AR2:AR2 Separable process

Let

$$\boldsymbol{Q}(\rho) = \begin{pmatrix} \rho & -2\rho & 1 & 0 & 0 & \dots \\ 0 & \rho & -2\rho & 1 & 0 & \dots \\ \vdots & & & \ddots & & \\ & & & \rho & -2\rho & 1 \end{pmatrix},$$

i.e precision matrix of an AR2 process. When $\rho = 1$, this corresponds to a RW2 process.

Then the Kronecker

$$\boldsymbol{Q}(\rho_t) \otimes \boldsymbol{Q}(\rho_x) = \begin{pmatrix} \rho_x\rho_t & -2\rho_x\rho_t & \rho_t & 0 & 0 & \dots & -2\rho_x\rho_t & 4\rho_x\rho_t & -2\rho_t & 0 \\ \dots & \rho_x & -2\rho_x & 1 & 0 & \dots & & & & \\ 0 & \rho_x\rho_t & -2\rho_x\rho_t & \rho_t & 0 & \dots & 0 & -2\rho_x\rho_t & 4\rho_x\rho_t & -2\rho_t \\ \dots & 0 & \rho_x & -2\rho_t & x & \dots & & & & \\ \vdots & & & & \ddots & & & & & \\ & & & & & & & & & \end{pmatrix}$$

is the precision matrix o fan AR2:AR2 process (marginal variances ignored).

The matrix applies penalty on

$$\begin{aligned} &\beta_{x,t} - 2\rho_x\beta_{x-1,t} + \rho_x\beta_{x-2,t} - 2\rho_t\beta_{x,t-1} + 4\rho_x\rho_t\beta_{x-1,t-1} - 2\rho_x\rho_t\beta_{x-2,t-1} + \rho_t\beta_{x,t-2} - 2\rho_x\rho_t\beta_{x-1,t-2} + \rho_x\rho_t\beta_{x-2,t-2} \\ =&\beta_{x,t} - \rho_x(2\beta_{x-1,t} - \beta_{x-2,t}) - \rho_t(2\beta_{x,t-1} - \beta_{x,t-2}) + 4\rho_x\rho_t\beta_{x-1,t-1} - 2\rho_x\rho_t\beta_{x-2,t-1} - 2\rho_x\rho_t\beta_{x-1,t-2} + \rho_x\rho_t\beta_{x-2,t-2} \\ =&\beta_{x,t} - \rho_x(2\beta_{x-1,t} - \beta_{x-2,t}) - \rho_t(2\beta_{x,t-1} - \beta_{x,t-2}) + \frac{1}{2}[2(2\beta_{x-1,t-1} - \beta_{x-2,t-1}) - (2\beta_{x-1,t-2} - \beta_{x-2,t-2})] + \frac{1}{2}[2(2\beta_{x-1,t-1} - \beta_{x-1,t-2}) - (2\beta_{x-2,t-1} - \beta_{x-2,t-2})] \end{aligned}$$

Therefore, the penalty draws the current $\beta_{x,t}$ towards a weighted average of the linearly extrapolated values in age, time dimensions. The first term is the linearly extrapolated value