CS 770: Assignment 5

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November 20, 2017

1 Exercise 1

```
function [C,A] = myDFT(f,X)
% discrete Fourier transform
% Output: C,A
% C contains the DFT coefficients
% A contains the DFT approxmiation
% Input: f,X
% f contains the function values
% X contains the X values which are to be approximated

    n = length(f);
    % n is the number of points

    C=zeros(1,n);
    % initialize the coefficients to 0

    i = sqrt(-1);
    % initialize i

    for k = 0:n-1
        for j = 0:n-1
            C(k+1) = C(k+1)+(1./n)*f(j+1)*exp(-2*pi*k*(j./n)*i);
        end
    end
    % Calculating the DFT coefficients

    N = length(X);
    A = zeros(1,N);
    for j = 0:N-1
        for k = 0:n-1
            A(j+1) = A(j+1) + C(k+1)*exp(2*pi*(k)*i*X(j+1));
```

```
            end
        end
        % Calculating the DFT approximations
end
```

2 Exercise 2

2.1 Exercise 2a

$f(x) = x$

2.2 Exercise 2b

$f(x) = exp(cos(2\pi x))$

2.3 Exercise 2c

$f(x) = ((x - 0.5)/0.5)^2$

2.4 Exercise 2d

$f(x) = ((x - 0.5)/0.5)^m$

3 Exercise 3

For Fourier series,

$$f(x) = \sum_{-\infty}^{\infty} C_k e^{2\pi i k x}$$

3.1 Exercise 3a

For $f(x) = (cos(8\pi x))^4$,

$$C_k = \int_0^1 (cos(8\pi x))^4 e^{-2\pi i k x}$$

3.2 Exercise 3b

For $f(x) = x$,

$$C_k = \int_0^1 x e^{-2\pi i k x}$$

$$\text{Let } u = x, \frac{dv}{dx} = e^{-2\pi i k x}$$

$$\text{Then } du = 1, v = \frac{1}{-2\pi i k} e^{-2\pi i k x}$$

$$\text{Then } \int x e^{-2\pi i k x} = \frac{x}{-2\pi i k} e^{-2\pi i k x} - \int \frac{1}{-2\pi i k} e^{-2\pi i k x}$$

$$\int xe^{-2\pi ikx} = \frac{x}{-2\pi ik}e^{-2\pi ikx} - \frac{1}{-4\pi^2k^2}e^{-2\pi ikx}$$

$$\text{Since } C_k = \int_0^1 xe^{-2\pi ikx}$$

$$\text{Then } C_k = (\frac{1}{-2\pi ik}e^{-2\pi ik} - \frac{1}{-4\pi^2k^2}e^{-2\pi ik}) - (0 - \frac{1}{-4\pi^2k^2})$$

$$C_k = \frac{\text{ffl}}{-4\pi^2k^2}$$

4 Exercise 4

When the function is periodic on $[a, b]$ instead of being periodic on the interval of $[0, 1]$

$$\text{Let } I = b - a\text{, then } C_k = \sum_0^{n-1} f(x_n)e^{-2\pi i\frac{k}{I}x_n}$$

When approximating the original function, we have

$$f(x) = \sum_0^{k-1} C_k e^{2\pi i\frac{k}{I}x}$$

For example,

$$\text{Let } f(x) = x - 5 \text{ on interval } [5, 10]$$

5 Exercise 5

6 Exercise 6