# Unit 2

**LOS 1**. Gaussian elimination

- form: $Ax = b$ where $A$ is an $m \times n$ matrix, $x$ is a $m$ column vector and b is a $m$ column vector
- augmented matrix: $[A \mid b]$
- steps for Gaussian elimination
  - choose a pivot (any element in the matrix): for example the underlined positions in the below matrix are pivots
    $$\begin{pmatrix} \underline{1} & 2 & 3 & 4 \\ 5 & \underline{6} & 7 & 8 \\ 6 & 7 & \underline{8} & 9 \end{pmatrix}$$
  - carry out row operations i.e. $R1 - R2$ such that the elements in the same column that are below the pivot becomes 0
  - repeat for a pivot on a different column until the $A$ part of the augmented matrix becomes an upper triangular matrix
  - back substitution

**LOS 2**. Row reduced row echelon form (RREF)

- steps:
  - after getting the upper triangular matrix, carry out further row operations such that in the pivot columns, the other elements other than the pivot are zero
  - divide the rows such that all pivots take the value of 1
  - pivot columns: columns containing pivot
  - non-pivot columns: columns not containing pivot

**LOS 3**. Computing inverses

- inverses are a combination of Gaussian elimination problems

  $AA^{-1} = I$

  $Aa_i^{-1} = e_i$ where $a_i$ is the $i$-th column of $A^{-1}$ and $e_i$ is the $i$-th column of $I$
- form:

  $$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

  $$\text{Aug}A = A = \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 6 & 0 & 1 & 0 \\ 7 & 8 & 9 & 0 & 0 & 1 \end{array}\right)$$
- Solve augmented matrix using RREF or Gaussian elimination

**LOS 4**. Elementary matrices

- identity matrix with one of the zeros replaced by a number
- Gaussian elimination is equivalent to multiplying $A$ by elementary matrices
- $M_3M_2M_1A = U$ where $U \in$ upper triangular matrix (the final result of Gaussian elimination)

**LOS 5**. LU Decomposition

- $A = LU$ where $L = M_1^{-1}M_2^{-1}M_3^{-1}$
- Solving $(LU)x = b$
    - Let $y = Ux$
    - Solve $Ly = b$ for $y$
    - Solve $Ux = y$ for $x$