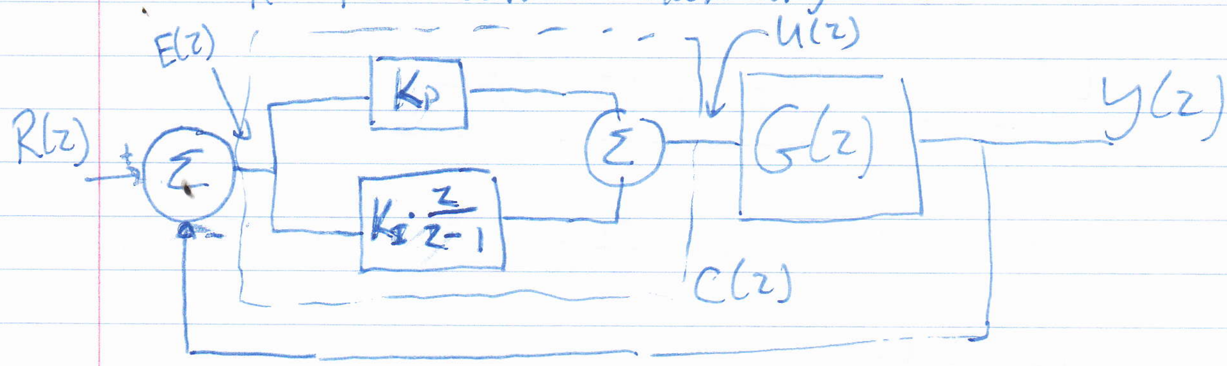
Derivation of $C(z)$ given K_p, K_I (or vice-versa) for PI controller using ECP Software.

* Note. The ECP controller uses a different convention for the "integral term" than the ECE 442/542 lecture. It is essential to use the convention contained here when determining your K_I and K_p

A "PI" controller block diagram *



* as implemented in ECP.

$$C(z) = \frac{U(z)}{E(z)} \; ; \; U(z) = K_p E(z) + K_I \frac{z}{z-1} E(z)$$

$$U(z) = E(z)\left(\frac{K_p(z-1)}{(z-1)} + \frac{K_I z}{(z-1)} \right) = \frac{(K_p + K_I)z - K_p}{z-1}$$

$$C(z) = \frac{U(z)}{E(z)} = \frac{(K_p+K_I)(z-\alpha)}{z-1} \; ; \; \text{where } \alpha = \frac{K_p}{K_p+K_I}$$

The structure of this $C(z)$ is the same as Dr Tharp's lecture.

$$\boxed{\frac{K(z-\alpha)}{z-1}}$$

Controller gain

adjustable zero

pole at $z=1$

But these factors translate into different K_p and K_I values (i.e. what you "plug in" to the controller).

$$\frac{K(z-\alpha)}{z-1} = \frac{(K_I+K_p)\left(z-\frac{K_p}{K_p+K_I}\right)}{z-1}$$

So $K = K_I + K_p$; $\alpha = \frac{K_p}{K_p+K_I} = \frac{K_p}{K} \Rightarrow K_p = \alpha \cdot K$

$K_I = K - K_p$

So given K, α, we can find K_p, K_I

$$\boxed{\begin{aligned} K_p &= \alpha \cdot K \\ K_I &= K - K_p \end{aligned}}$$

given K_p, K_I, we can find K, α

$$\boxed{\begin{aligned} K &= K_p + K_I \\ \alpha &= \frac{K_p}{K_p+K_I} \end{aligned}}$$

where $C(z) = \frac{K(z-\alpha)}{(z-1)}$

(generic PI controller)