Assignment #1, due September 27th, 2017
AMATH 740, CS 770, CM 750
Fall 2017

Reading:
Optional: C. Moler "Numerical Computing with Matlab" (online, free download), especially Ch. 1.7
Required: M. Overton "Numerical Computing with IEEE Floating Point Arithmetic" (posted 2.5 pages on integer arithmetic)

1. (a) Consider an approximation of the first derivative of $f(x)$

$$\frac{df}{dx} \approx \frac{f(x+h) - f(x)}{h}.$$

The error in this appximation is $e(x)$

$$e(x, h) = \frac{f(x+h) - f(x)}{h} - f'(x). \tag{1}$$

Using the Taylor series expansion of $f(x+h)$ about x, obtain

$$e(x, h) = \frac{h}{2} \frac{d^2 f}{dx^2}(x + \xi).$$

Plot $e(x, h)$ in (1) on the log-log scale for h ranging from 0.1 to 10^{-20} for the function of your choosing at a point x of your choosing. In an exact arithmetic we would expect e(x,h) to linearly decay with h. However with the floating point computer arithmetic you will find that the error begins to increase starting at some h. Explain this phenomenon and give a rough estimate (the order of magnitude is sufficient) for this turning point.

   (b) Repeat the part above for

$$\frac{d^2 f}{dx^2} \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

2. Give your own examples that would illustrate inexactness of algebraic operations, noncommutativeness of algebraic operations, and cancellation errors in IEEE floating point arithmetics (see Moler p.40 for an example). Include a print-out of your Matlab session or your code.

3. Exercise 1.38 in Moler

4. Exercises 3.1 - 3.6 in Overton