Assignment #4, due November 11th, 2017, IN CLASS
AMATH 740, CS 770, CM 750
Fall 2017

1. (a) Derive explicit formulas for natural cubic spline interpolation, i.e. give the linear system that needs to be solved to find spline coefficients. You can assume that points x_i are equidistributed.

   (b) Implement the above algorithm. You can use the Matlab build-in linear solver or write your own. Submit the printout of your code.

2. Construct the intepolating polynomial passing through the points (-1,-5), (0,1), (1,1),(2,1) using the monomial, Lagrange and Newton bases.

3. (a) Consider linear interpolation of $f(x) = x^3$ at $x_0 = 0$ and $x_1 = 1$. Find the value of ξ for the error function $E_n f(x)$ derived in class.

   (b) Repeat for $f(x) = (2x - 1)^4$.

4. Interpolate $f(x) = sin(\pi x)$, $f(x) = 1/(1 + 25x^2)$, $f(x) = |x|$ on $[-1, 1]$ with degree 10 polynomials using the equidistant and Chebyshev points, and with cubic splines with 11 nodes. Compare results and make a meaningful conclusion. Show your work.

5. (a) Show that
$$\int_0^1 \frac{1}{1+x^2} = \pi$$

   (b) Approximate the integral using the 6 point Gauss-Legendre quadrature and a composite trapezoidal rule with the same number of function evaluations. Compare results and make a meaningful conclusion. Show your work. Nodes and weights can be found in Abramowitz and Stegun: Handbook of Mathematical Functions (available online) or other electronic resorces.

6. Let Q(n) be the composite trapezoidal rule approximation to $\int_a^b f(x)dx$ with $[a, b]$ divided into n subintervals. Show that for $f(x)$ of sufficient smoothness
$$\frac{Q(n) - Q(2n)}{Q(2n) - Q(4n)} \to 4, \quad n \to \infty$$

7. Derive the composite Simpson's Rule and find the expression for its error.