- Interpolated fx as well
- Fitted 5x5 and simulated 1x1 assuming constant values within age groups and periods
- Fitted splines to the thiele parameters, fx and mx
  - fitted P-splines around the LQ derived values for the child and old age mortality component (i.e. ϕ, ψ, A and B), with spline coefficients given MVN priors

$$\boldsymbol{\beta}_i \sim N(\mathbf{0}, \sigma_i^2 \boldsymbol{I}), \qquad i \in \{\phi, \psi, A, B\}$$

  - fitted P-splines to the parameters of the hump component (i.e. λ, δ and ϵ), given RW1 priors

$$\begin{aligned} \boldsymbol{D}\boldsymbol{\beta}_i &\sim N(\mathbf{0}, \sigma_i^2 \boldsymbol{I}) \\ \beta_{i0} &\sim N(\hat{\mu}_i, \tilde{\varepsilon}_i), \qquad i \in \{\lambda, \delta, \epsilon\} \end{aligned}$$

  - fitted 2-D tensor P-splines to the fertility rates around the interpolated WPP fx estimates, spline coefficients given MVN prior

$$\boldsymbol{\beta}_{f_x} \sim N(\mathbf{0}, \sigma_{f_x}^2 \boldsymbol{I})$$

  - fitted 2-D tensor P-splines to the migration proportions, given 1st order penalties along the age,time and the cross age-time dimensions, on top of shrinkage of each parameters towards 0

$$\boldsymbol{\beta}_{m_x} \sim N(\mathbf{0}, \boldsymbol{P}^{-1}) \qquad ,$$

    where $\boldsymbol{P} = \lambda_x(\boldsymbol{I}_t \otimes \boldsymbol{D'_x}\boldsymbol{D_x}) + \lambda_t(\boldsymbol{D'_t}\boldsymbol{D_t} \otimes \boldsymbol{I}_x) + \lambda_{xt}(\boldsymbol{D_x} \otimes \boldsymbol{D_t})'(\boldsymbol{D_x} \otimes \boldsymbol{D_t}) + \tau^2 \boldsymbol{I_{xt}}$

- Initially had number of basis = 15 in age ($\sim$ 7 ages per knot) and time ($\sim$ 5 years per knot), and number of basis = 7 in age for fertility ($\sim$ 8 ages per knot)
  - pushed to having number of basis = 30 in age ($\sim$ 3 ages per knot) and time ($\sim$ 2 years per knot), and number of basis = 14 for fertility age ($\sim$ 3 ages per knot)
- Took about an hour to converge with 'naive' initial values, e.g. fixing spline coefficients to be all 0
- Only fitted to Zimbabwe and Namibia so far
- Estimated migration lower than those estimated from 2x2?

Namibia

```
## [1] "Census Females"

## # A tibble: 87 x 4
##       age `1991` `2001` `2011`
##     <dbl>  <dbl>  <dbl>  <dbl>
##  1      0 24851. 23909. 32157
##  2      1 21976. 23883. 28690.
##  3      2 21726. 24603. 28220
##  4      3 21259. 25041. 27500.
##  5      4 21247. 25458. 27077
##  6      5 20771. 25430. 26048.
##  7      6 20065. 25264. 24803.
##  8      7 19362. 25259. 23924.
##  9      8 18677. 25248. 23376.
## 10      9 18357. 25517. 23940.
## # ... with 77 more rows

## [1] "Census Males"

## # A tibble: 87 x 4
##       age `1991` `2001` `2011`
##     <dbl>  <dbl>  <dbl>  <dbl>
##  1      0 24861. 23791. 31976
##  2      1 21883. 24016. 28452.
##  3      2 21600. 24629. 28036.
##  4      3 21144. 24966. 27342.
##  5      4 21149. 25281. 26940.
##  6      5 20679. 25177. 25925.
##  7      6 19965. 24978. 24627.
##  8      7 19163. 24795. 23618.
##  9      8 18386. 24574. 23047
## 10      9 17972. 24684. 23621.
## # ... with 77 more rows
```

Thiele log-Normal Hump Spline

```
## [1] "relative convergence (4)"

##           log_tau2_logpop_f           log_tau2_logpop_f
##                   6.3819610                   4.0893856
##           log_tau2_logpop_m           log_tau2_logpop_m
##                   6.4394130                   4.1790129
##                log_tau2_fx                log_tau2_gx_f
##                   5.1275869                   3.6305660
##              log_tau2_gx_m          log_lambda_gx_age_f
##                   3.3015230                   7.8968090
##         log_lambda_gx_age_m         log_lambda_gx_time_f
##                   7.9207862                   8.1001565
##        log_lambda_gx_time_m      log_lambda_gx_agetime_f
##                   8.1891921                   7.4611406
##     log_lambda_gx_agetime_m                log_lambda_tp
##                   6.9077507                   3.5659551
```

```
## log_lambda_tp_0_inflated_sd            log_dispersion_f
##                    0.3246860                   0.8152099
##              log_dispersion_m       log_marginal_prec_phi_f
##                    0.9745773                   4.6849502
##       log_marginal_prec_psi_f  log_marginal_prec_lambda_f
##                    4.6381213                   1.6820184
##     log_marginal_prec_delta_f log_marginal_prec_epsilon_f
##                    2.8822772                   3.4625452
##         log_marginal_prec_A_f         log_marginal_prec_B_f
##                    6.9157718                   7.0424717
##       log_marginal_prec_phi_m       log_marginal_prec_psi_m
##                    4.7171337                   4.6494609
##    log_marginal_prec_lambda_m    log_marginal_prec_delta_m
##                    1.6568263                   2.9618588
## log_marginal_prec_epsilon_m         log_marginal_prec_A_m
##                    3.2933916                   6.9173784
##         log_marginal_prec_B_m
##                    6.9307313
```

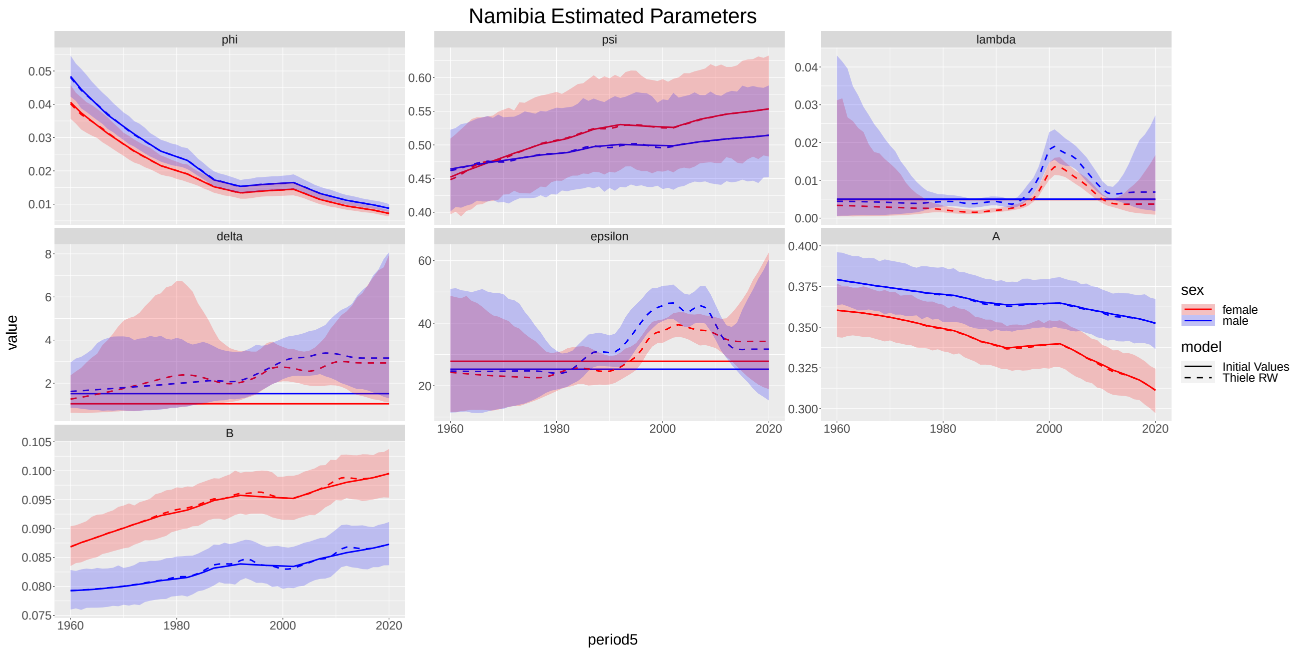

Figure 1: Estimated parameters

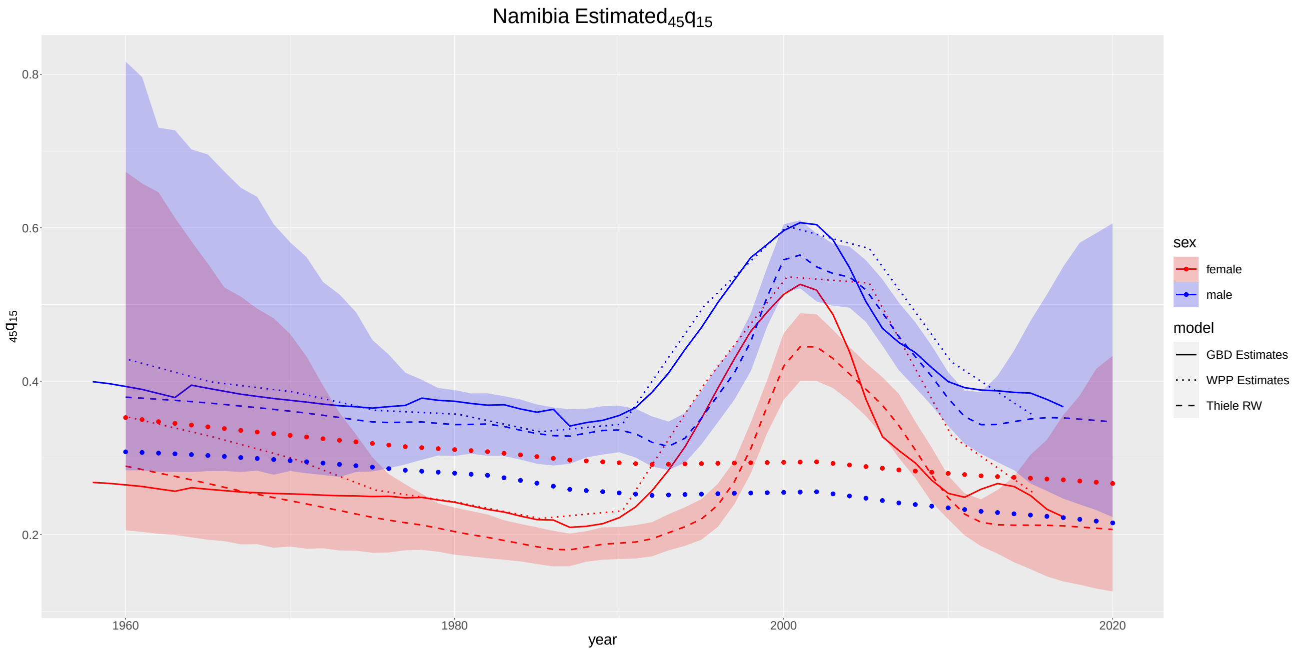

Figure 2: Estimated $_{45}q_{15}$

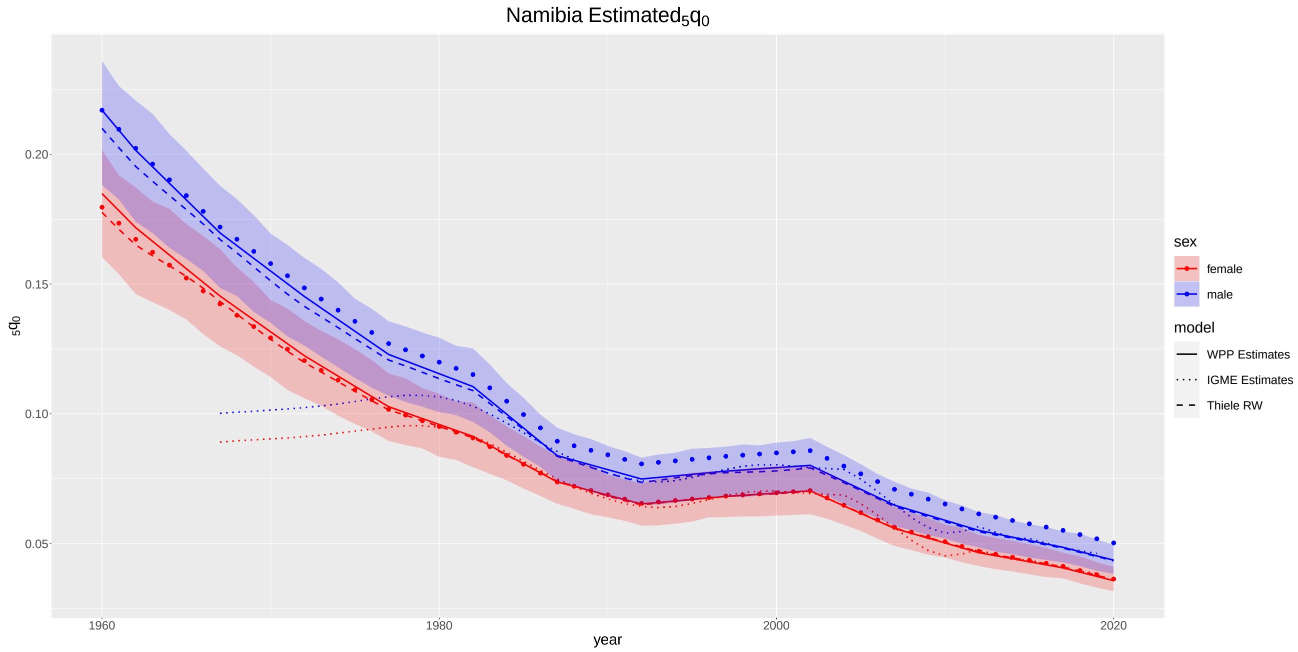

Figure 3: Estimated $_5q_0$

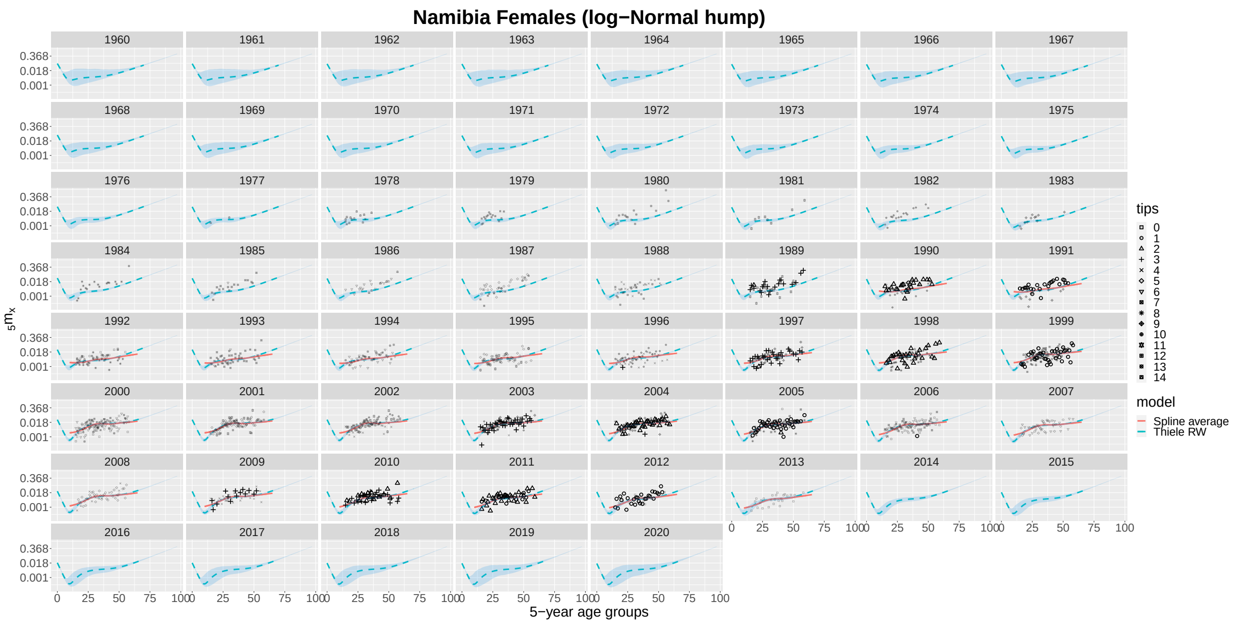

Figure 4: Mortality Schedules

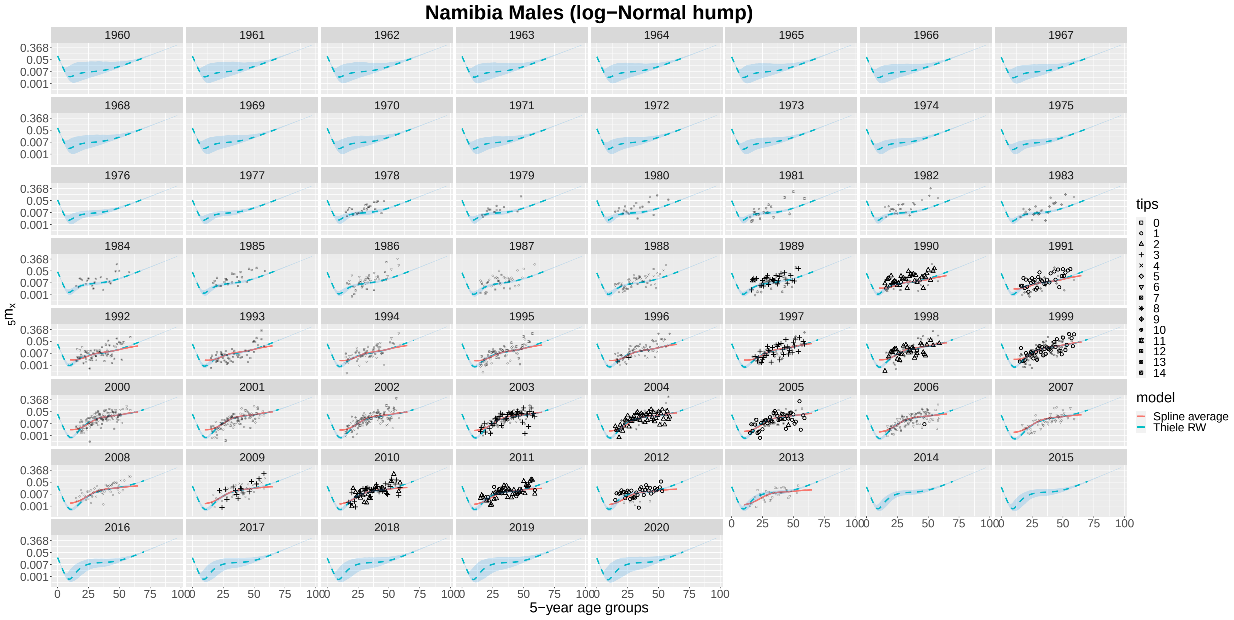

Figure 5: Mortality Schedules

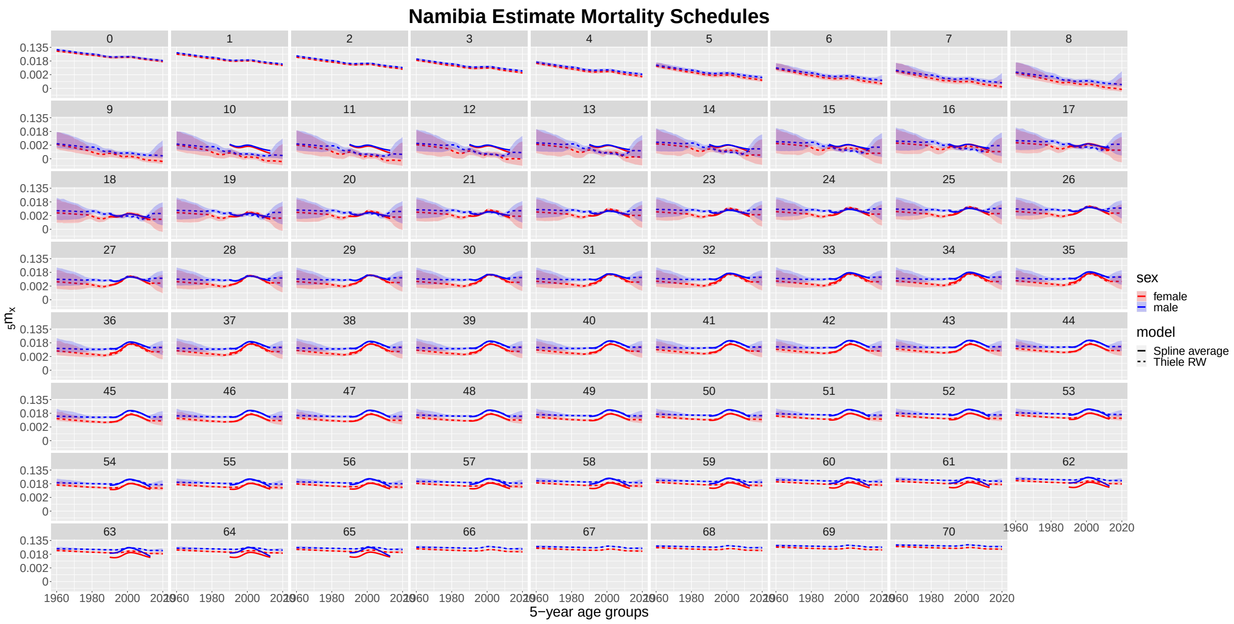

Figure 6: Mortality Schedules

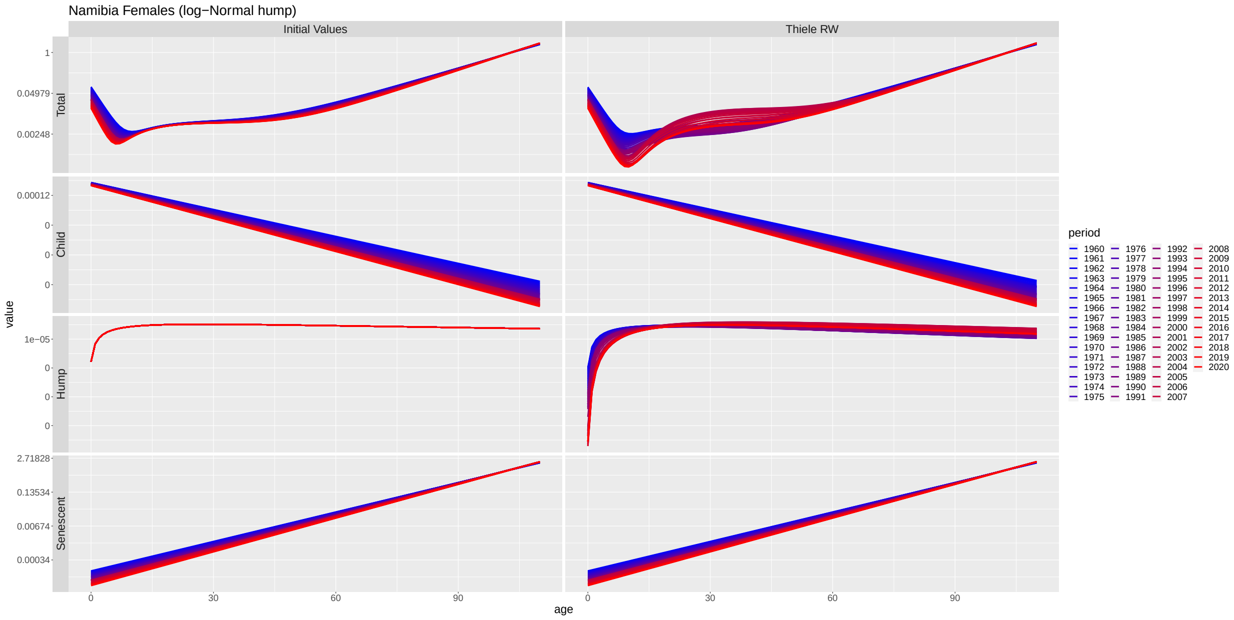

Figure 7: Thiele Decomposed

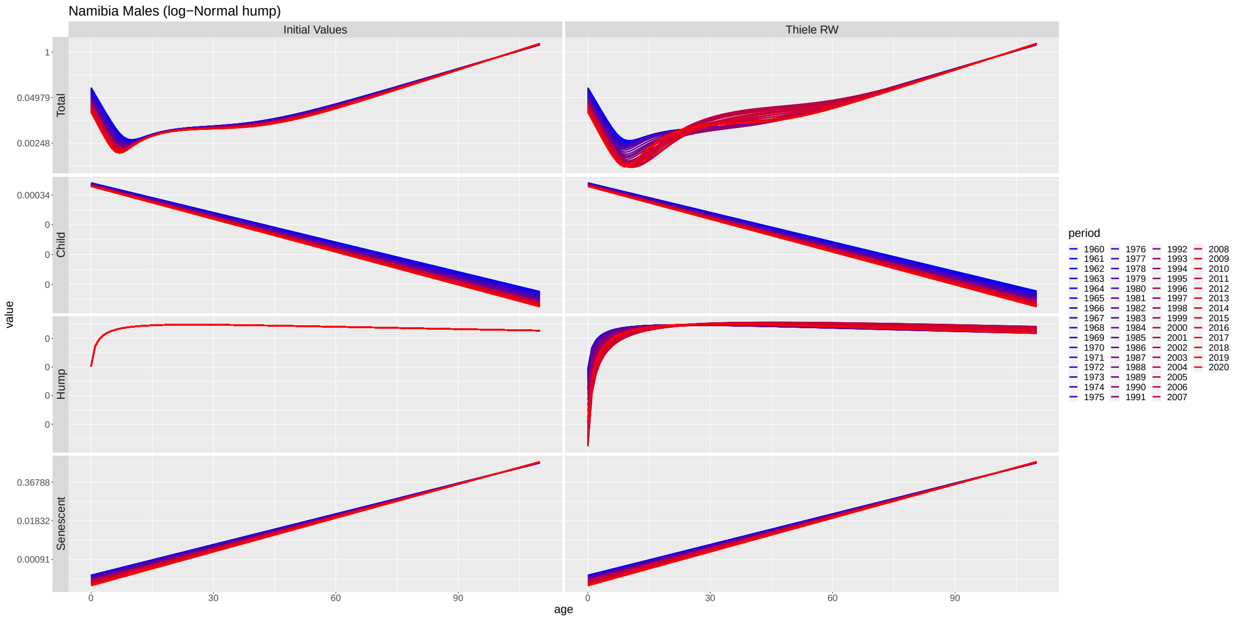

Figure 8: Thiele Decomposed

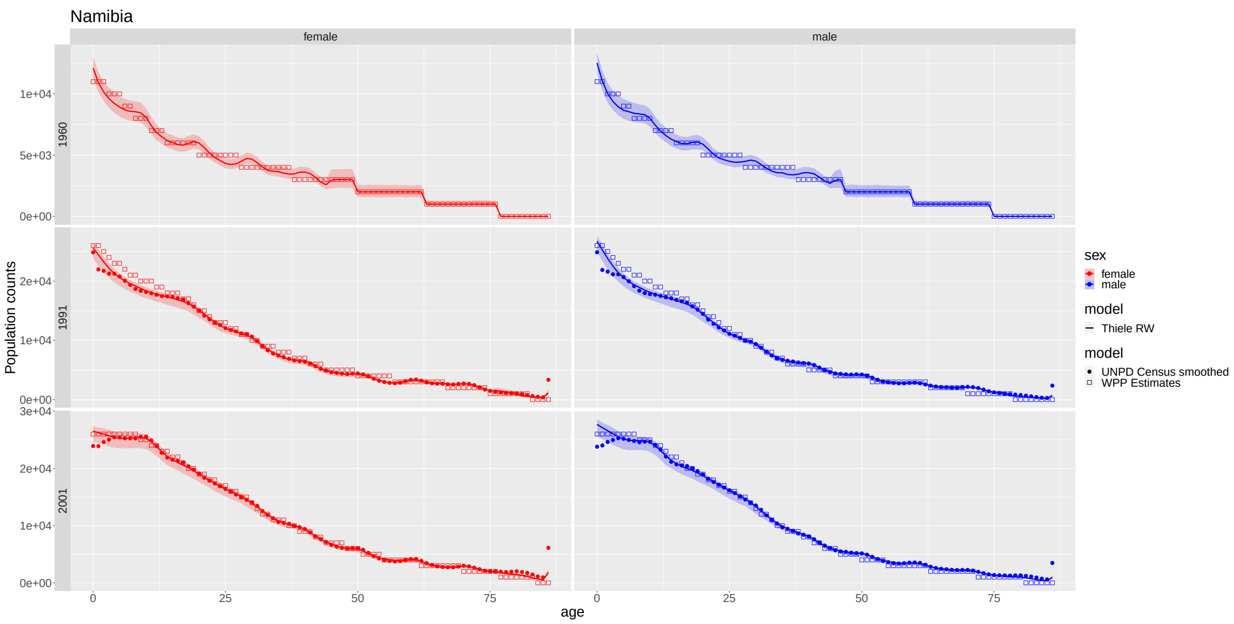

Figure 9: Population

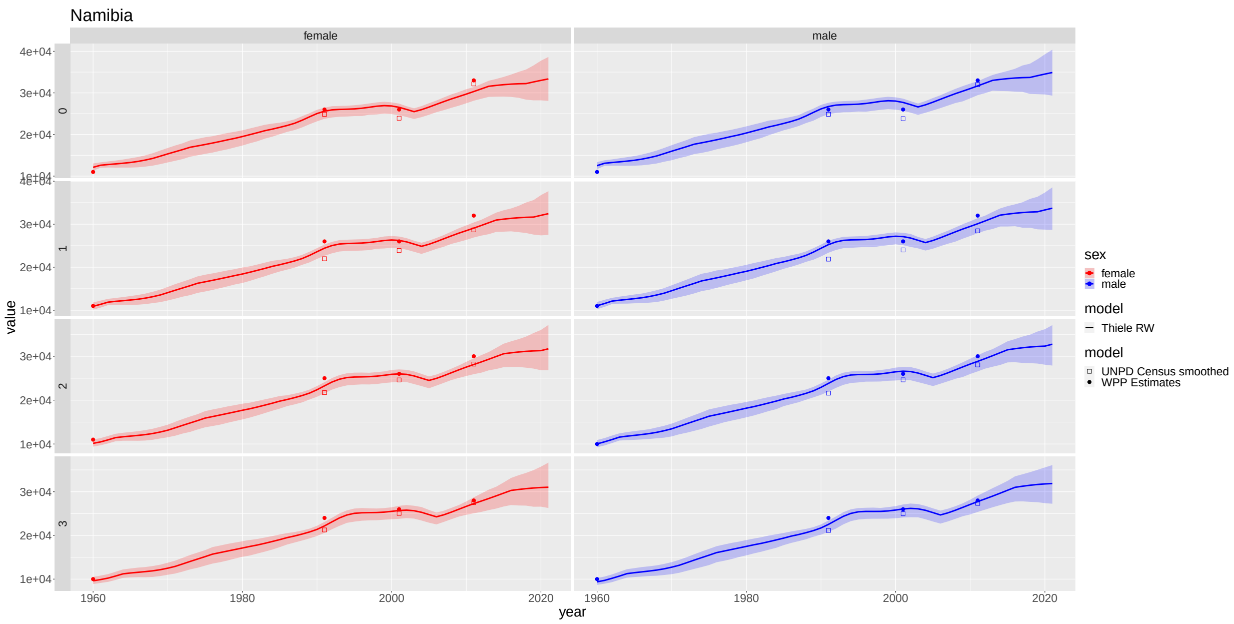

Figure 10: Population

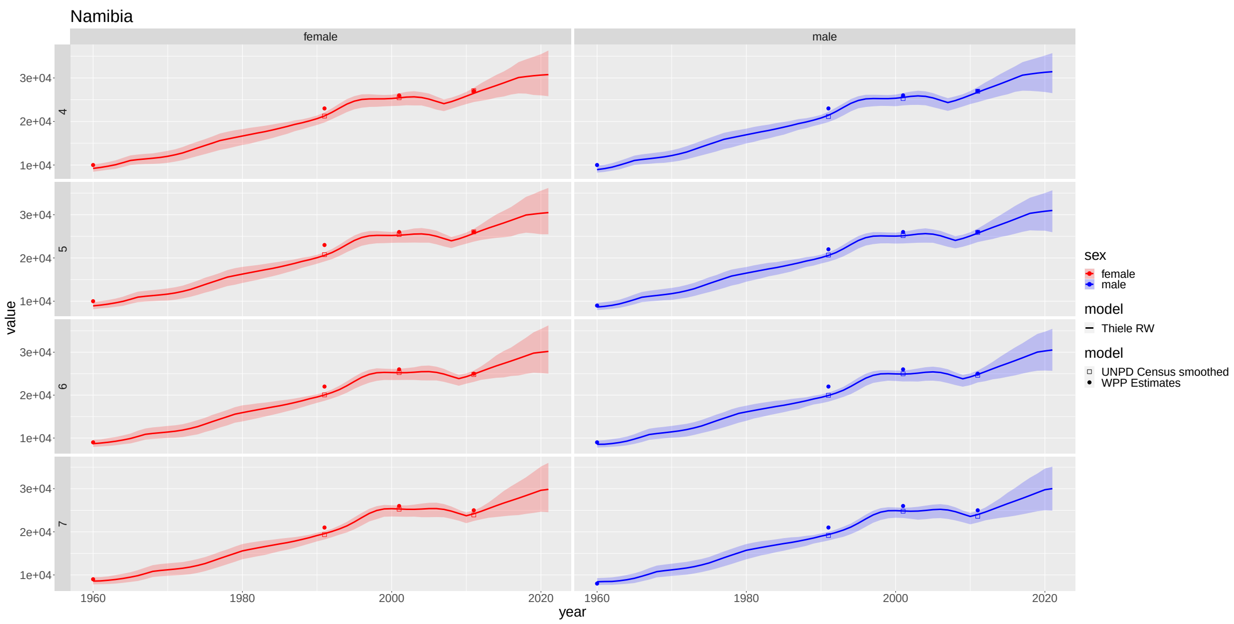

Figure 11: Population

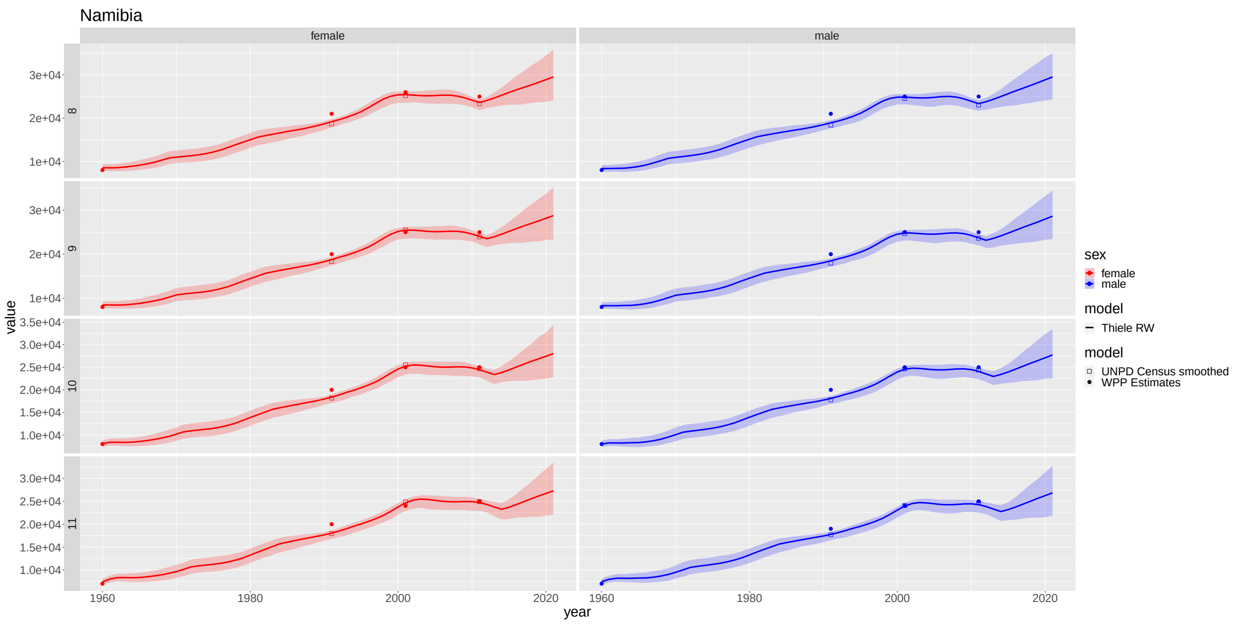

Figure 12: Population

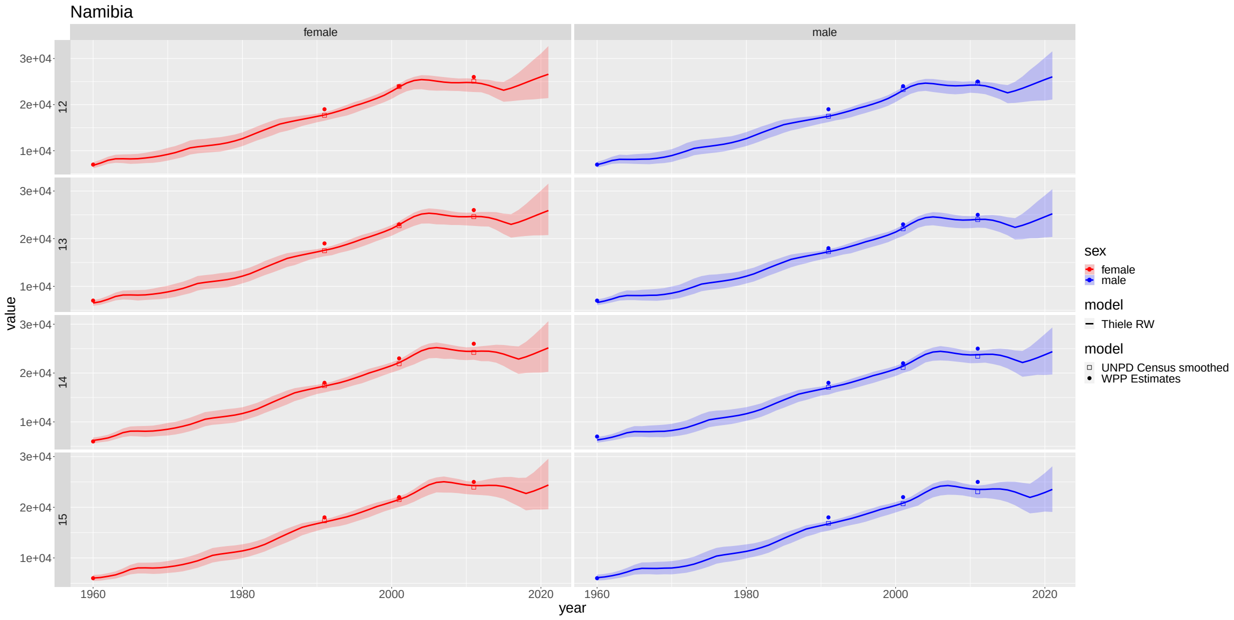

Figure 13: Population

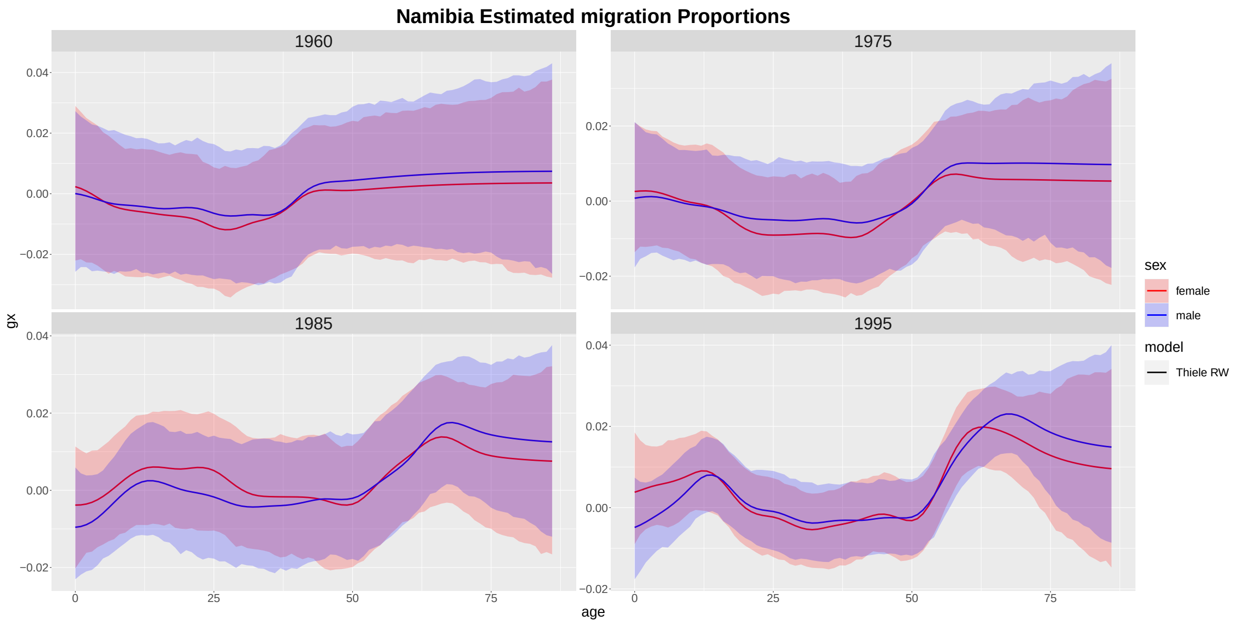

Figure 14: Migration

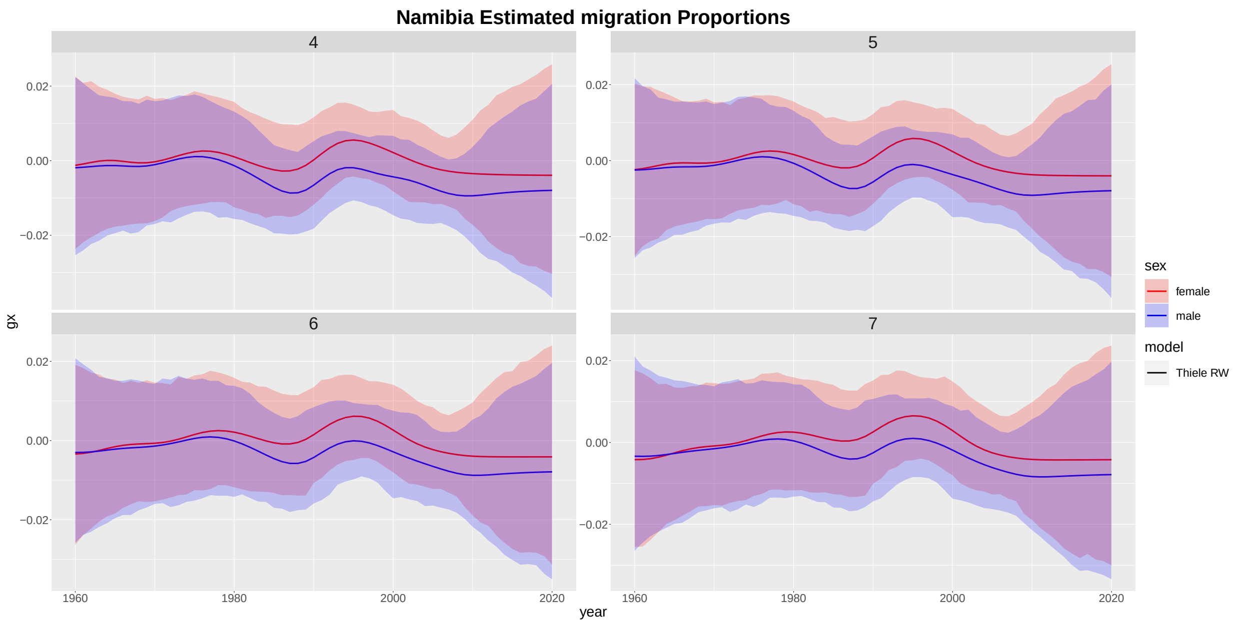

Figure 15: Migration

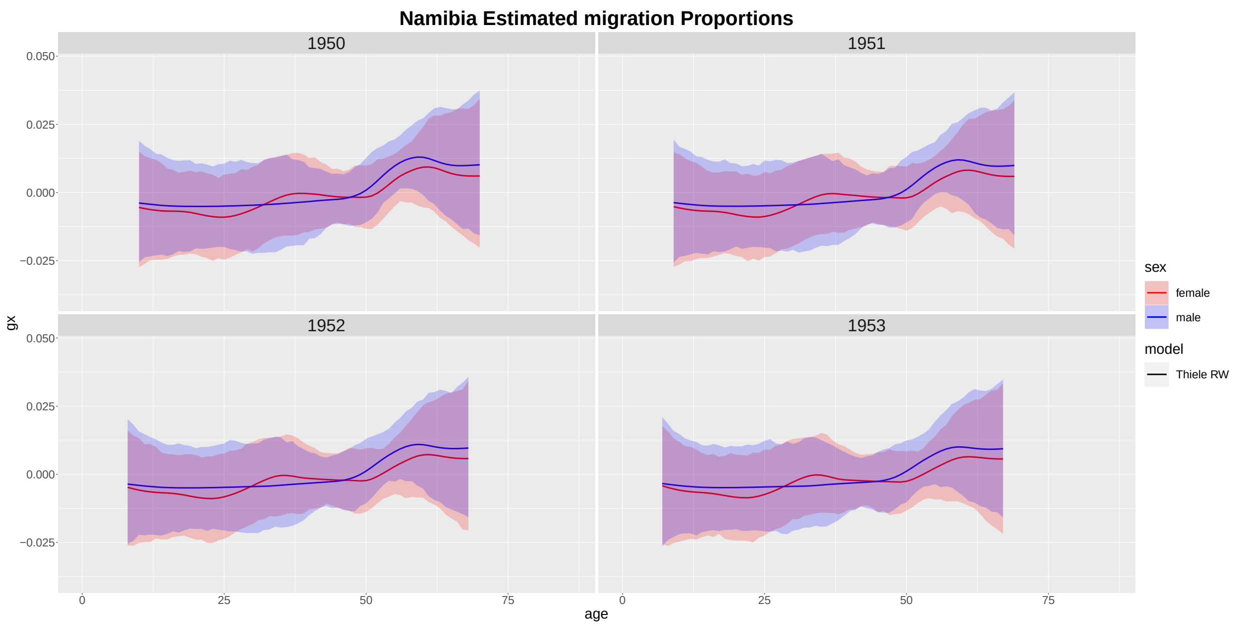

Figure 16: Migration

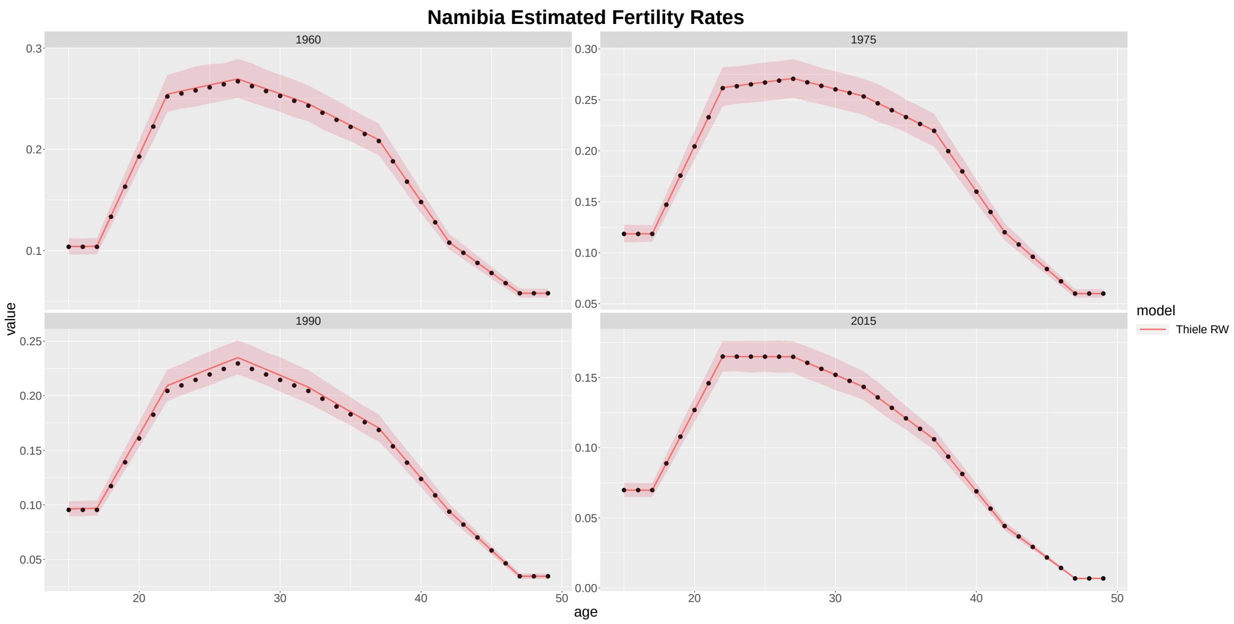

Figure 17: Fertility

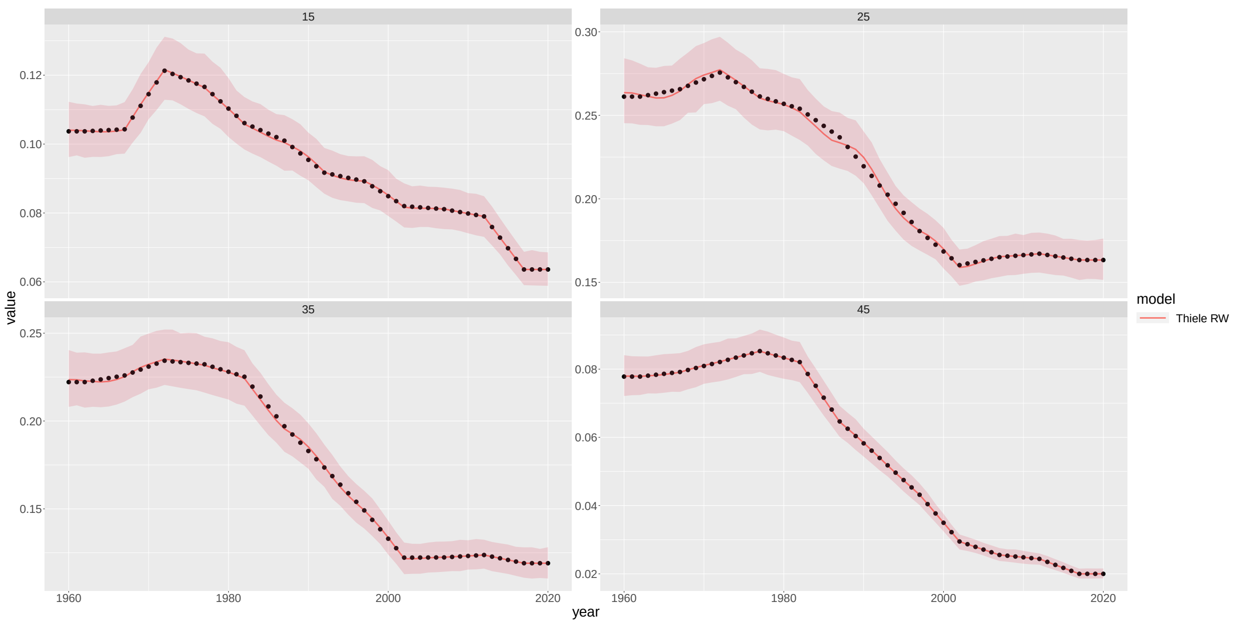

Figure 18: Fertility

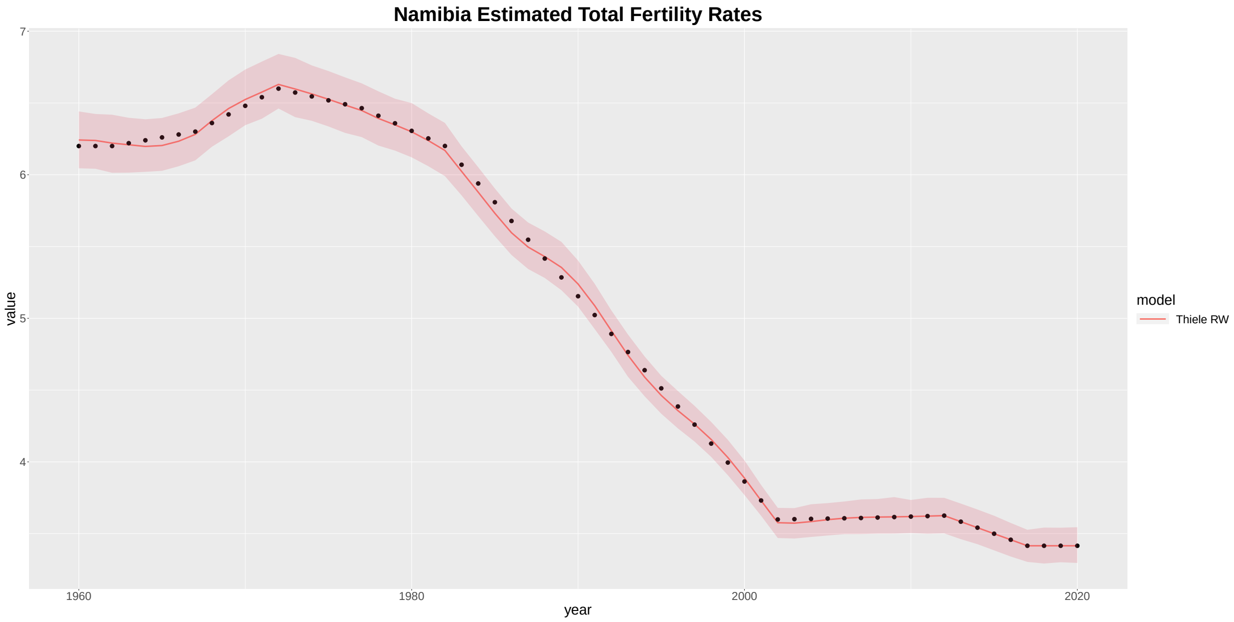

Figure 19: Total Fertility