Voronoi based clustering

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impa

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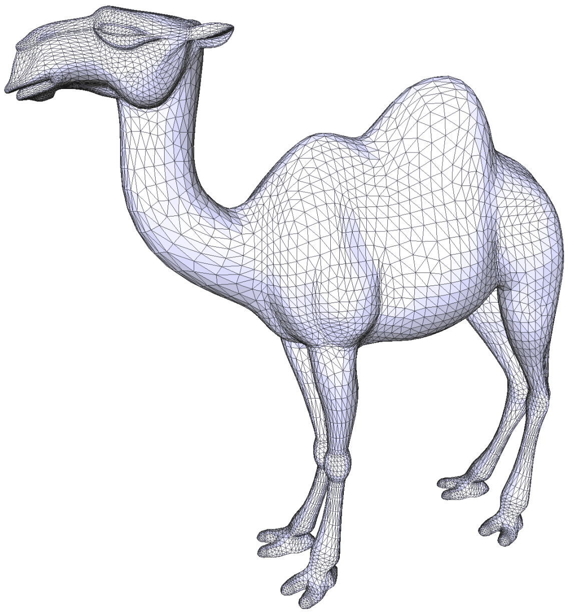
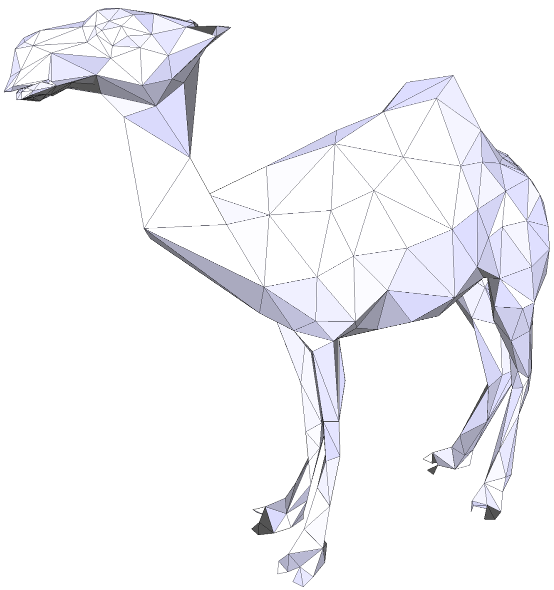
SÃO CARLOS

Summary

- Motivation
- Voronoi clustering
- Topological clustering
- Results
- Directions

Motivation





Given a mesh M, we want to obtain another mesh that approximates M with a lower number of elements (vertices / faces)

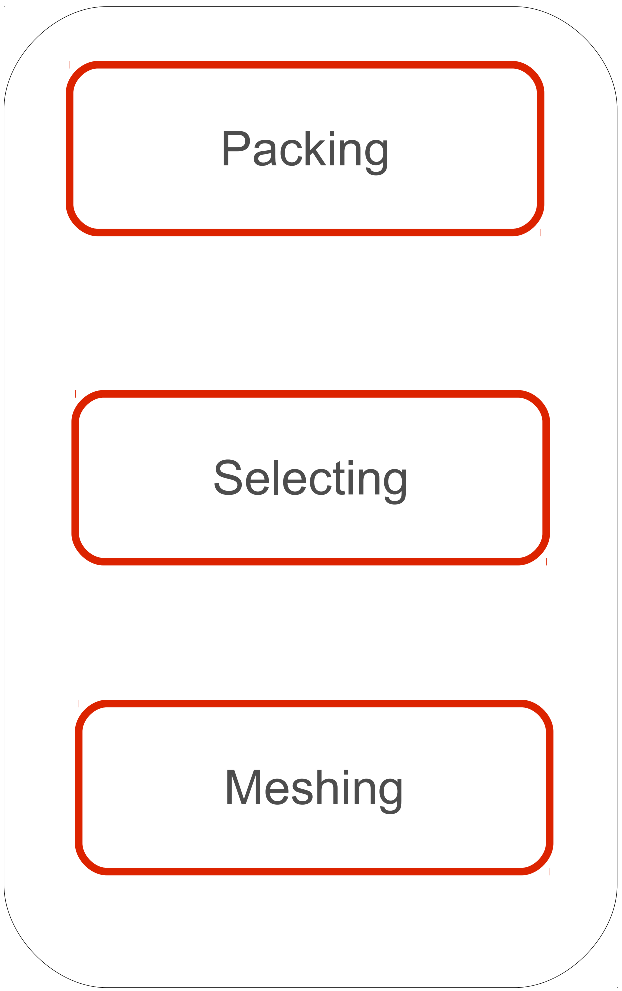
Motivation

Manifold base mesh construction

Given a mesh M, we want to obtain another mesh that approximates M with a lower number of elements (vertices / faces)

Motivation

- Clustering



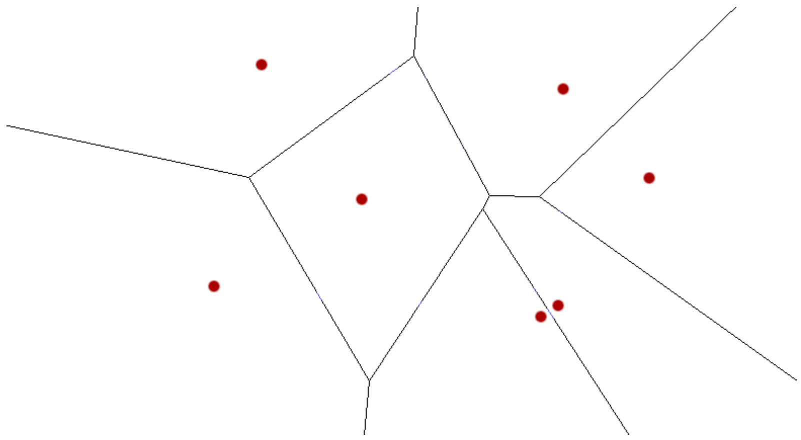
Summary

- Motivation
- Voronoi clustering
    - Overview
    - Lloyd's relaxation
    - Some extensions for unstructured meshes
- Topological clustering
- Results
- Directions

Voronoi Diagram

- Continuous setting:

  Given an open set Ω of $\mathbb{R}^d$, and $(x_i)_{i=1}^n$ points (seeds), a Voronoi tesselation is defined by



$$C_i = \{w \in \Omega : \|w - x_i\| \leq \|w - x_j\|, j = 1, 2, \ldots, n, j \neq i\}$$

Voronoi Diagram

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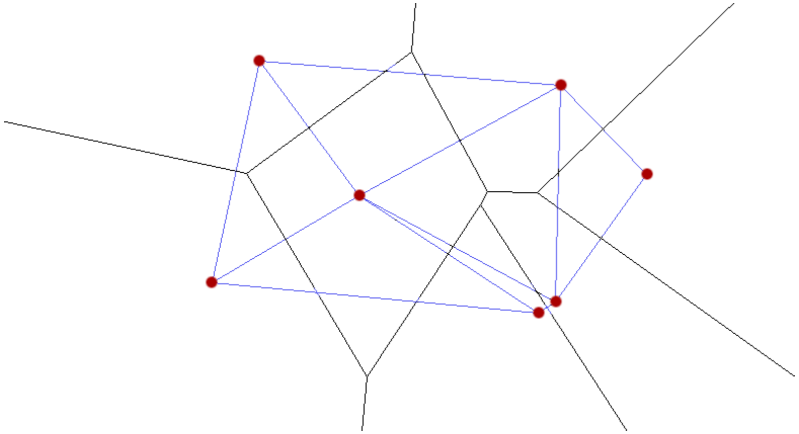
- 2D case: Dual of a triangulation



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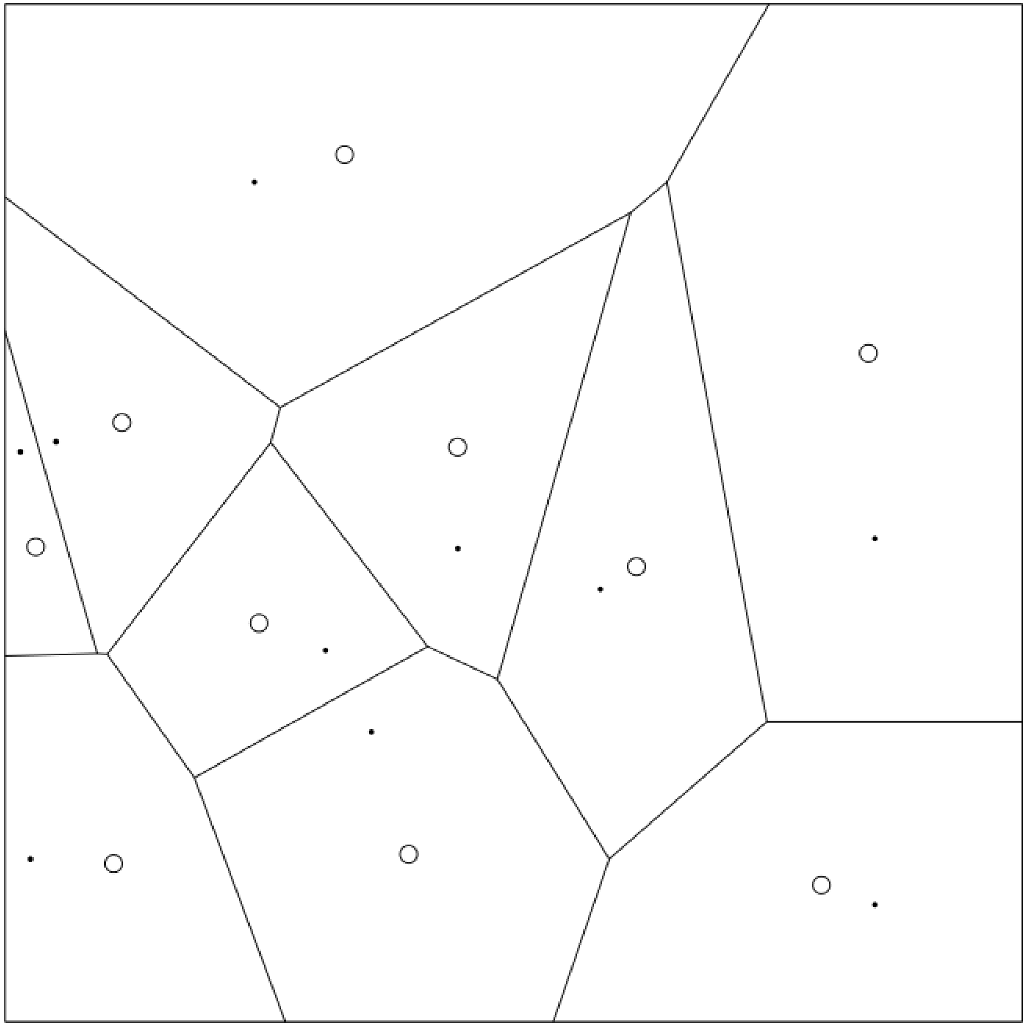
Centroidal Voronoi Diagram (CVD)

- How to avoid skinny triangles?

  An intuition…



Centroidal Voronoi Diagram (CVD)

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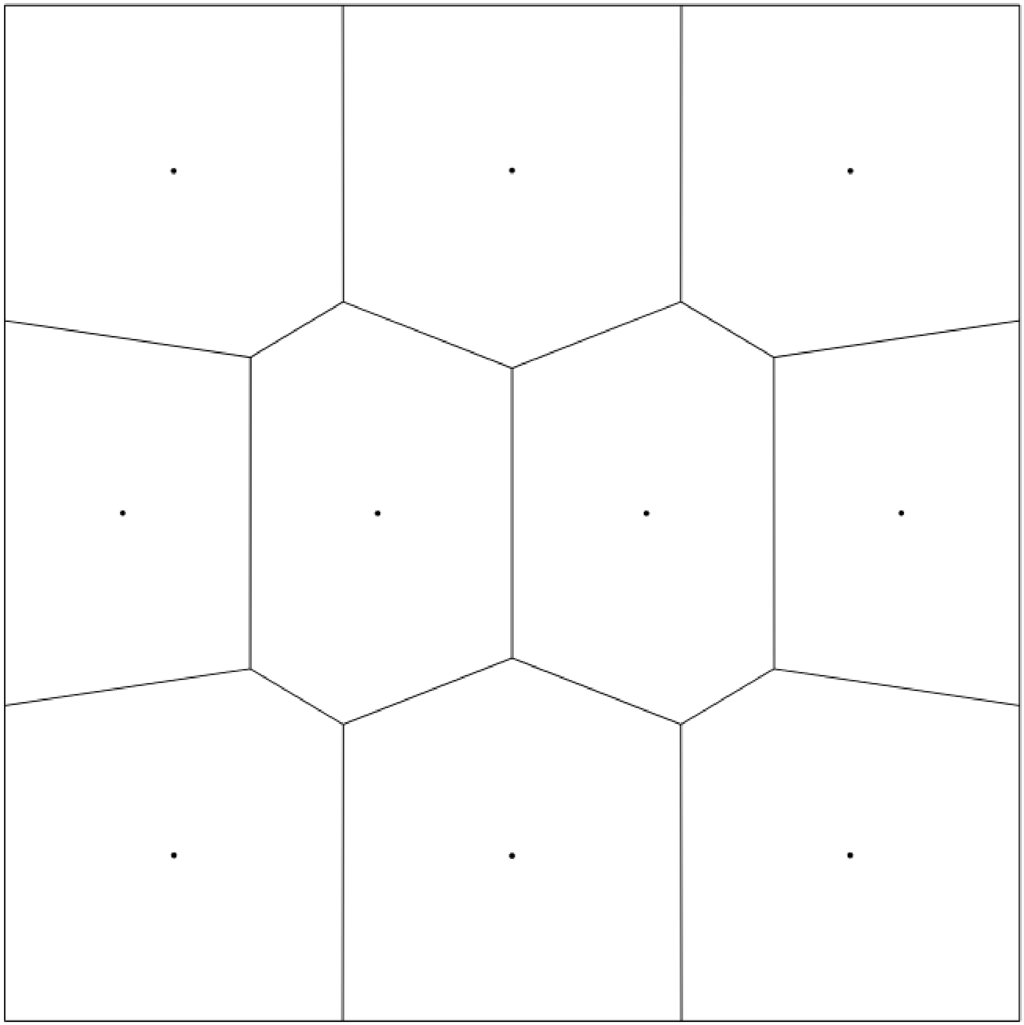

Centroidal Voronoi Diagram (CVD)

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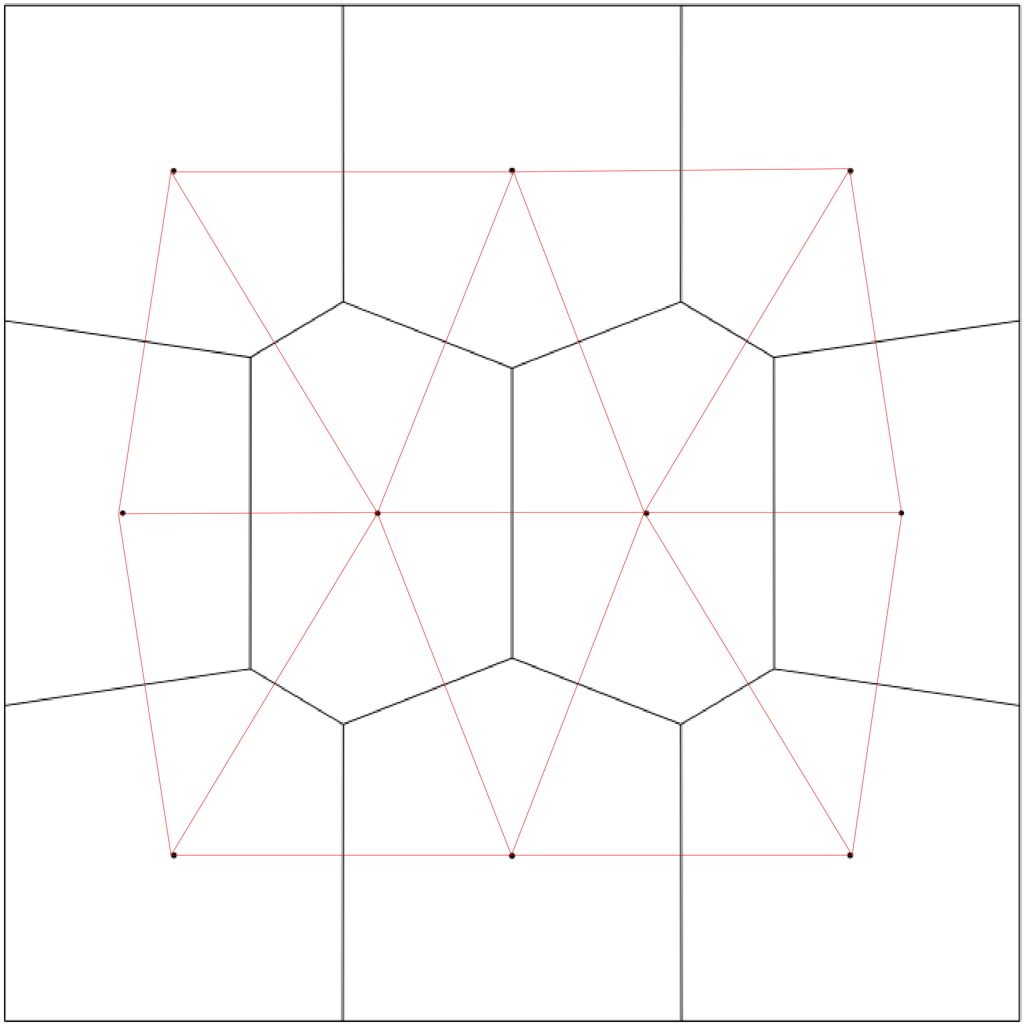


Centroidal Voronoi Diagram (CVD)

- More formally...

Given a region $C \subset \mathbb{R}^n$. The *mass centroid* z^* minimizes the functional

$$\int_C \|z^* - y\| dy$$

and is given by

$$z^* = \frac{\int_C y dy}{\int_C dy}$$

Centroidal Voronoi Diagram (CVD)

- More generally...

Given a density function ρ, the mass centroid is given by

$$z^* = \frac{\int_C y\rho(y)dy}{\int_C \rho(y)dy}$$

and minimizes

$$\int_C \rho(y)\|z^* - y\|dy$$

Centroidal Voronoi Diagram (CVD)

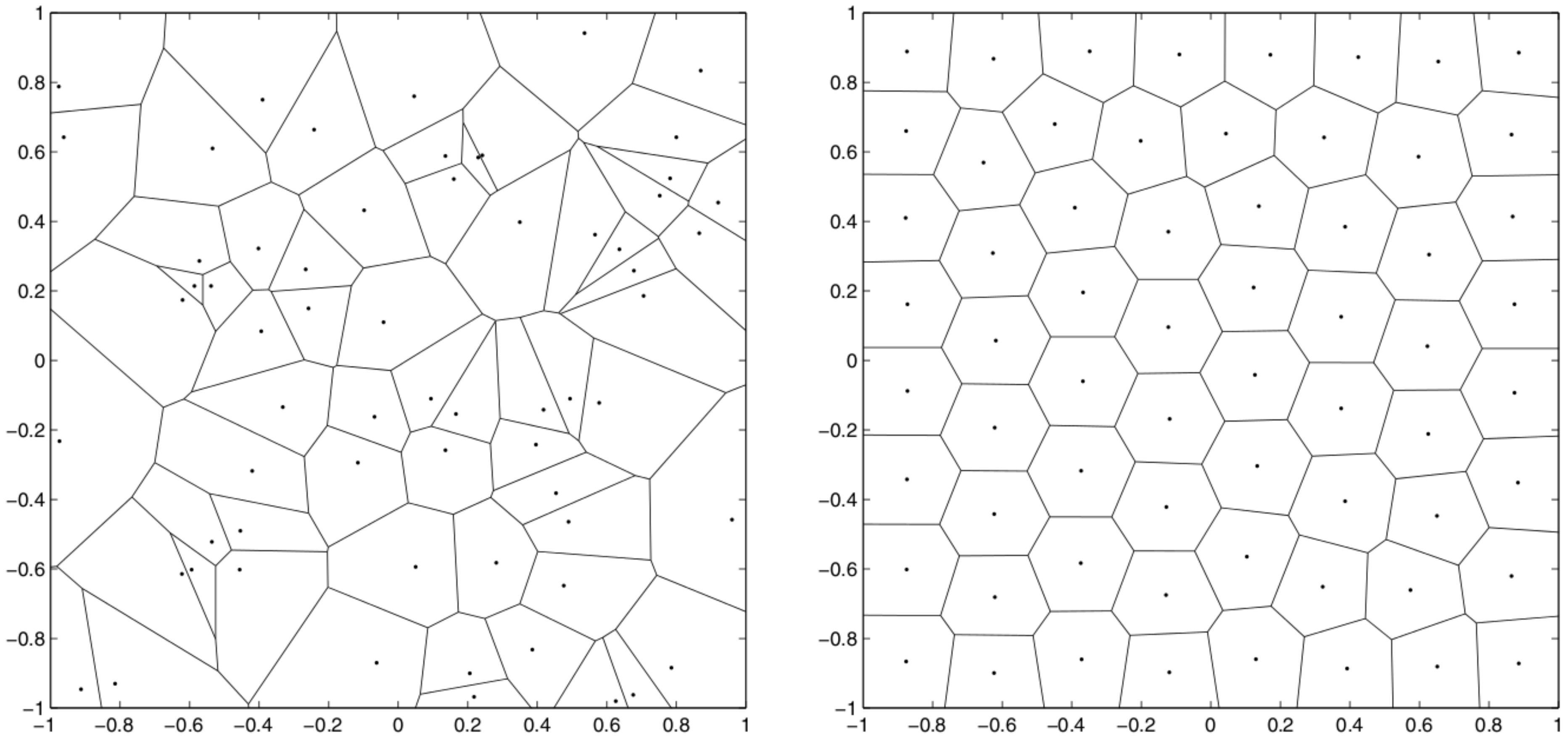
- Definition: A CVD of Ω tesselates it in disjoint regions

$$C_i = \{w \in \Omega : \|w - z_i\| \leq \|w - z_j\|, j = 1, 2, \ldots, n, j \neq i\}$$

with
$$z_i = \frac{\int_{C_i} y\rho(y)dy}{\int_{C_i} \rho(y)dy}$$

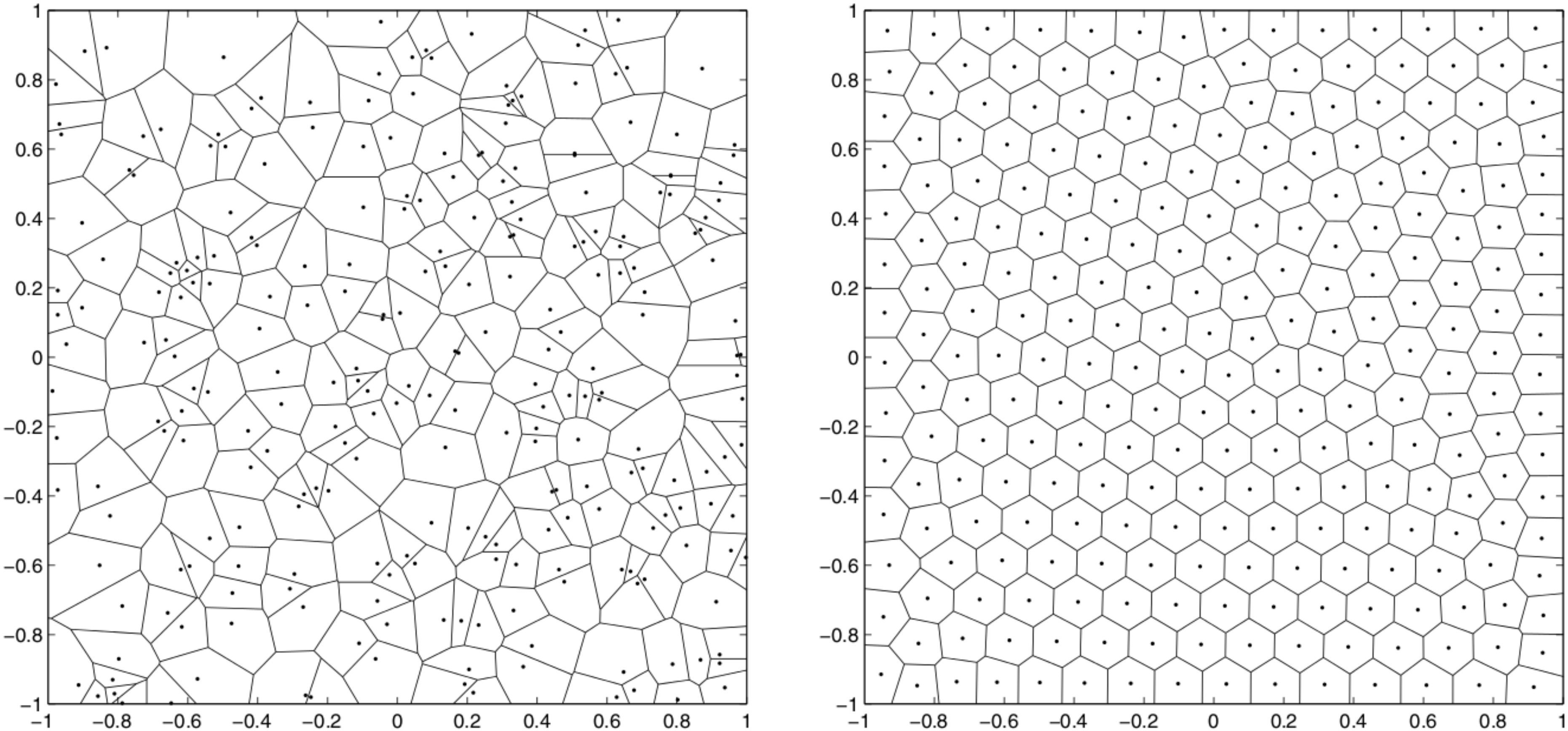
minimizing
$$E = \sum_{i}^{n} \int_{C_i} \rho(y)\|z_i - y\|dy$$

Centroidal Voronoi Diagram (CVD)



$$\rho \equiv 1$$

Centroidal Voronoi Diagram (CVD)



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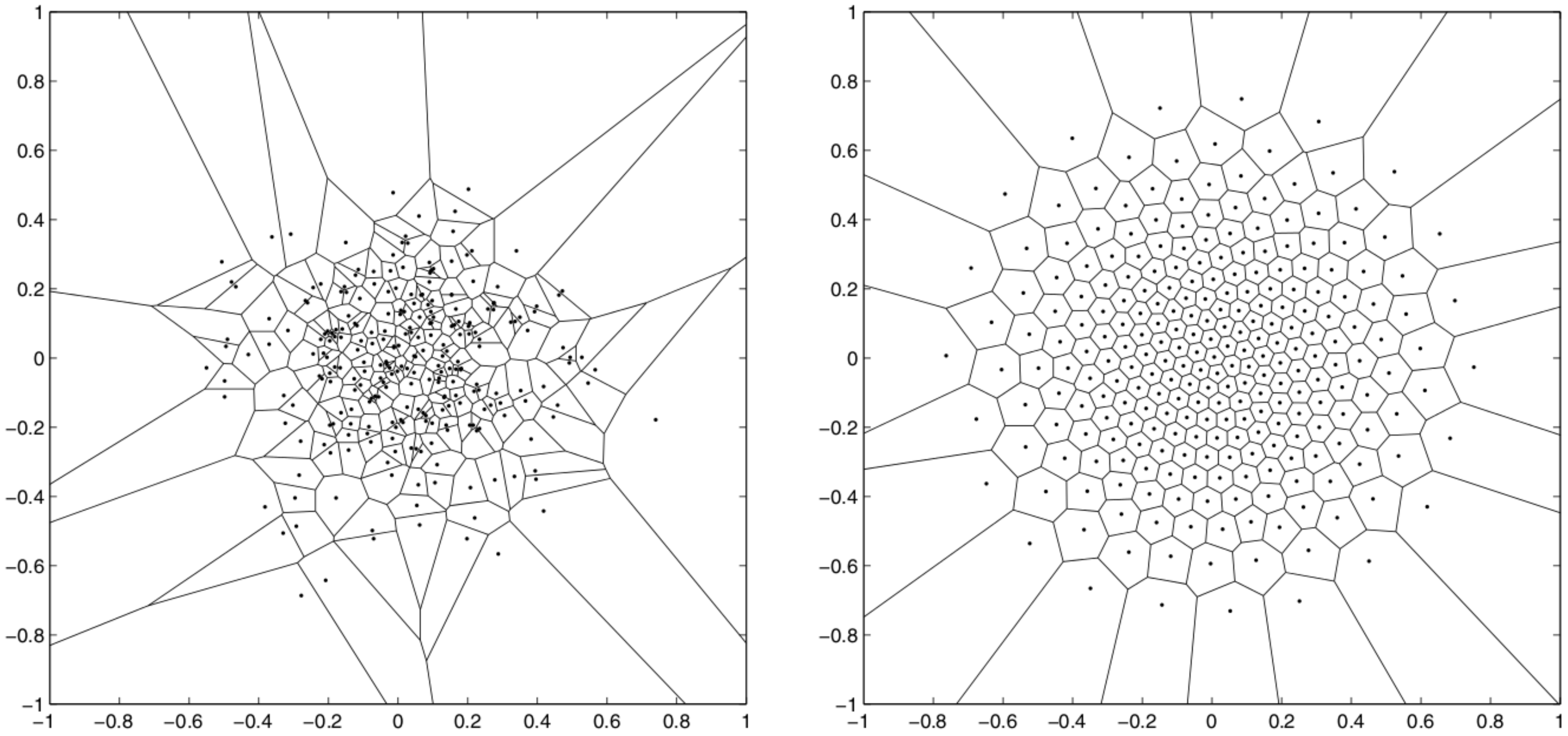


$$\rho(x, y) = \exp^{-10(x^2+y^2)}$$

Centroidal Voronoi Diagram (CVD)

- Lloyd's algorithm:
  1. Select an initial set of k points (seeds);
  2. Construct a Voronoi tesselation associated with these seeds;
  3. Compute the mass centroids of each region;
  4. If this new set of points meets some convergence criterion, terminate; otherwise, return to step 1.

Centroidal Voronoi Diagram (CVD)

- Wide range of applications
  - Clustering
  - Optimal quadrature rules
  - Image processing

  (...)

Centroidal Voronoi Diagram (CVD)

- And for meshes?
  - Constrained Centroidal Voronoi Diagram (CCVD)

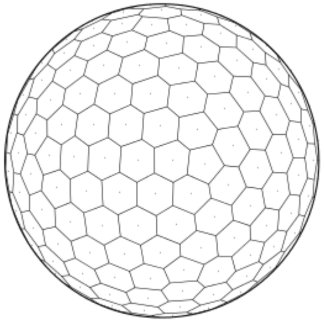
    [Du 2003]

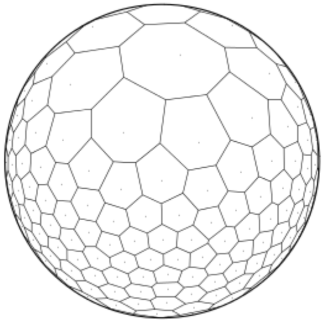
    - Centroids problem

  - Geodesic metric on the mesh

    [Cedrim 2011]

    - High cost for a relaxation process





Centroidal Voronoi Diagram (CVD)

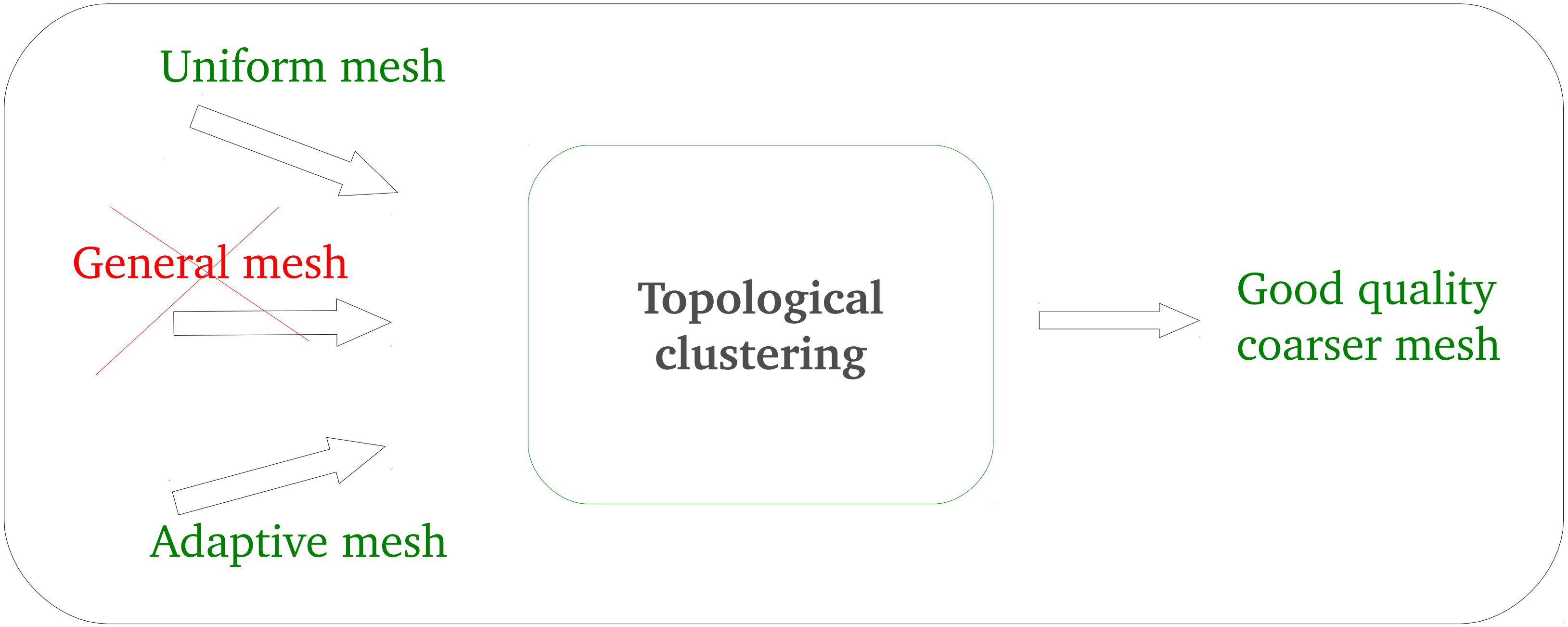
- And for meshes?
  - Valette [2008]
    - Initial clustering by average densities over n clusters
    - Cleaning process for disconnected cells
    - Adjusts cells according to tests on its boundaries
    - Centroids defined in order to minimize QEM
    - Polynomial fitting on local neighbourhood for densities

Summary

- Motivation
- Voronoi clustering
- Topological clustering
    - Cell definition
    - Centroids
    - Relaxation
    - Quality control
- Results
- Directions

Topological clustering

- Our approach aims to
  - Generate a base (coarse) mesh
  - Maintain properties of previously processed meshes

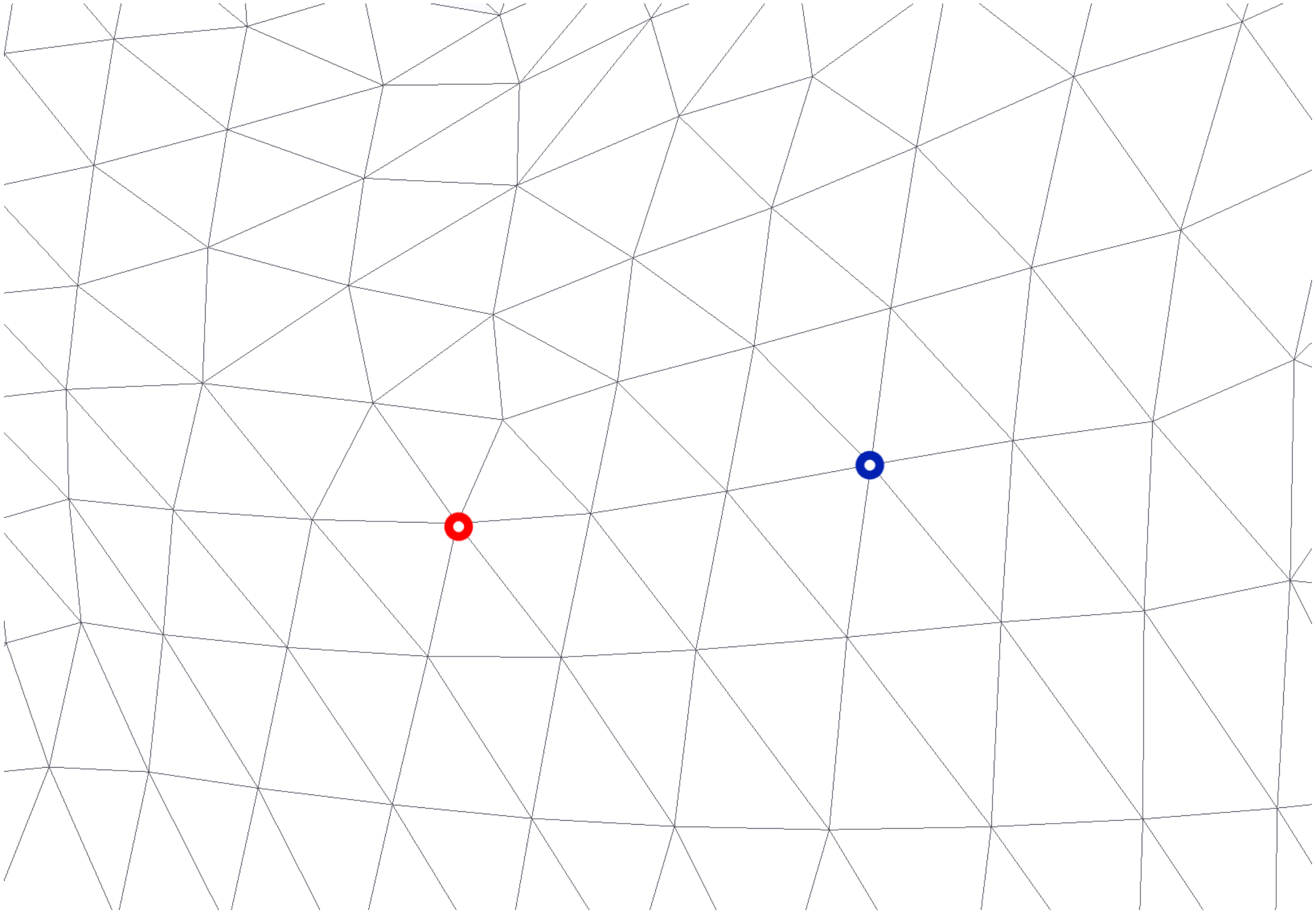


Topological clustering

- Main idea
  1. Define a cell decomposition
  2. Calculate each cell centroid
  3. Nested convergence
     1. Relaxation process
     2. Quality control

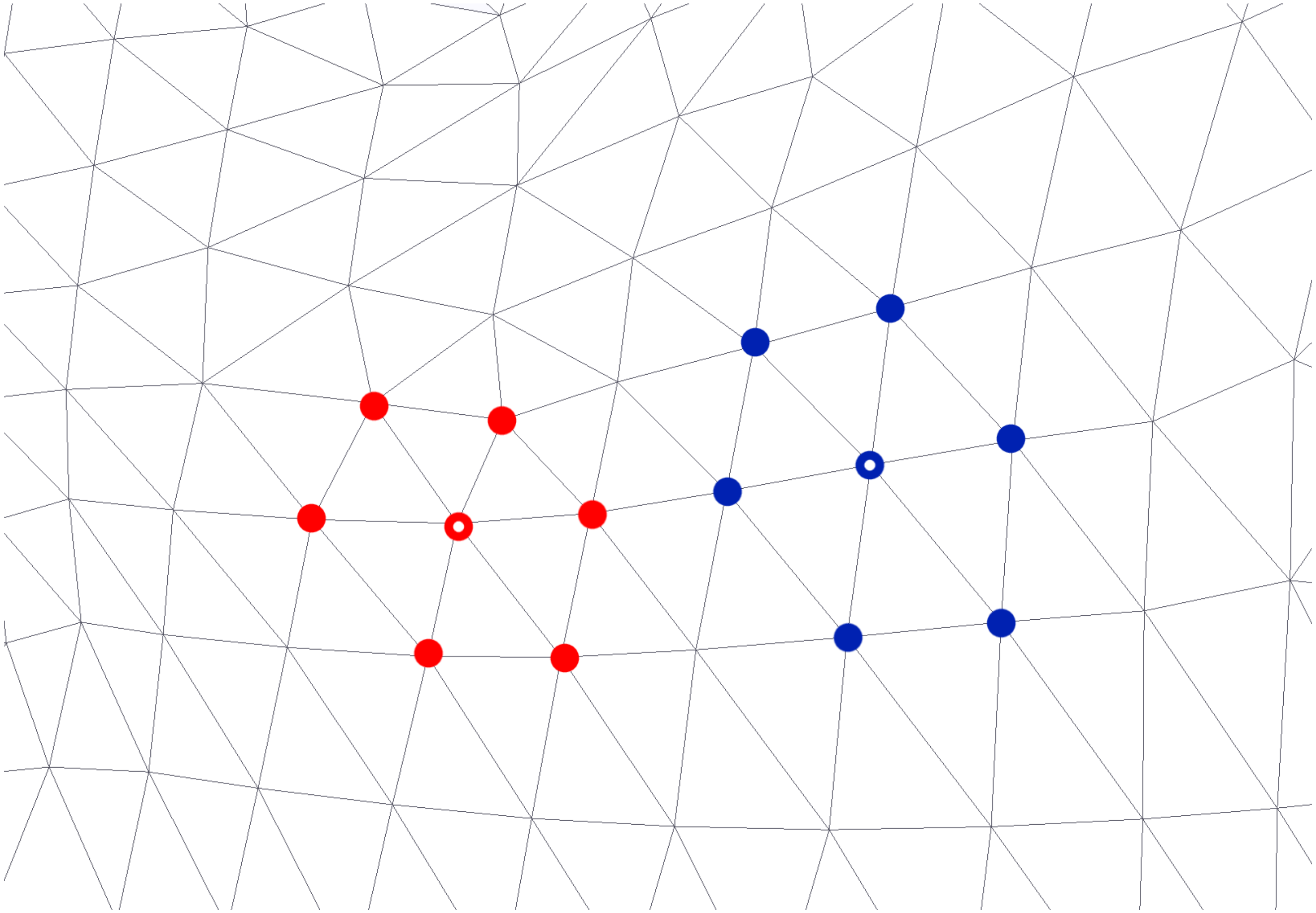
Topological clustering

- Cell definition



Topological clustering

- Cell definition



Topological clustering

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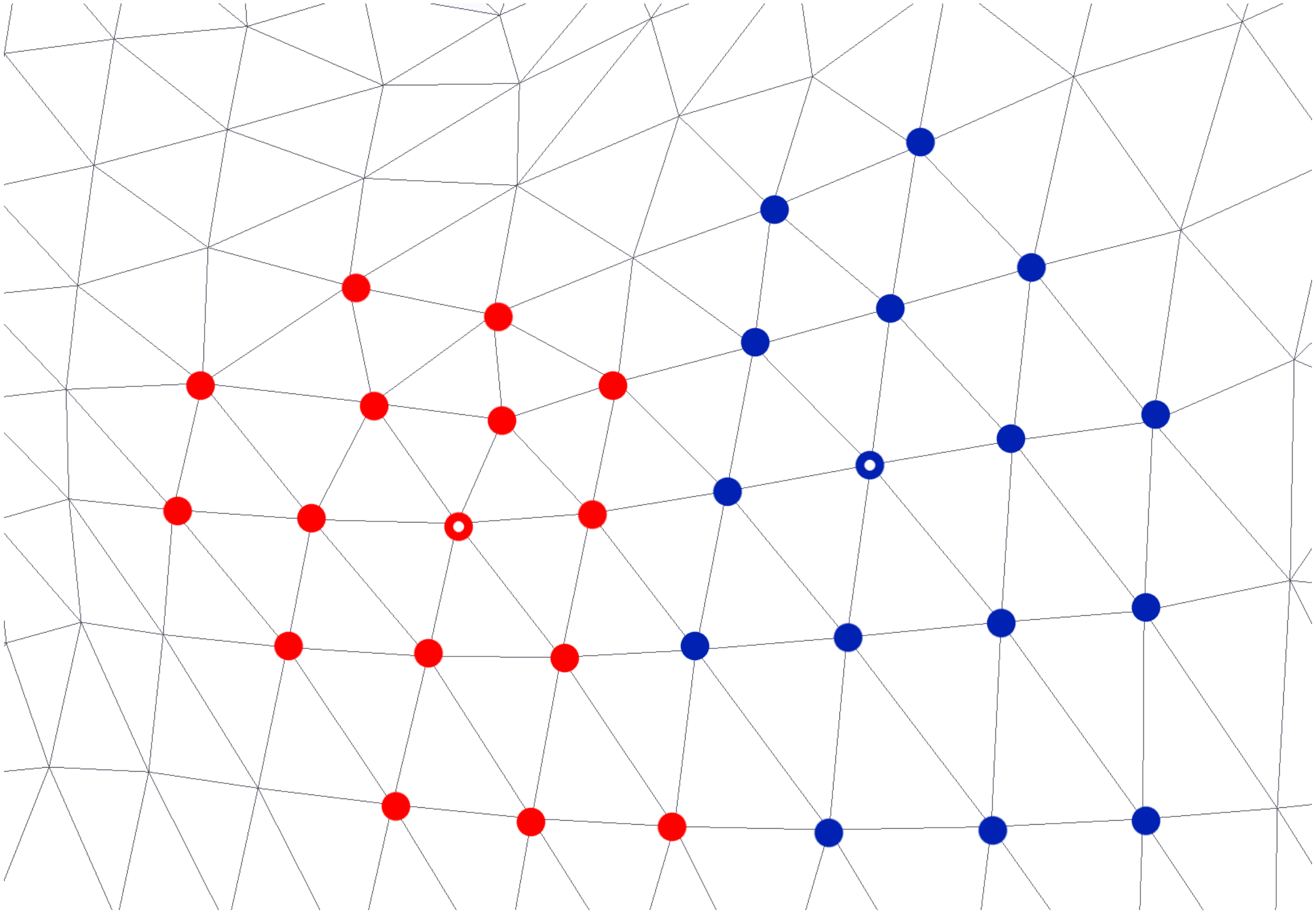

Topological clustering

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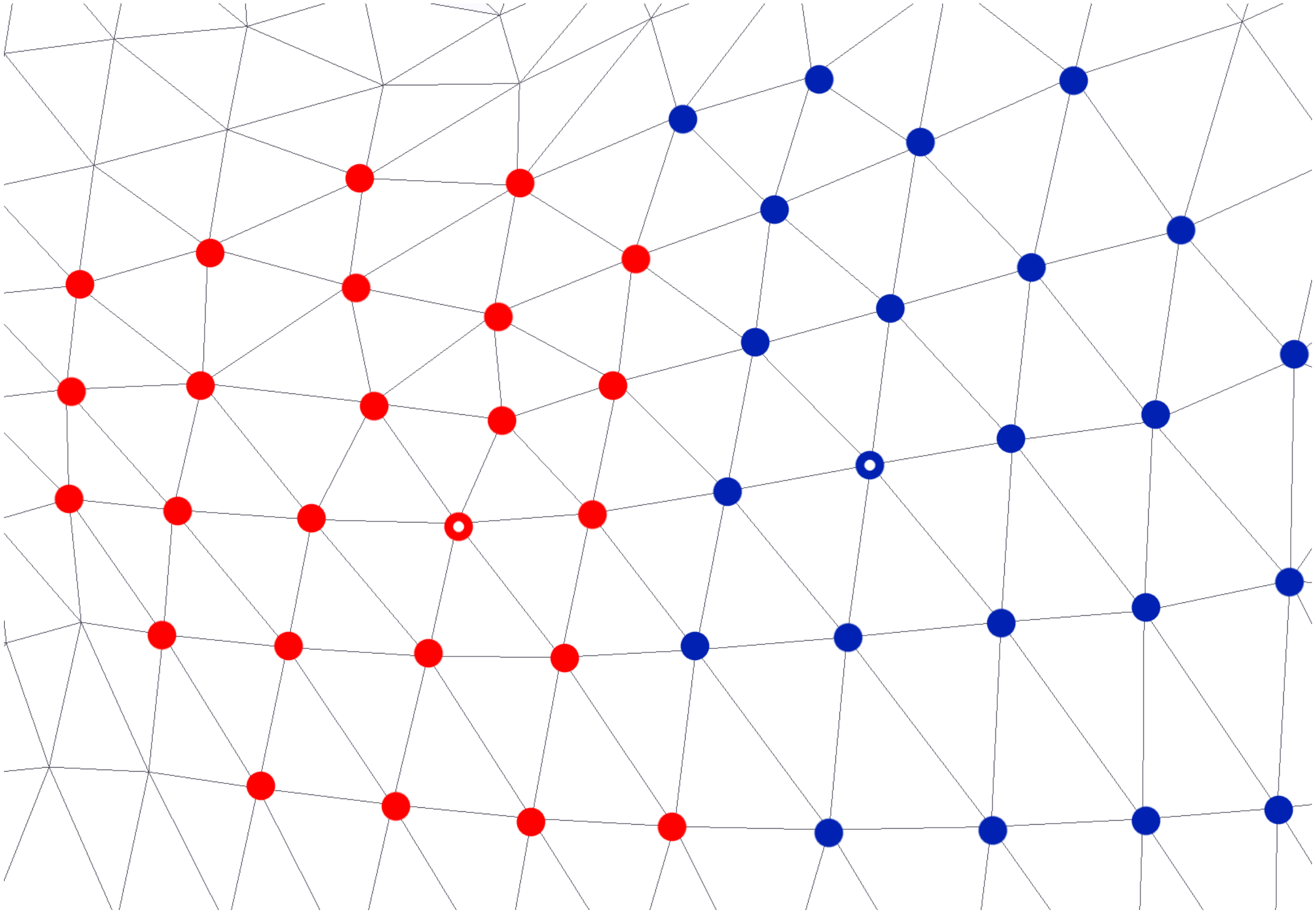

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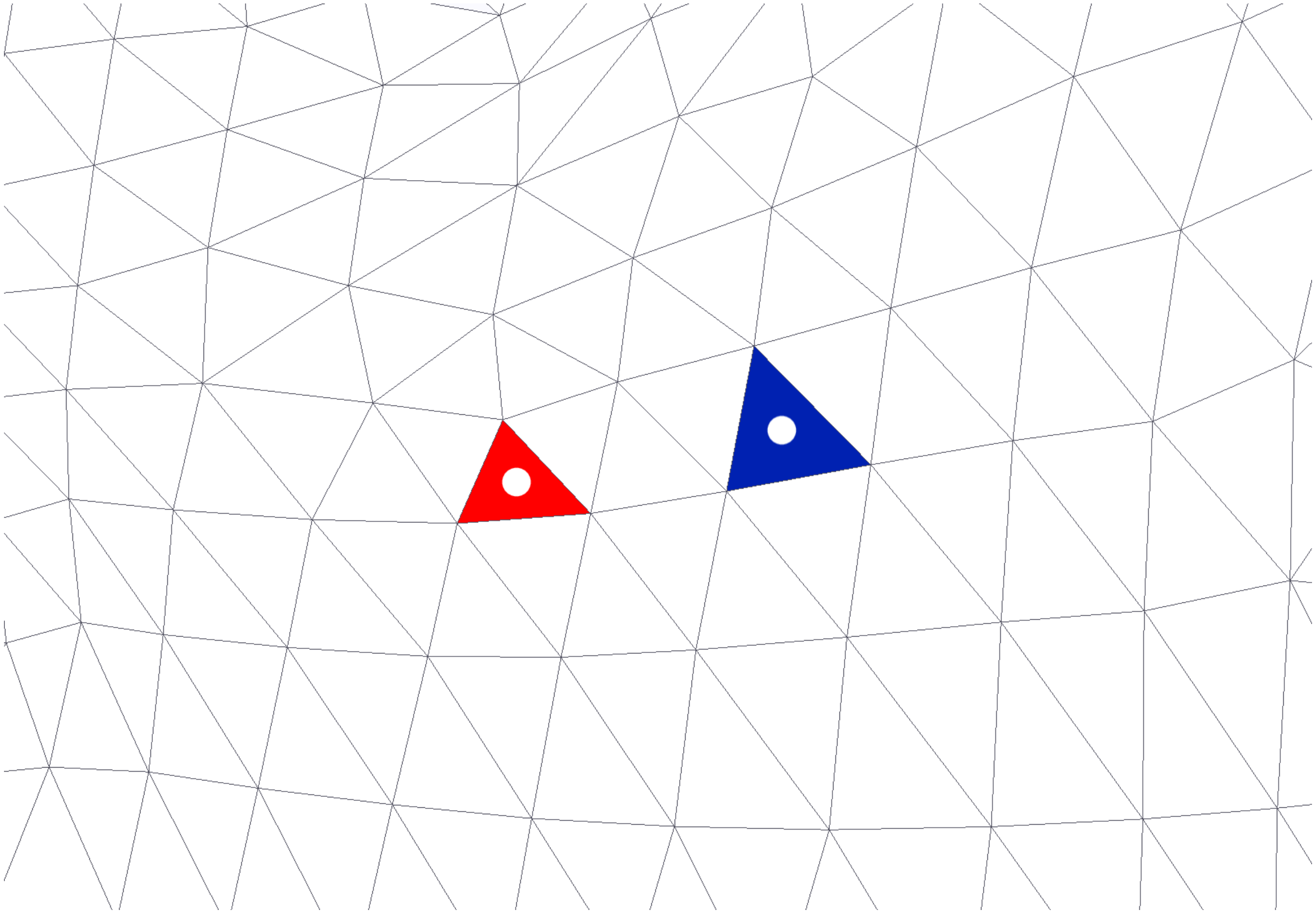

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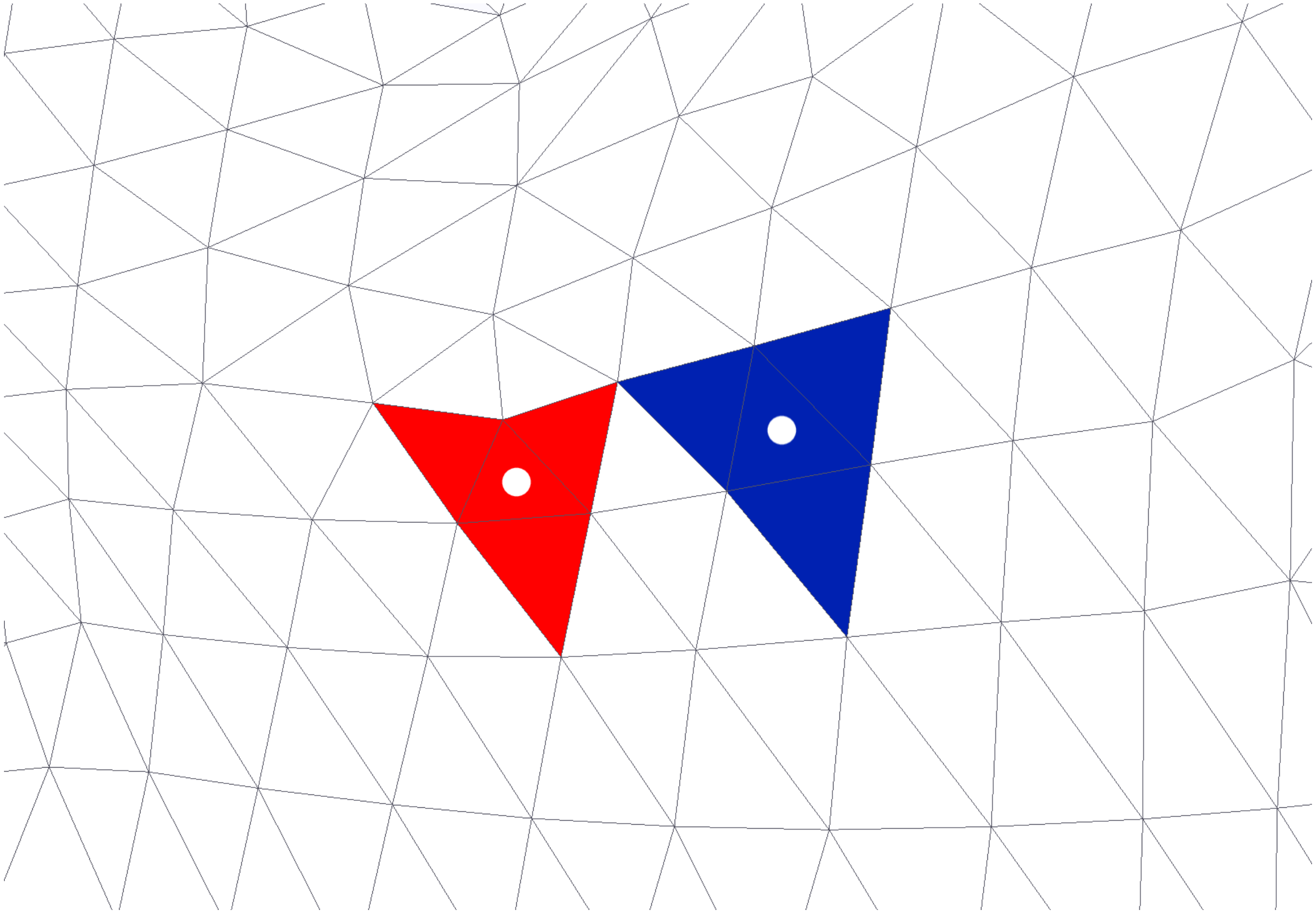

Topological clustering

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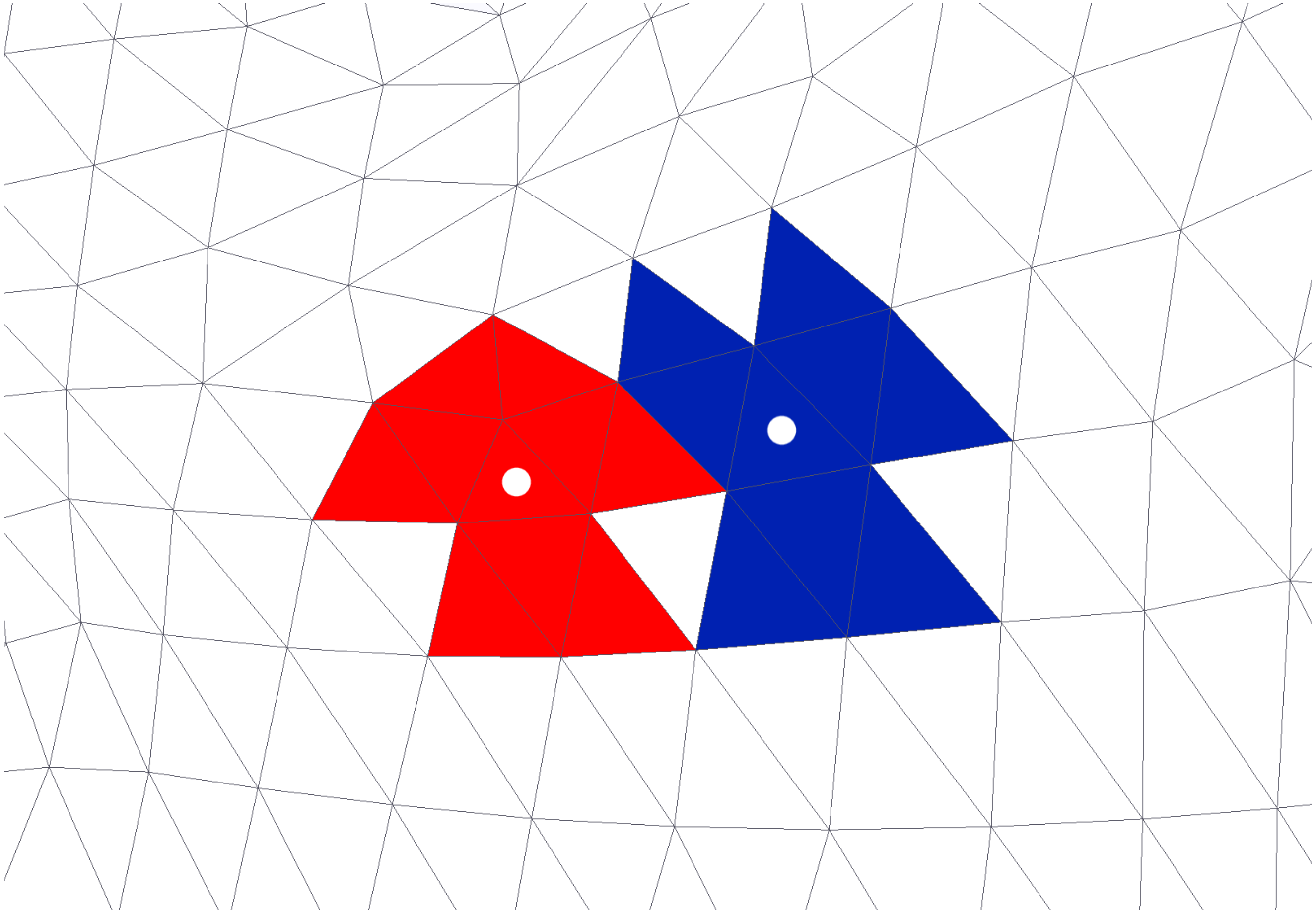

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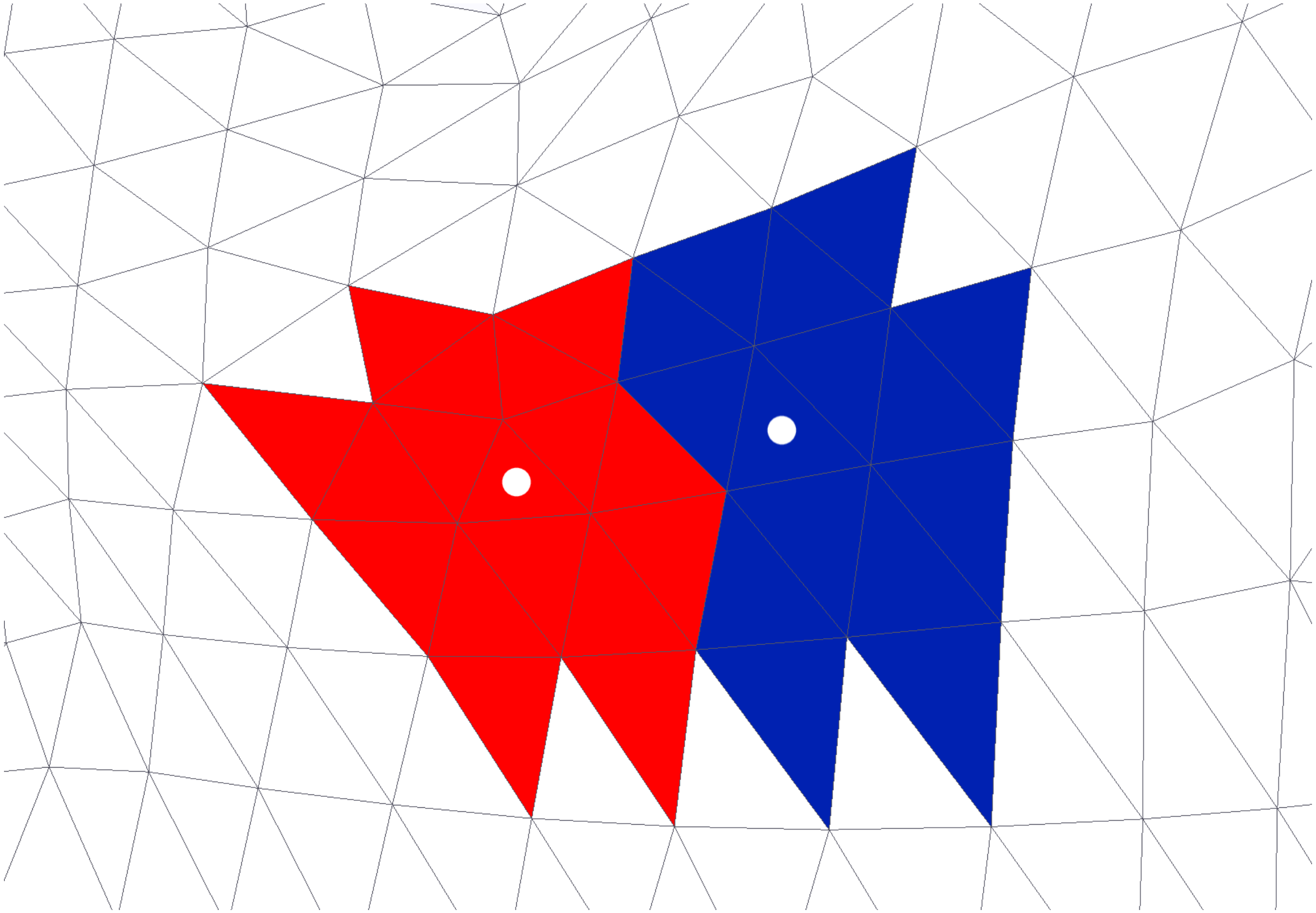

Topological clustering

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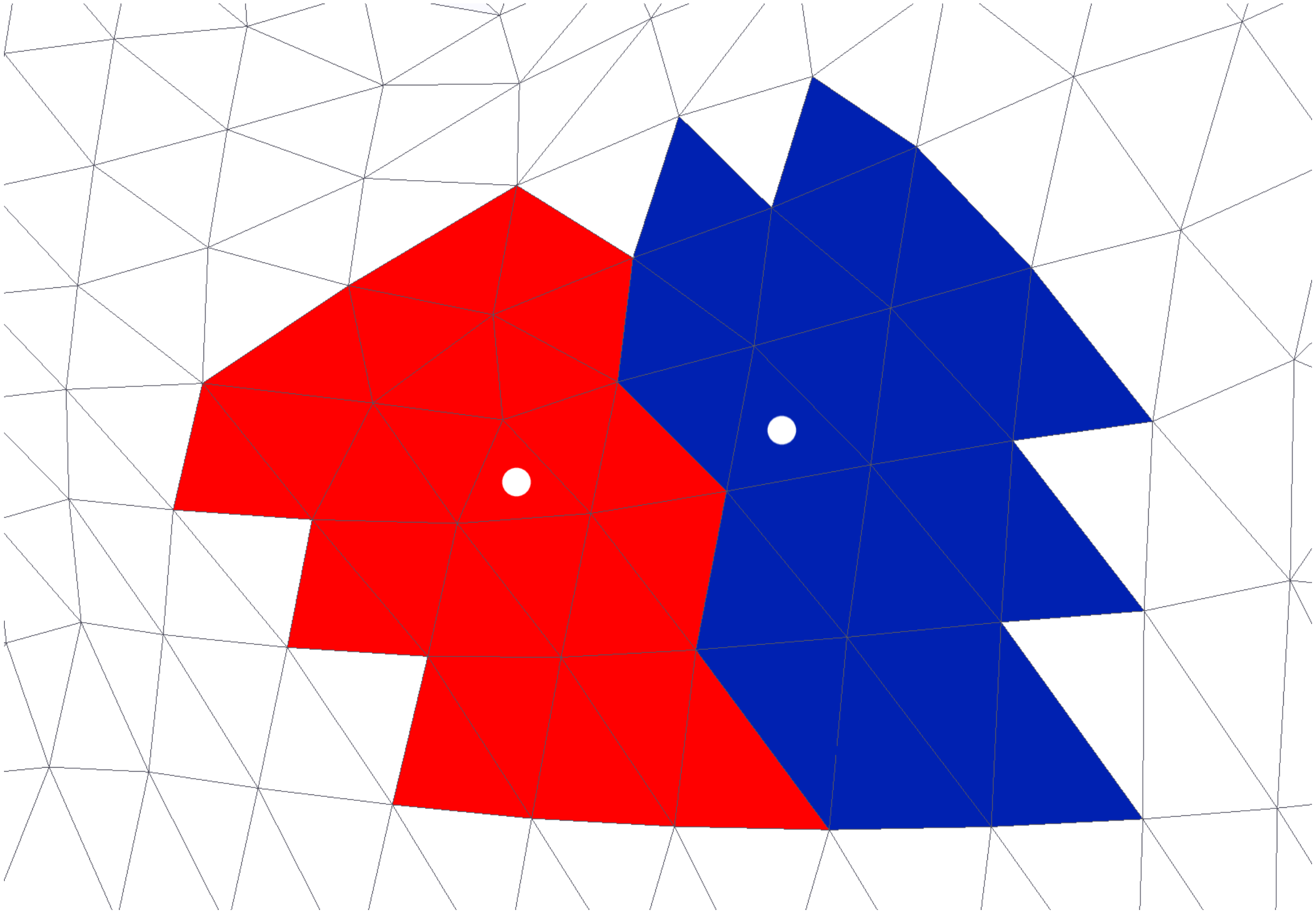

Topological clustering

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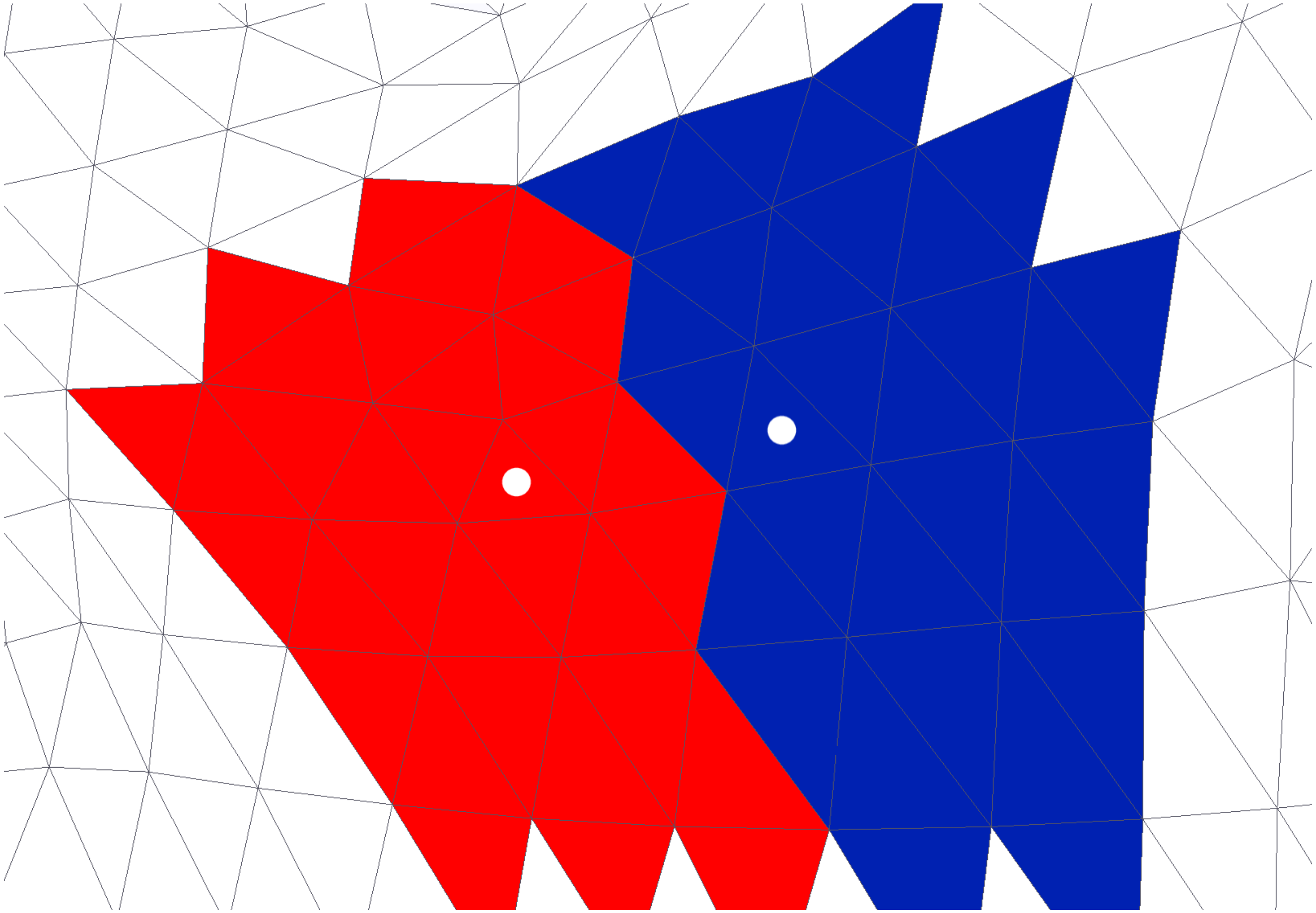

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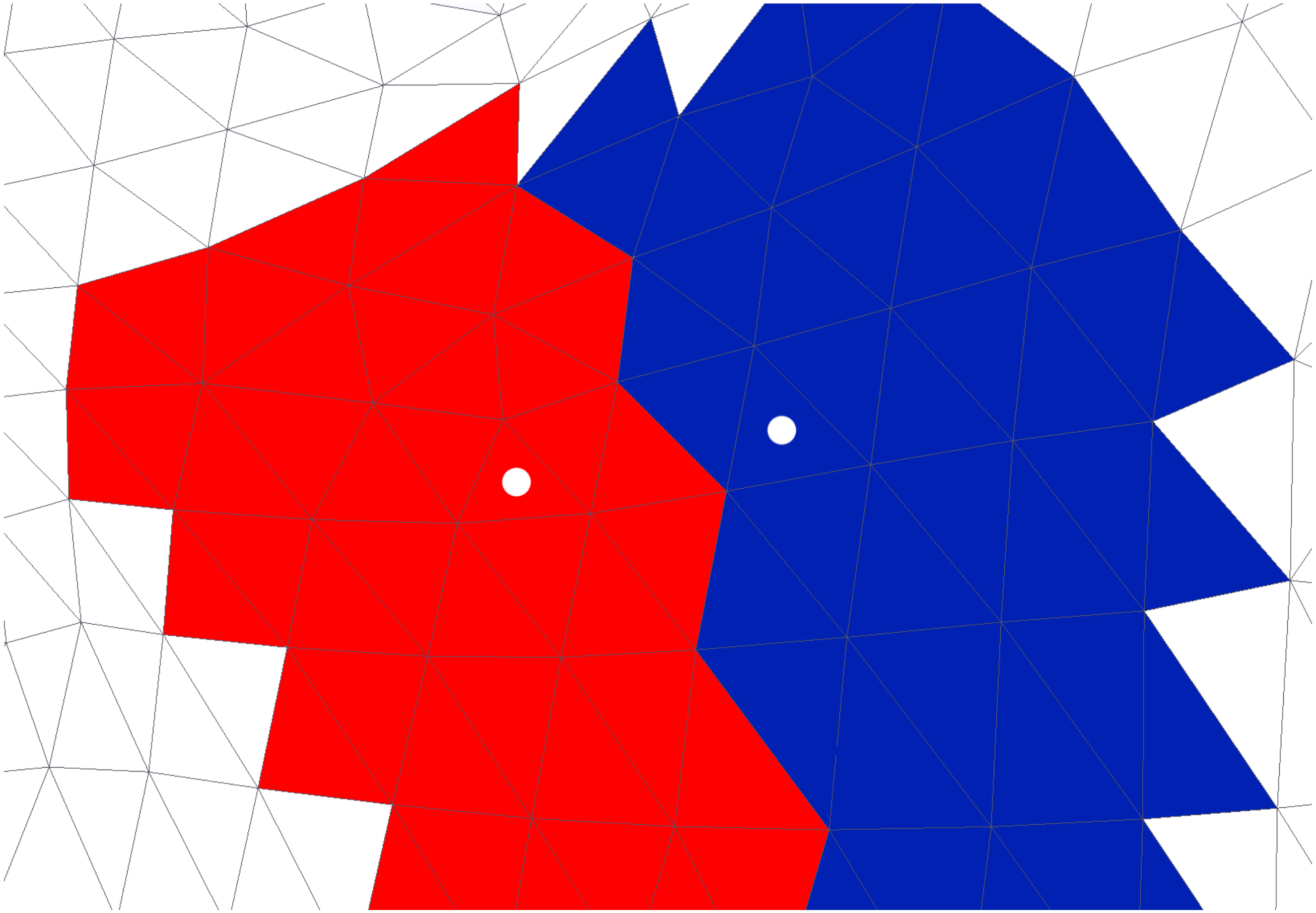
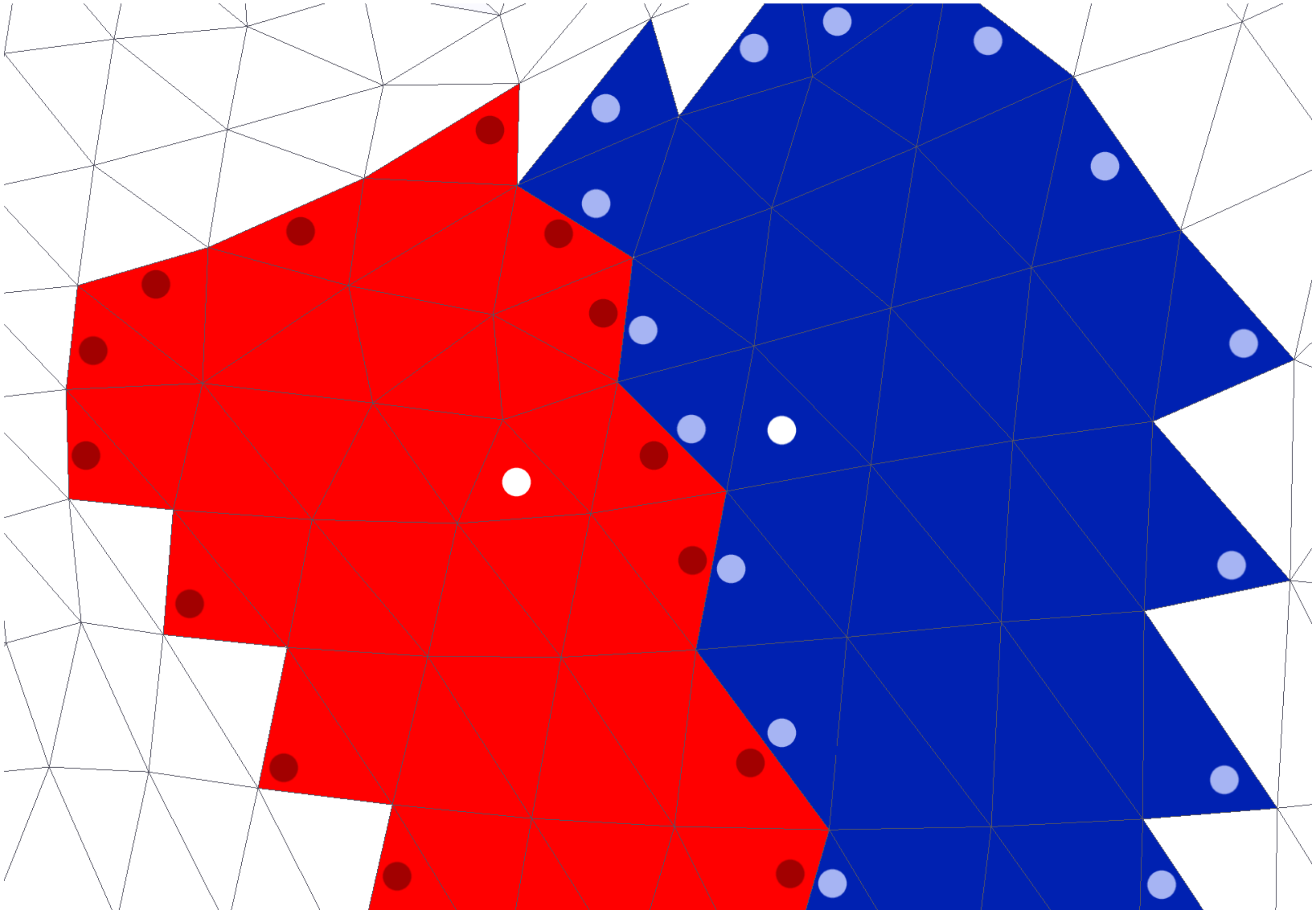


Topological clustering

- Centroids



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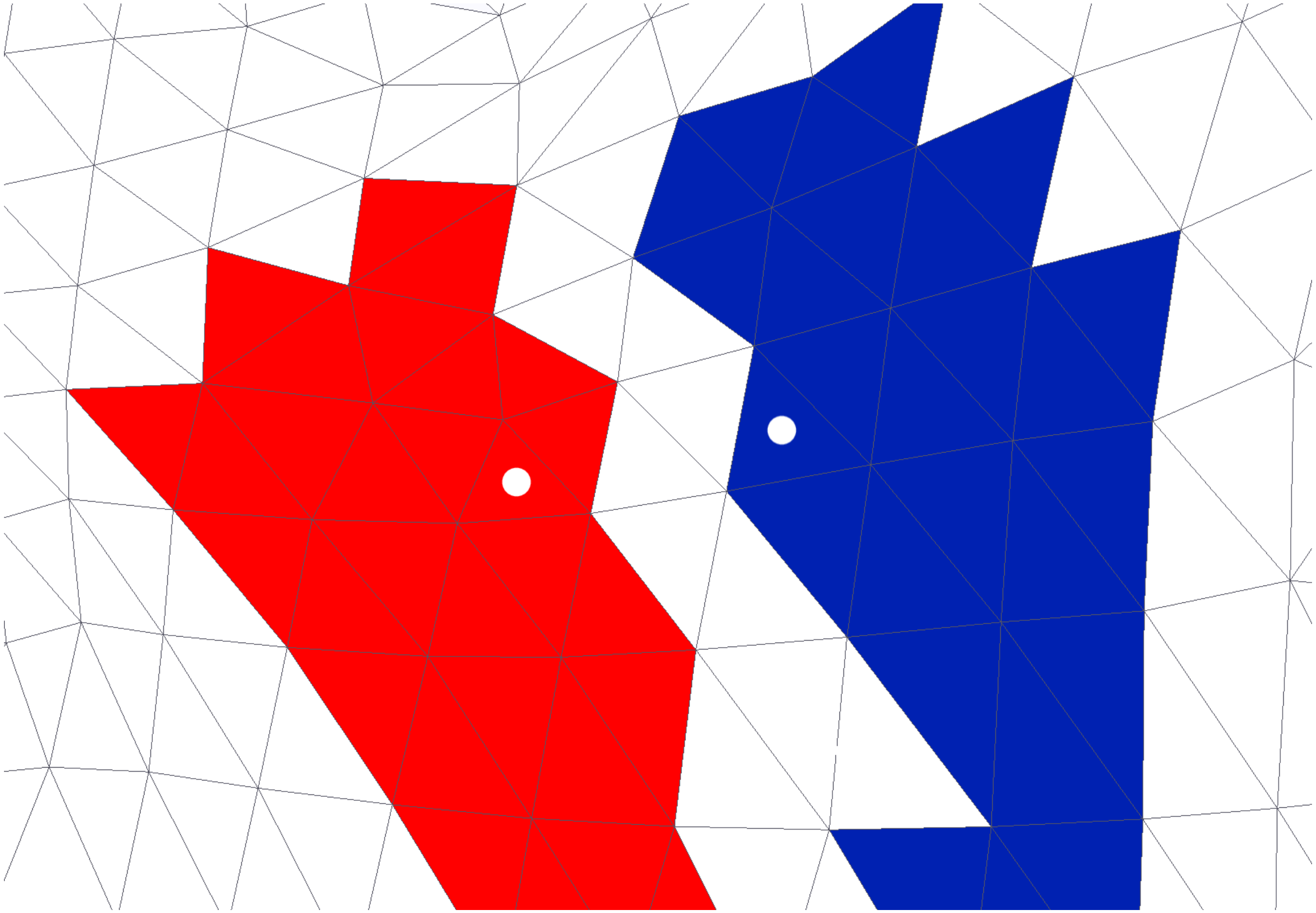

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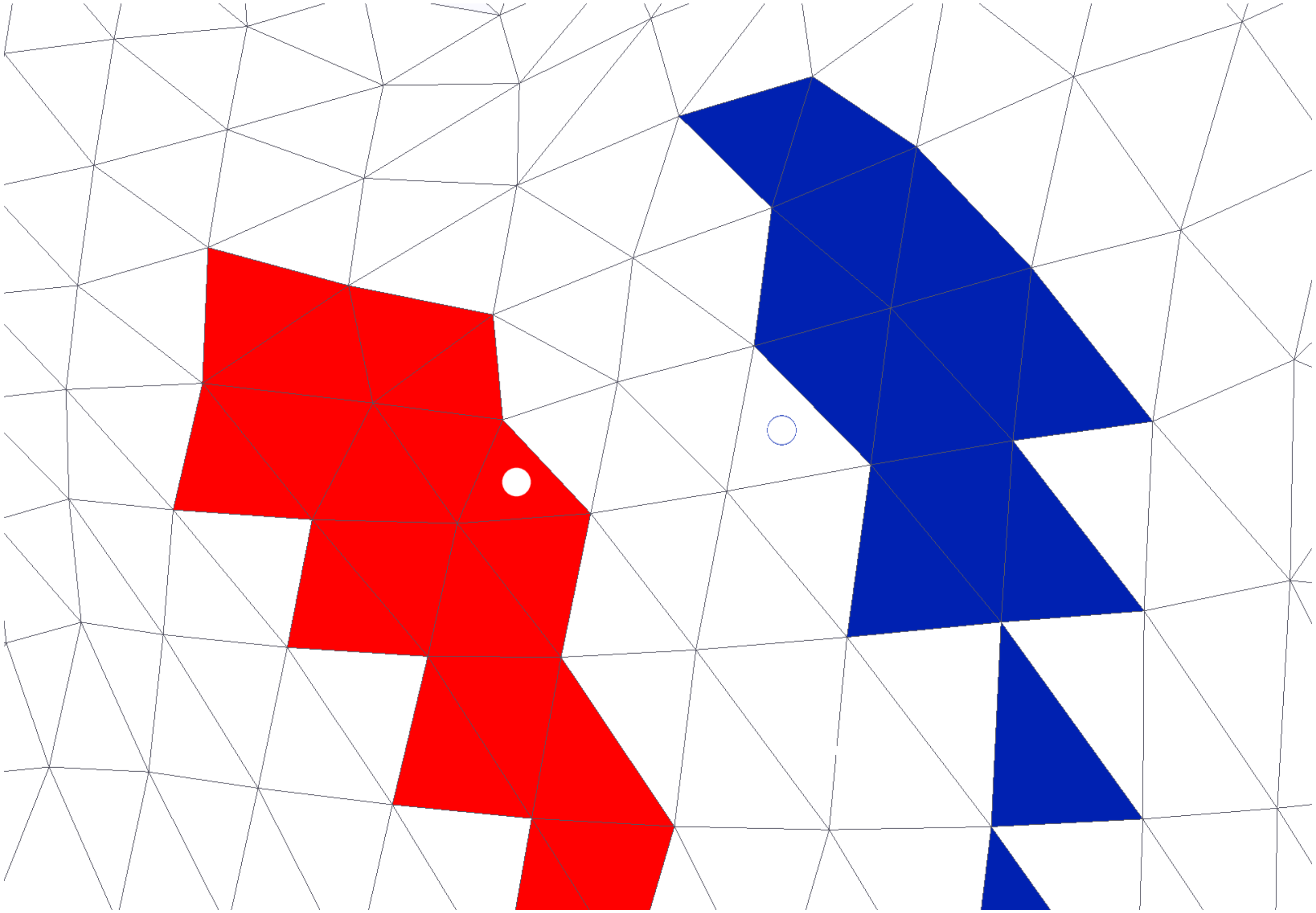

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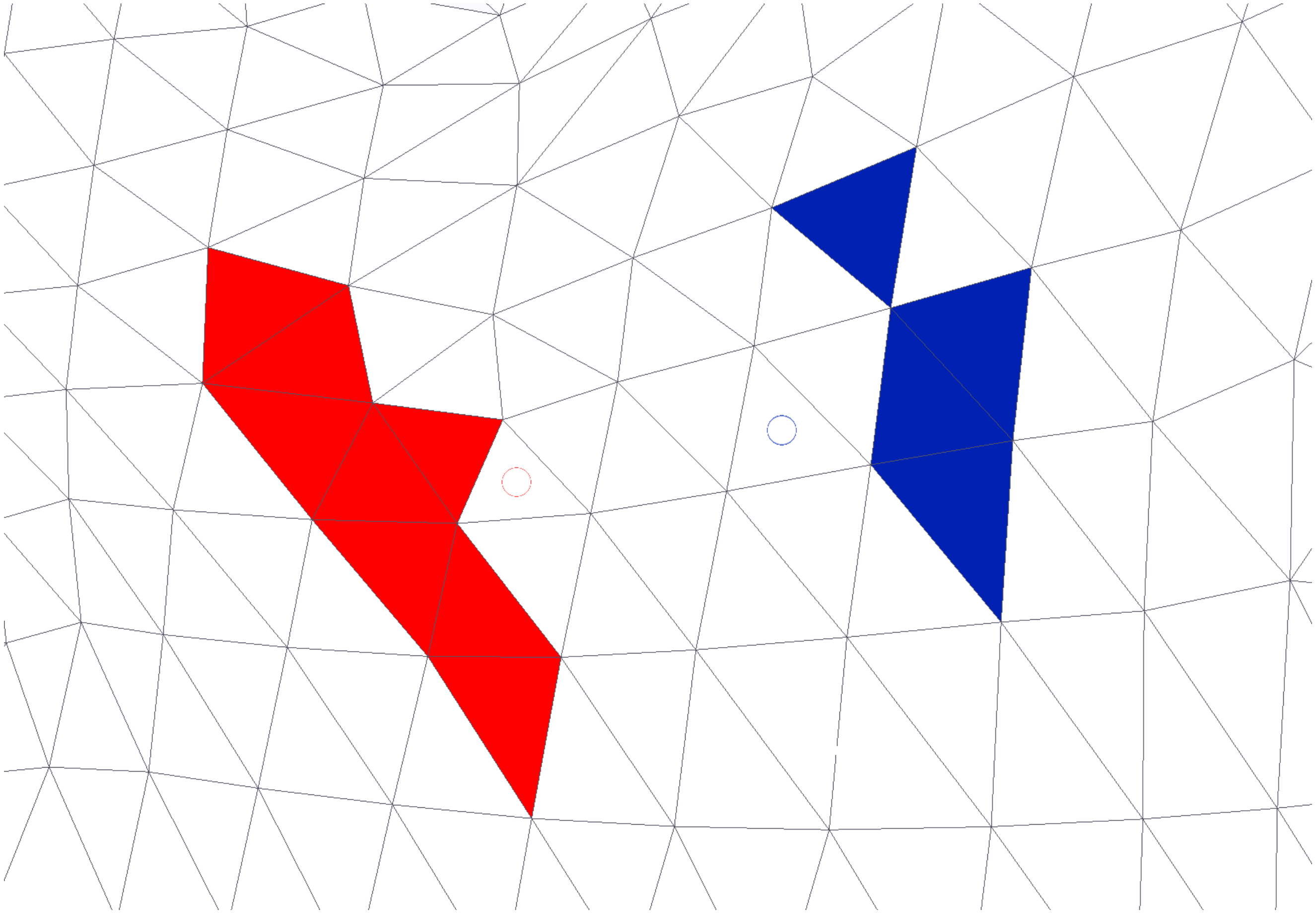

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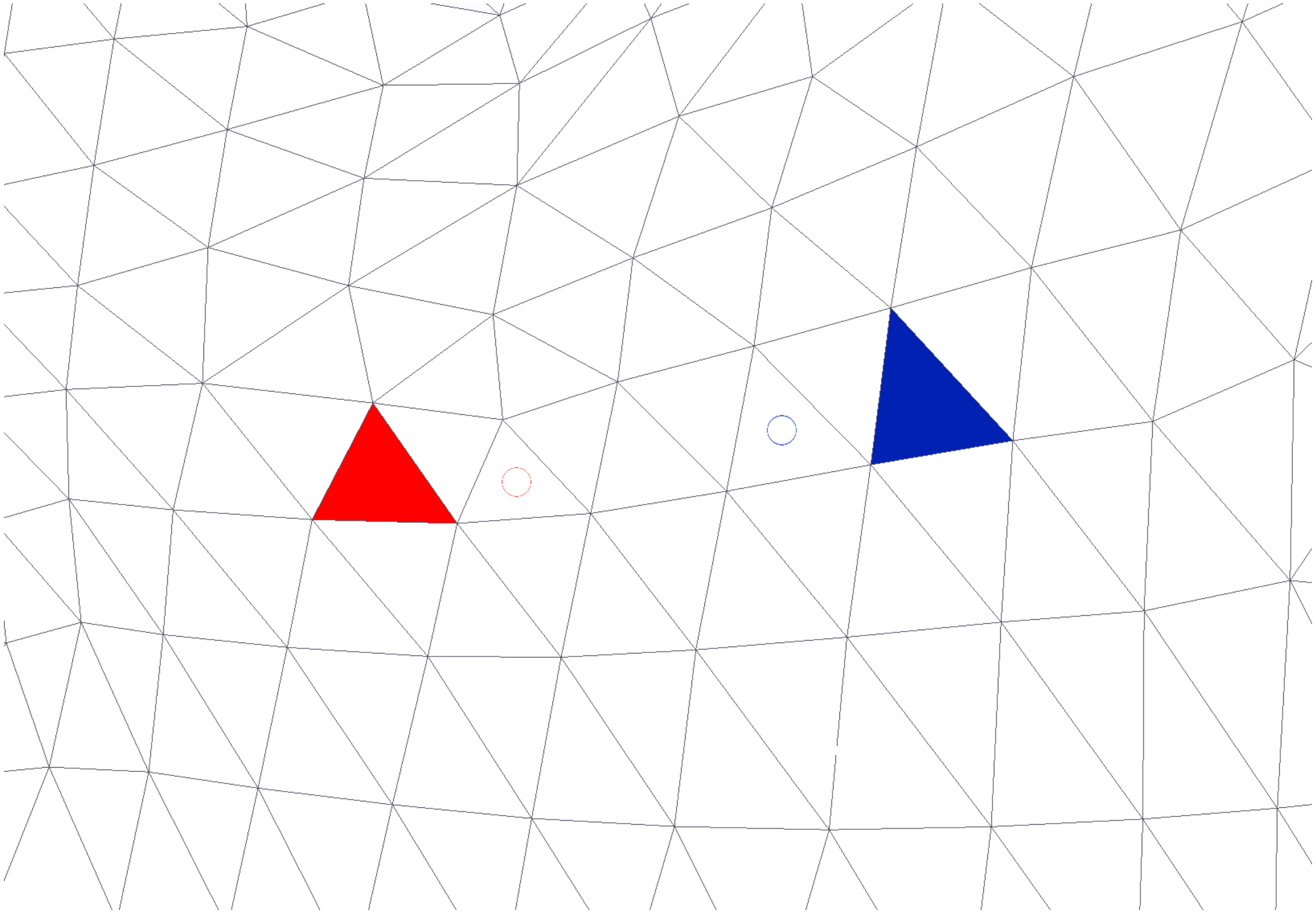

Topological clustering

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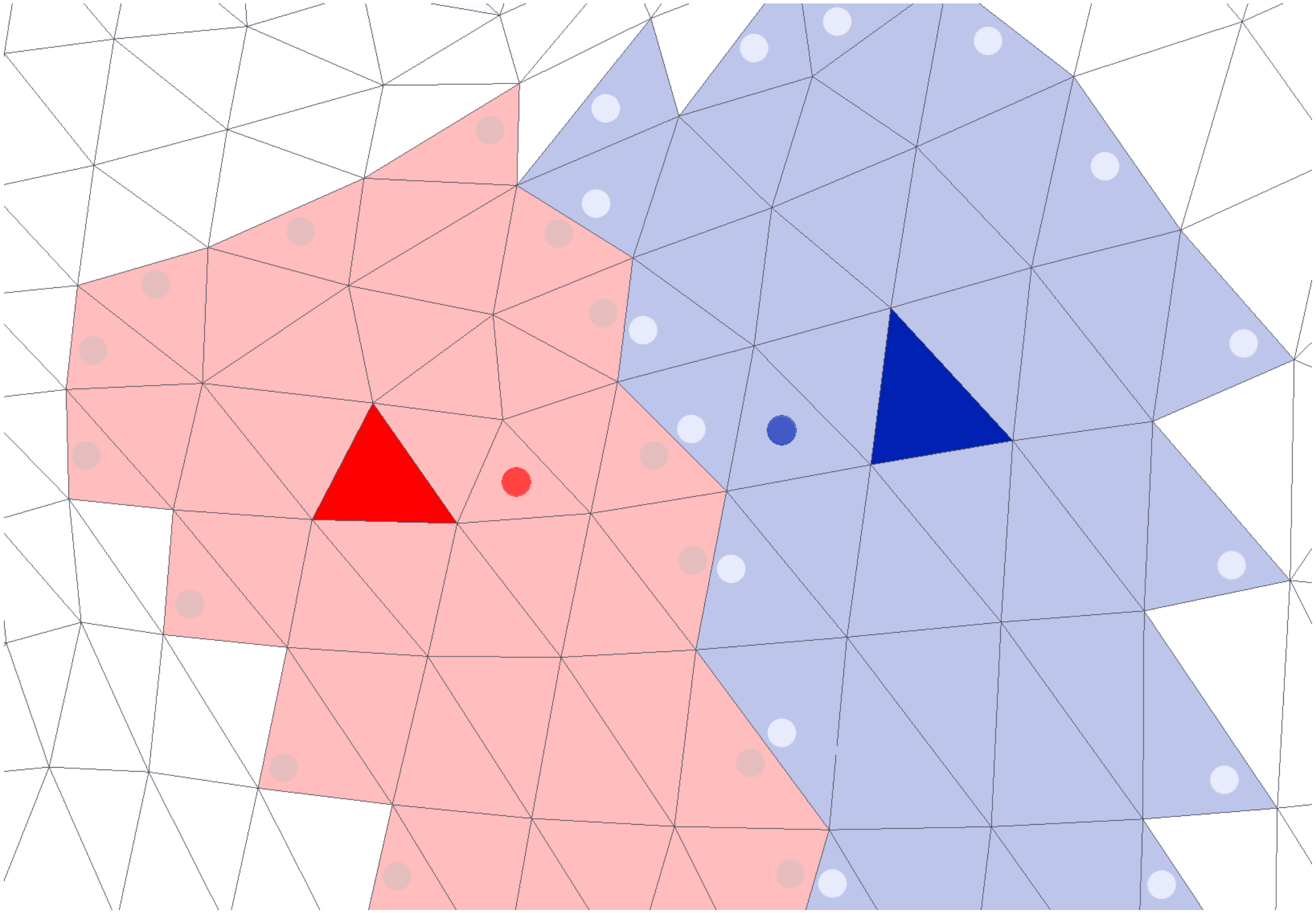
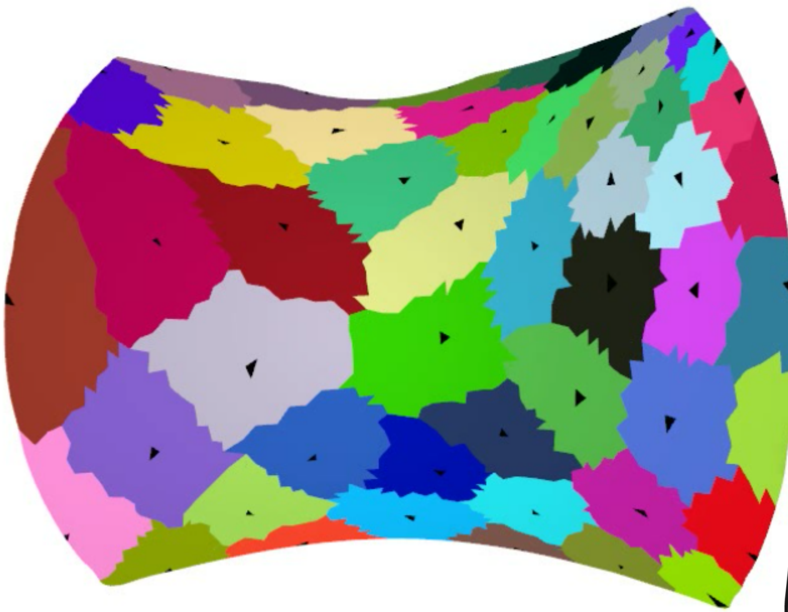


Topological clustering

- Centroids
  - Remain as elements of the mesh
  - Connected cells



file:spread

Topological clustering

- Relaxation process

  "(...)meets some convergence criterion(...)"

Topological clustering

- Relaxation process

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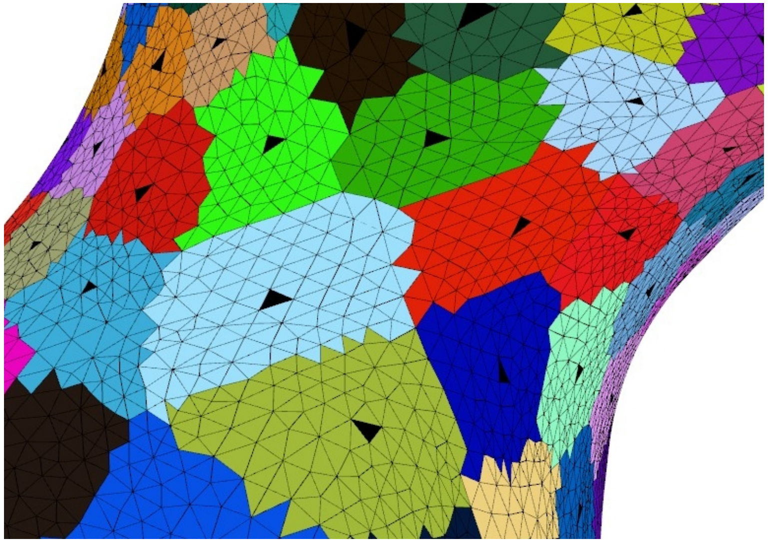
- Cell converges locally if centroids and seeds became neighbours since *k* previous iterations



Topological clustering

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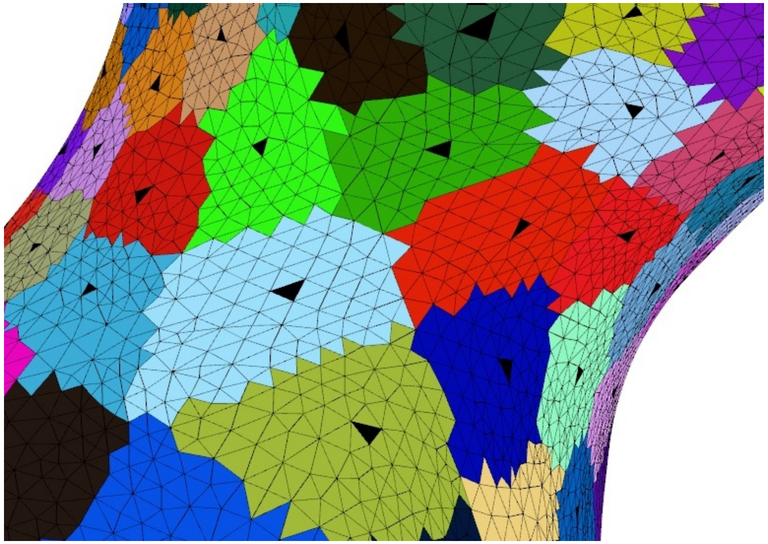
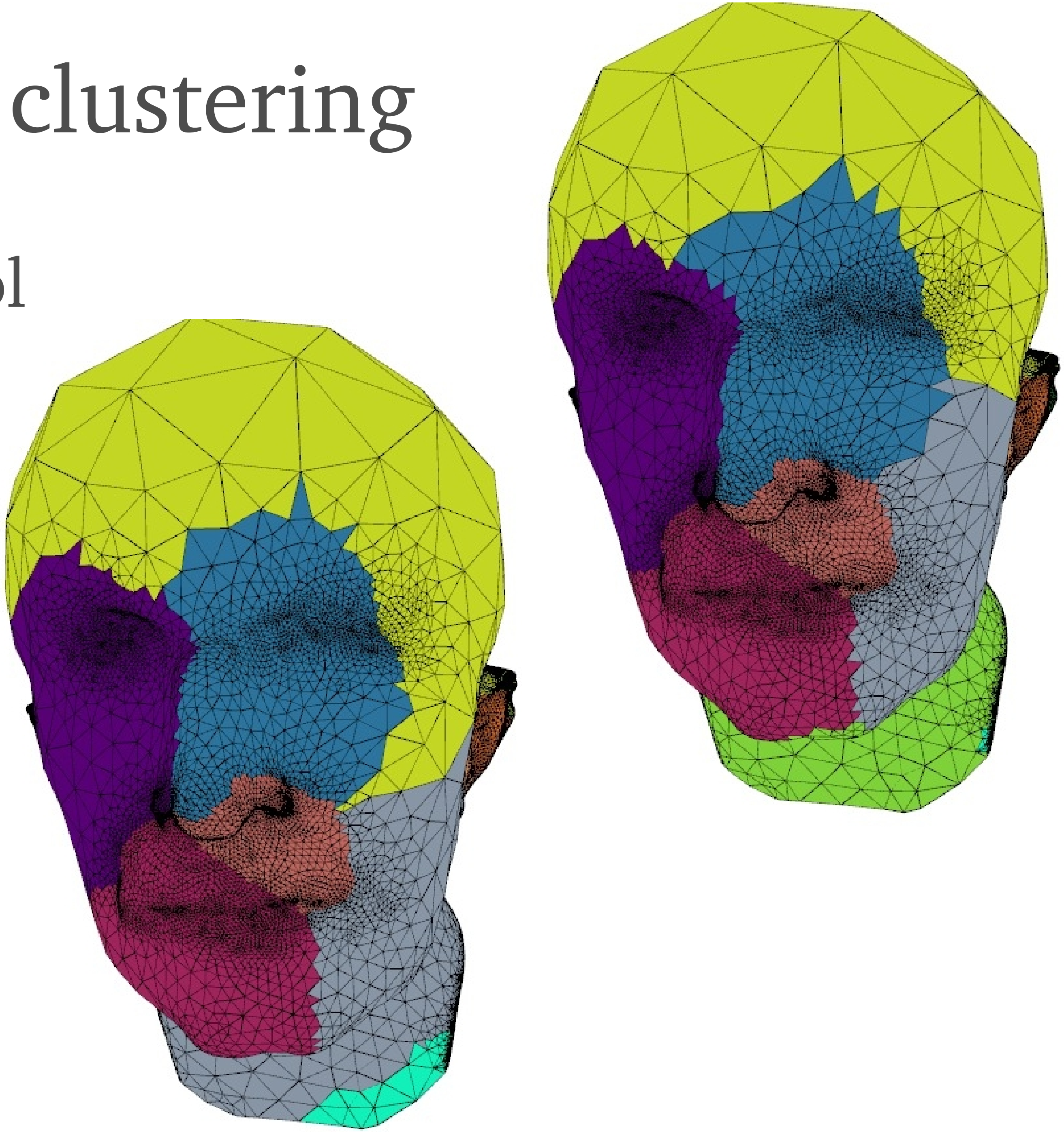


Topological clustering

- Quality control
  - Average area deviation

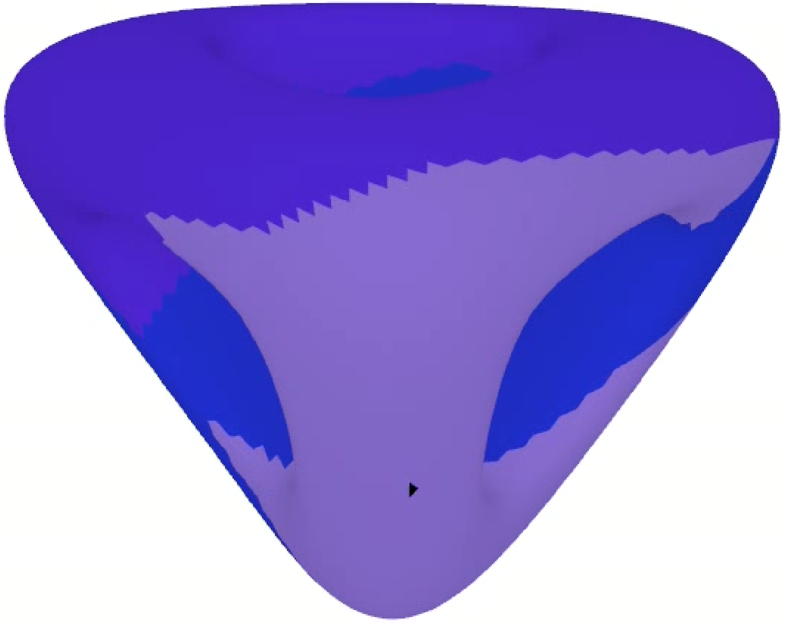


Topological clustering

- Quality control
  - Average area deviation
  - Elements per cluster
  - Number of clusters

Results

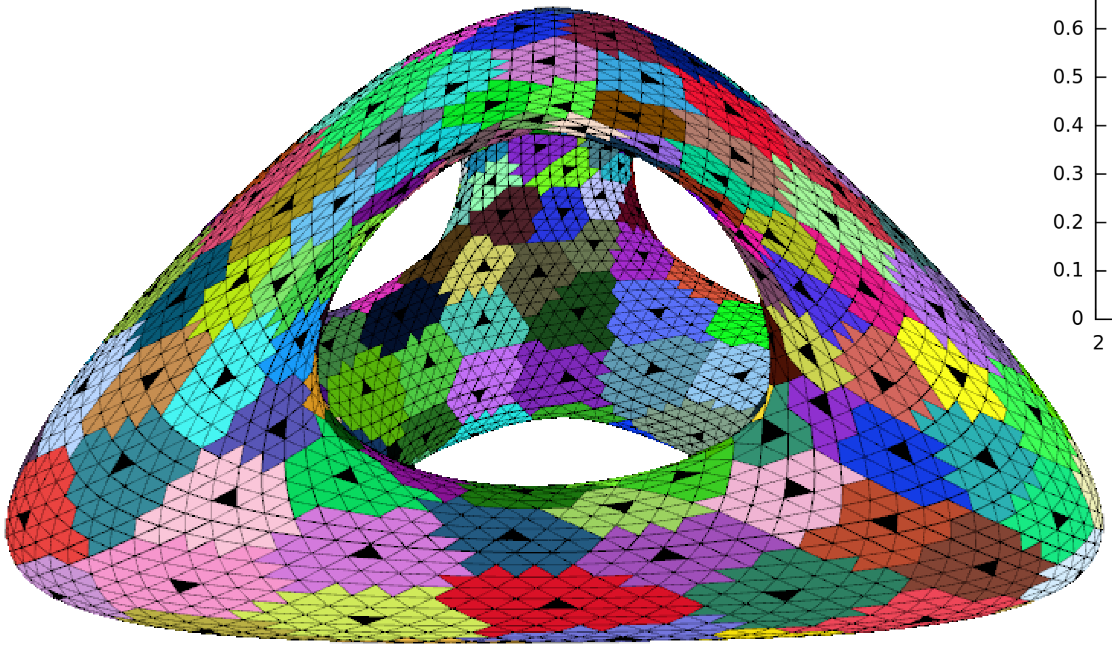
- Uniform input mesh (1.9x average area)

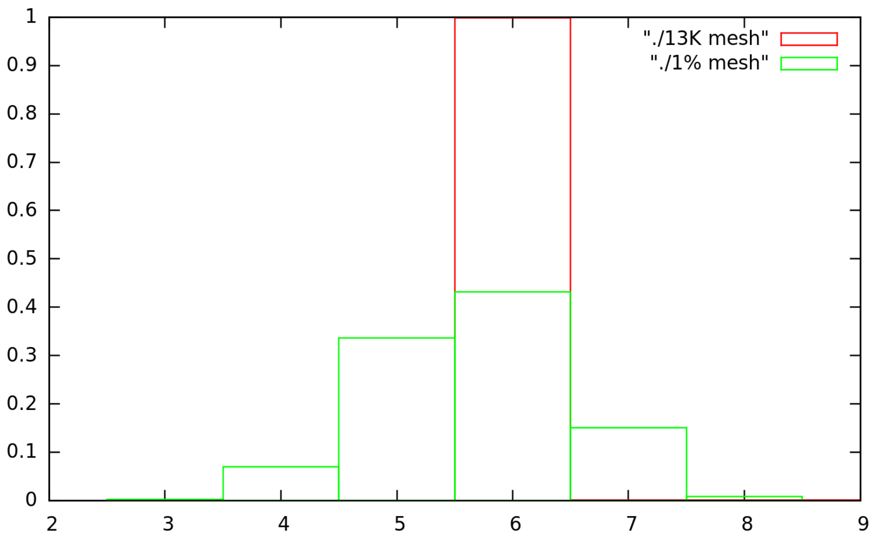


file:genus3_1.9_24c

Results

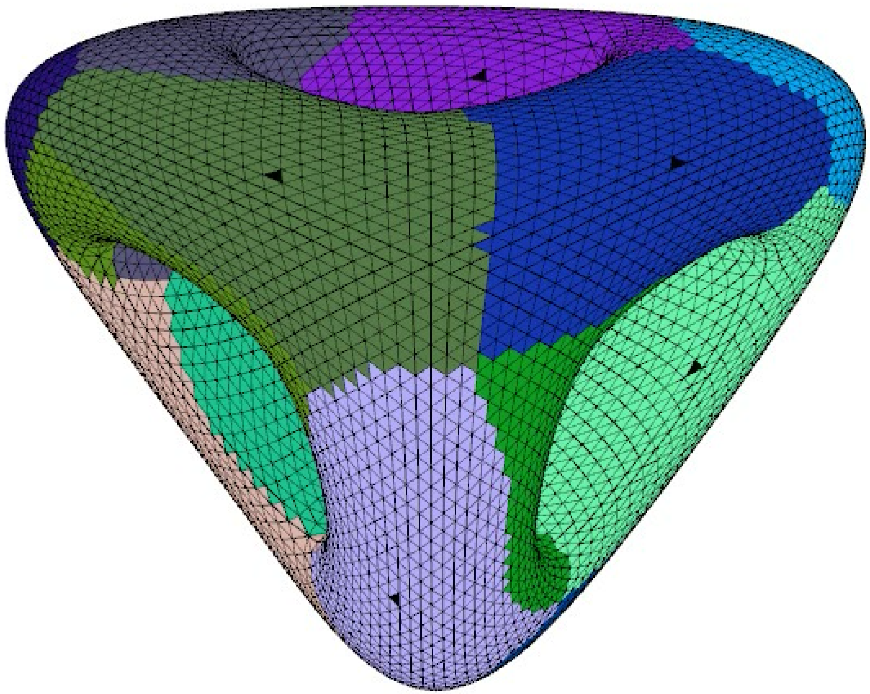
- Uniform input mesh (1.5x average area)





Results

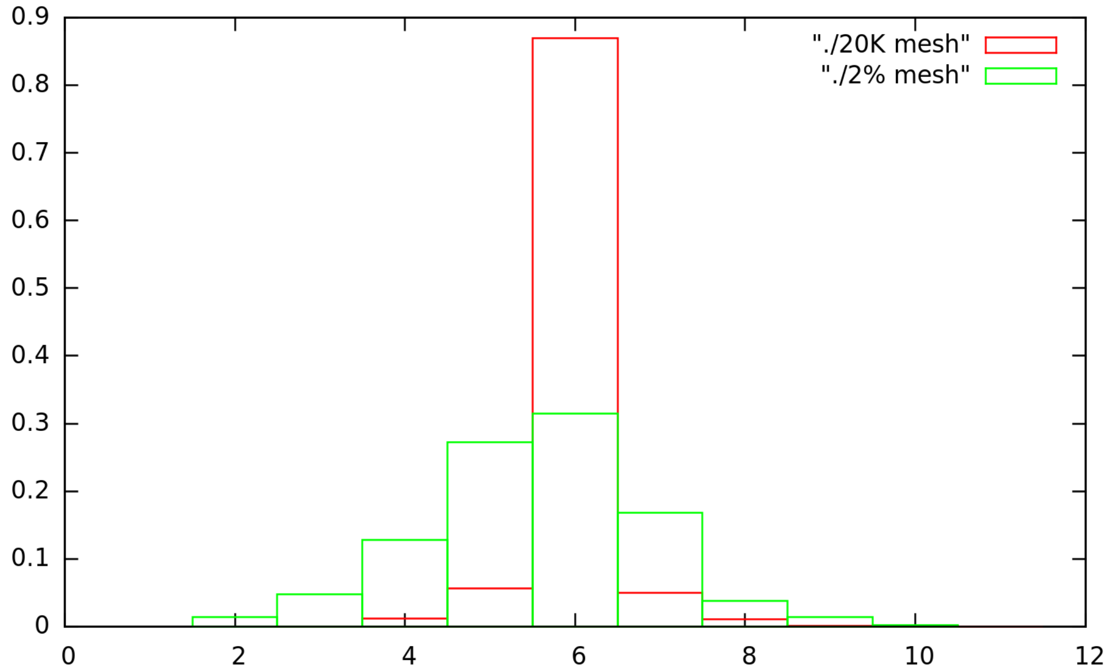
- Uniform input mesh (500 tri per cluster)



file:genus3_500

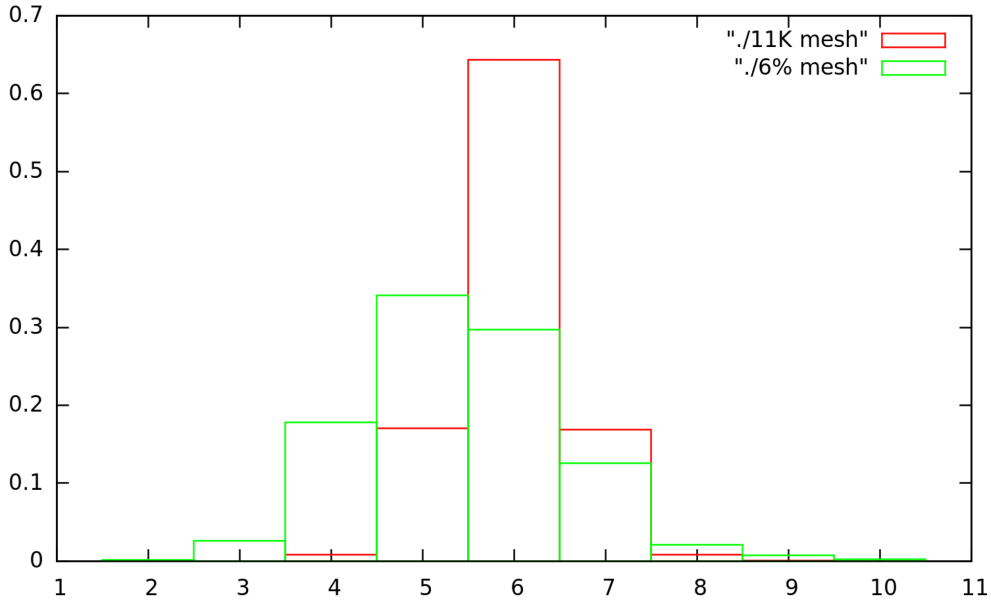
Results

- Adaptive input mesh (100 tri per cluster)



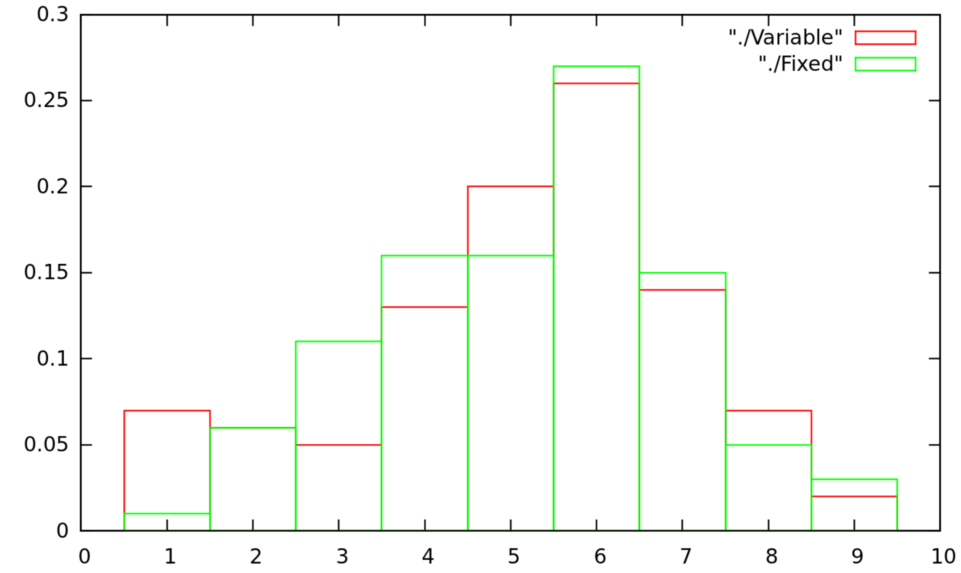
Results

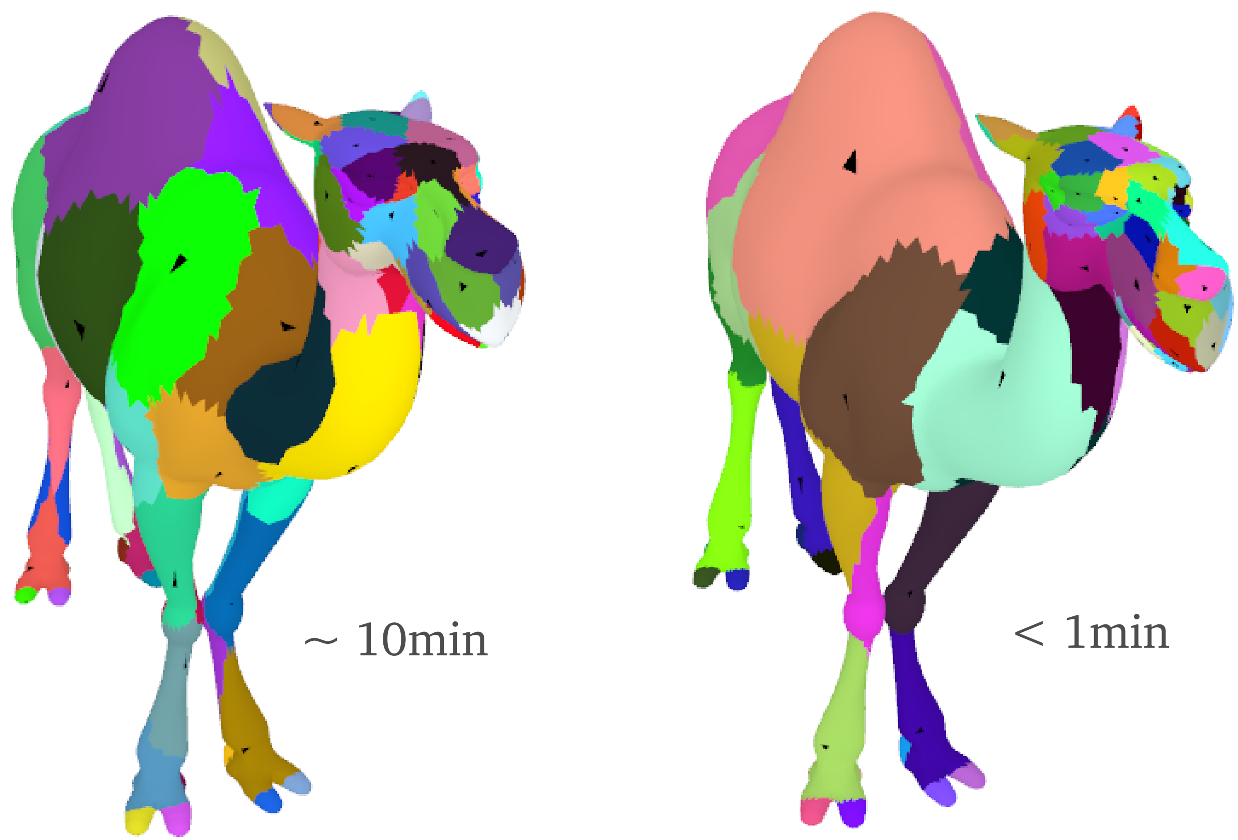
- Adaptive input mesh (3.5x average area)



Results

- Seeds size
  Variable X Fixed
  - 0.5% from original mesh
- Isotropy and anisotropy



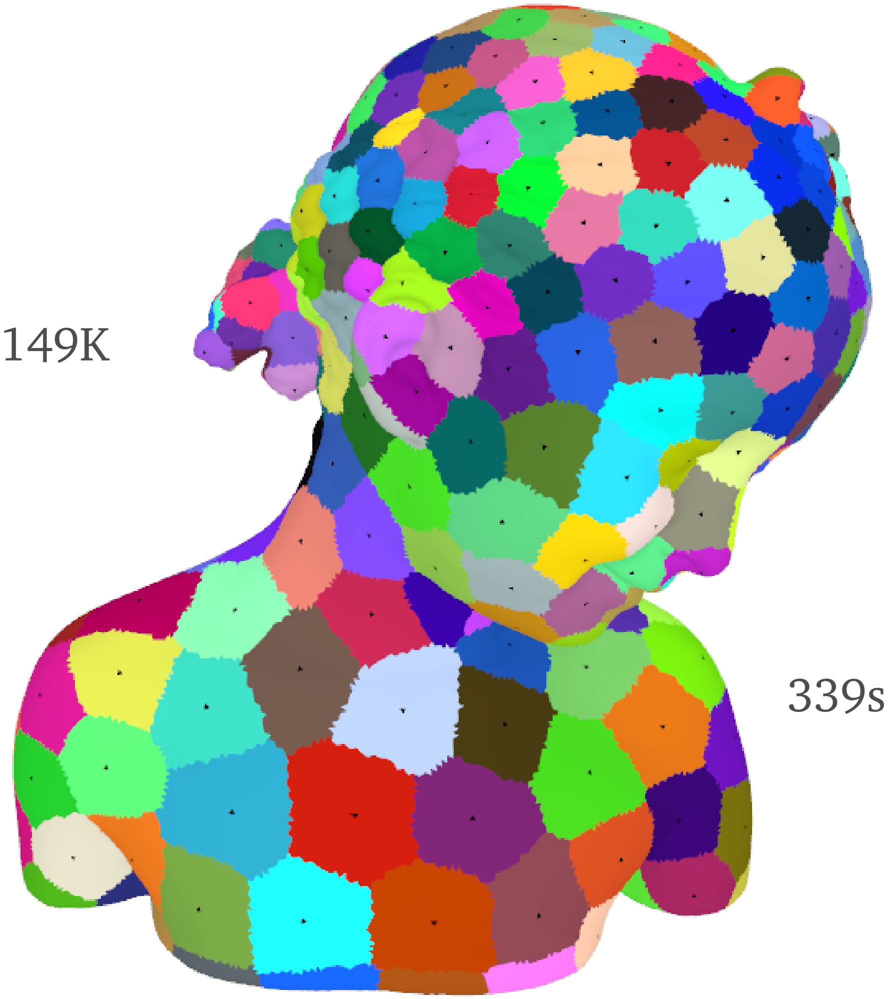


100 clusters

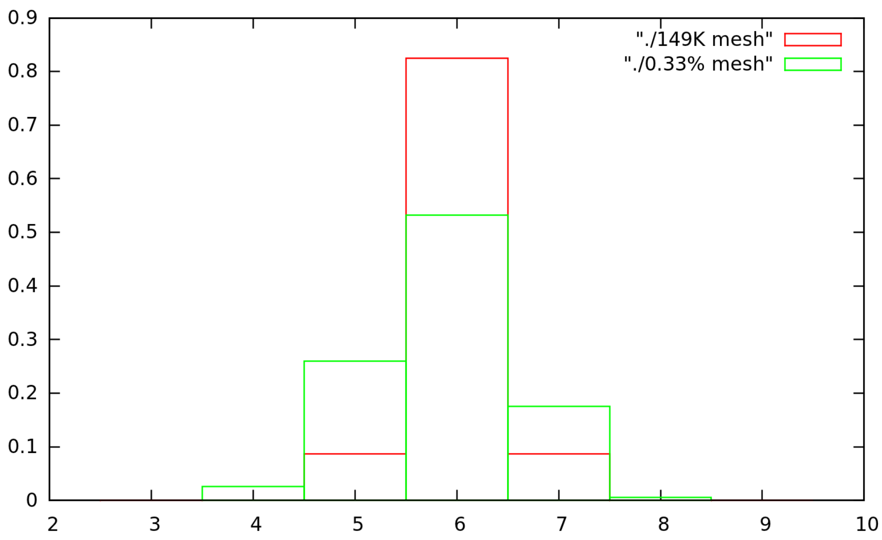
Results

- Uniform input mesh (500 clusters – 0.33% of original)

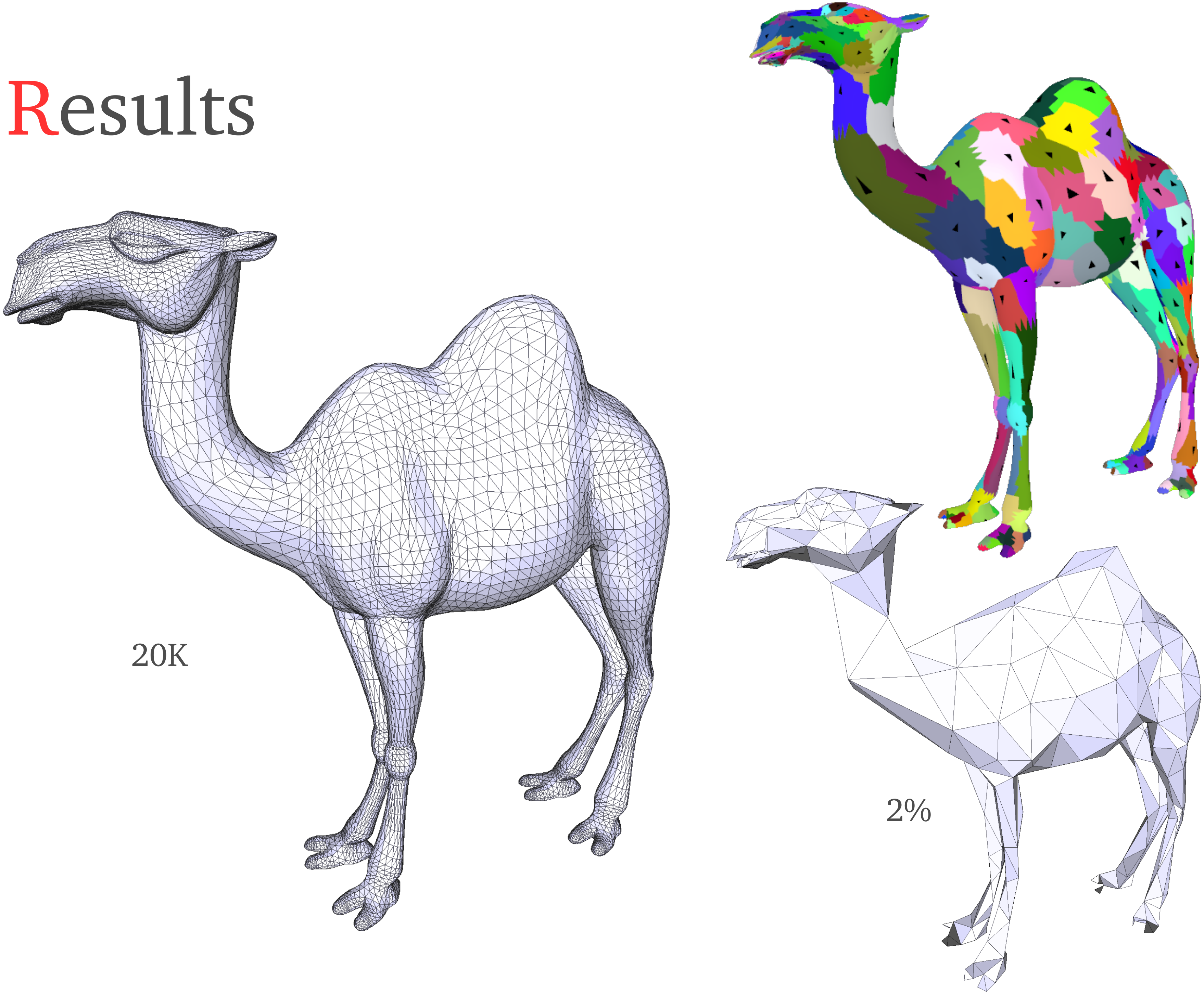
149K



339s



Results



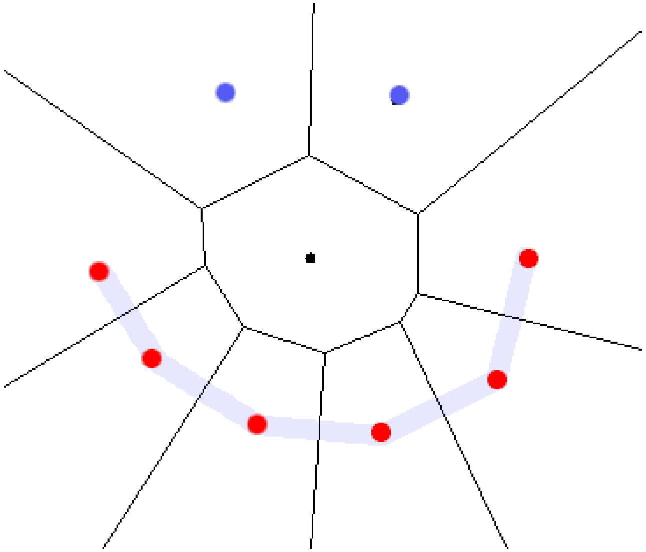
Directions

- New ways to control quality
- Speedup process
- Topological guarantees

Main references

- **VALETTE, S. et al.** - *A generic remeshing of 3D triangular meshes with metric-dependent discrete Voronoi diagrams* – IEEE Transactions on Visualization and Computer Graphics (2008)
- **BOUBEKEUR, T. and ALEXA, M.** - *Simplification by Stochastic Sampling and Topological Clustering* – Computer and Graphics (2009)
- **DU, Q. et al.** - *Centroidal Voronoi Tesselations: Applications and Algorithms* – SIAM Review (1999)
- **DU, Q. et al.** - *Constrained Centroidal Voronoi Tesselations for Surfaces* – SIAM Review (2003)
- **CEDRIM, D. et al.** - *Simplificação de malhas triangulares baseada no diagrama de Voronoi intrínseco* – Sibgrapi2011 - XXIV Conference on Graphics, Patterns and Images - Workshop of Theses and Dissertations (2011)
- **SACHT, L. K. & PEREIRA, T. S.** - *Centroidal Voronoi Tesselation on Meshes*

Thank's for your attention!



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