Text Exercise 5

Part (a)

Answer:

Given:

$$\beta_2 = 0.914$$

$$\frac{\partial \Pr[\text{resp}_i = 1]}{\partial \text{active}_i} \frac{\text{active}_i}{\Pr[\text{resp}_i = 1]} = \Pr[\text{resp}_i = 0]\text{active}_i \beta_2$$

For $\text{age} = 50,\ \text{active} = 1,\ \text{male} = 1:$

$$\Pr[\text{resp}_i = 0] = \frac{1}{1 + \exp(\beta_0 + \beta_1 \text{male}_i + \beta_2 \text{active}_i + \beta_3 \text{age}_i + \beta_4 (\text{age}_i/10)^2)}$$

$$\Pr[\text{resp}_i = 0] = 0.240$$

$$\text{Elasticity} = \Pr[\text{resp}_i = 0]\text{active}_i \beta_2 = \mathbf{0.219}$$

For $\text{age} = 50,\ \text{active} = 0,\ \text{male} = 1:$

$$\text{Elasticity} = \Pr[\text{resp}_i = 0]\text{active}_i \beta_2 = \mathbf{0}$$

Part (b)

Answer:

Given:

$$\frac{\Pr[\text{resp}_i = 1 \mid \text{active}_i = 1] - \Pr[\text{resp}_i = 1 \mid \text{active}_i = 0]}{\Pr[\text{resp}_i = 1 \mid \text{active}_i = 0]}$$

Let:

$$\gamma = \beta_0 + \beta_1 \text{male}_i + \beta_2 \text{active}_i + \beta_3 \text{age}_i + \beta_4 (\text{age}_i/10)^2$$

Such that:

$$\Pr[\text{resp}_i = 0] = \frac{1}{1 + \exp(\gamma + \beta_2 \text{active}_i)}$$

Therefore:

$$\frac{\Pr[\text{resp}_i = 1 \mid \text{active}_i = 1] - \Pr[\text{resp}_i = 1 \mid \text{active}_i = 0]}{\Pr[\text{resp}_i = 1 \mid \text{active}_i = 0]} =$$

$$\frac{\frac{\exp(\gamma+\beta_2)}{1+\exp(\gamma+\beta_2)}}{\frac{\exp(\gamma)}{1+\exp(\gamma)}} - 1 =$$

$$\frac{\exp(\gamma)\exp(\beta_2)}{1 + \exp(\gamma + \beta_2)} \frac{1 + \exp(\gamma)}{\exp(\gamma)} - 1 =$$

$$\frac{\exp(\beta_2) + \exp(\gamma + \beta_2)}{1 + \exp(\gamma + \beta_2)} - \frac{1 + \exp(\gamma + \beta_2)}{1 + \exp(\gamma + \beta_2)} =$$

$$\frac{\exp(\beta_2)}{1 + \exp(\gamma + \beta_2)} - \frac{1}{1 + \exp(\gamma + \beta_2)} =$$

$$(\exp(\beta_2) - 1)\frac{1}{1 + \exp(\gamma + \beta_2)} =$$

$$(\exp(\beta_2) - 1)\Pr[\text{resp}_i = 0 \mid \text{active}_i = 1]$$

Part (c)

Answer:

Given:

$$\text{Elasticity} = (\exp(\beta_2) - 1)\Pr[\text{resp}_i = 0 \mid \text{active}_i = 1]$$

For $\text{age} = 50,\ \text{active} = 1,\ \text{male} = 1:$

$$\Pr[\text{resp}_i = 0 \mid \text{active}_i = 1] = \frac{1}{1 + \exp(\beta_0 + \beta_1 \text{male}_i + \beta_2 + \beta_3 \text{age}_i + \beta_4 (\text{age}_i/10)^2)}$$

$$\Pr[\text{resp}_i = 0 \mid \text{active}_i = 1] = 0.240$$

$$\text{Elasticity} = (\exp(\beta_2) - 1)\Pr[\text{resp}_i = 0 \mid \text{active}_i = 1] = \mathbf{0.358}$$