CS770: Assignment 4

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1 Question 1

1.1 Question 1a

For cubic spline, let the function $S(x)$ define the spline function
where $a = x_0 < x_1 < x_2 < x_3 < ...... < x_n = b$

$$S(X) = \begin{cases} S_0(x), & x_0 < x < x_1 \\ S_1(x), & x_1 < x < x_2 \\ ...... \\ S_i(x), & x_i < x < x_{i+1} \\ ...... \\ S_{n-1}(x), & x_{n-1} < x < x_n \end{cases} \tag{1}$$

Where each $S_i(x)$ has degree 3 in this case.
In cubic spline, $S(x)$ satisfies

$$\begin{cases} S_i(x_i) = S_{i+1}(x_i) \\ S_i'(x_i) = S_{i+1}'(x_i) \\ S_i''(x_i) = S_{i+1}''(x_i) \\ S_i(x_i) = y_i \end{cases}$$

Where each $i = 0, 1, 2, ..., n-2$
By the definition of natural cubic spline, we have two additional constraints,

$$\begin{cases} S_0''(x_0) = 0 \\ S_{n-1}''(x_{n-1}) = 0 \end{cases}$$

1.2 Question 1b

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function [coeffs] = nSpline(X,y)
%This function returns the coefficients of the natural cubic spline
%X and y are the input points where f(X(i)) = y(i)
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%Each spline function on each interval has degree 3

%Si = a+bx+cx^2+dx^3
%We have n such Si's, where is n = length(X)-1
%coeffs should be [a1;b1;c1;d1;a2;b2;.....;an;bn;cn;dn]
%coeffs is a 4n by 1 vector

    numP = length(X);
    %numP is the number of points
    n =numP - 1;
    %n is the number of spline functions

    X = [];
    %initialize the matrix to be empty
    A = [];
    %A contains all the known values
    %X*coeffs = A

    %we construct the matrix using for loop
    for i = 1:numP
        a = (i-1)*4+1;
        b = a+1;
        c = b+1;
        d = c+1;
        %a,b,c,d are indices for the convinience of calculation

        if i==1
            tempX = zeros(1,4*n);
            tempX(1,c) = 2;
            tempX(1,d) = 6*X(i);
            X = [X;tempX];
            A = [A;0];
            tempX = zeros(1,4*n);
            tempX(1,a) = 1;
            tempX(1,b) = X(i);
            tempX(1,c) = X(i)^2;
            tempX(1,d) = X(i)^3;
            X = [X;tempX];
            A = [A;y(i)];
        end


        if i==numP
```

```
        tempX = zeros(1,4*n);
        tempX(1,c-4) = 2;
        tempX(1,d-4) = 6*X(i);
        X = [X;tempX];
        A = [A;0];
        tempX = zeros(1,4*n);
        tempX(1,a-4) = 1;
        tempX(1,b-4) = X(i);
        tempX(1,c-4) = X(i)^2;
        tempX(1,d-4) = X(i)^3;
        X = [X;tempX];
        A = [A;y(i)];
end
%these the special end points contraints for natural cubic constraint

if i>1 && i<numP
        tempX = zeros(1,4*n);
        tempX(1,c-4) = 2;
        tempX(1,d-4) = 6*X(i);
        X = [X;tempX];
        A = [A;0];
        tempX = zeros(1,4*n);
        tempX(1,a-4) = 1;
        tempX(1,b-4) = X(i);
        tempX(1,c-4) = X(i)^2;
        tempX(1,d-4) = X(i)^3;
        X = [X;tempX];
        A = [A;y(i)];
end
%this is the constraint for S(xi) = yi

if i>1 && i<numP
        tempX = zeros(1,4*n);
        tempX(1,b-4) = 1;
        tempX(1,c-4) = 2*X(i);
        tempX(1,d-4) = 3*X(i)^2;
        tempX(1,b) = -1;
        tempX(1,c) = -2*X(i);
        tempX(1,d) = -3*X(i)^2;
        X = [X;tempX];
        A = [A;0];
        %this is the constraint for Si'(xi) = Si+1'(xi)
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        tempX = zeros(1,4*n);
        tempX(1,c-4) = 2;
        tempX(1,d-4) = 6*X(i);
        tempX(1,c) = -2;
        tempX(1,d) = -6*X(i);
        X = [X;tempX];
        A = [A;0];
        %this is the constraint for Si''(xi) = Si+1''(xi)
    end
  end

  coeffs = X\A;

end
```

2 Question 2

3 Question 3

3.1 Question 3a

3.2 Question 3b

4 Question 4

5 Question 5

5.1 Question 5a

5.2 Question 5b

6 Question 6

7 Question 7