95exp(-0.02t)	A1
			[6]
		[SR: ln (100 - x) for -ln (100 - x). If no other error and x = 100 - 95exp(0.02t) or equivalent obtained, give M1A0A1M1A0A1√]	
	(iii)	State that x tends to 100 as t becomes very large	B1
			[1]
8	**(i)**	State derivative $\frac{1}{x} - \frac{2}{x^2}$, or equivalent	B1
		Equate 2-term derivative to zero and attempt to solve for x	M1
		Obtain coordinates of stationary point (2, ln 2 +1), or equivalent	A1+A1
		Determine by any method that it is a minimum point, with no incorrect work seen	A1
			[5]
	(ii)	State or imply the equation $\alpha = \frac{2}{3 - \ln\alpha}$	B1
		Rearrange this as $3 = \ln\alpha + \frac{2}{\alpha}$ (or *vice versa)*	B1
			[2]
	(iii)	Use the iterative formula correctly at least once	M1
		Obtain final answer 0.56	A1
		Show sufficient iterations to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.555, 0.565)	A1
			[3]
9	**(i)**	State or imply a correct normal vector to either plane, e.g. **i** + 2**j** - 2**k** or 2**i** - 3**j** + 6**k**	B1
		Carry out correct process for evaluating the scalar product of both the normal vectors	M1
		Using the correct process for the moduli, divide the scalar product of the two normals by the product of their moduli and evaluate the inverse cosine of the result	M1
		Obtain answer 40.4° (or 40.3°) or 0.705 (or 0.704) radians	A1
		[Allow the obtuse answer 139.6° or 2.44 radians]	
			[4]

(ii) *EITHER*	Carry out a complete strategy for finding a point on l		M1
	Obtain such a point e.g. (0, 3, 2)		A1
	EITHER	Set up two equations for a direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ of l, e.g. $a + 2b - 2c = 0$ and $2a - 3b + 6c = 0$	B1
		Solve for one ratio, e.g. $a:b$	M1
		Obtain $a:b:c$ = 6: -10: -7, or equivalent	A1
		State a correct answer, e.g. $\mathbf{r} = 3\mathbf{j} + 2\mathbf{k} + \lambda(6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k})$	A1√
	OR	Obtain a second point on l, e.g. (6, -7, -5)	A1
		Subtract position vectors to obtain a direction vector for l	M1
		Obtain $6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k}$, or equivalent	A1
		State a correct answer, e.g. $\mathbf{r} = 3\mathbf{j} + 2\mathbf{k} + \lambda(6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k})$	A1√
	OR	Attempt to find the vector product of the two normal vectors	M1
		Obtain two correct components	A1
		Obtain $6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k}$, or equivalent	A1
		State a correct answer, e.g. $\mathbf{r} = 3\mathbf{j} + 2\mathbf{k} + \lambda(6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k})$	A1√
OR	Express one variable in terms of a second		M1
	Obtain a correct simplified expression, e.g. $x = (9 - 3y)/5$		A1
	Express the same variable in terms of the third and form a three term equation		M1
	Incorporate a correct simplified expression, e.g. $x = (12 - 6z)/7$ in this equation		A1
	Form a vector equation for the line		M1
	State a correct answer, e.g. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ -5/3 \\ -7/6 \end{pmatrix}\lambda$, or equivalent		A1√
OR	Express one variable in terms of a second		M1
	Obtain a correct simplified expression, e.g. $y = (9 - 5x)/3$		A1
	Express the third variable in terms of the second		M1
	Obtain a correct simplified expression, e.g. $z = (12 - 7x)/6$		A1
	Form a vector equation for the line		M1
	State a correct answer, e.g. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -5/3 \\ -7/6 \end{pmatrix}$, or equivalent		A1√
			[6]

10 (i) *EITHER*	Make relevant use of the correct sin 2A formula	M1
	Make relevant use of the correct cos 2A formula	M1
	Derive the given result correctly	A1
OR	Make relevant use of the tan 2A formula	M1
	Make relevant use of $1 + \tan^2 A = \sec^2 A$ or $\cos^2 A + \sin^2 A = 1$	M1
	Derive the given result correctly	A1
		[3]

(ii)		State or imply indefinite integral is ln sin x, or equivalent	B1
		Substitute correct limits correctly	M1
		Obtain given exact answer correctly	A1
			[3]
(iii)	*EITHER*	State indefinite integral of cos $2x$ is of the form k ln sin $2x$	M1
		State correct integral ½ ln sin $2x$	A1
		Substitute limits correctly throughout	M1
		Obtain answer ¼ 1n 3, or equivalent	A1
	OR	State or obtain indefinite integral of cosec $2x$ is of the form k ln tan x, or equivalent	M1
		State correct integral ½ ln tan x, or equivalent	A1
		Substitute limits correctly	M1
		Obtain answer ¼ ln 3, or equivalent	A1
			[4]