
# CS489/698: Intro to ML

## Lecture 12: Training Neural Networks and CNNs part 1

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# Outline

- Training

- Convolutional Layers

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- Training

- Convolutional Layers

# SGD Formalized

**Algorithm 8.1** Stochastic gradient descent (SGD) update at training iteration $k$

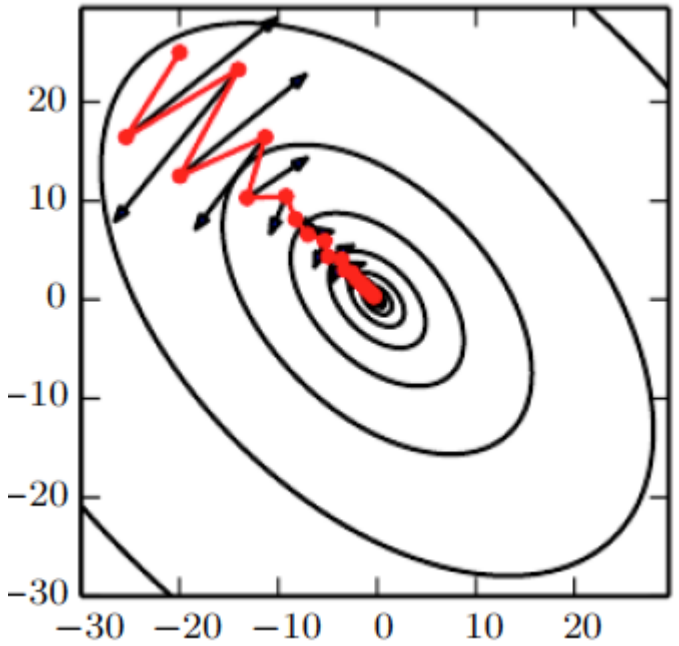
**Require:** Learning rate $\epsilon_k$.
**Require:** Initial parameter $\boldsymbol{\theta}$
  **while** stopping criterion not met **do**
    Sample a minibatch of $m$ examples from the training set $\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(m)}\}$ with corresponding targets $\boldsymbol{y}^{(i)}$.
    Compute gradient estimate: $\hat{\boldsymbol{g}} \leftarrow +\frac{1}{m} \nabla_{\boldsymbol{\theta}} \sum_i L(f(\boldsymbol{x}^{(i)}; \boldsymbol{\theta}), \boldsymbol{y}^{(i)})$
    Apply update: $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \epsilon \hat{\boldsymbol{g}}$
  **end while**

# Momentum

- Adds a velocity term
- $$\boldsymbol{v} \leftarrow \alpha \boldsymbol{v} - \epsilon \nabla_{\boldsymbol{\theta}} \left( \frac{1}{m} \sum_{i=1}^{m} L(\boldsymbol{f}(\boldsymbol{x}^{(i)}; \boldsymbol{\theta}), \boldsymbol{y}^{(i)}) \right),$$

  $$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \boldsymbol{v}.$$

- Adds a speedup of at most $\frac{\epsilon ||\boldsymbol{g}||}{1 - \alpha}$.




where 0 < m < 1 is a new global parameter which must be determined by trial and error. Momentum simply adds a fraction m of the previous weight update to the current one. When the gradient keeps pointing in the same direction, this will increase the size of the steps taken towards the minimum. It is otherefore often necessary to reduce the global learning rate $\mu$ when using a lot of momentum (m close to 1). If you combine a high learning rate with a lot of momentum, you will rush past the minimum with huge steps

- Momentum constant usually 0.9, 0.5, 0.99 corresponding to a 10x, 2x, 100x increase in max speed



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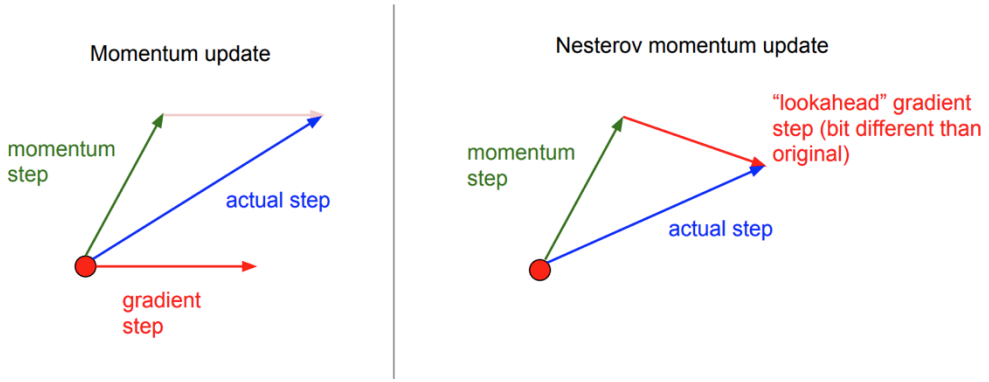
# Nesterov Momentum

- Applies momentum first then gradient

$$\boldsymbol{v} \leftarrow \alpha \boldsymbol{v} - \epsilon \nabla_{\boldsymbol{\theta}} \left( \frac{1}{m} \sum_{i=1}^{m} L(\boldsymbol{f}(\boldsymbol{x}^{(i)}; \boldsymbol{\theta}), \boldsymbol{y}^{(i)}) \right),$$

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \boldsymbol{v}.$$

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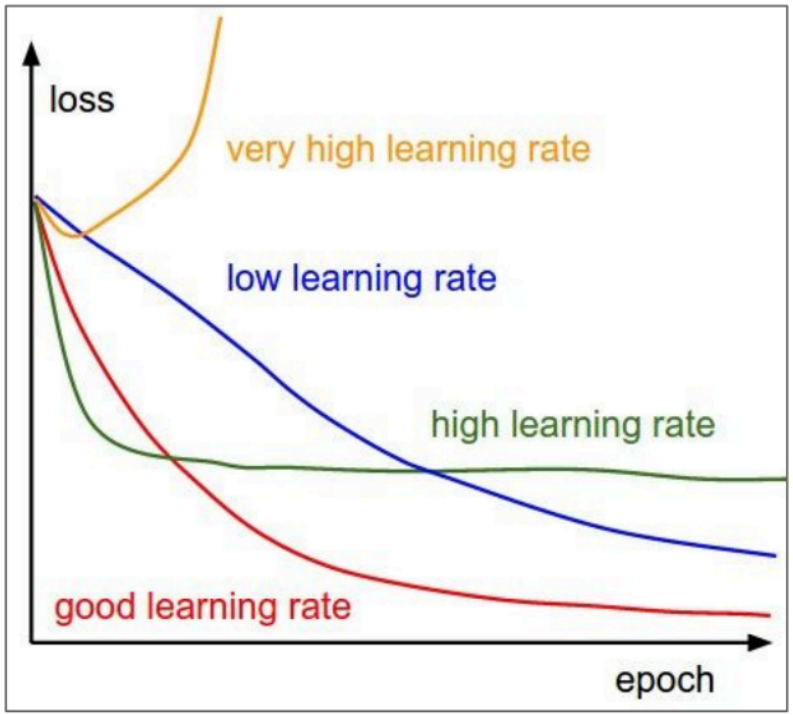
# Setting the learning rate




**=> Learning rate decay over time!**

**step decay:**
e.g. decay learning rate by half every few epochs.

**exponential decay:**

$$\alpha = \alpha_0 e^{-kt}$$

**1/t decay:**

$$\alpha = \alpha_0 / (1 + kt)$$

- Also good to try a setting for 100 iterations and see which is best on validation set

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 http://cs231n.stanford.edu/slides/2016/winter1516_lecture6.pdf

# Adagrad

- Adapting the learning rate per parameter
- If gradient is always high, decreases learning rate
- If gradient is low, increases learning rate
- meant for fast convergence on convex problems
- suffers from premature decay

**AdaGrad is an optimization method that allows different step sizes for different features. It increases the influence of rare but informative features**

# Adagrad

**Algorithm 8.4** The AdaGrad algorithm

**Require:** Global learning rate $\epsilon$

**Require:** Initial parameter $\boldsymbol{\theta}$

**Require:** Small constant $\delta$, perhaps $10^{-7}$, for numerical stability

Initialize gradient accumulation variable $\boldsymbol{r} = \mathbf{0}$

**while** stopping criterion not met **do**

Sample a minibatch of $m$ examples from the training set $\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(m)}\}$ with corresponding targets $\boldsymbol{y}^{(i)}$.

Compute gradient: $\boldsymbol{g} \leftarrow \frac{1}{m} \nabla_{\boldsymbol{\theta}} \sum_i L(f(\boldsymbol{x}^{(i)}; \boldsymbol{\theta}), \boldsymbol{y}^{(i)})$

Accumulate squared gradient: $\boldsymbol{r} \leftarrow \boldsymbol{r} + \boldsymbol{g} \odot \boldsymbol{g}$

Compute update: $\Delta\boldsymbol{\theta} \leftarrow -\frac{\epsilon}{\delta + \sqrt{\boldsymbol{r}}} \odot \boldsymbol{g}$. (Division and square root applied element-wise)

Apply update: $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \Delta\boldsymbol{\theta}$

**end while**

# RMSProp

- Replaces average with exponential decay
- Prevents premature decay

**Algorithm 8.6** RMSProp algorithm with Nesterov momentum

**Require:** Global learning rate $\epsilon$, decay rate $\rho$, momentum coefficient $\alpha$.

**Require:** Initial parameter $\boldsymbol{\theta}$, initial velocity $\boldsymbol{v}$.

Initialize accumulation variable $\boldsymbol{r} = \boldsymbol{0}$

**while** stopping criterion not met **do**

Sample a minibatch of $m$ examples from the training set $\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(m)}\}$ with corresponding targets $\boldsymbol{y}^{(i)}$.

Compute interim update: $\tilde{\boldsymbol{\theta}} \leftarrow \boldsymbol{\theta} + \alpha \boldsymbol{v}$

Compute gradient: $\boldsymbol{g} \leftarrow \frac{1}{m} \nabla_{\tilde{\boldsymbol{\theta}}} \sum_i L(f(\boldsymbol{x}^{(i)}; \tilde{\boldsymbol{\theta}}), \boldsymbol{y}^{(i)})$

Accumulate gradient: $\boldsymbol{r} \leftarrow \rho \boldsymbol{r} + (1 - \rho) \boldsymbol{g} \odot \boldsymbol{g}$

Compute velocity update: $\boldsymbol{v} \leftarrow \alpha \boldsymbol{v} - \frac{\epsilon}{\sqrt{\boldsymbol{r}}} \odot \boldsymbol{g}$. ($\frac{1}{\sqrt{\boldsymbol{r}}}$ applied element-wise)

Apply update: $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \boldsymbol{v}$

**end while**

# Adam - Adaptive Momentum

- Applies momentum with parameterized learning rates

**Adaptive Moment Estimation (Adam) [15] is another method that computes adaptive learning rates for each parameter. In addition to storing an exponentially decaying average of past squared gradients vtvt like Adadelta and RMSprop, Adam also keeps an exponentially decaying average of past gradients mtmt, similar to momentum:**

**Algorithm 8.7** The Adam algorithm

**Require:** Step size $\epsilon$ (Suggested default: 0.001)
**Require:** Exponential decay rates for moment estimates, $\rho_1$ and $\rho_2$ in $[0, 1)$. (Suggested defaults: 0.9 and 0.999 respectively)
**Require:** Small constant $\delta$ used for numerical stabilization. (Suggested default: $10^{-8}$)
**Require:** Initial parameters $\boldsymbol{\theta}$
  Initialize 1st and 2nd moment variables $\boldsymbol{s} = \boldsymbol{0}$, $\boldsymbol{r} = \boldsymbol{0}$
  Initialize time step $t = 0$
  **while** stopping criterion not met **do**
    Sample a minibatch of $m$ examples from the training set $\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(m)}\}$ with corresponding targets $\boldsymbol{y}^{(i)}$.
    Compute gradient: $\boldsymbol{g} \leftarrow \frac{1}{m} \nabla_{\boldsymbol{\theta}} \sum_i L(f(\boldsymbol{x}^{(i)}; \boldsymbol{\theta}), \boldsymbol{y}^{(i)})$
    $t \leftarrow t + 1$
    Update biased first moment estimate: $\boldsymbol{s} \leftarrow \rho_1 \boldsymbol{s} + (1 - \rho_1)\boldsymbol{g}$
    Update biased second moment estimate: $\boldsymbol{r} \leftarrow \rho_2 \boldsymbol{r} + (1 - \rho_2)\boldsymbol{g} \odot \boldsymbol{g}$
    Correct bias in first moment: $\hat{\boldsymbol{s}} \leftarrow \frac{\boldsymbol{s}}{1 - \rho_1^t}$
    Correct bias in second moment: $\hat{\boldsymbol{r}} \leftarrow \frac{\boldsymbol{r}}{1 - \rho_2^t}$
    Compute update: $\Delta\boldsymbol{\theta} = -\epsilon \frac{\hat{\boldsymbol{s}}}{\sqrt{\hat{\boldsymbol{r}}} + \delta}$ (operations applied element-wise)
    Apply update: $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \Delta\boldsymbol{\theta}$
  **end while**

# How to choose?

- SGD < SGD+Momentum < SGD+Nesterov Momentum
- Adam is a good default
- RMSProp is good for RNNs, but also good default
- SGD + Nesterov momentum is best if you have time/ resources to optimize learning rate
- More of an Art

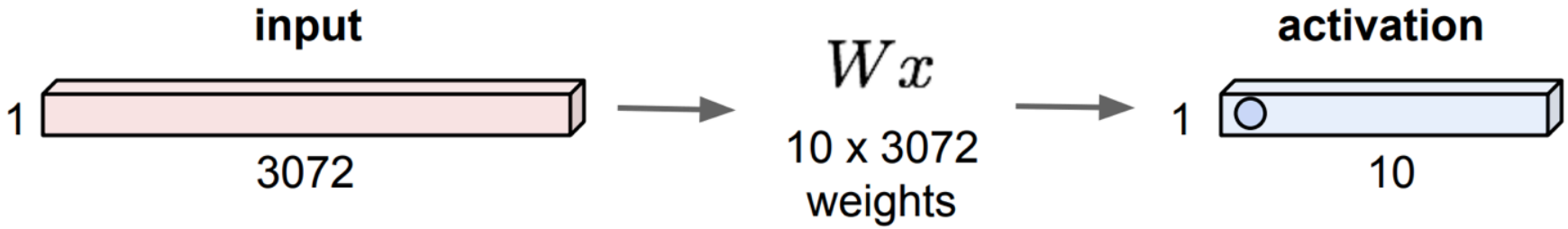
# Outline

- Training

- Convolutional Layers

# Fully Connected Layer

32x32x3 image -> stretch to 3072 x 1

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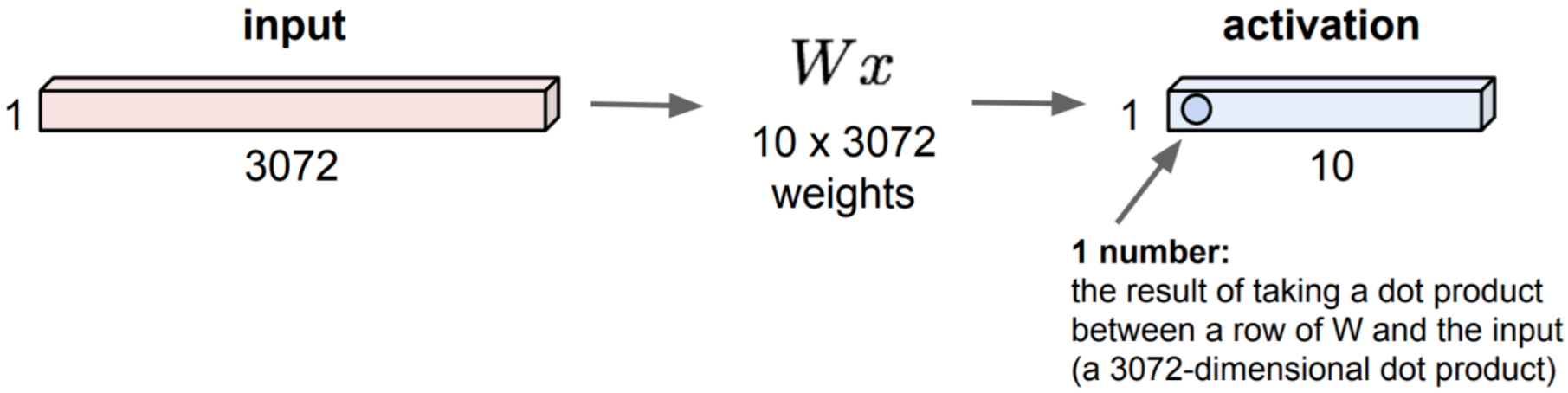
# Fully Connected Layer

32x32x3 image -> stretch to 3072 x 1

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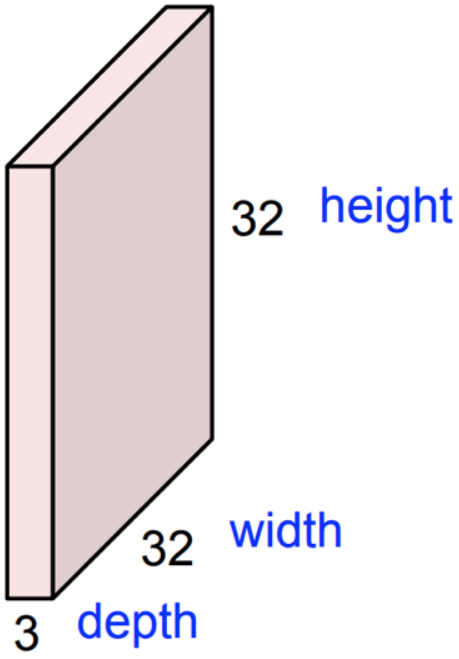
# Convolutional Layer

32x32x3 image -> preserve spatial structure

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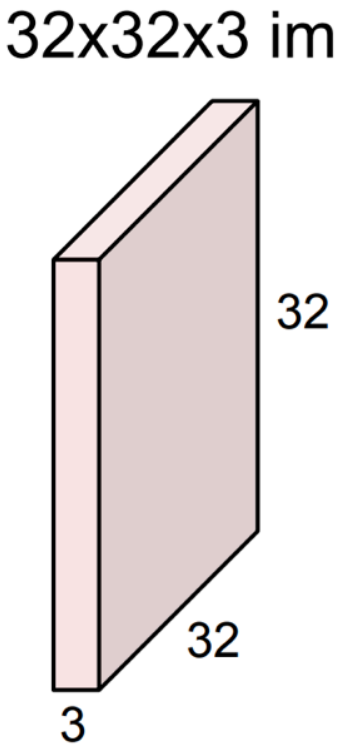
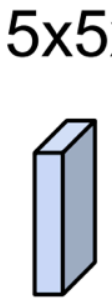
# Convolutional Layer




**Convolve** the filter with the image
i.e. “slide over the image spatially,
computing dot products”

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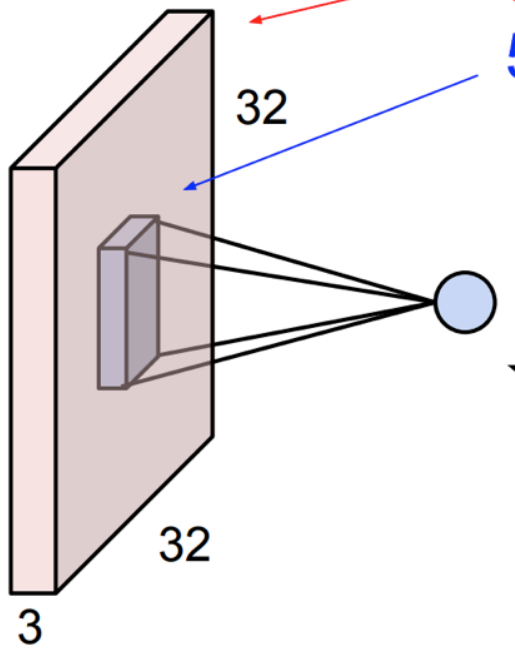
# Convolutional Layer

## Convolution Layer




32x32x3 image

5x5x3 filter $w$

**1 number:**
the result of taking a dot product between the filter and a small 5x5x3 chunk of the image
(i.e. 5*5*3 = 75-dimensional dot product + bias)

$$w^T x + b$$

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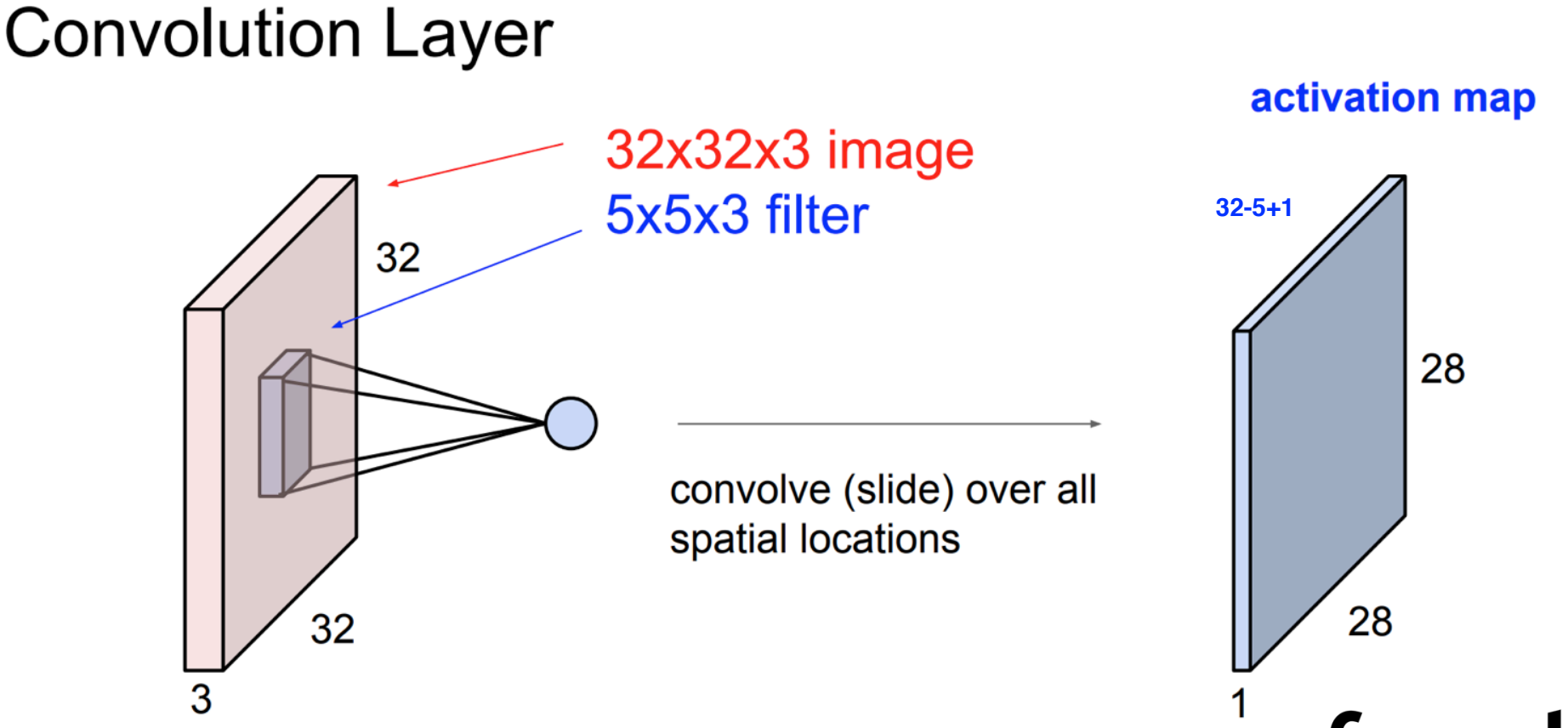
# Convolutional Layer

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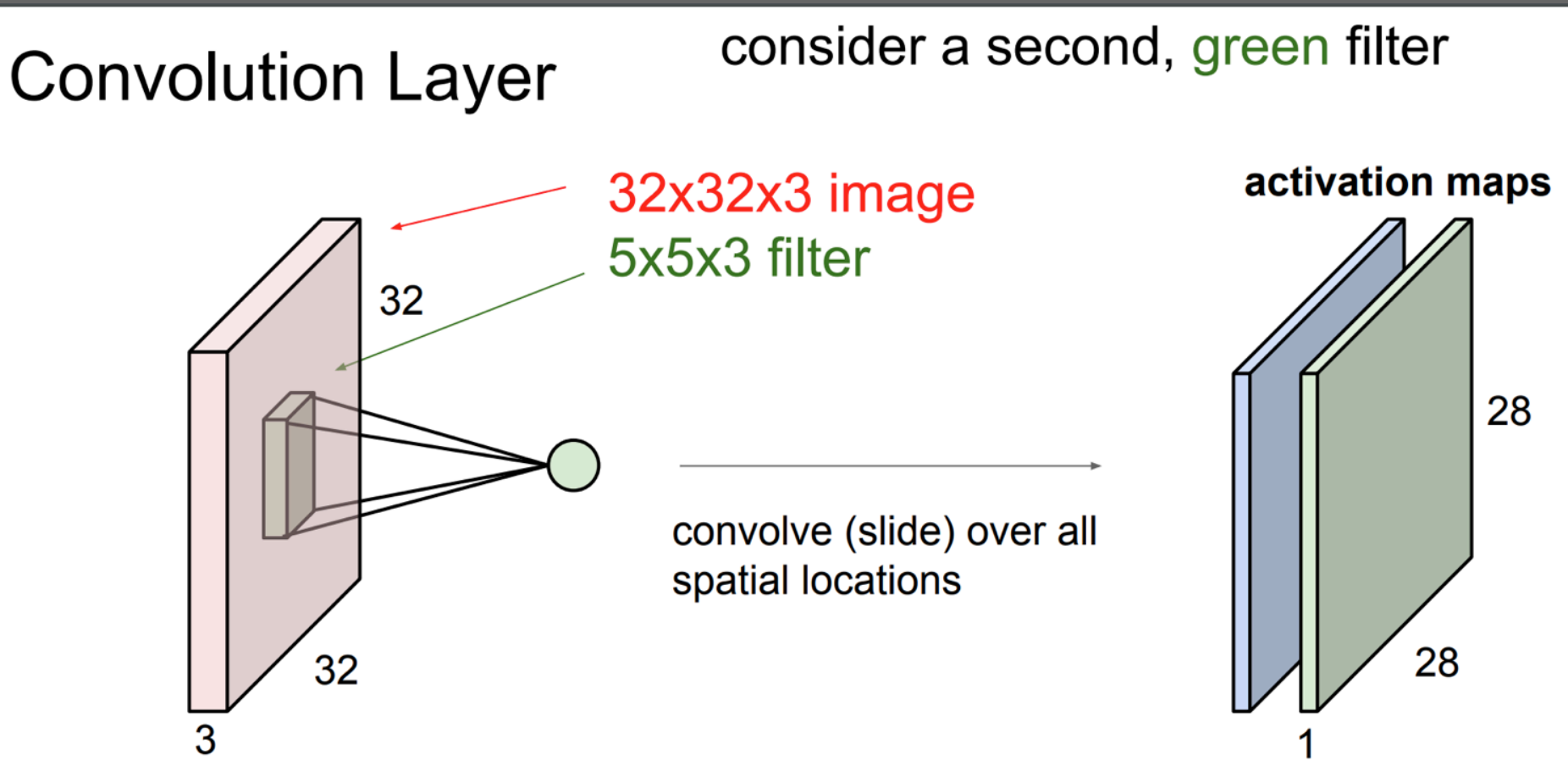
# Convolutional Layer

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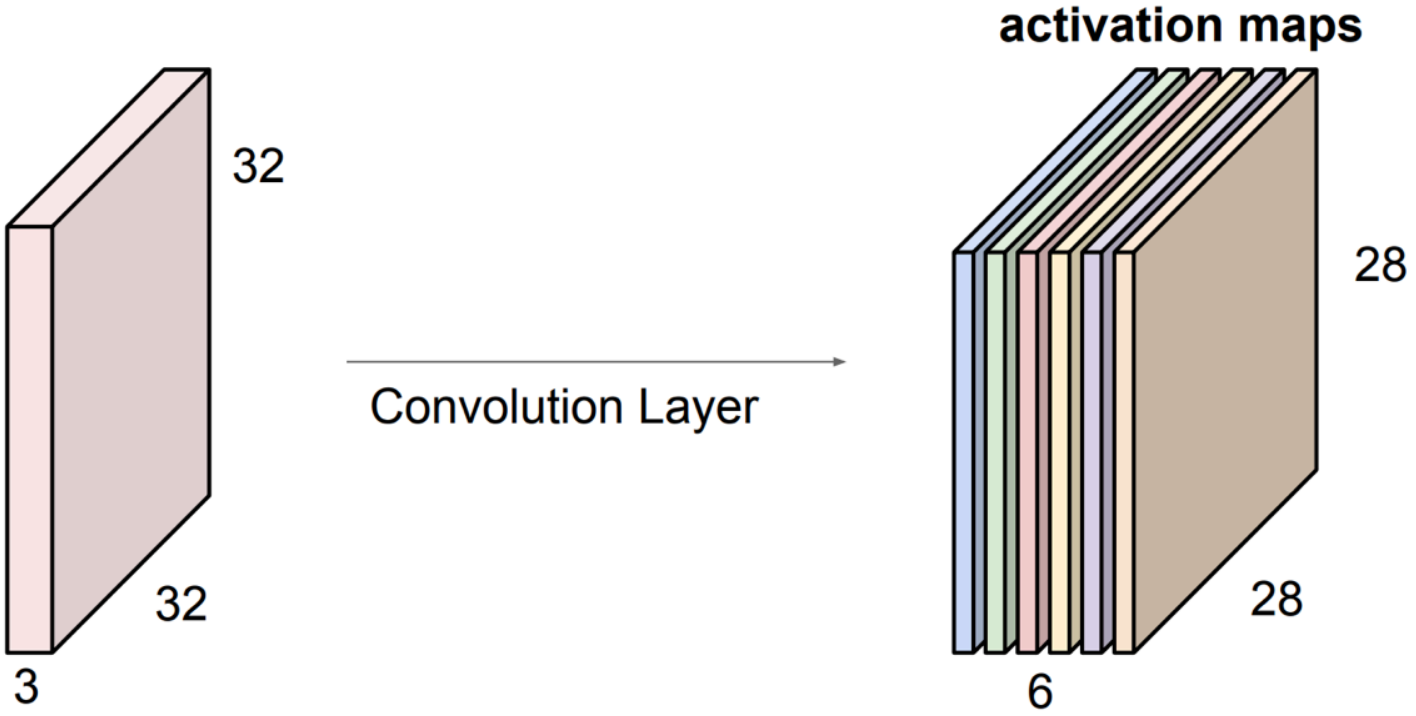
# Convolutional Layer

For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:




We stack these up to get a "new image" of size 28x28x6!

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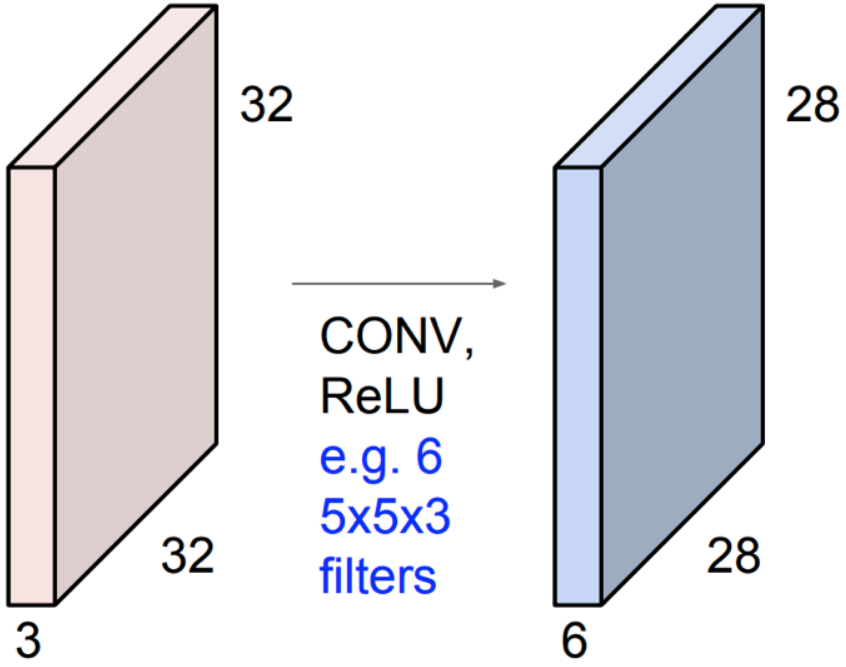
# Convolutional Layer

**Preview:** ConvNet is a sequence of Convolution Layers, interspersed with activation functions

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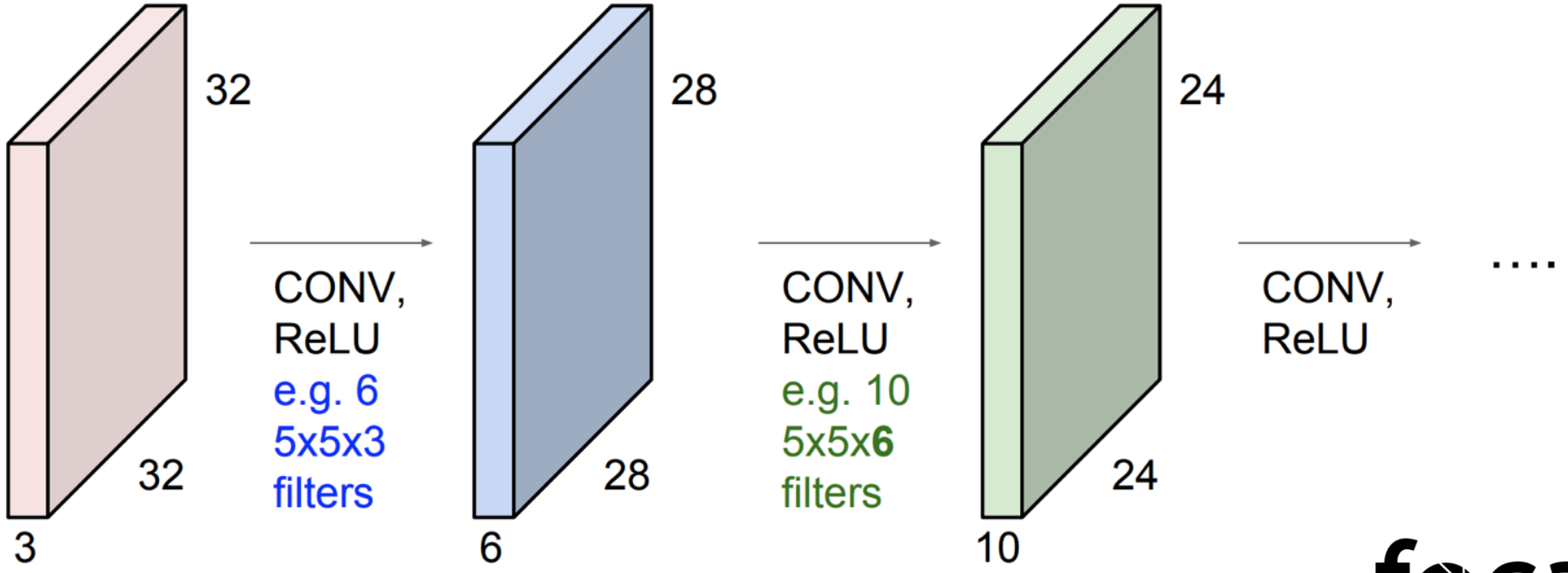
# Convolutional Layer

**Preview:** ConvNet is a sequence of Convolutional Layers, interspersed with activation functions

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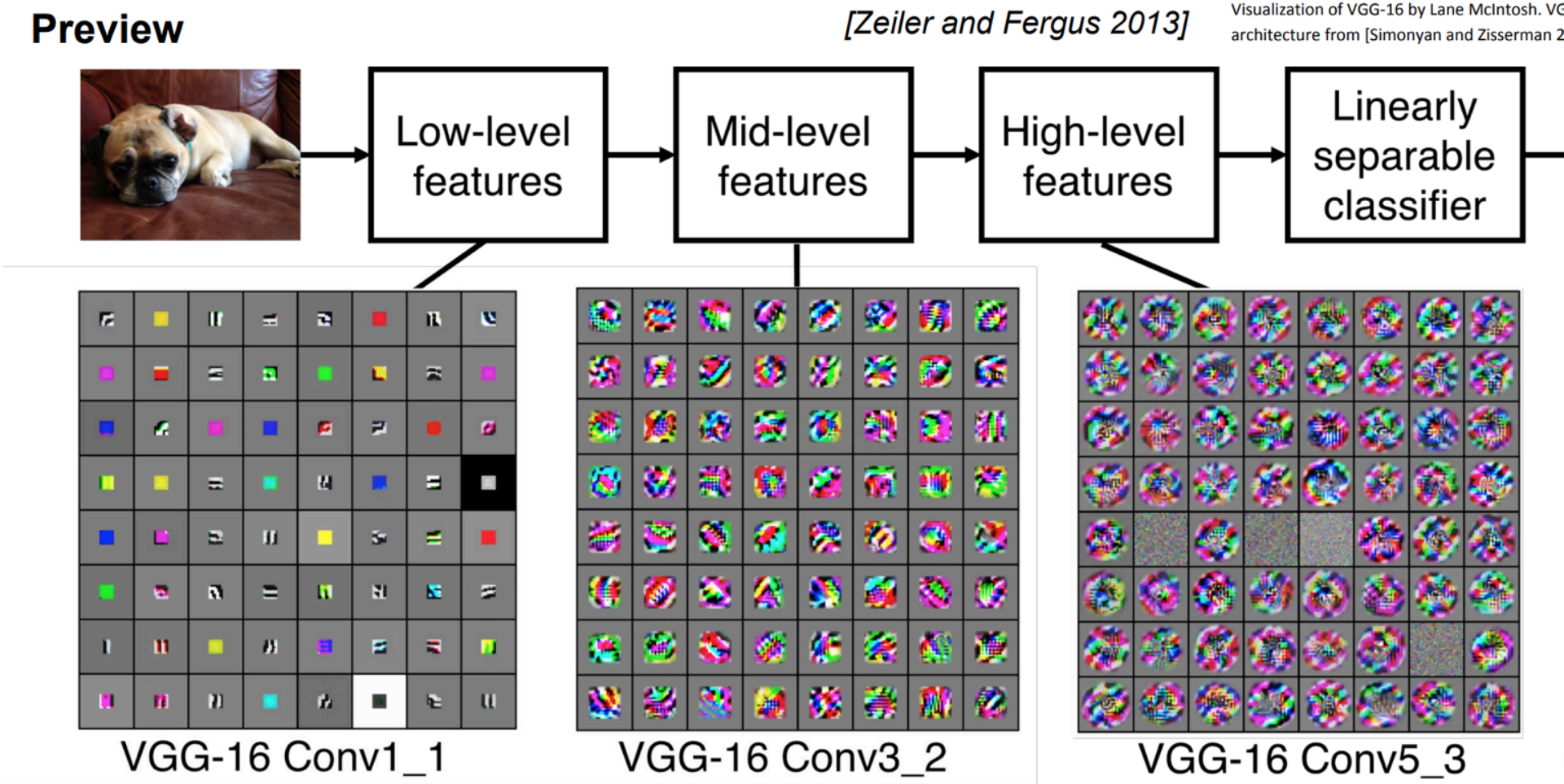
# Convolutional Layer

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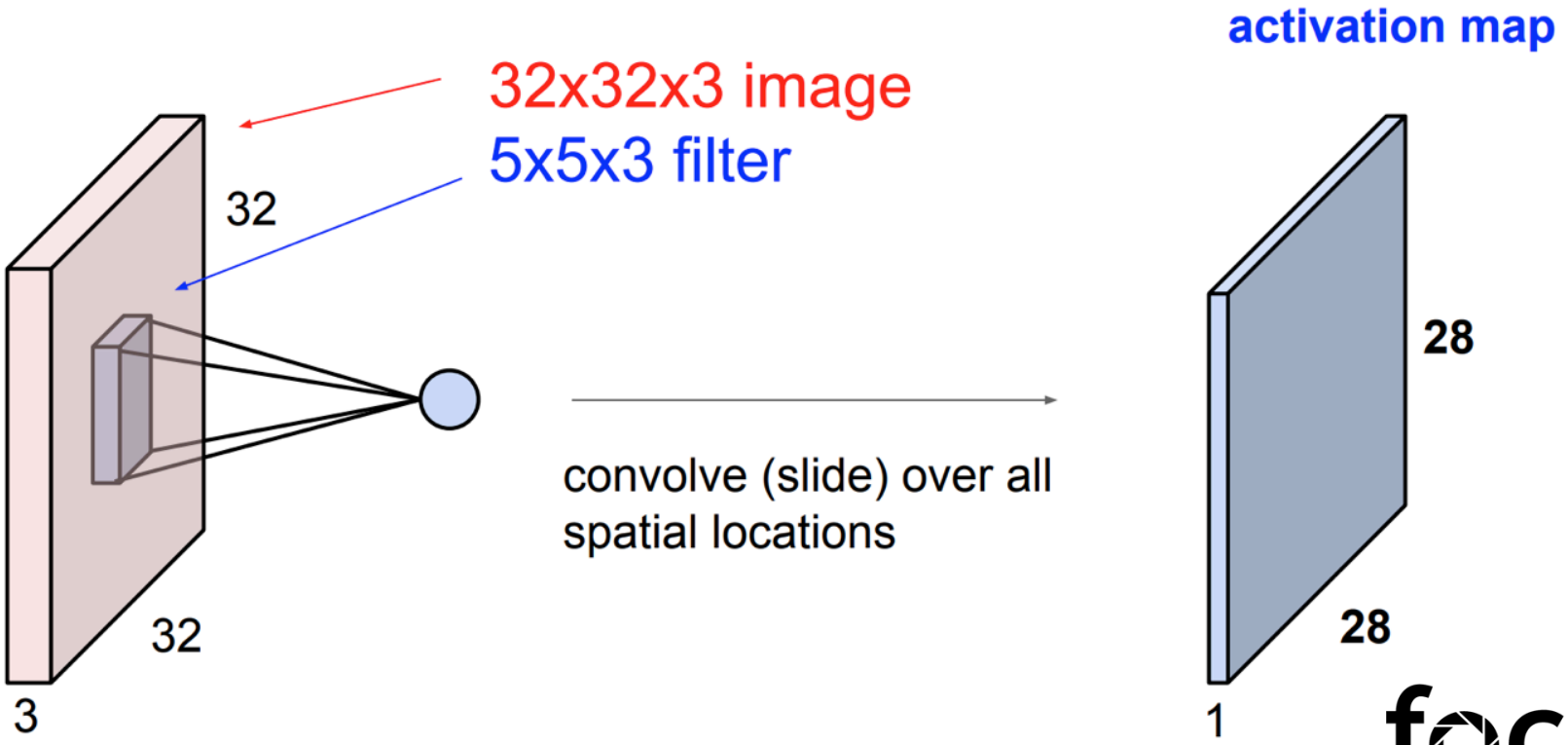
# Convolutional Layer

A closer look at spatial dimensions:

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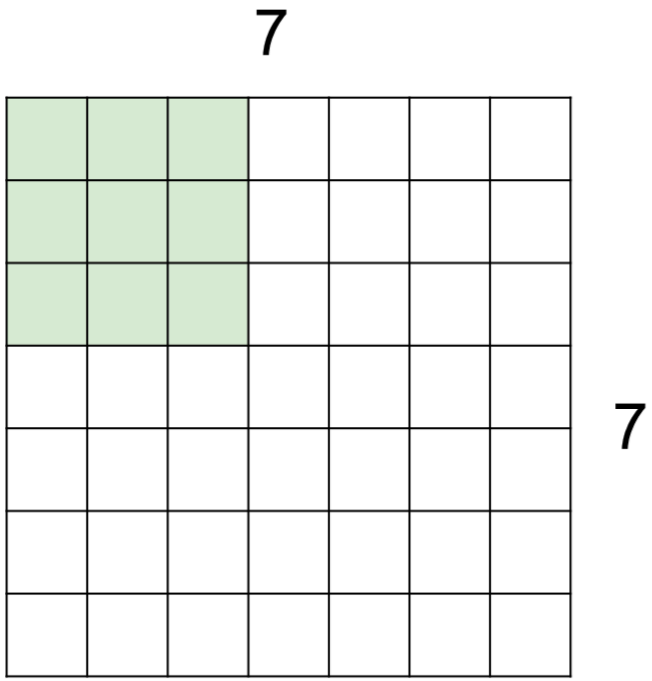
# Convolutional Layer

A closer look at spatial dimensions:




7x7 input (spatially)
assume 3x3 filter

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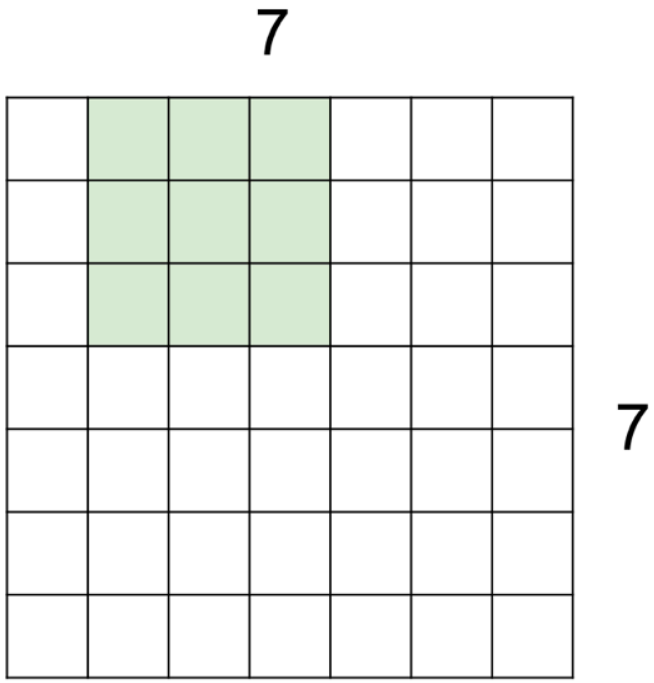
# Convolutional Layer

A closer look at spatial dimensions:




7x7 input (spatially)
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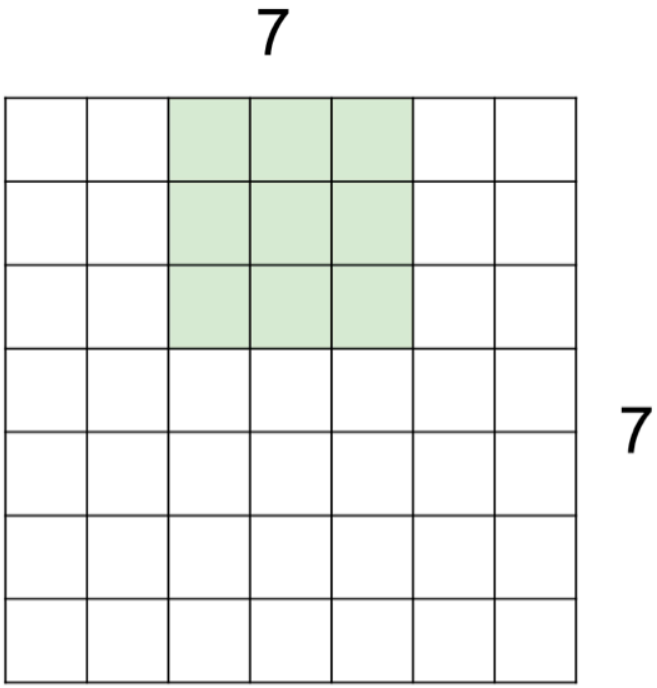
# Convolutional Layer

A closer look at spatial dimensions:




7x7 input (spatially)
assume 3x3 filter

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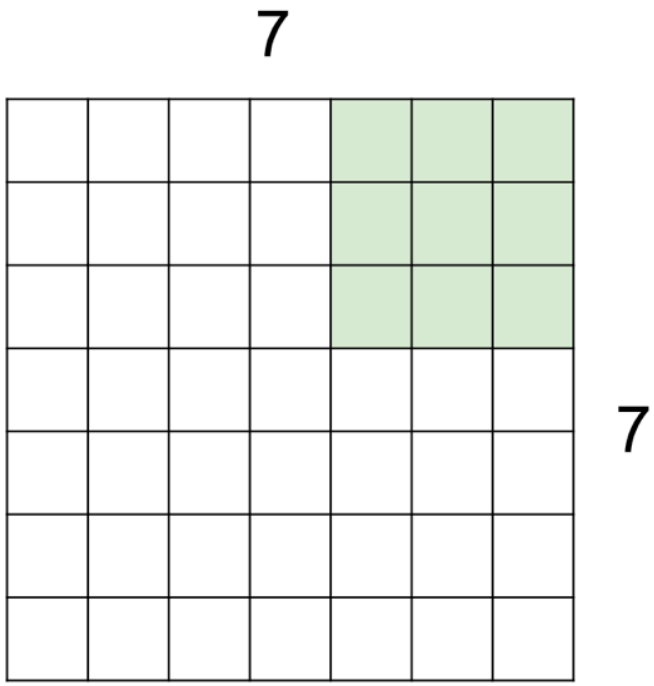
# Convolutional Layer

A closer look at spatial dimensions:




7x7 input (spatially)
assume 3x3 filter

**=> 5x5 output**

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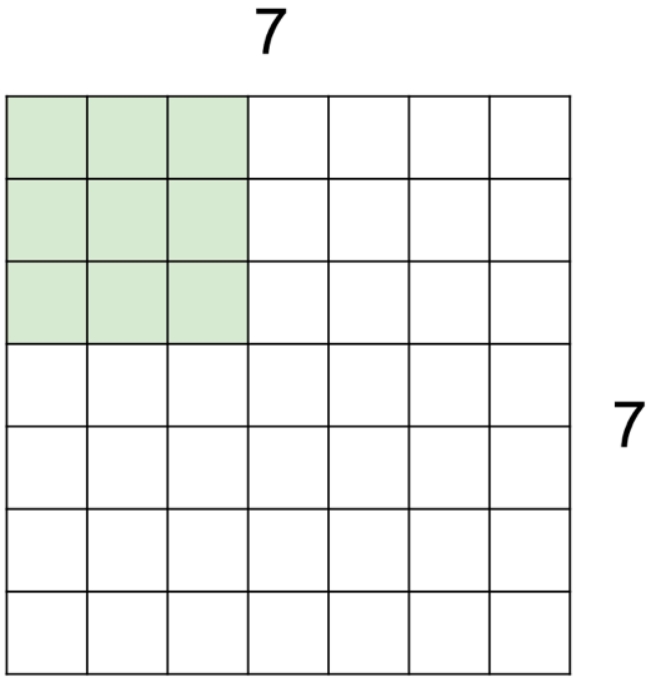
# Convolutional Layer

A closer look at spatial dimensions:




7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

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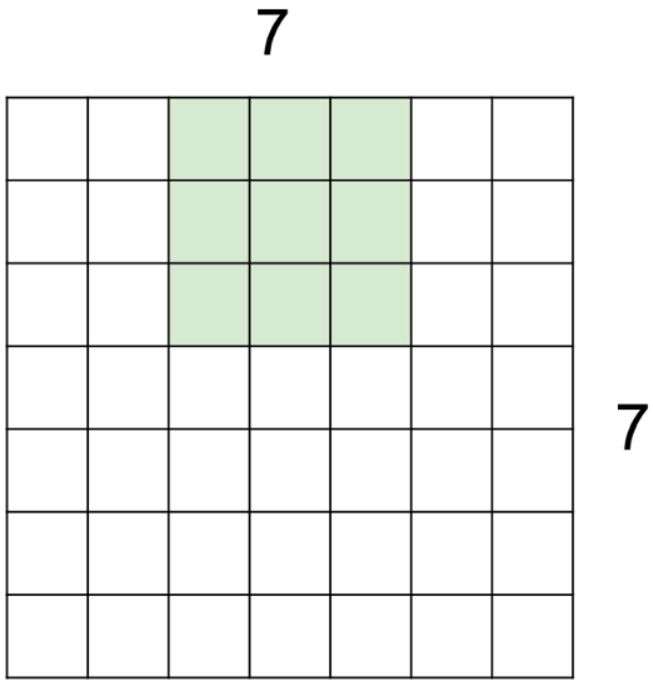
# Convolutional Layer

A closer look at spatial dimensions:

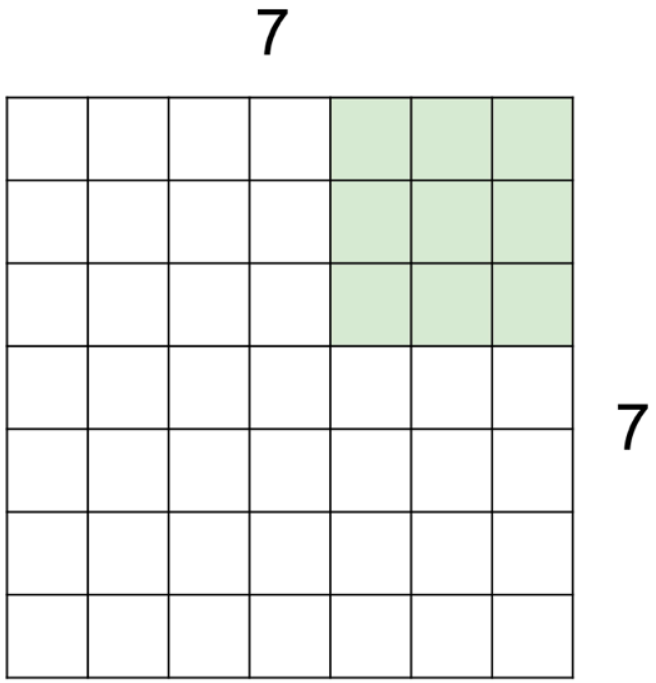



7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

http://cs231n.stanford.edu/slides/2017/cs231n_2017_lecture5.pdf

# Convolutional Layer

A closer look at spatial dimensions:




7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**
**=> 3x3 output!**

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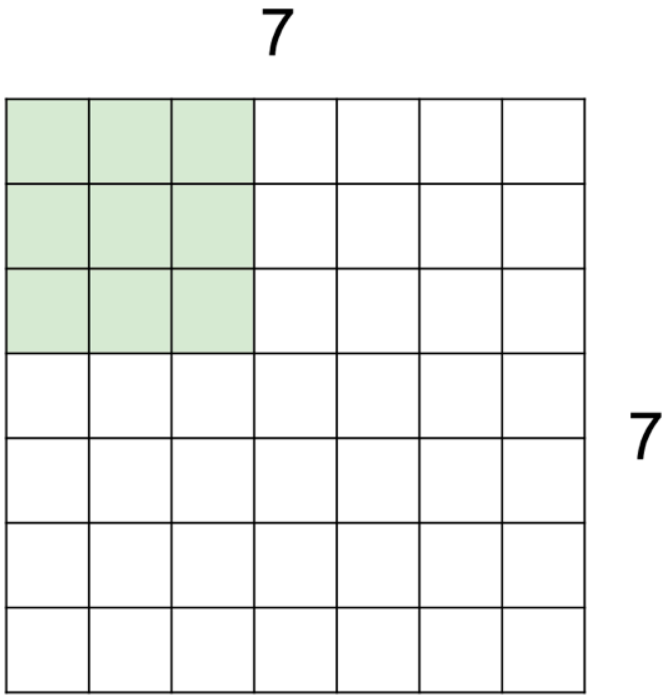
# Convolutional Layer

A closer look at spatial dimensions:




7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

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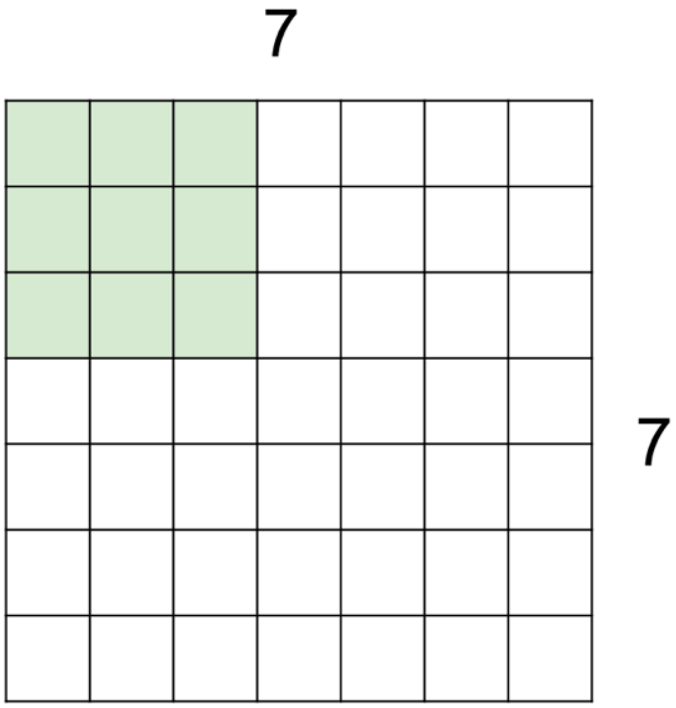
# Convolutional Layer

A closer look at spatial dimensions:




7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

**doesn't fit!**
cannot apply 3x3 filter on
7x7 input with stride 3.

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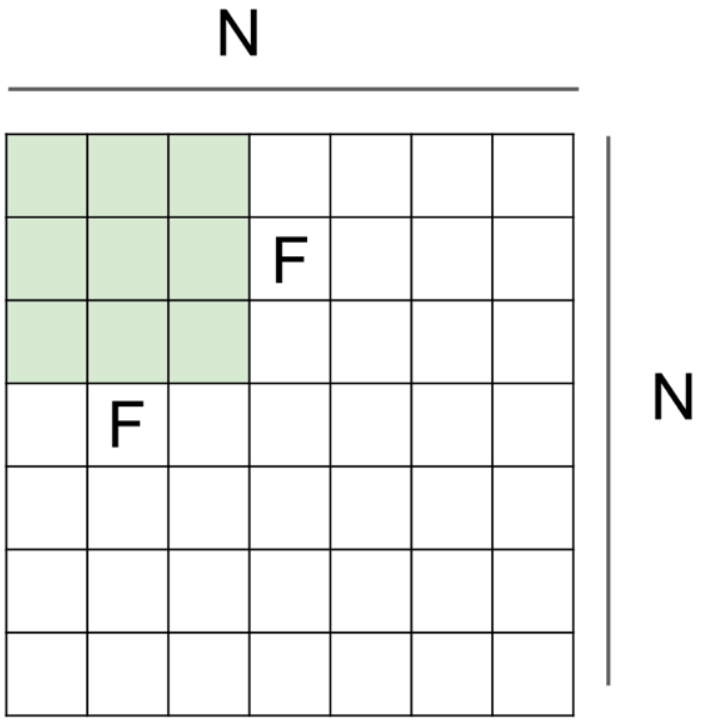
# Convolutional Layer




Output size:
**(N - F) / stride + 1**

e.g. N = 7, F = 3:
stride 1 => (7 - 3)/1 + 1 = 5
stride 2 => (7 - 3)/2 + 1 = 3
stride 3 => (7 - 3)/3 + 1 = 2.33 :\

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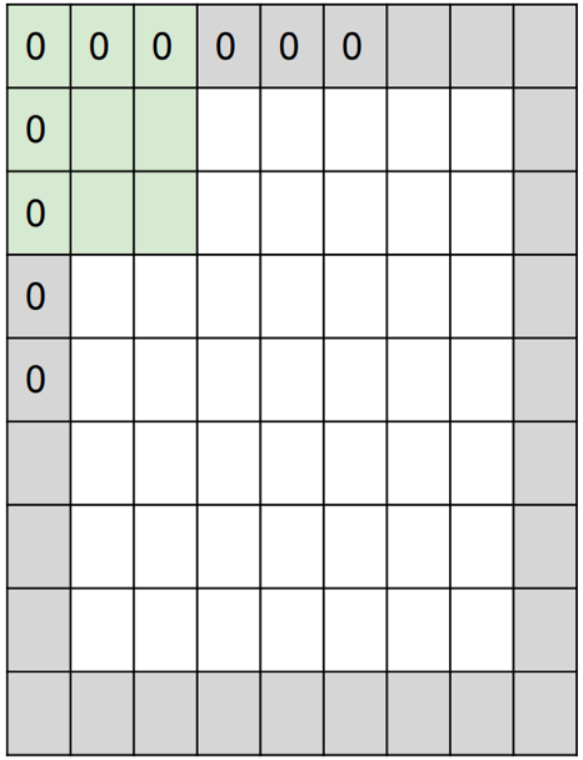
# Convolutional Layer

## In practice: Common to zero pad the border




e.g. input 7x7
**3x3** filter, applied with **stride 1**
**pad with 1 pixel** border => what is the output?

(recall:)
(N - F) / stride + 1

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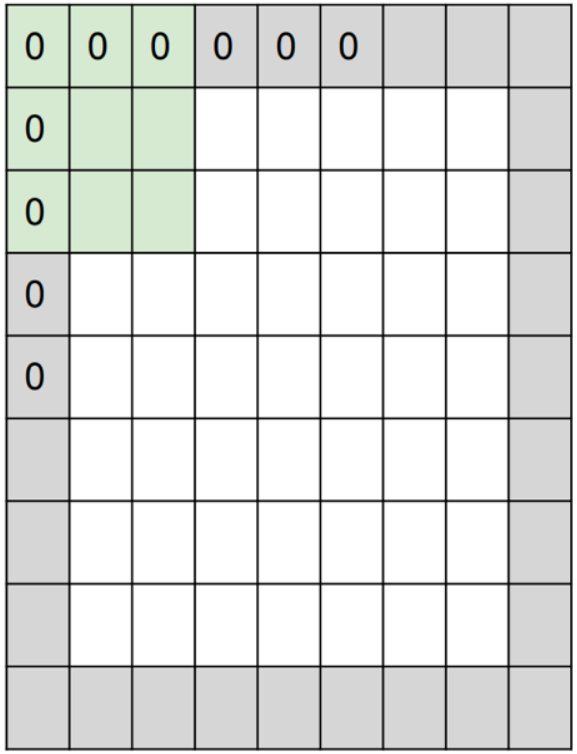
# Convolutional Layer

## In practice: Common to zero pad the border




e.g. input 7x7
**3x3** filter, applied with **stride 1**
**pad with 1 pixel** border => what is the output?

**7x7 output!**

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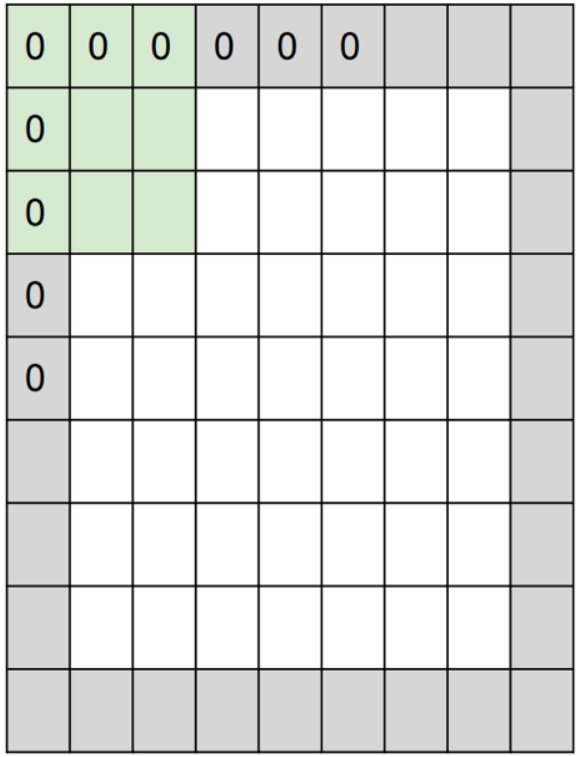
# Convolutional Layer

## In practice: Common to zero pad the border




e.g. input 7x7
**3x3** filter, applied with **stride 1**
**pad with 1 pixel** border => what is the output?

**7x7 output!**
in general, common to see CONV layers with stride 1, filters of size FxF, and zero-padding with (F-1)/2. (will preserve size spatially)
e.g. F = 3 => zero pad with 1
F = 5 => zero pad with 2
F = 7 => zero pad with 3

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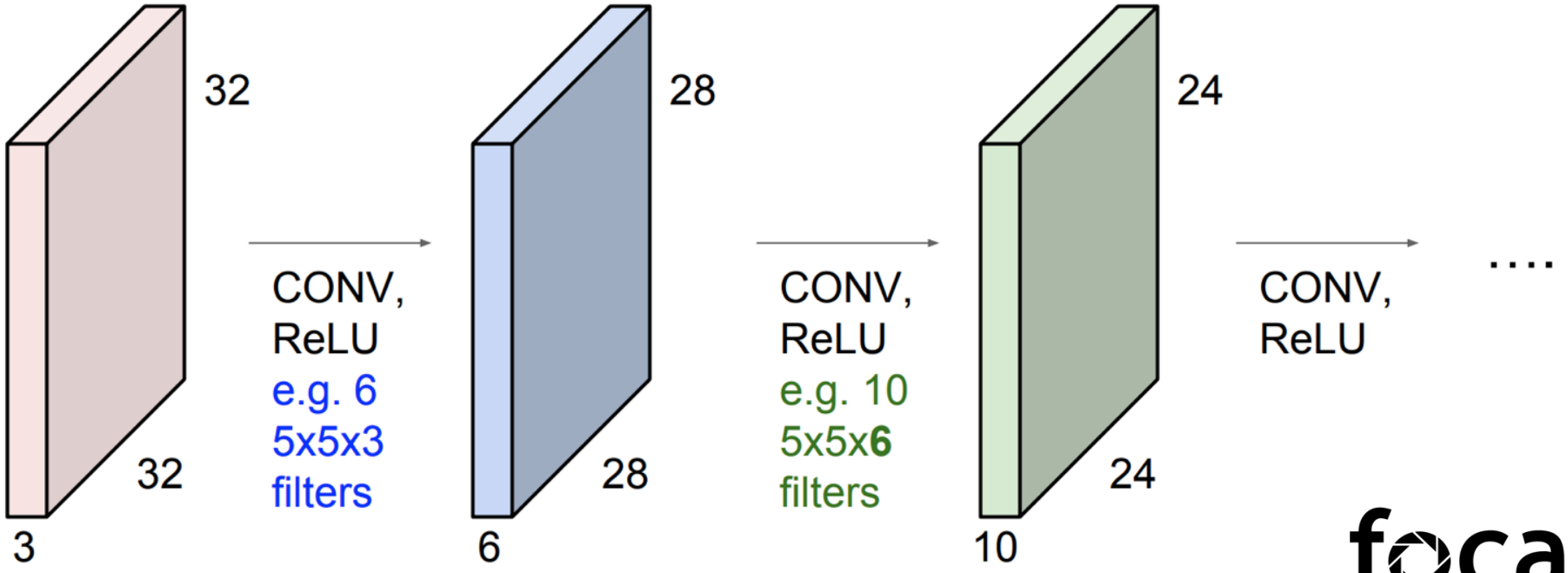
# Convolutional Layer

**Remember back to…**

E.g. 32x32 input convolved repeatedly with 5x5 filters shrinks volumes spatially! (32 -> 28 -> 24 ...). Shrinking too fast is not good, doesn't work well.

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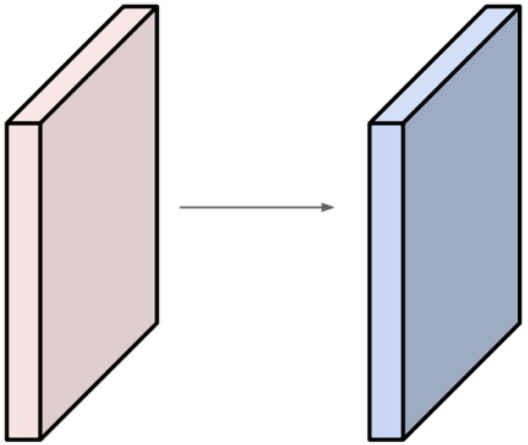
# Convolutional Layer

Examples time:



Input volume: **32x32x3**
10 5x5 filters with stride 1, pad 2

Output volume size: ?

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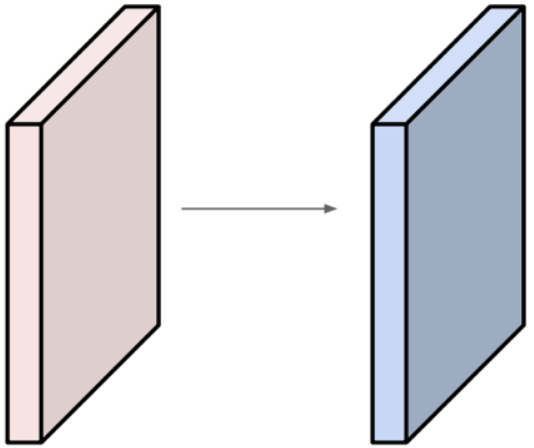
# Convolutional Layer

Examples time:



Input volume: **32x32x3**
10 5x5 filters with stride 1, pad 2

Output volume size:
(32+2*2-5)/1+1 = 32 spatially, so
**32x32x10**

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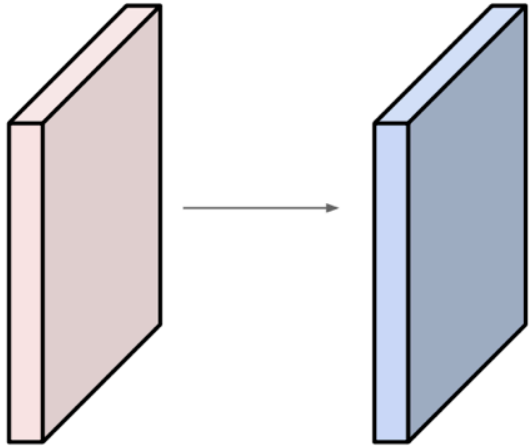
# Convolutional Layer

Examples time:



Input volume: **32x32x3**
10 5x5 filters with stride 1, pad 2

Number of parameters in this layer?

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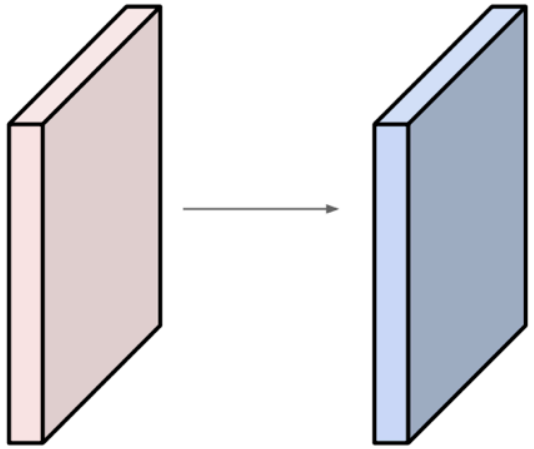
# Convolutional Layer

Examples time:



Input volume: **32x32x3**
10 5x5 filters with stride 1, pad 2

Number of parameters in this layer?
each filter has 5*5*3 + 1 = 76 params (+1 for bias)
=> 76*10 = **760**

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# Convolutional Layer

**Summary**. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
  - Number of filters $K$,
  - their spatial extent $F$,
  - the stride $S$,
  - the amount of zero padding $P$.
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F + 2P)/S + 1$
  - $H_2 = (H_1 - F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
  - $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and $K$ biases.
- In the output volume, the $d$-th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the $d$-th filter over the input volume with a stride of $S$, and then offset by $d$-th bias.

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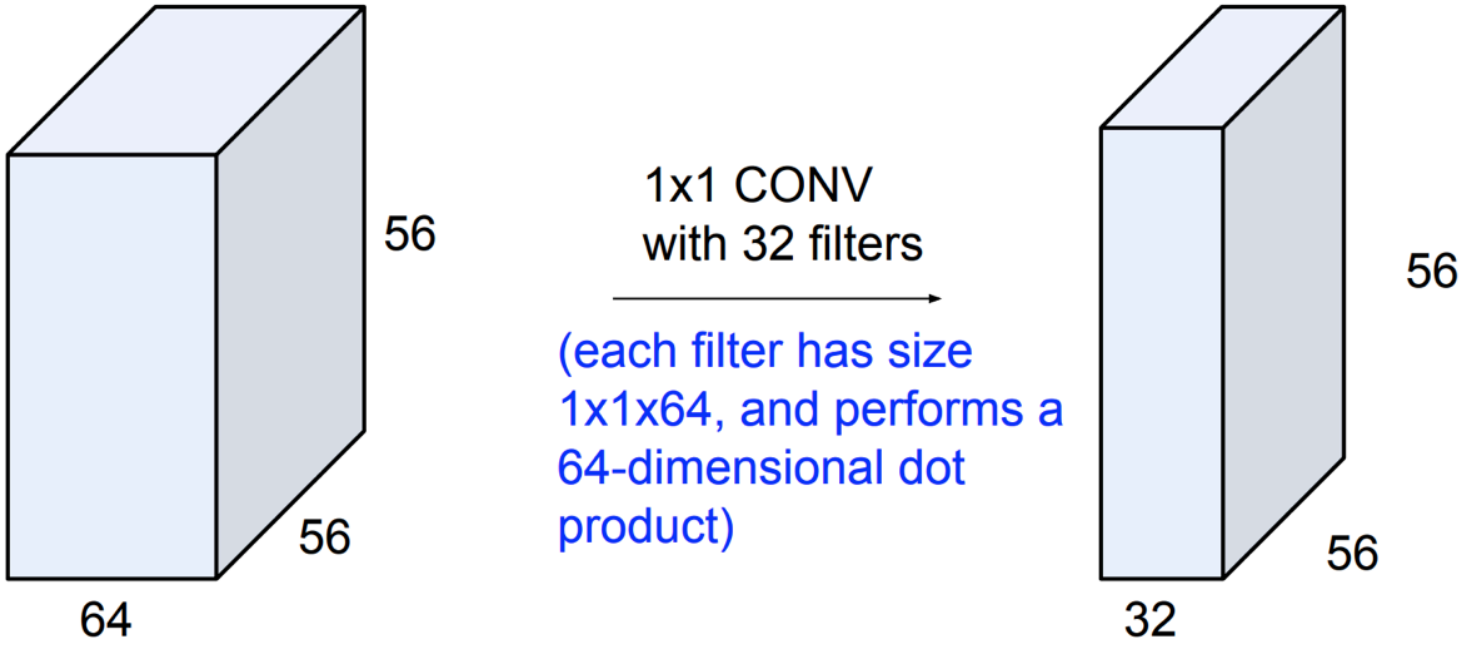
# Convolutional Layer

(btw, 1x1 convolution layers make perfect sense)

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# Deep Learning for Retail