Exercise 1.2

(a) Show that in the regression model $\log(y_i) = \alpha + \beta \log(x_i) + \epsilon_i$, the elasticity of y with respect to x is equal to β (that is, does not depend on the values of x_i and y_i).

Answer:

$$\begin{aligned}
&\log(y_i) = \alpha + \beta \log(x_i) + \epsilon_i \\
&y_i = e^{\alpha+\beta\log(x_i)+\epsilon_i} \\
&\frac{y_i}{x_i} = \frac{\beta}{x_i} e^{\alpha+\beta\log(x_i)+\epsilon_i}
\end{aligned}$$

$$\begin{aligned}
\text{Elasticity} = \frac{dy}{dx} \times \frac{x}{y} &= \left(\frac{\beta}{x_i} e^{\alpha+\beta\log(x_i)+\epsilon_i}\right) \times \frac{x_i}{y_i} \\
&= \left(\frac{\beta}{x_i} y_i\right) \times \frac{x_i}{y_i} = \beta
\end{aligned}$$

(b) Determine the elasticity of y with respect to x in the model $y_i = \alpha + \beta \log(x_i) + \epsilon_i$

Answer:

$$\begin{aligned}
\text{Elasticity} &= \frac{dy}{dx} \times \frac{x}{y} \\
&= \frac{1}{x_i} \beta \frac{x_i}{y_i} \\
&= \frac{\beta}{y_i}
\end{aligned}$$

(c) Determine the elasticity of y with respect to x in the model $\log(y_i) = \alpha + \beta x_i + \epsilon_i$

Answer:

$$\begin{aligned}
&\log(y_i) = \alpha + \beta x_i + \epsilon_i \\
&y_i = e^{\alpha+\beta x_i+\epsilon_i}
\end{aligned}$$

$$
\begin{aligned}
\text{Elasticity} &= \frac{dy}{dx} \times \frac{x}{y} \\
&= \beta e^{\alpha+\beta x_i+\epsilon_i} \frac{x_i}{y_i} \\
&= \beta y_i \frac{x_i}{y_i} \\
&= \beta x_i
\end{aligned}
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