Assignment #5, due November 22th or 27th, 2017, IN CLASS
AMATH 740, CS 770, CM 750
Fall 2017

Reading:
C. Moler "Numerical Computing with Matlab" (online, free download), Ch. 8

1. Implement either the DFT or the FFT algorithm as well as construction of the Fourier approximation of a function from its Fourier coefficients. Make sure to modify the formulas to take into account that Matlab indexing starts at 1. Submit a print-out, as usual

2. Apply your code to the following functions, plot the results of the Fourier approximation and the original function. Choose your own sampling rate n, possibly several ones.

   (a) $f(x) = x$. This function is discontinuous and you will see that a Fourier approximation will oscillate near the discontinuity. This is called the Gibbs phenomenon.

   (b) $f(x) = exp(cos(2\pi x))$. You should expect a good approximation here.

   (c) $f(x) = ((x - 0.5)/0.5)^2$. This function is continuous, but approximation is not as good as in (b). Why?

   (d) $f = ((x - 0.5)/0.5)^m$, $m = 5, 10, 25, 50, ..$ with a fixed number of sampling points. Explain the observed behavior. It might be easier to plot the error here.

3. (a) Derive by hand the continuous Fourier Series of $f(x) = (cos(8\pi x))^4$. Compare the discrete and continuous Fourier coefficients with $n = 5, 11, 21$. Explain your findings.

   (b) Repeat with $f(x) = x$.

4. Derive formulas for using the DFT when f is sampled on an arbitrary interval $[a, b]$. Modify your code and submit an example of a Fourier approximant on $[a, b]$ for same simple function.

5. (bonus) Come up with a sequence of functions $\{f_k\}$ that are k times differentiable and periodic with period one. Try to see numerically that the error in Fourier approximation decays at about n^{-k} rate.

6. (bonus) Problems 8.1, 8.2 in Moler. File touchtone.zip is posted on Learn.