CS 770: Assignment 5

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1 Exercise 1

```
function [C,A] = myDFT(f,X)
% discrete Fourier transform
% Output: C,A
% C contains the DFT coefficients
% A contains the DFT approxmiation
% Input: f,X
% f contains the function values
% X contains the X values which are to be approximated

    n = length(f);
    % n is the number of points

    C=zeros(1,n);
    % initialize the coefficients to 0

    i = sqrt(-1);
    % initialize i

    for k = 0:n-1
        for j = 0:n-1
            C(k+1) = C(k+1)+(1./n)*f(j+1)*exp(-2*pi*k*(j./n)*i);
        end
    end
    % Calculating the DFT coefficients

    N = length(X);
    A = zeros(1,N);
    for j = 0:N-1
        for k = 0:n-1
            A(j+1) = A(j+1) + C(k+1)*exp(2*pi*(k)*i*X(j+1));
```

```
        end
    end
    % Calculating the DFT approximations
end
```

2 Exercise 2

2.1 Exercise 2a

$f(x) = x$

2.2 Exercise 2b

$f(x) = exp(cos(2\pi x))$

2.3 Exercise 2c

$f(x) = ((x - 0.5)/0.5)^2$

2.4 Exercise 2d

$f(x) = ((x - 0.5)/0.5)^m$

3 Exercise 3

For Fourier series,

$$a_k = 2\int_0^1 f(x)cos(2\pi kx)dx,\ b_k = 2\int_0^1 f(x)sin(2\pi kx)dx$$

3.1 Exercise 3a

For $f(x) = (cos(8\pi x))^4$,

$$a_k = 2\int_0^1 (cos(8\pi x))^4 cos(2\pi kx)dx,\ b_k = 2\int_0^1 (cos(8\pi x))^4 sin(2\pi kx)dx$$

$$\text{Since } cos(2x) = 2cos(x)^2 - 1$$

$$\text{Then } cos(8\pi x)^4 = \tfrac{1}{4}(\tfrac{cos(32\pi x)}{2} + 2cos(16\pi x) + 1)$$

By the orthogonality property,

$$a_8 = \tfrac{1}{4},\ a_{16} = 1,\ \text{All } b_k \text{ is } 0$$

3.2 Exercise 3b

For $f(x) = x$,

$$a_k = \int_0^1 xcos(2\pi kx)dx,\ b_k = \int_0^1 xsin(2\pi kx)dx$$

4 Exercise 4

When the function is periodic on $[a, b]$ instead of being periodic on the interval of $[0, 1]$

$$\text{Let } I = b - a, \text{ then } C_k = \sum_0^{n-1} f(x_n)e^{-2\pi i \frac{k}{I} x_n}$$

When approximating the original function, we have

$$f(x) = \sum_0^{k-1} C_k e^{2\pi i \frac{k}{I} x}$$

For example,

$$\text{Let } f(x) = x - 5 \text{ on interval } [5, 10]$$

5 Exercise 5

6 Exercise 6

6.1 8.1

The phone number is

6.2 8.2