- Stan is still giving divergent transitions warnings whenever DEF is included, will come back to that later
- Tried fixing the same LQ k for all periods, while allowing DEF to vary over time as suggested, and fitted to BF female DHS data

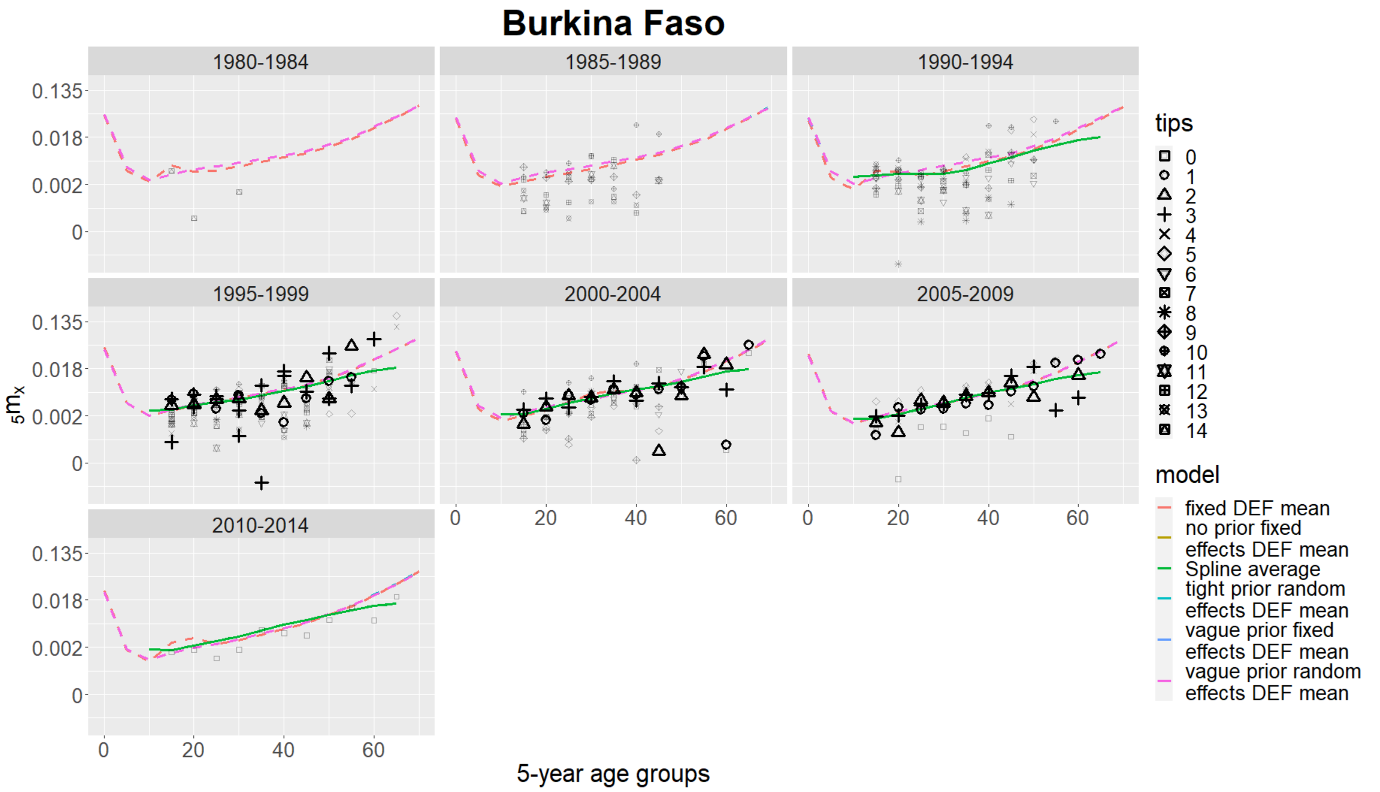
$$k \sim N(0,1)$$

$$\begin{aligned} \log(h) &= IGME_h + \sigma_h AR1(\rho_h), \\ \sigma_h &\sim IG(1, 0.0109), \\ \rho_h &\sim N(0,10), \end{aligned}$$

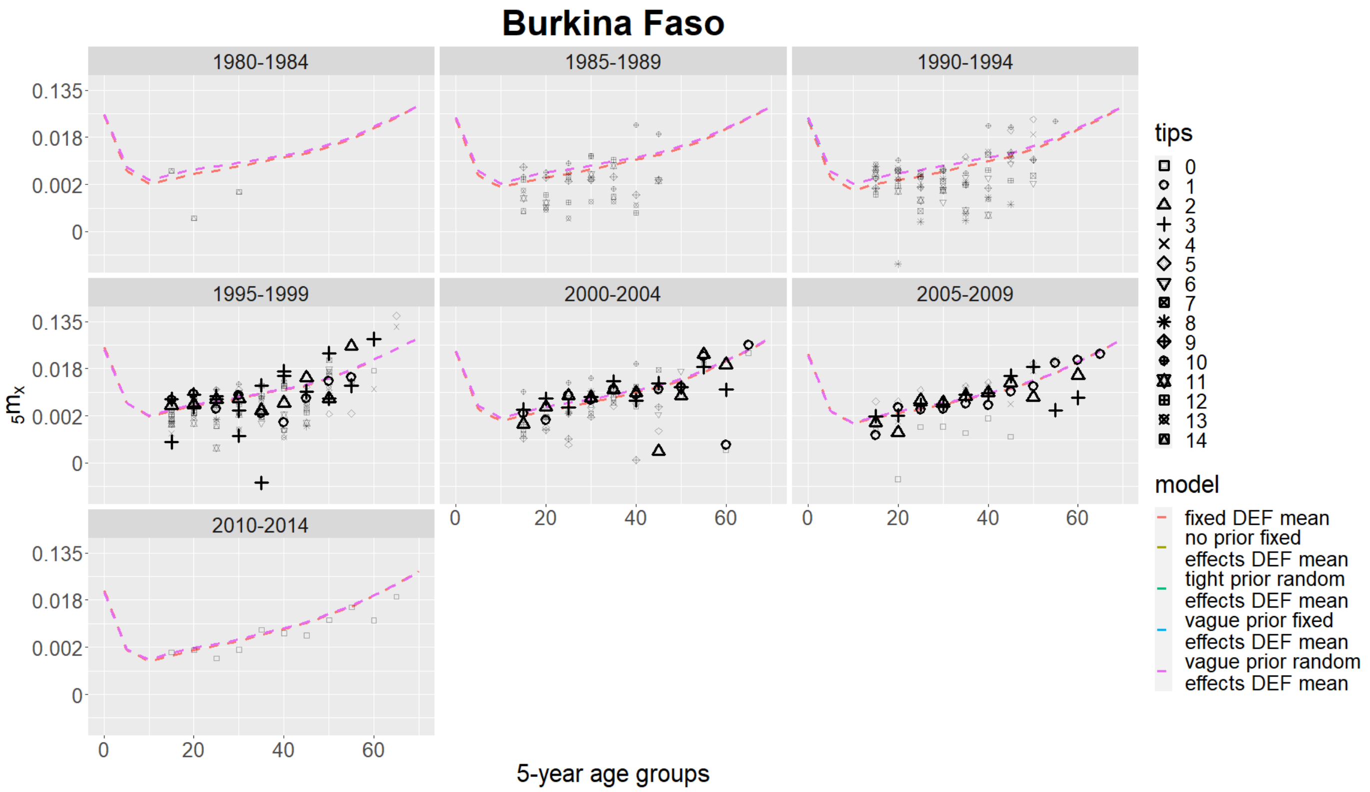
$$\begin{aligned} \log(i) &= \mu_i + \sigma_i AR1(\rho_i), \\ \sigma_i &\sim IG(1,5), \\ \text{logit}(\rho_i) &\sim N(0,10), \qquad i \in \{D, E, F\} \end{aligned}$$

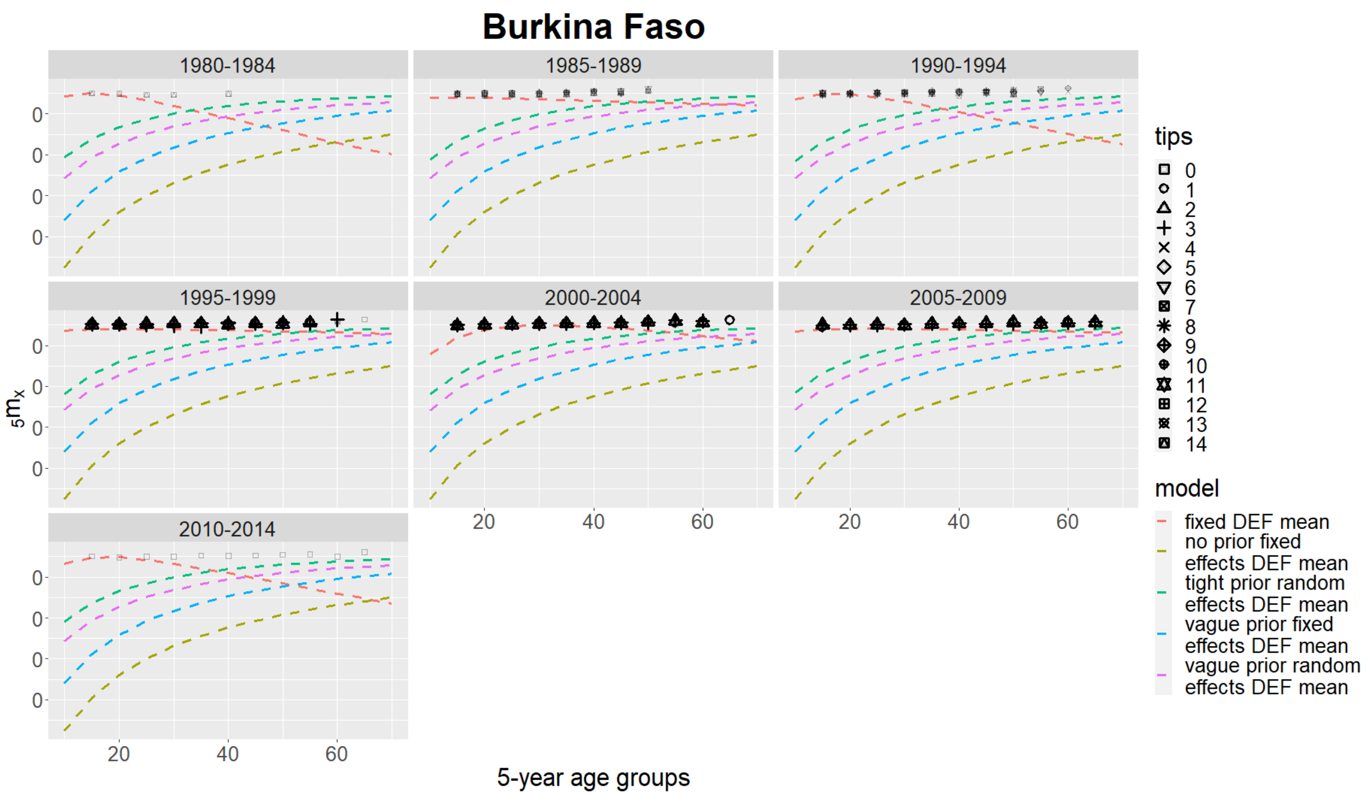
  - rather sensitive to initial values, wasted much time trying to see what works ☹
    1) μ_i estimated as fixed effects without prior $\rightarrow$ $\mu_D \approx \log(0.0006)$, $\mu_E \approx \log(4)$, $\mu_F \approx \log(641)$, obviously insensible results
    2) tried μ_i treated as constants and fixed at $\mu_D = \log(0.005)$, $\mu_E = \log(8)$, $\mu_F = \log(35)$
       $\rightarrow$ still gives high values of $F(\approx 120)$, often wouldn't converge unless E fixed at higher values
    3) tried μ_i treated as constants and fixed at $\mu_D = \log(0.0005)$, $\mu_E = \log(8)$, $\mu_F = \log(35)$
       $\rightarrow$ gives more sensible $F(\approx 16 \text{ to } 33)$ but wider spread E
    4) given vague normal priors on $\mu_i \sim N(\eta_i, 30)$ with $\eta_D = \log(0.0005)$, $\eta_E = \log(8)$, $\eta_F = \log(35)$
       $\rightarrow$ μ_i estimated as random effects, $F \approx 164$
       $\rightarrow$ μ_i estimated as fixed effects, $F \approx 204$
    5) given tight normal priors on $\mu_F \sim N(log(35), 0.25)$ (tighter than that wouldn't converge)
       $\rightarrow$ μ_i estimated as random effects, $F \approx 100$
       $\rightarrow$ μ_i estimated as fixed effects, wouldn't converge
    6) given slightly tighter normal priors on $\mu_F \sim N(log(35), 0.2)$, wouldn't converge
  - estimated LogQuad k remains quite consistently ≈ -0.76 in the above cases (except when μ_i are fixed at constants)
  - priors on σ_i have too strong effects on the estimated values? (even when estimated $\log(i)$ are very close to the estimated μ_i, σ_i are still relatively high)
  - results from the above cases pointing towards no additional DEF needed for females?
  - restrict possible range for F? or maybe F needs to be re-scaled?
- Fit to males data converged quite stably in different cases as mentioned before, with estimated peak of the DEF hump at around age 45 (except when vague priors are given to μ_i and they are estimated as random effects)

Female estimated mortality schedule



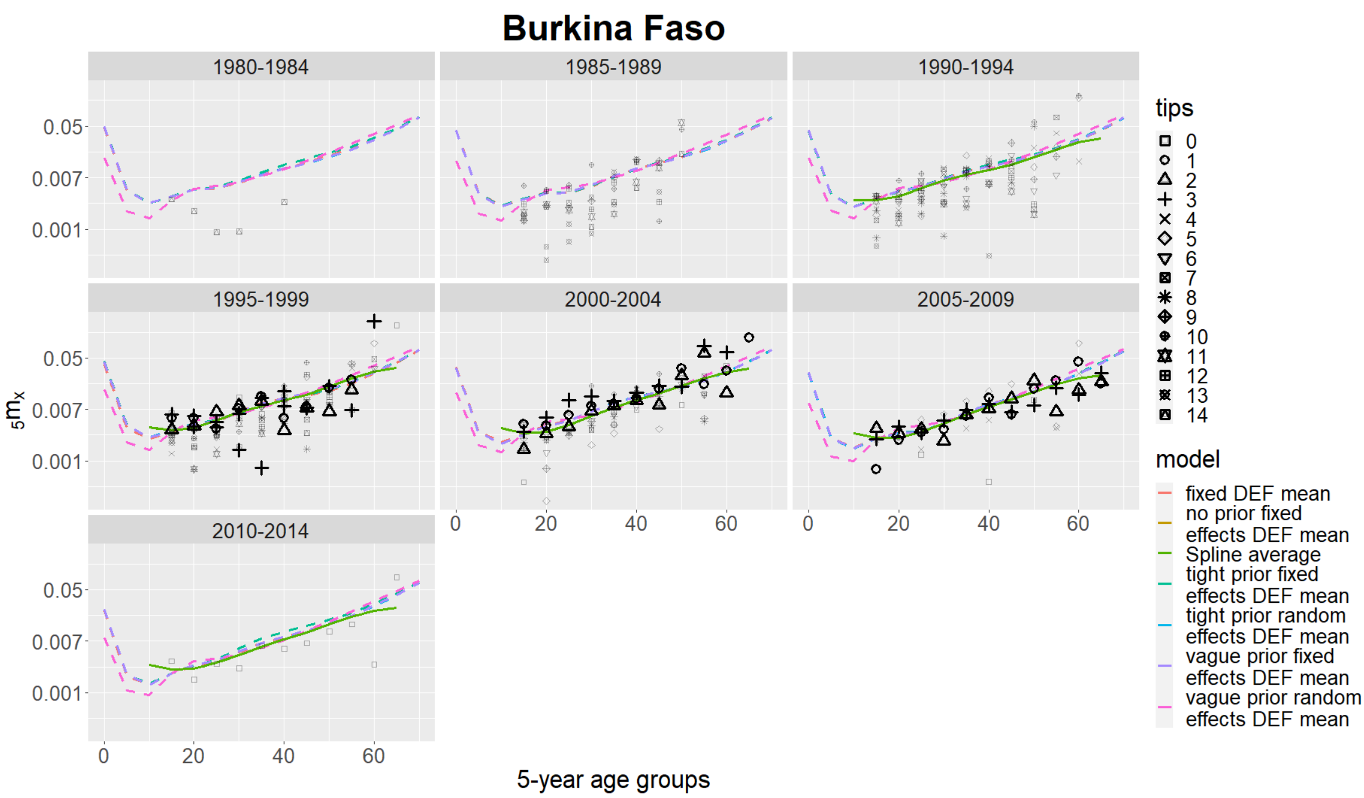
Females LQ and DEF decomposed



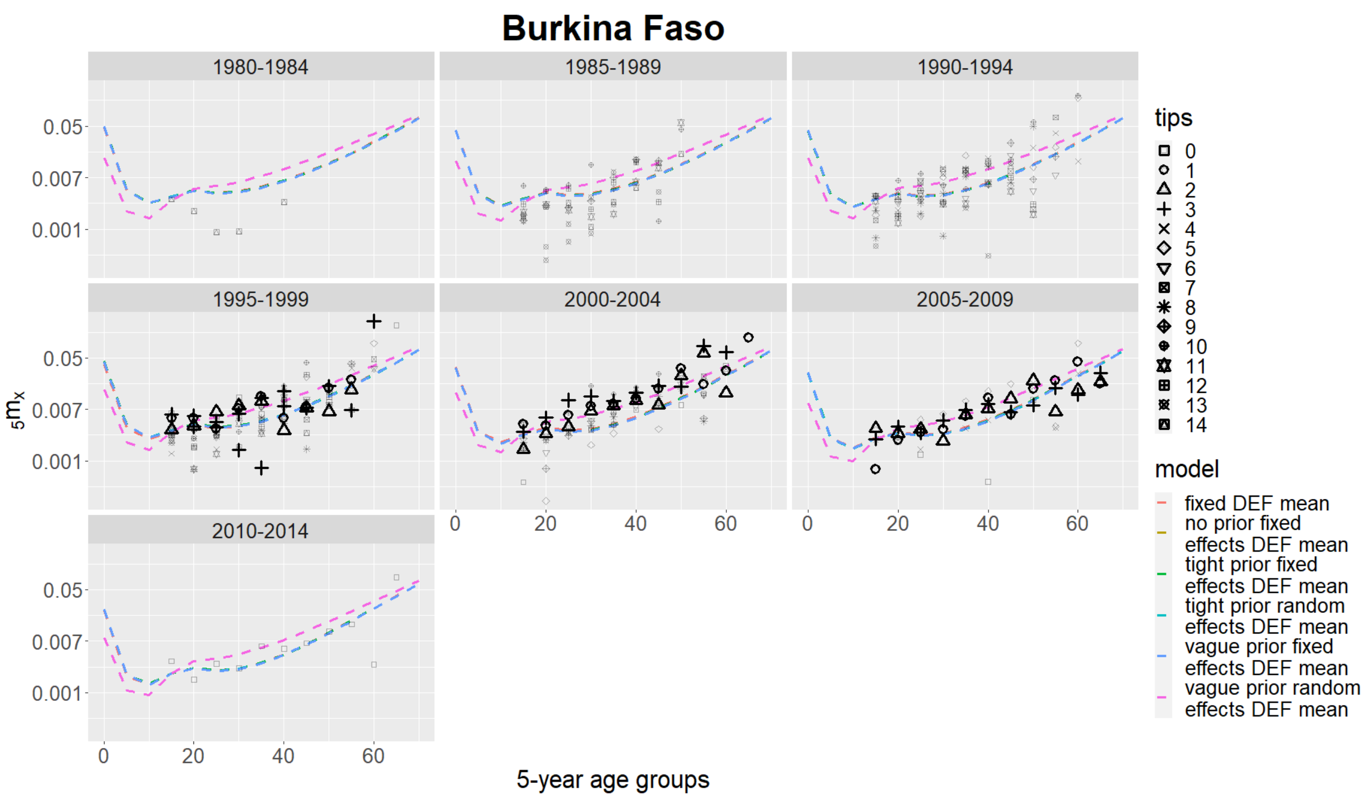


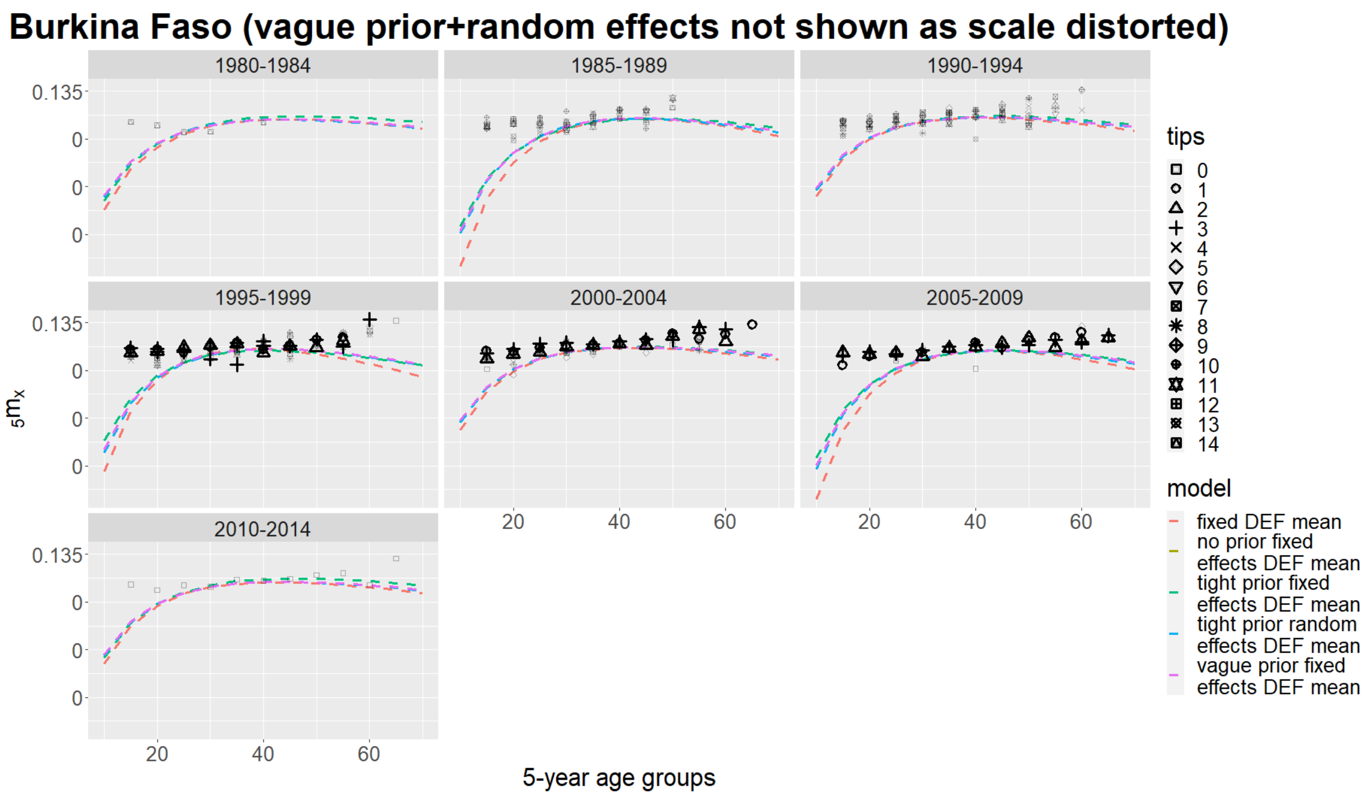
- DEF peaks at crazy old ages in most cases

Males estimated mortality schedule



Males LQ and DEF decomposed





- DEF peaks at around age 45 in most cases, except when μ_i are given vague priors and treated as random effects

- Tried to fit the Thiele model which has the following form:

$$m(x) = \underbrace{\varphi e^{-\psi x}}_{\text{negative exponential}} + \underbrace{\lambda e^{-\delta(x-\epsilon)^2}}_{\text{normal}} + \underbrace{Ae^{Bx}}_{\text{Gompertz}} \qquad \varphi, \psi, \lambda, \delta, \epsilon > 0$$

  compared to the HP-8 model:

$$\frac{q_x}{1-q_x} = A^{(x+B)^C} + De^{-E(lnx-lnF)^2} + GH^x$$

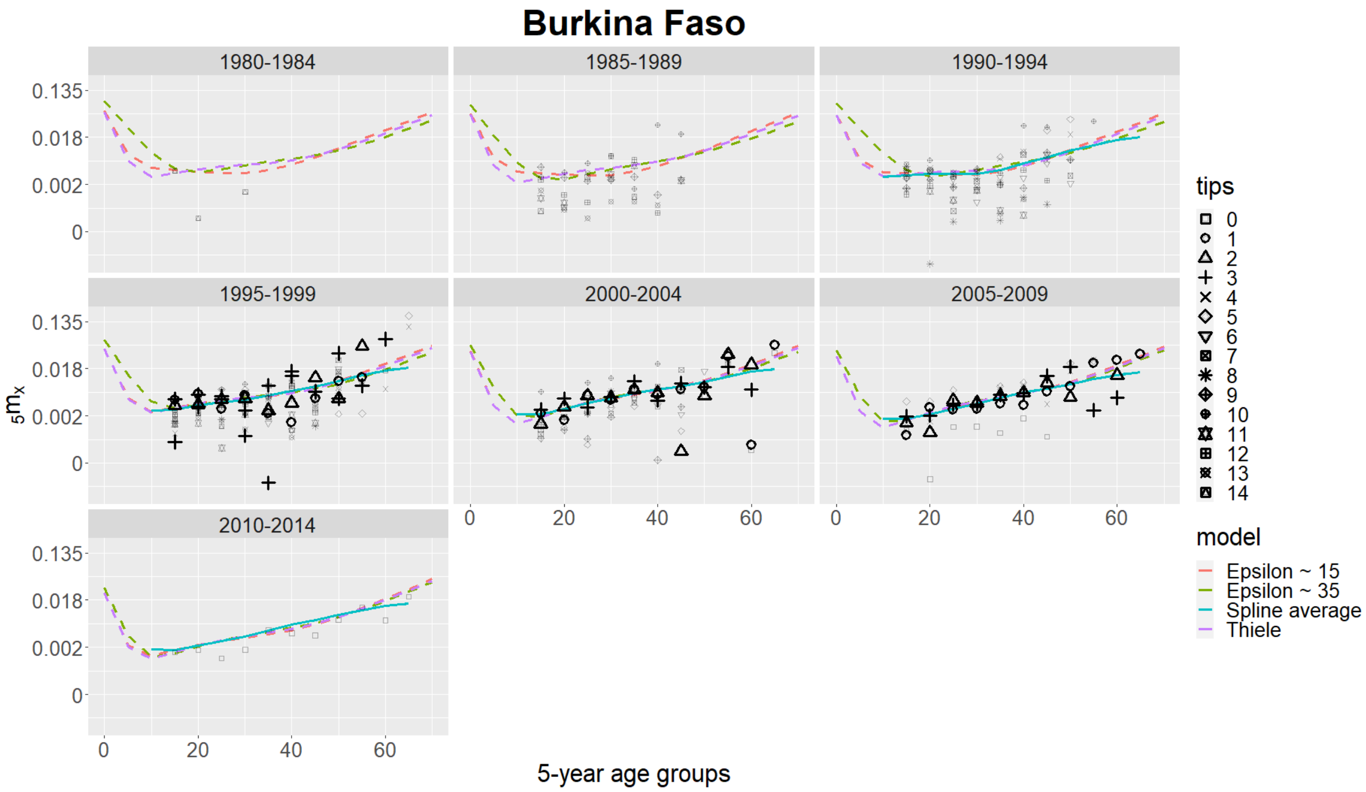
  - modelled on m_x directly
  - 7 parameters
  - priors for the parameters of the Thiele model are obtained by regressing the model on the mortality schedules from the LogQuad models fixing h at IGME estimates and $k = 0$
    1) set age = seq(2, 92, by = 5)
    2) regressed $\varphi e^{-\psi x}$ on LogQuad m_x from age 0-4 to 10-14, with penalties such that m_0 is consistent with the mortality rate implied by the IGME estimates
    3) regressed $\lambda e^{-\delta(x-\epsilon)^2}$ on Logquad m_x from age 10-14 to 40-44
    4) regressed Ae^{Bx} on LogQuad m_x from age 65-69 to 90-94
    5) regressed the whole model $\varphi e^{-\psi x} + \lambda e^{-\delta(x-\epsilon)^2} + Ae^{Bx}$ on LogQuad m_x from age 0-4 to age 90-94, with penalties such that estimated m_0 are close the IGME implied levels
  - all parameters are given log-normal priors with the regressed values as prior means and standard deviation $\sigma_i \sim IG(1, 0.01)$
  - the estimated hump seems to still have quite a strong influence at the youngest ages relative to the mid ages, should I use $(lnx - ln\epsilon)^2$ as in the HP-8 model for the hump in Thiele?

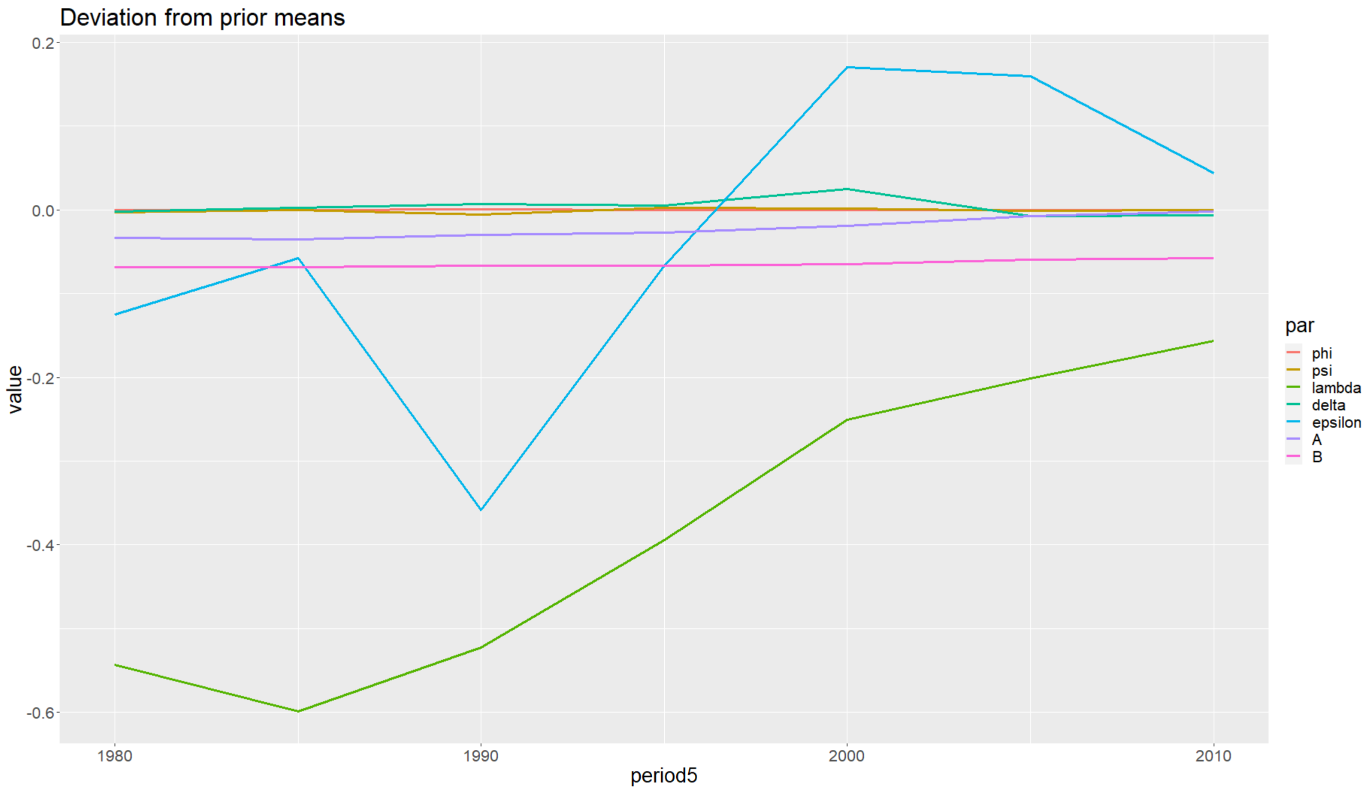
- Initial values when fitting to males sometimes need a bit tweaking (but still easier then LQ + DEF in my experience), especially the precision parameters, or maybe I was just lucky with females

- Modelled all parameters as AR(1) processes around the prior means, yet to check the estimated ρ_i

- Have done a simple robustness test on ϵ by setting different prior means manually

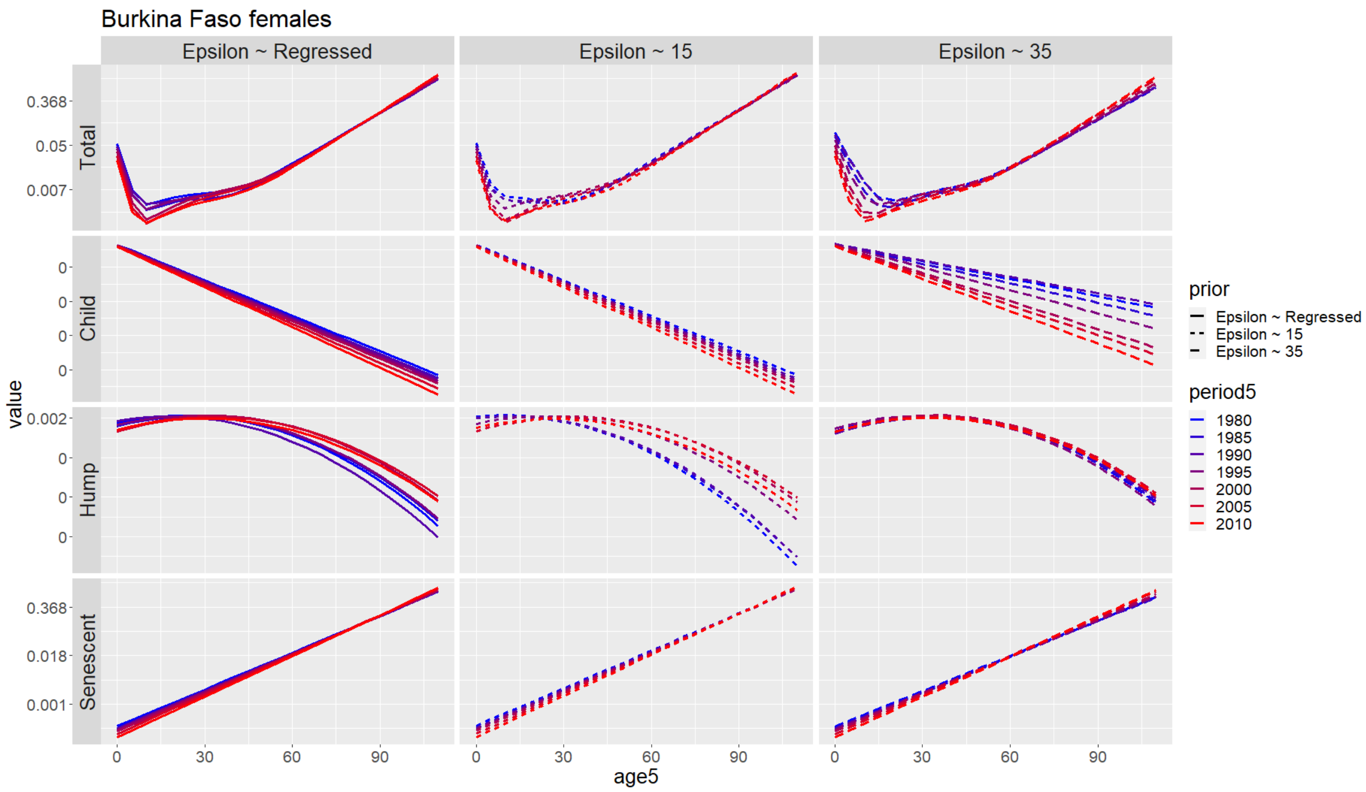
Females estimated mortality schedules





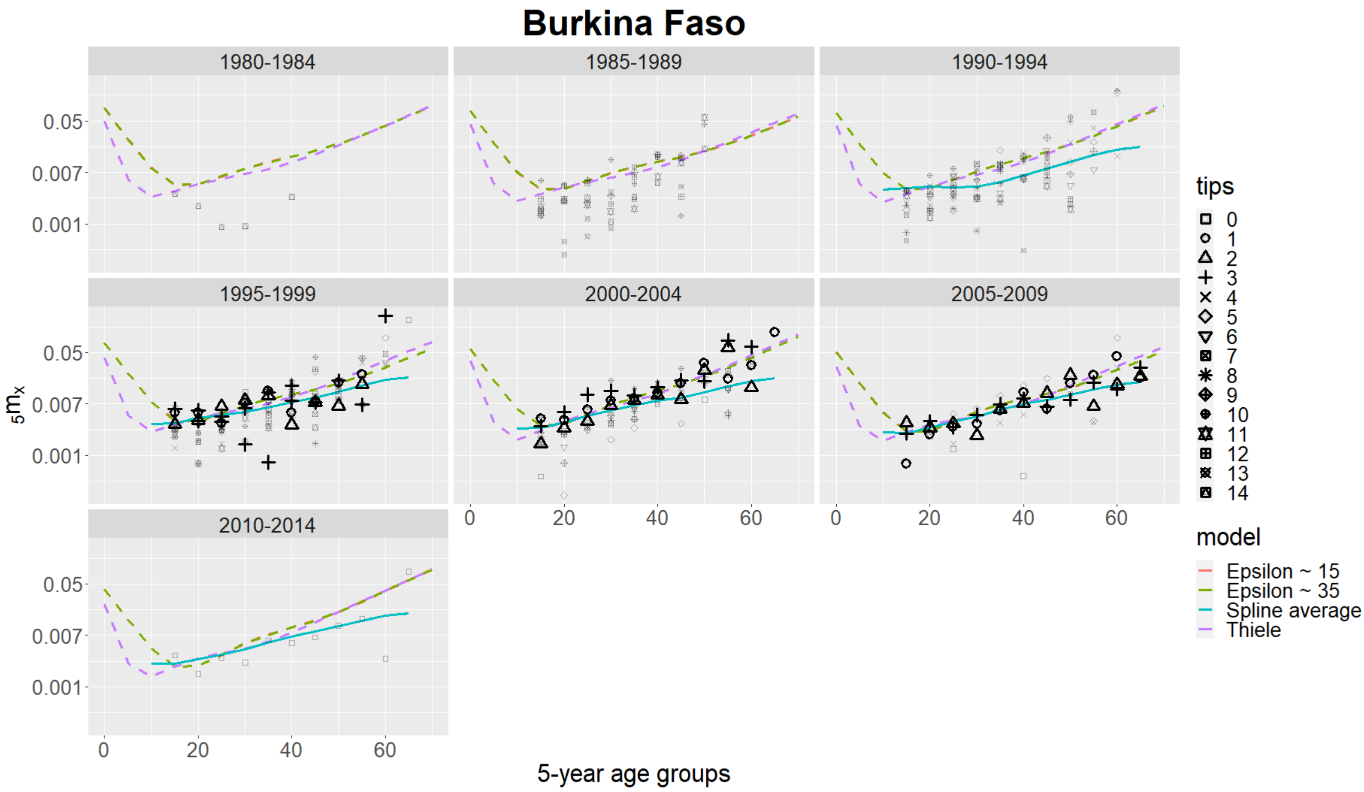
- most variations seen in λ and ϵ

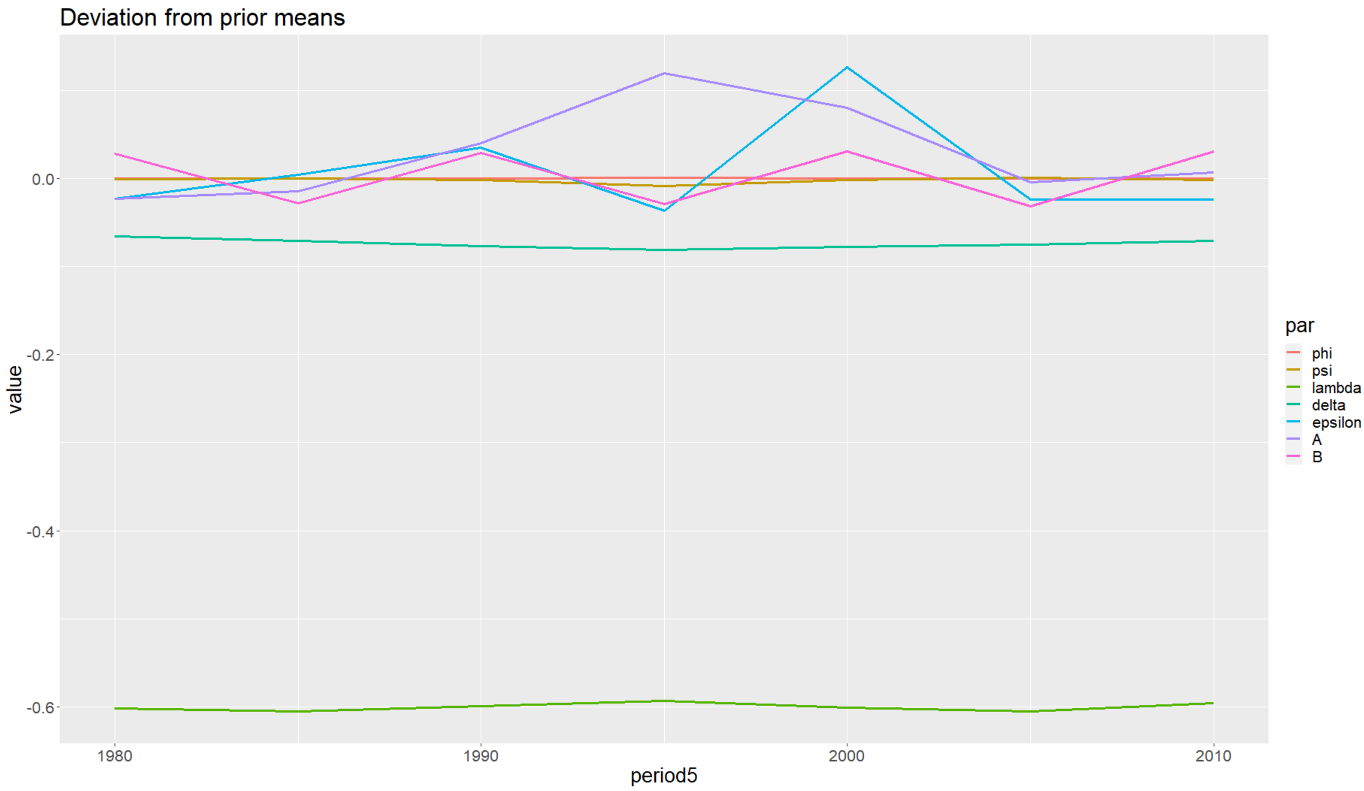
Females Thiele Decomposed



- slight shift in estimated ϵ to older ages
- when prior means for ϵ set to 35 doesnt seem to shift

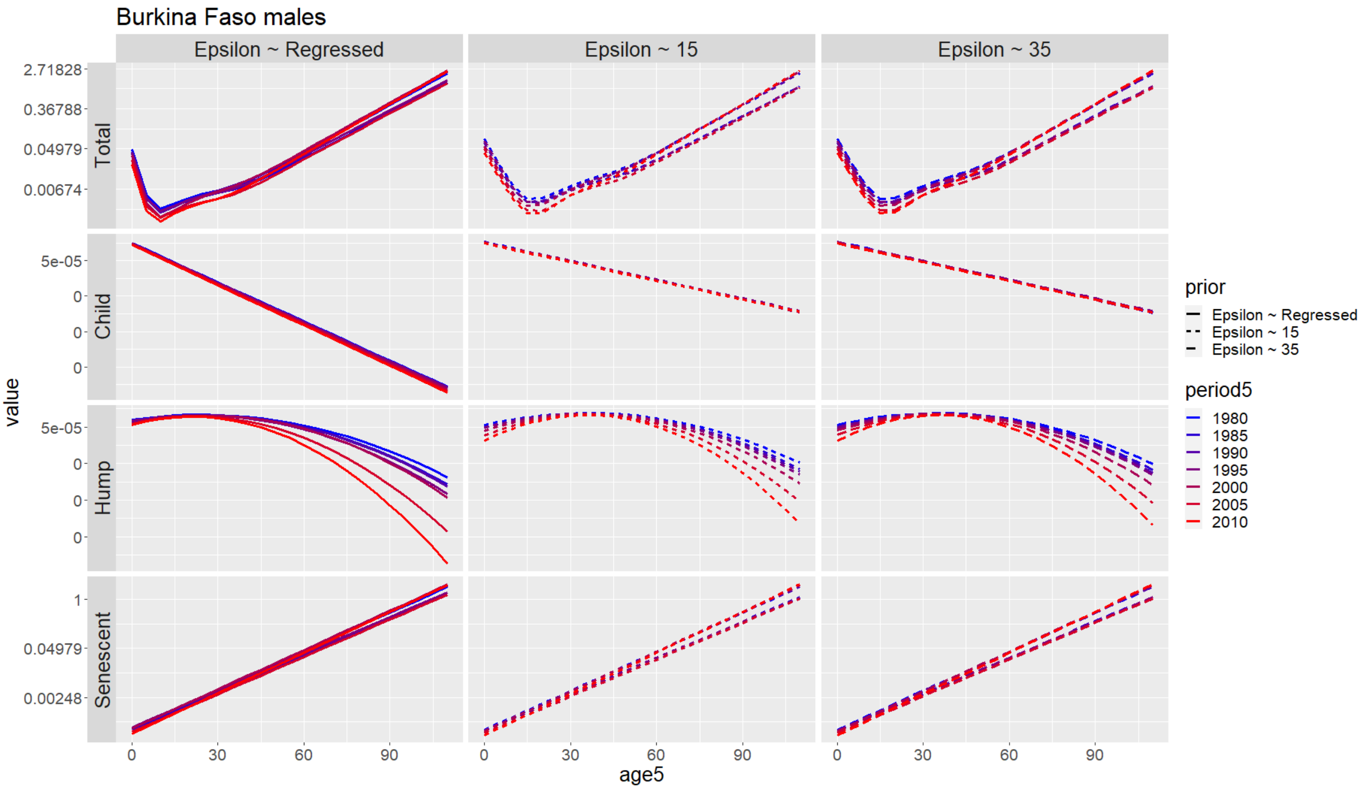
Males estimated mortality schedules





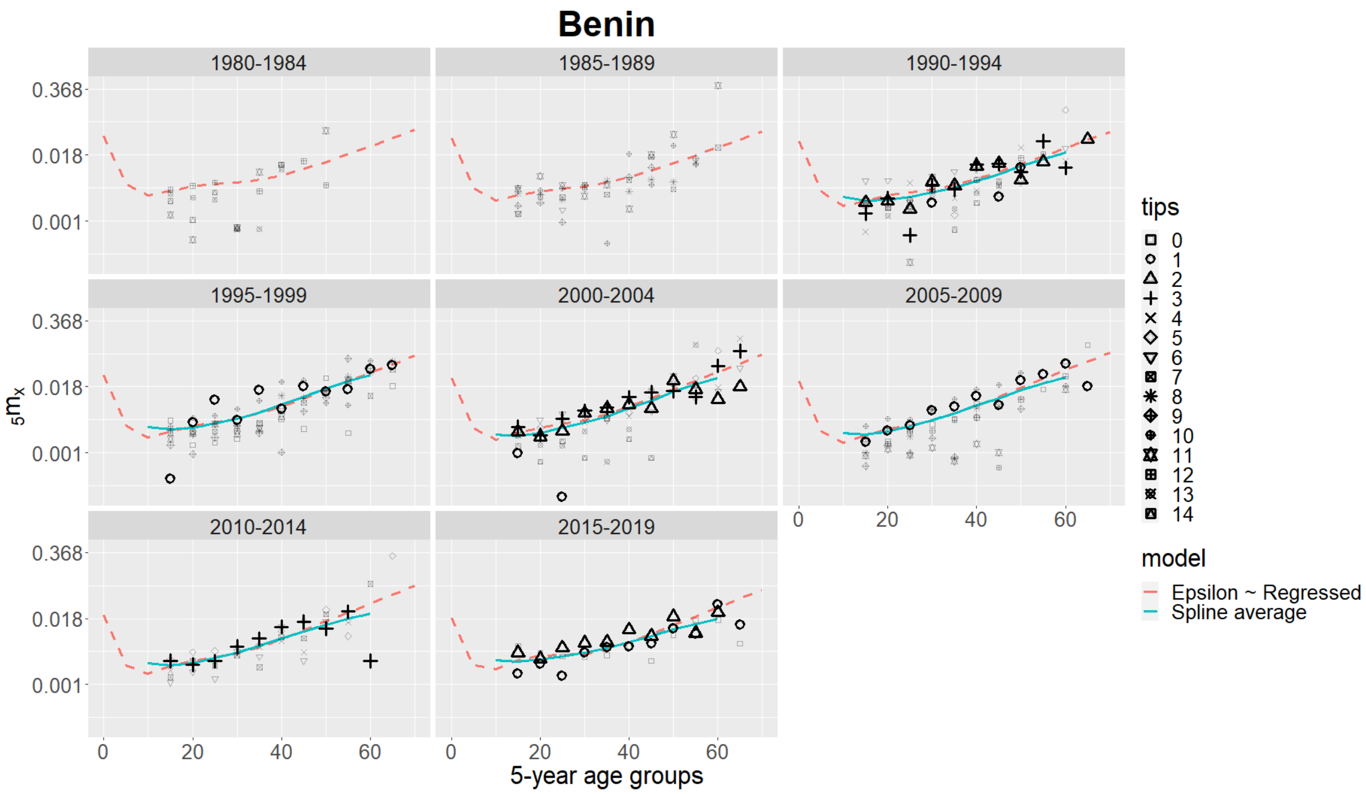
- most variations seen in λ

Males Thiele Decomposed

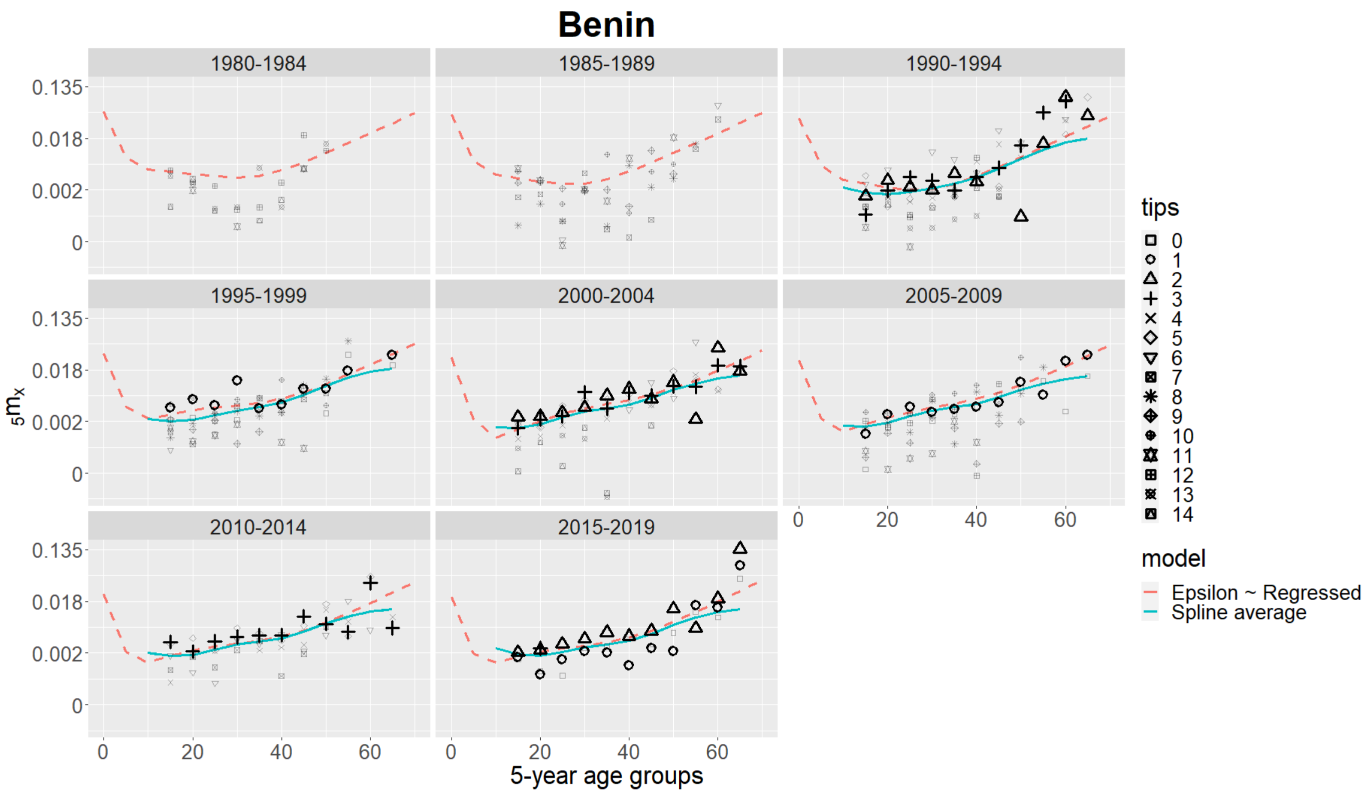


- slight shift in estimated ϵ to younger ages (whereas in females it moves to older ages)
- seem to be more robust for males as the shapes are more consistent even with different prior means on ϵ, possibly because of a more identifiable hump?
- Female spline estimates plotted by mistake!

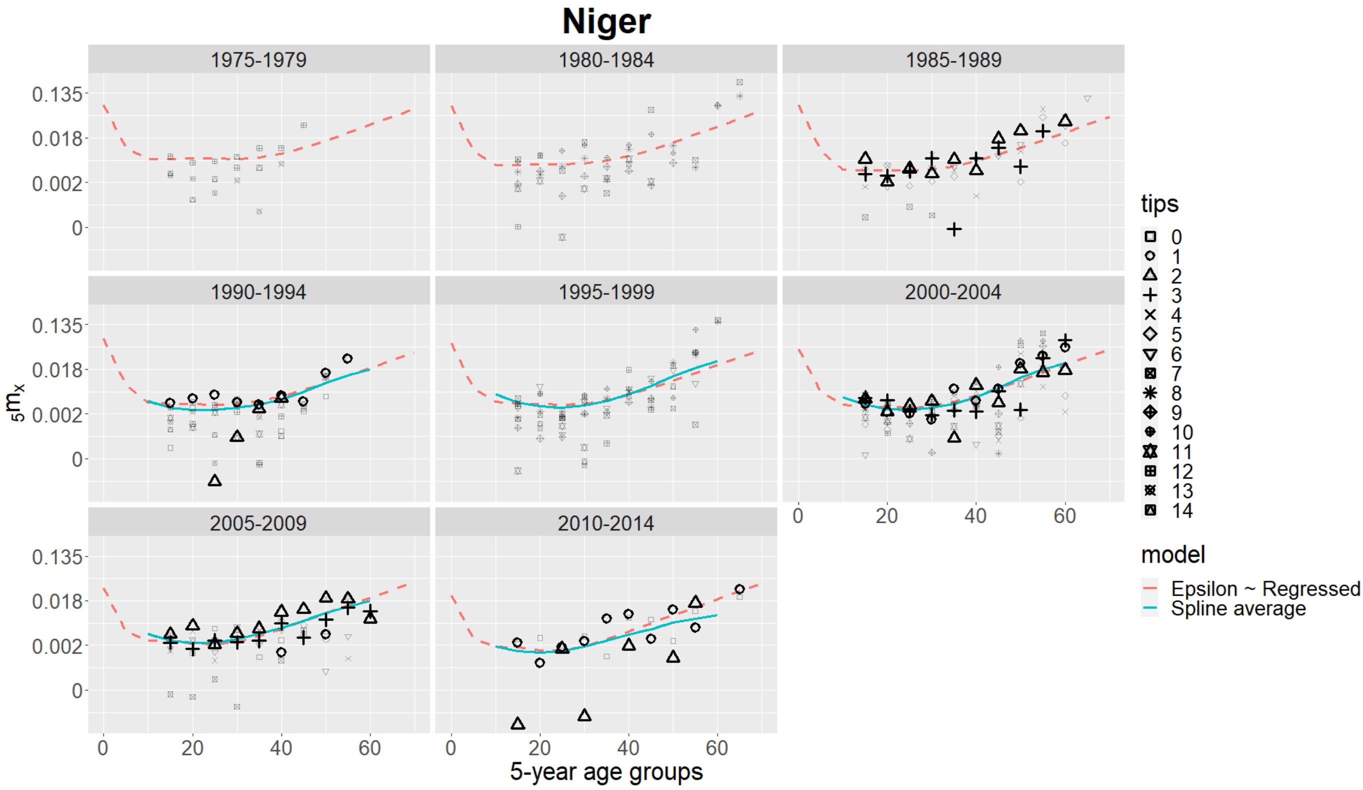
Also tried to fit to some selected countries



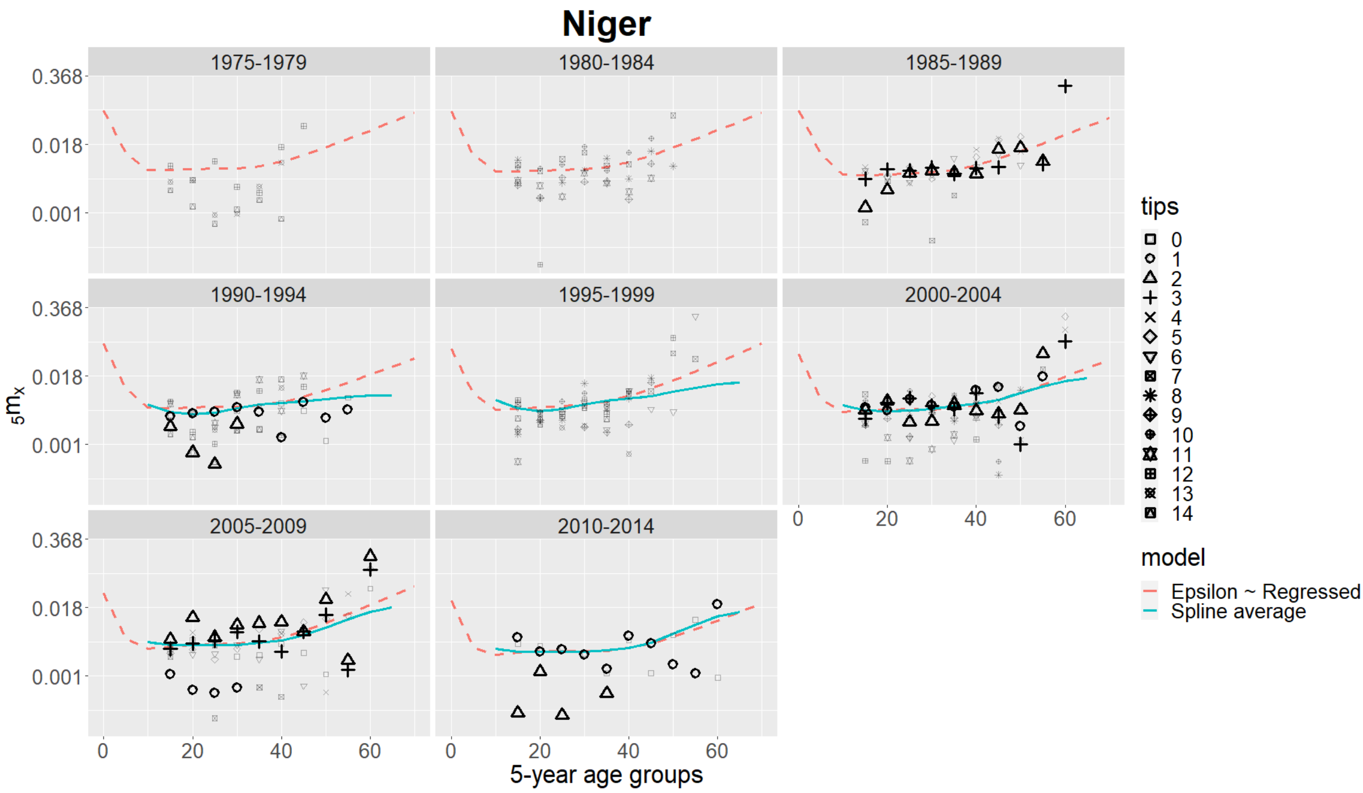
Males



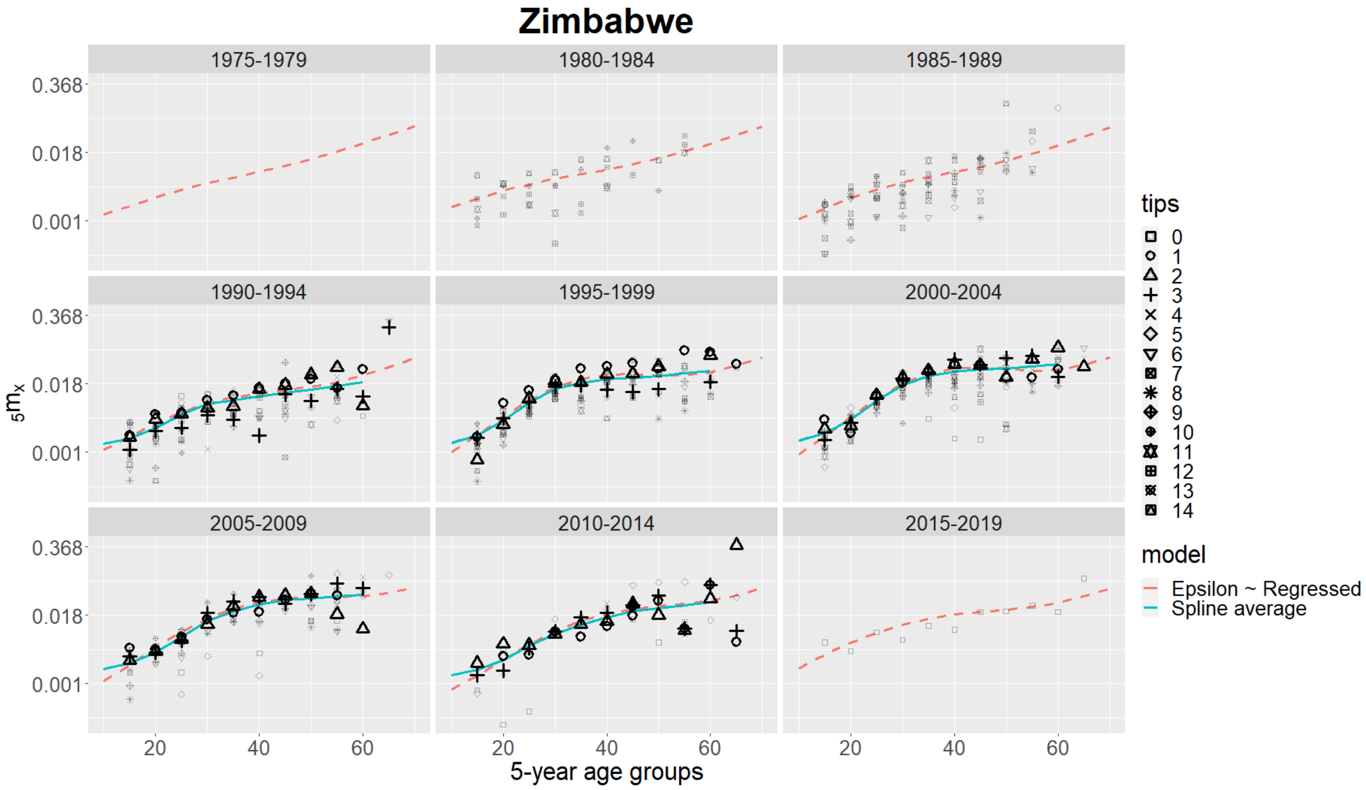
Females



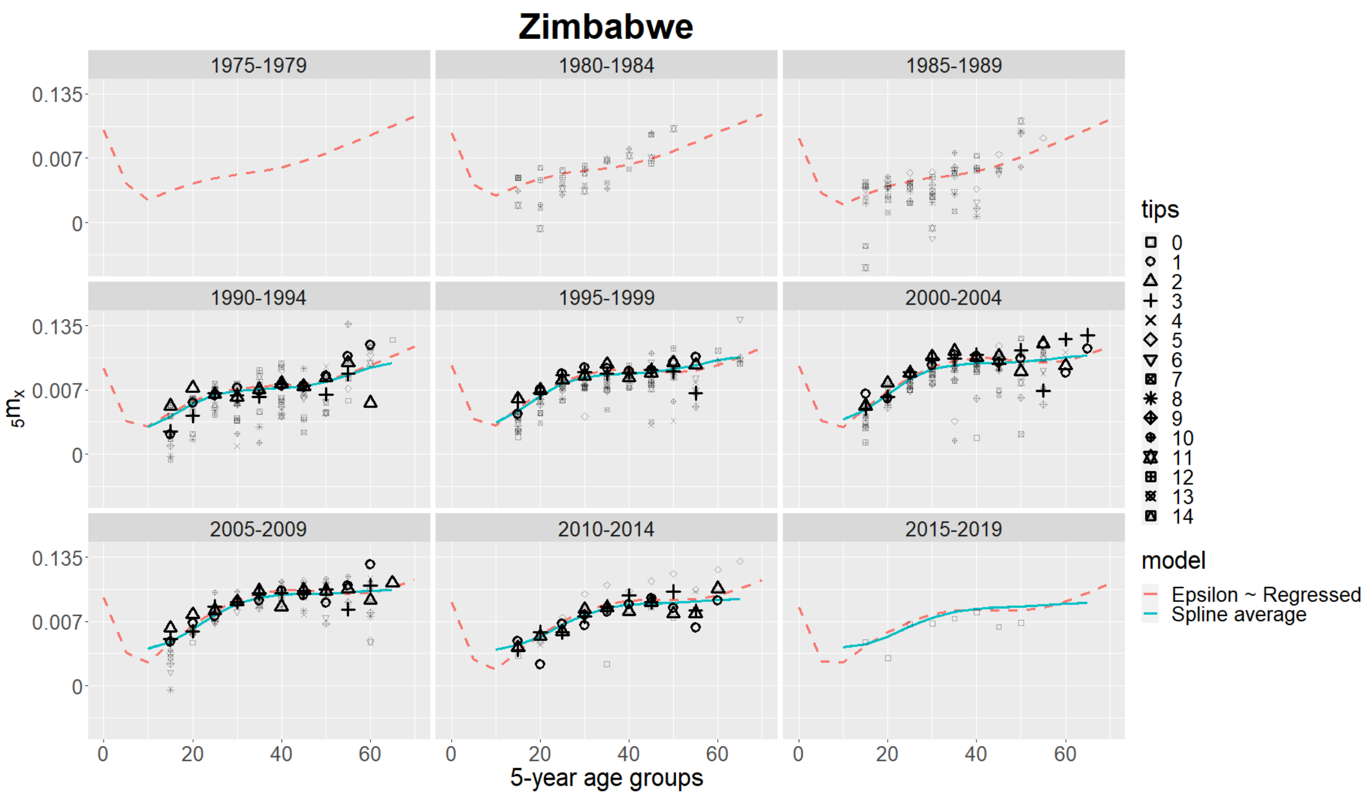
Males



Females



Males



Females

- Splines seem to over-smooth the hump