

BIOS 545 Lecture 3

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Factors

- R supports factors which are a special data type for managing categories of data
- Identifying categorical variables is usually straightforward. These are the variables by which you might want to summarize some continuous data
- Categorical variables usually take on a definite number of values
- Many times R functions will convert character labels to factors by default but not always
- Storing data as factors insures that R modeling functions will treat such data correctly

Factors

Let's say we have some automobile data that tells us if a car has an automatic transmission (0) or a manual transmission (1). We store this into a vector called `transvec`

```
transvec <- c(1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,0,0,0,0,0,1,1,1,1,1,1,1)
```

```
table(transvec)      #How many are Auto and Manual ? Count 'em up.
```

```
transvec
```

```
0  1
```

```
19 13
```

```
mytransfac <- factor(transvec, levels = c(0,1), labels = c("Auto","Man"))
```

```
levels(mytransfac)
```

```
[1] "Auto" "Man"
```

```
mytransfac
```

```
[1] Man  Man  Man  Auto Auto Auto Auto Auto Auto Auto Auto Auto Auto Auto Auto
```

```
[16] Auto Auto Man  Man  Man  Auto Auto Auto Auto Auto Man  Man  Man  Man  Man
```

```
[31] Man  Man
```

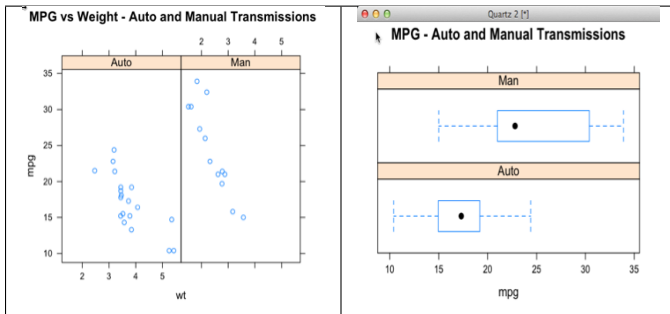
```
Levels: Auto Man
```

Factors

R knows how to handle factors when doing plots. Here we get an X/Y plot and a Box Plot with very little work since R knows that `mytransfac` is a factor

```
library(lattice)
xyplot(mpg~wt | mytransfac, mtcars,
       main="MPG vs Weight - Auto and Manual Transmissions")

bwplot(~mpg|mytransfac, mtcars,
       main="MPG - Auto and Manual Transmissions", layout=c(1,2))
```



Factors - aggregation preview

With our knowledge of factors and vectors we can do some basic aggregation using the `tapply` command. We have a factor vector called `mytransfac`.

Let's summarize some MPG data that corresponds to the automobiles used in the `mytransfac` vector. So for each car we have its MPG figure and whether it has an automatic or manual transmission.

```
mympg <- c(21,21,22.8,21.4,18.7,18.1,14.3,24.4,22.8,19.2,17.8,  
           16.4,17.3,15.2,10.4,10.4,14.7,32.4,30.4,33.9, 21.5,  
           15.5,15.2,13.3,19.2,27.3,26,30.4,15.8,19.7,15,21.4)  
  
# tapply( continuous_value_to_summarize,  factor_or_grouping variable,  
          function_for_summary)  
  
tapply(mympg, mytransfac, mean)  
      Auto      Man  
17.14737 24.39231
```

Factors - cut

It is sometimes useful to take a continuous variable and chop it up into intervals or categories for purposes of summary or grouping. R has a function to do this called "cut" to accomplish this.

Let's work through some examples to understand what is going on. Let's cut up the numbers between 0 and 10 into 4 distinct intervals

```
cut(0:10,breaks=4)
```

```
[1] (-0.01,2.5] (-0.01,2.5] (-0.01,2.5] (2.5,5]      (2.5,5]  (2.5,5]  (5,7.5]
     (5,7.5]      (7.5,10]      (7.5,10]  (7.5,10]
Levels: (-0.01,2.5] (2.5,5] (5,7.5] (7.5,10]
```

```
table(cut(0:10,breaks=4))
```

```
(-0.01,2.5]      (2.5,5]      (5,7.5]      (7.5,10]
              3              3              2              3
```

```
table(cut(0:10,breaks=4))
```

```
(-0.01,2.5]      (2.5,5]      (5,7.5]      (7.5,10]
              3              3              2              3
```

Factors - cut

Well that was cool but since we are creating 4 intervals we should probably name them

```
my.cut <- cut(0:10,breaks=4,labels=c("Q1","Q2","Q3","Q4"))  
[1] Q1 Q1 Q1 Q2 Q2 Q2 Q3 Q3 Q4 Q4 Q4  
Levels: Q1 Q2 Q3 Q4
```

```
table(my.cut)  
my.cut  
Q1 Q2 Q3 Q4  
 3  3  2  3
```

But you can to take finer-grained control over how the intervals are made.

```
quantile(0:10)  
 0%  25%  50%  75% 100%  
0.0  2.5  5.0  7.5 10.0
```

```
table(cut(0:10,breaks=quantile(0:10),include.lowest=TRUE))  
  
[0,2.5]  (2.5,5]  (5,7.5]  (7.5,10]  
      3         3         2         3
```

Factors - cut

Let's summarize some exam scores according to a typical grading system.
F: < 60, D: 60-70: C: 70-80: B:80-90 A:90-100

```
set.seed(123)
exam.score <- runif(25,50,100) # Simulate some scores

cut(exam.score,breaks=c(50,60,70,80,90,100))
[1] (60,70] (80,90] (70,80] (90,100] (90,100] (50,60] (70,80] (90,100]
[9] (70,80] (70,80] (90,100] (70,80] (80,90] (70,80] (50,60] (90,100]
[17] (60,70] (50,60] (60,70] (90,100] (90,100] (80,90] (80,90] (90,100]
[25] (80,90]
Levels: (50,60] (60,70] (70,80] (80,90] (90,100]
```

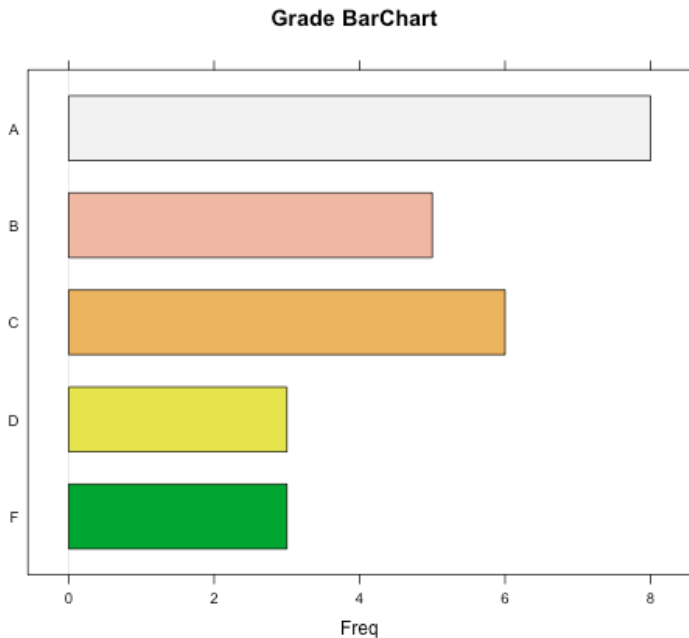
```
cut(exam.score,breaks=c(50,60,70,80,90,100),labels=c("F","D","C","B","A"))
[1] D B C A A F C A C C A C B C F A D F D A A B B A B
Levels: F D C B A
```

```
my.table <- table(cut(exam.score,breaks=c(50,60,70,80,90,100),
                      labels=c("F","D","C","B","A")))
```

```
F D C B A
3 3 6 5 8
```

```
barchart(my.table,main="Grade BarChart",col=terrain.colors(5))
```


Factors - cut



Factors - cut

- But the intervals don't exactly match the grading scheme
- A score of 90 will get a B when it should get an A
- Note the [and) characters show us if the interval boundary includes a score
- There is an argument that can help **right**)

```
set.seed(123)
```

```
cut(exam.score,breaks=c(50,60,70,80,90,100))
```

```
[1] (60,70] (80,90] (70,80] (90,100] (90,100] (50,60] (70,80] (90,100]  
[9] (70,80] (70,80] (90,100] (70,80] (80,90] (70,80] (50,60] (90,100]  
[17] (60,70] (50,60] (60,70] (90,100] (90,100] (80,90] (80,90] (90,100]  
[25] (80,90]  
Levels: (50,60] (60,70] (70,80] (80,90] (90,100]
```

```
cut(exam.score,breaks=c(50,60,70,80,90,100),right=F)
```

```
[1] [60,70) [80,90) [70,80) [90,100) [90,100) [50,60) [70,80) [90,100)  
[9] [70,80) [70,80) [90,100) [70,80) [80,90) [70,80) [50,60) [90,100)  
[17] [60,70) [50,60) [60,70) [90,100) [90,100) [80,90) [80,90) [90,100)  
[25] [80,90)  
Levels: [50,60) [60,70) [70,80) [80,90) [90,100)
```

Factors - cut

So if you don't think that the cut command is cool then here is how you would have had to solve the problem - ughhhh !

```
set.seed(123)
exam.score <- runif(25,50,100)

account <- 0
bcount <- 0
ccount <- 0
dcount <- 0
fcount <- 0
exam.score <- runif(25,50,100)
for (ii in 1:length(exam.score)) {
  if (exam.score[ii] < 60) {fcount = fcount + 1} else
    if ((exam.score[ii] >= 60) & (exam.score[ii] < 70)) {dcount = dcount + 1} else
      if ((exam.score[ii] >= 70) & (exam.score[ii] < 80)) {ccount = ccount + 1} else
        if ((exam.score[ii] >= 80) & (exam.score[ii] < 90)) {bcount = bcount + 1} else
          if ((exam.score[ii] >= 90) & (exam.score[ii] <= 100)) {account = account + 1}
}
cat("account bcount ccount dcount fcount")
cat(account,bcount,ccount,dcount,fcount)
account bcount ccount dcount fcount
8 5 7 3 2
```

Factors - cut

Sometimes we want our factors to be ordered. For example, we intuitively know that January comes before February and so on. Can we get R to create ordered factors ?

```
mons <- c("Jan","Feb","Mar","Apr","May","Jun","Jan",  
          "Feb","May","Jun", "Apr","Mar")
```

```
my.fact.mons <- factor(mons)  
[1] Jan Feb Mar Apr May Jun Jan Feb May Jun Apr Mar  
Levels: Apr Feb Jan Jun Mar May
```

```
my.fact.mons[1] < my.fact.mons[2]
```

Warning message:

```
In Ops.factor(my.fact.mons[1], my.fact.mons[2]) :  
  < not meaningful for factors
```

```
levels(my.fact.mons)  
[1] "Apr" "Feb" "Jan" "Jun" "Mar" "May"
```

<http://www.stat.berkeley.edu/classes/s133/factors>

Factors - cut

Sometimes we want our factors to be ordered. For example, we intuitively know that January comes before February and so on. Can we get R to create ordered factors ?

```
my.fact.mons <- factor(mons, labels=c("Jan","Feb","Mar","Apr","May","Jun"),
                        ordered=TRUE)
```

```
my.fact.mons
```

```
[1] Mar Feb May Jan Jun Apr Mar Feb Jun Apr Jan May
```

```
Levels: Jan < Feb < Mar < Apr < May < Jun
```

```
my.fact.mons[1] < my.fact.mons[2]
```

```
[1] FALSE
```

```
table(my.fact.mons)
```

```
my.fact.mons
```

```
Jan Feb Mar Apr May Jun
```

```
  2   2   2   2   2   2
```

```
levels(my.fact.mons)
```

```
# This is what we want !
```

```
[1] "Jan" "Feb" "Mar" "Apr" "May" "Jun"
```

<http://www.stat.berkeley.edu/classes/s133/factors>

Factors - cut

Let's do an AOV on the mtcars data set variables MPG and number of gears the latter of which takes on the values 3,4,5. So it is well suited to be a factor.

```
mtcars$gear <- factor(mtcars$gear) # Turn gear into a factor
aov.ex1 = aov(mpg ~ gear,mtcars)
summary(aov.ex1)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
factor(gear)	2	483.24	241.622	10.901	0.0002948 ***
Residuals	29	642.80	22.166		

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
print(model.tables(aov.ex1,"means"))
Tables of means
Grand mean

20.09062

gear
      3      4      5
16.11 24.53 21.38
rep 15.00 12.00  5.00
```

Factors - cut

Let's do an AOV on the mtcars data set variables MPG and number of gears the latter of which takes on the values 3,4,5. So it is well suited to be a factor.

```
my.tukey <- TukeyHSD(aov.ex1,"gear") # Tukey Multiple Comparisons
my.tukey
```

```
  Tukey multiple comparisons of means
    95% family-wise confidence level
```

```
Fit: aov(formula = mpg ~ gear, data = mtcars)
```

```
$gear
```

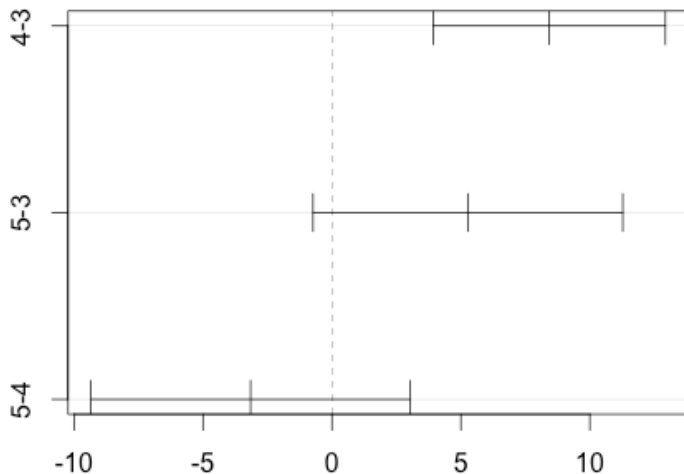
	diff	lwr	upr	p adj
4-3	8.426667	3.9234704	12.929863	0.0002088
5-3	5.273333	-0.7309284	11.277595	0.0937176
5-4	-3.153333	-9.3423846	3.035718	0.4295874

```
plot(my.tukey)
```

Differences between Gears are significant at the five percent level if the confidence interval around the estimation of the difference does not contain zero

Factors - cut

95% family-wise confidence level



Differences in mean levels of gear

Matrices

- Matrices are important mathematical structures for which R has excellent support
- Matrices are ideal for storing information on scientific data
- Think of a matrix as being a vector with dimensions
- There are two common ways to create a matrix

Matrices - Creating

1) Using the **dim()** function. You can think of the following vector as being a matrix with one row and twelve columns.

```
myvec <- c(1:12)
```

To create, for example, a 3x4 matrix use the **dim()** function to adjust the dimensions of the vector

```
dim(myvec) <- c(3,4)
```

```
myvec
```

	[,1]	[,2]	[,3]	[,4]
[1,]	1	4	7	10
[2,]	2	5	8	11
[3,]	3	6	9	12

Note that columns are "filled" before rows. Note also that the requested dimension must make sense with the available number of elements

```
dim(myvec <- c(5,4))
```

```
Error in dim(myvec = c(5, 4)) :
```

```
supplied argument name 'myvec' does not match 'x'
```

Matrices - Creating

2) Using the **matrix()** function.

```
mymat <- matrix( c(7, 4, 2, 4, 7, 2), nrow=3, ncol=2)
      [,1] [,2]
[1,]    7    4
[2,]    4    7
[3,]    2    2
```

You can use the **nrow** and **ncol** arguments to explicitly specify the desired number of rows and columns. You can also request that the rows get filled first as opposed to the columns:

```
mymat <- matrix( c(7, 4, 2, 4, 7, 2), nrow=3, ncol=2, byrow=TRUE)
mymat
      [,1] [,2]
[1,]    7    4
[2,]    2    4
[3,]    7    2
```

Matrices - Naming Rows and Columns

It is useful to name the rows and columns of matrices

```
set.seed(123)
X <- matrix(rpois(20,1.5),nrow=4)
```

```
X
      [,1] [,2] [,3] [,4] [,5]
[1,]    1    4    1    2    1
[2,]    2    0    1    2    0
[3,]    1    1    4    0    1
[4,]    3    3    1    3    4
```

Let's say that these refer to four trials. We want to label the rows "Trial.1", "Trial.2", etc.

```
rownames(X) <- paste("Trial",1:nrow(X),sep=".")
```

```
X
      [,1] [,2] [,3] [,4] [,5]
Trial.1    1    4    1    2    1
Trial.2    2    0    1    2    0
Trial.3    1    1    4    0    1
Trial.4    3    3    1    3    4
```

Matrices - Naming Rows and Columns

We can something similar with the columns

```
colnames(X) <- paste("P",1:ncol(X),sep=".")
```

```
X
```

	P.1	P.2	P.3	P.4	P.5
Trial.1	1	4	1	2	1
Trial.2	2	0	1	2	0
Trial.3	1	1	4	0	1
Trial.4	3	3	1	3	4

Example taken from "The R Book - Michael J. Crawley

Matrices - Naming Rows and Columns

We aren't restricted to naming things according to a pattern

```
drug.names <- c("aspirin","paracetamol","nurofen","hedex","placebo")
```

```
colnames(X) <- drug.names
```

X

	aspirin	paracetamol	nurofen	hedex	placebo
Trial.1	1	4	1	2	1
Trial.2	2	0	1	2	0
Trial.3	1	1	4	0	1
Trial.4	3	3	1	3	4

Example taken from "The R Book - Michael J. Crawley

Matrices - Indexing

Names provide an intuitive way to index into a matrix structure

X

	aspirin	paracetamol	nurofen	hedex	placebo
Trial.1	1	4	1	2	1
Trial.2	2	0	1	2	0
Trial.3	1	1	4	0	1
Trial.4	3	3	1	3	4

```
X['Trial.1',] # Gets all columns for Trial #1
```

aspirin	paracetamol	nurofen	hedex	placebo
1	4	1	2	1

```
# Get's the nurofen column for Trial.1
```

```
X['Trial.1','nurofen']  
[1] 1
```

Example taken from “The R Book - Michael J. Crawley

Matrices - Indexing

Names provide an intuitive way to index into a matrix structure

X

	aspirin	paracetamol	nurofen	hedex	placebo
Trial.1	1	4	1	2	1
Trial.2	2	0	1	2	0
Trial.3	1	1	4	0	1
Trial.4	3	3	1	3	4

```
X[, 'nurofen'] # Get all Trials for nurofen
```

Trial.1	Trial.2	Trial.3	Trial.4
1	1	4	1

```
X[, 'nurofen', drop=FALSE] # Preserves matrix structure if desired
```

	nurofen
Trial.1	1
Trial.2	1
Trial.3	4
Trial.4	1

Matrices - Indexing

It is more common to use numeric indexing especially if you are accessing parts of a matrix from a program

```
set.seed(123)
```

```
X <- matrix(rpois(9,1.5),nrow=3)
```

```
X
```

```
      [,1] [,2] [,3]  
[1,]     1     3     1  
[2,]     2     4     3  
[3,]     1     0     1
```

```
X[1,1]      # First row, First Column  
[1] 1
```

```
X[2,2]      # Second row, Second Column  
[1] 4
```

```
X[3,3]      # Third row, Third column  
[1] 1
```

```
diag(X)      # Ah, there is a function that gets the diagonal.  
[1] 1 4 1     # Always check to see if there is already a function  
              # to do what you want
```

Example taken from “The R Book - Michael J. Crawley

Matrices - Indexing

Extracting information from a matrix is an important skill. Practice makes perfect

X

	[,1]	[,2]	[,3]
[1,]	1	3	1
[2,]	2	4	3
[3,]	1	0	1

```
X[1:2,1]      # Gets First and second rows and the first column  
[1] 1 2
```

```
X[1:2,2]      # Gets First and second rows and the second column  
[1] 3 4
```

```
X[1:2,]       # Gets First and second rows and ALL columns  
  [,1] [,2] [,3]  
[1,]   1   3   1  
[2,]   2   4   3
```

Example taken from "The R Book - Michael J. Crawley

Matrices - Indexing

Extracting information from a matrix is an important skill. Practice makes perfect

X

	[,1]	[,2]	[,3]
[1,]	1	3	1
[2,]	2	4	3
[3,]	1	0	1

X[,c(1,3)] # Get all rows but only columns 1 and 3

	[,1]	[,2]
[1,]	1	1
[2,]	2	3
[3,]	1	1

X[,-2] # Same effect as above. Get all rows and columns except 2

	[,1]	[,2]
[1,]	1	1
[2,]	2	3
[3,]	1	1

Example taken from "The R Book - Michael J. Crawley

Matrices - Indexing

Remember that a matrix is just a vector with dimensions so you could index into a matrix using single bracket notation.

```
X
```

```
      [,1] [,2] [,3]  
[1,]    1    3    1  
[2,]    2    4    3  
[3,]    1    0    1
```

```
X[1:4]
```

```
[1] 1 2 1 3
```

```
X >= 2
```

```
      [,1] [,2] [,3]  
[1,] FALSE  TRUE FALSE  
[2,]  TRUE  TRUE  TRUE  
[3,] FALSE FALSE FALSE
```

```
X[X >= 2]    # Returns which values are greater or equal to 2
```

```
[1] 2 3 4 3
```

```
which(X >= 2)    # Returns which elements are greater or equal to 2
```

```
[1] 2 4 5 8
```

Matrices - Indexing

Remember that a matrix is just a vector with dimensions so you could index into a matrix using single bracket notation.

```
      [,1] [,2] [,3]  
[1,]    1    3    1  
[2,]    2    4    3  
[3,]    1    0    1
```

```
X %% 2 == 0      # Returns a logical matrix
```

```
      [,1] [,2] [,3]  
[1,] FALSE FALSE FALSE  
[2,]  TRUE  TRUE  FALSE  
[3,] FALSE  TRUE  FALSE
```

```
X[ X %% 2 == 0 ]  # Finds the actual element values  
[1] 2 4 0
```

Matrices - Indexing

Remember that a matrix is just a vector with dimensions so you could index into a matrix using single bracket notation.

X

	[,1]	[,2]	[,3]
[1,]	1	3	1
[2,]	2	4	3
[3,]	1	0	1

```
X[X %% 2 == 0] <- 99
```

X

	[,1]	[,2]	[,3]
[1,]	1	3	1
[2,]	99	99	3
[3,]	1	99	1

Matrices - Indexing

There are two functions called `row` and `col` that return the numeric row and column, respectively of the matrix.

`X`

	[,1]	[,2]	[,3]
[1,]	1	3	1
[2,]	2	4	3
[3,]	1	0	1

`row(X)`

	[,1]	[,2]	[,3]
[1,]	1	1	1
[2,]	2	2	2
[3,]	3	3	3

The values correspond to the actual row number

`col(X)`

	[,1]	[,2]	[,3]
[1,]	1	2	3
[2,]	1	2	3
[3,]	1	2	3

The values correspond to the actual col number

Matrices - Indexing

There are two functions called `row` and `col` that return the numeric row and column, respectively of the matrix.

```
row(X) == col(X)
```

```
      [,1] [,2] [,3]  
[1,]  TRUE FALSE FALSE  
[2,] FALSE  TRUE FALSE  
[3,] FALSE FALSE  TRUE
```

```
X[row(X) == col(X)]
```

```
[1] 1 4 1
```

```
X[row(X) == col(X)] <- 0 # Put zeroes in the diagonal
```

```
      [,1] [,2] [,3]  
[1,]    0    3    1  
[2,]    2    0    3  
[3,]    1    0    0
```


Matrices - rbind/cbind

Sometimes we need to add rows and columns to a matrix. There are two commands to do this: `rbind` and `cbind`.

```
set.seed(123)
```

```
X <- matrix(rpois(9,1.5),nrow=3)
```

```
colnames(X) <- c("aspirin","paracetamol","nurofen")
```

```
rownames(X) <- paste("Trial",1:3,sep=".")
```

```
rbind(X,Trial.4=c(4,7,5))
```

	aspirin	paracetamol	nurofen
Trial.1	1	3	1
Trial.2	2	4	3
Trial.3	1	0	1
Trial.4	4	7	5

Matrices - rbind/cbind

Binding columns works pretty much the same way:

```
X
      aspirin paracetamol nurofen
Trial.1      1          3       1
Trial.2      2          4       3
Trial.3      1          0       1
```

```
rowSums(X)
Trial.1 Trial.2 Trial.3
      5      9      2
```

```
cbind(X, rowsums = rowSums(X))
      aspirin paracetamol nurofen rowsums
Trial.1      1          3       1       5
Trial.2      2          4       3       9
Trial.3      1          0       1       2
```

Matrices - Calculations

Let's look at some examples involving calculations on matrices:

```
set.seed(123)
```

```
X <- matrix(rpois(9,1.5),nrow=3)
```

```
colnames(X) <- c("aspirin","paracetamol","nurofen")
```

```
rownames(X) <- paste("Trial",1:3,sep=".")
```

```
X
```

	aspirin	paracetamol	nurofen
Trial.1	1	3	1
Trial.2	2	4	3
Trial.3	1	0	1

```
mean(X[,3]) # Mean of the 3rd column  
[1] 1.666667
```

```
var(X[3,]) # Variance of the 3rd row  
[1] 0.3333333
```

Matrices - Calculations

There are functions optimized specifically for use with matrices. They are very fast and work well on large matrices

```
X
      aspirin paracetamol nurofen
Trial.1      1          3       1
Trial.2      2          4       3
Trial.3      1          0       1
```

```
rowSums(X)
Trial.1 Trial.2 Trial.3
      5      9      2
```

```
colSums(X)
aspirin paracetamol      nurofen
      4          7          5
```

Matrices - Calculations

There are functions optimized specifically for use with matrices. They are very fast and work well on large matrices.

However be careful if the matrix has missing data in which case you will need to include the **na.rm=TRUE** argument.

```
rowMeans(X)
```

```
  Trial.1  Trial.2  Trial.3  
1.6666667 3.0000000 0.6666667
```

```
colMeans(X,na.rm=TRUE)
```

```
  aspirin paracetamol    nurofen  
1.333333  2.333333  1.666667
```

```
colMeans(X)[3]
```

```
  nurofen  
1.666667
```

Matrices - Calculations

There are functions optimized specifically for use with matrices. They are very fast and work well on large matrices.

However be careful if the matrix has missing data in which case you will need to include the **na.rm=TRUE** argument.

```
X[1,2] <- NA
```

```
X
```

	aspirin	paracetamol	nurofen
Trial.1	1	NA	1
Trial.2	2	4	3
Trial.3	1	0	1

```
colMeans(X)
```

aspirin	paracetamol	nurofen
1.333333	NA	1.666667

```
colMeans(X, na.rm=TRUE)
```

aspirin	paracetamol	nurofen
1.333333	2.000000	1.666667

Matrices - apply

There is a function called **apply** which simplifies looping over the rows or columns of a matrix. The “apply family” of functions is important in R.

```
X
      aspirin paracetamol nurofen
Trial.1      3          1       3
Trial.2      2          2       2
Trial.3      2          0       5
```

```
apply(X,1,range)      # 1 is for rows
      Trial.1 Trial.2 Trial.3
[1,]      1      2      0
[2,]      3      4      1
```

```
apply(X,2,range)      # 2 is for columns
      aspirin paracetamol nurofen
[1,]      1          0       1
[2,]      2          4       3
```

Matrices - Linear Algebra

R supports common linear algebra operations

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 8 \\ 2 & 9 \end{bmatrix} \quad t(A) = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 8 & 9 \end{bmatrix} \quad (1)$$

```
A = matrix(c(1,3,2,2,8,9),3,2)
```

```
A
```

	[,1]	[,2]
[1,]	1	2
[2,]	3	8
[3,]	2	9

```
t(A)
```

	[,1]	[,2]	[,3]
[1,]	1	3	2
[2,]	2	8	9

Matrices - Linear Algebra

B

	[,1]	[,2]
[1,]	5	4
[2,]	8	2

```
diag(B)           # Fetches the diagonal  
[1] 5 2
```

```
diag(c(1,2,3))    # Creates a matrix with 1,2,3 on the diagonal
```

	[,1]	[,2]	[,3]
[1,]	1	0	0
[2,]	0	2	0
[3,]	0	0	3

```
diag(1,4)         # Creates a 4 x 4 Identity matrix
```

	[,1]	[,2]	[,3]	[,4]
[1,]	1	0	0	0
[2,]	0	1	0	0
[3,]	0	0	1	0
[4,]	0	0	0	1

Matrices - Linear Algebra

The inverse of a $n \times n$ matrix A is the matrix B (which is also $n \times n$) that when multiplied by A gives the identity matrix

```
A <- matrix(1:4,2,2)
```

A

```
      [,1] [,2]  
[1,]     1     3  
[2,]     2     4
```

```
B = solve(A)
```

B

```
      [,1] [,2]  
[1,]    -2  1.5  
[2,]     1 -0.5
```

```
A %% B      # We get the identity matrix
```

```
      [,1] [,2]  
[1,]     1     0  
[2,]     0     1
```

Matrices - Linear Algebra

Suppose we have the following system of equations

$$\begin{array}{rcl} x_1 + 3x_2 & = & 7 \\ 2x_1 + 4x_2 & = & 10 \end{array}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 10 \end{bmatrix} \text{ i.e. } Ax = b$$

A

```
      [,1] [,2]  
[1,]     1     3  
[2,]     2     4
```

b = c(7,10)

```
x = solve(A) %*% b
```

x

```
      [,1]  
[1,]     1  
[2,]     2
```

<http://bendixcarstensen.com/APC/linalg-notes-BxC.pdf>

Matrices - Eigen values

Eigen values/vectors show up a lot in math - like with Principal Components

```
my.wines <- read.csv("http://steviep42.bitbucket.org/YOUTUBE.DIR/wines.csv",
                     header=T)

my.scaled.wines <- scale(my.wines[,-1])  # Scale the data
my.cov <- cor(my.wines[,-1])             # Get the covariance matrix
my.eigen <- eigen(my.cov)                 # Now find the eigen values/vectors

options(digits=3)

my.eigen                                     # Check out the Eigen values and vectors
$values

[1] 4.76e+00 1.81e+00 3.53e-01 7.44e-02 3.73e-16 -2.61e-16 -2.99e-16

$vectors

      [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
[1,] -0.3965  0.1149  0.80247  0.0519 -1.46e-01  0.00e+00 -4.02e-01
[2,] -0.4454 -0.1090 -0.28106 -0.2745  4.84e-01 -5.18e-01 -3.64e-01
[3,] -0.2646 -0.5854 -0.09607  0.7603  5.41e-16  3.75e-16 -1.16e-15
[4,]  0.4160 -0.3111  0.00734 -0.0939  3.24e-01  4.88e-01 -6.15e-01
[5,] -0.0485 -0.7245  0.21611 -0.5474 -2.16e-01 -3.23e-02  2.80e-01
[6,] -0.4385  0.0555 -0.46576 -0.1687 -5.67e-01  3.86e-01 -2.97e-01
[7,] -0.4547  0.0865  0.06430 -0.0835  5.20e-01  5.85e-01  4.01e-01

$loadings <- my.eigen$vectors
```

Matrices - Eigen values

Eigen values/vectors show up a lot in math - like with Principal Components. The loadings derived in the previous slide are the principal components

```
loadings <- my.eigen$vector
```

The scores are the product of the matrix multiplication between the scaled.wines and the loadings. This takes the original wine data and re-expresses it in terms of the "principal components".

```
scores <- my.scaled.wines %*% loadings
```

Matrices - Clustering

Matrices are also used a lot in cluster analysis. Let's look at a matrix of 32 cars and attempt to cluster them according to their various attributes such as MPG, Number of Cylinders, Gears, Weight, etc. This data set (mtcars) is internal to R so you can refer to it easily.

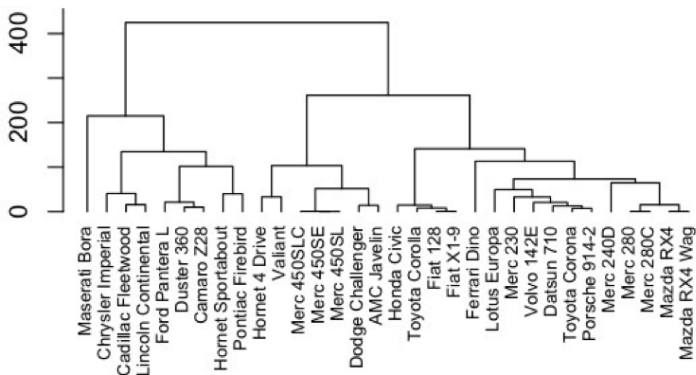
	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
Mazda RX4	21.0	6	160.0	110	3.90	2.62	16.5	0	1	4	4
Mazda RX4 Wag	21.0	6	160.0	110	3.90	2.88	17.0	0	1	4	4
Datsun 710	22.8	4	108.0	93	3.85	2.32	18.6	1	1	4	1
Hornet 4 Drive	21.4	6	258.0	110	3.08	3.21	19.4	1	0	3	1
Hornet Sportabout	18.7	8	360.0	175	3.15	3.44	17.0	0	0	3	2
..											
..											

```
# We first compute a distance between the rows and then cluster them.
```

```
hc <- hclust(dist(mtcars[,2:11])) # The first column is a label.  
plot(hc, hang = -1,cex=0.7)
```

Matrices - Eigen values

Cluster Dendrogram



Matrices - replicate

- We can create matrices using the replicate command
- Useful when doing repeated sampling activity like bootstrapping

```
replicate(4,rnorm(5))
```

	[,1]	[,2]	[,3]	[,4]
[1,]	-1.181720384	0.2717525	-1.4716542	2.26654104
[2,]	0.268970133	0.3423814	0.9553185	0.07749788
[3,]	0.007413904	-1.2102476	0.2273662	-0.46742459
[4,]	1.726961040	0.9977138	2.0491924	0.77174367
[5,]	0.950821481	-1.8599874	-0.8587209	0.95906263

```
some.population <- rnorm(1000)
```

```
replicate(4,sample(some.population, 5, replace=TRUE))
```

	[,1]	[,2]	[,3]	[,4]
[1,]	-0.4680211	0.27727612	-0.5346220	0.94161600
[2,]	0.3138391	0.36105532	0.1108916	0.35186402
[3,]	-1.8416441	-0.05812402	1.3535505	0.05288187
[4,]	-0.9483933	-0.24572418	1.6950778	1.30636068
[5,]	1.0369327	-0.66983941	0.3055545	1.57318148

Matrices - Functions

Operation	Description
$A * B$	Element-wise multiplication
$A^t(A)$	Transpose
$\text{diag}(x)$	Creates diagonal matrix with elements of x in diagonal
$\text{diag}(A)$	Returns a vector with the elements of the diagonal
$\text{diag}(k)$	Creates a $k \times k$ identity matrix
$\text{solve}(A,b)$	Returns vector x in the equation $b = Ax$
$\text{solve}(A)$	Inverse of A where A is a square matrix
$y \leftarrow \text{eigen}(A)$	Gets eigenvalues and eigenvectors of A
$y \leftarrow \text{svd}(A)$	Single value decomposition of A
$y \leftarrow \text{qr}(A)$	QR decomposition of A
$\text{cbind}(A,B,...)$	Combine matrices/vectors horizontally
$\text{rbind}(A,B,...)$	Combine matrices/vectors vertically
$\text{rowMeans}(A)$	Returns vector of row means
$\text{rowSums}(A)$	Returns vector of row sums
$\text{colMeans}(A)$	Returns vector of column means
$\text{colSums}(A)$	Returns vector of column means

Lists

- Lists provide a way to store information of different types within a single data structure
- Remember that vectors and matrices restrict us to only one data type at a time.
- That is we cannot mix, for example, characters and numbers within a vector or matrix.
- Many functions in R return information stored in lists
- Consider the following example wherein we store information about a family. Not all this information is of the same type

Lists

```
family1 <- list(husband="Fred", wife="Wilma", numofchildren=3,  
               agesofkids=c(8,11,14))
```

```
length(family1)  # Has 4 elements  
[1] 4
```

```
family1  
$husband  
[1] "Fred"
```

```
$wife  
[1] "Wilma"
```

```
$numofchildren  
[1] 3
```

```
$agesofkids  
[1] 8 11 14
```

```
str(family1)  
List of 4  
 $ husband      : chr "Fred"  
 $ wife          : chr "Wilma"  
 $ numofchildren: num 3  
 $ agesofkids    : num [1:3] 8 11 14
```

Lists - Creating

If possible, always create named elements. It is easier for humans to index into a named list

```
family1 <- list(husband="Fred", wife="Wilma", numofchildren=3,  
               agesofkids=c(8,11,14))
```

If the list elements have names then use "\$" to access the element

```
family1$agesofkids  
[1] 8 11 14
```

```
family1$agesofkids[1:2]  
[1] 8 11
```

Checkout the sapply command

```
sapply(family1,class)  
      husband      wife numofchildren  agesofkids  
"character"  "character"   "numeric"   "numeric"
```

```
sapply(family1,length)  
      husband      wife numofchildren  agesofkids  
           1           1           1           3
```

Lists - Creating

If the list elements have no names then you have to use numeric indexing

```
family2 <- list("Barney","Betty",2,c(4,6))
```

```
[[1]]
```

```
[1] "Barney"
```

```
[[2]]
```

```
[1] "Betty"
```

```
[[3]]
```

```
[1] 2
```

```
[[4]]
```

```
[1] 4 6
```

Lists - Creating

If the list elements have no names then you have to use numeric indexing

```
family2 <- list("Barney","Betty",2,c(4,6))
```

```
family2[4]      # Accesses the 4th index and associated element  
[[1]]  
[1] 4 6
```

```
family2[[4]]    # Accesses the 4th element value only - more direct  
[1] 4 6
```

```
family2[3:4]    # Get 3rd and 4th indices and associate values  
[[1]]  
[1] 2  
  
[[2]]  
[1] 4 6
```

Lists - unlist()

Use the “unlist” function to turn a list into a vector. Since the list usually has mixed data types then the elements will be converted to a single type.

```
family1 <- list(husband="Fred", wife="Wilma", numofchildren=3,  
               agesofkids=c(8,11,14))
```

```
unlist(family1)
```

husband	wife	numofchildren	agesofkids1	agesofkids2	agesofkids3
"Fred"	"Wilma"	"3"	"8"	"11"	"14"

```
as.numeric(unlist(family1))
```

```
[1] NA NA 3 8 11 14
```

Lists - Uses

As newcomers to R we usually doesn't create lists except in two major cases:

- We are writing a function that does some interesting stuff and we want to return to the user a structure that has information of varying types
- As a precursor to creating a a data frame, which represents a hybrid object with characteristics of a list and a matrix

Lists - Functions

As an example of the first case, R has lots of statistical functions that return lists of information.

```
data(mtcars)    # Load mtcars into the environment
```

```
mylm <- lm(mpg ~ wt, data = mtcars)
```

```
print(mylm)
```

Call:

```
lm(formula = mpg ~ wt, data = mtcars)
```

Coefficients:

(Intercept)	wt
37.285	-5.344

```
# But there is a lot more information
```

```
typeof(mylm)  
[1] "list"
```

Lists - Functions

```
str(mylm,give.attr=F) # Lots of stuff here
```

List of 12

```
$ coefficients : Named num [1:2] 37.29 -5.34
$ residuals    : Named num [1:32] -2.28 -0.92 -2.09 1.3 -0.2 ...
$ effects      : Named num [1:32] -113.65 -29.116 -1.661 1.631 0.111 ...
$ rank         : int 2
$ fitted.values: Named num [1:32] 23.3 21.9 24.9 20.1 18.9 ...
$ assign       : int [1:2] 0 1
$ qr           :List of 5
..$ qr        : num [1:32, 1:2] -5.657 0.177 0.177 0.177 0.177 ...
..$ qraux     : num [1:2] 1.18 1.05
..$ pivot     : int [1:2] 1 2
..$ tol       : num 1e-07
..$ rank      : int 2
$ df.residual  : int 30
$ xlevels      : Named list()
$ call         : language lm(formula = mpg ~ wt, data = mtcars)
$ terms        :Classes 'terms', 'formula' length 3 mpg ~ wt
$ model        : 'data.frame': 32 obs. of 2 variables:
..$ mpg       : num [1:32] 21 21 22.8 21.4 18.7 18.1 14.3 24.4 22.8 19.2 ...
..$ wt        : num [1:32] 2.62 2.88 2.32 3.21 3.44 ...
```

Lists - Functions

```
names(mylm)
```

```
[1] "coefficients" "residuals"    "effects"      "rank"
[5] "fitted.values" "assign"        "qr"           "df.residual"
[9] "xlevels"      "call"          "terms"        "model"
```

```
mylm$effects
```

```
(Intercept)      wt
-113.6497374 -29.1157217 -1.6613339  1.6313943  0.1111305 -0.3840041
-3.6072442  4.5003125  2.6905817  0.6111305 -0.7888695  1.1143917
 0.2316793 -1.6061571  1.3014525  2.2137818  6.0995633  7.3094734
 2.2421594  6.8956792 -2.2010595 -2.6694078 -3.4150859 -3.1915608
 2.7346556  0.8200064  0.5948771  1.7073457 -4.2045529 -2.4018616
-2.9072442 -0.6494289
```

```
# Some use the $ notation to extract desired information they want straight
# from the function call
```

```
lm(mpg ~ wt, data = mtcars)$coefficients
```

```
(Intercept)      wt
 37.285126    -5.344472
```

Lists - Functions

Some other basic R functions will return a list - such as some of the character functions:

```
mystring <- "This is a test"
```

```
mys <- strsplit(mystring, " ")
```

```
str(mys)
```

```
List of 1
```

```
$ : chr [1:4] "This" "is" "a" "test"
```

```
mys
```

```
[[1]]
```

```
[1] "This" "is"    "a"     "test"
```

```
mys[[1]][1]
```

```
[1] "This"
```

```
mys[[1]][1:2]
```

```
[1] "This" "is"
```

```
unlist(mys)
```

```
[1] "This" "is"    "a"     "test"
```

Lists - Functions

When we create our own functions we can return a list

```
my.summary <- function(x) {  
  return.list <- list()  
  return.list$mean <- mean(x)  
  return.list$sd <- sd(x)  
  return.list$var <- var(x)  
  return(return.list)  
}
```

```
my.summary(1:10)
```

```
$mean
```

```
[1] 5.5
```

```
$sd
```

```
[1] 3.02765
```

```
$var
```

```
[1] 9.166667
```

```
names(my.summary(1:10))
```

```
[1] "mean" "sd"   "var"
```

```
my.summary(1:10)$var
```

```
[1] 9.166667
```

Lists - sapply/lapply

Remember the sapply command ? We use it to apply a function over each element of a list or a vector. This helps us avoid having to write a “for-loop” every time we want to process a list or a vector.

```
# sapply( vector_or_list, function_to_apply_to_each element)
```

```
family1 <- list(husband="Fred", wife="Wilma", numofchildren=3,  
               agesofkids=c(8,11,14))
```

```
sapply(family1,class)
```

husband	wife	numofchildren	agesofkids
"character"	"character"	"numeric"	"numeric"

```
sapply(family1,length)
```

husband	wife	numofchildren	agesofkids
1	1	1	3

Lists - `sapply/lapply`

sapply tries to return a "simplified" version of the output (either a vector or matrix) hence the "s" in the "sapply". If you don't use something like `sapply` then the example on the previous slide would look this:

```
# sapply( vector_or_list, function_to_apply_to_each element)

family1 <- list(husband="Fred", wife="Wilma", numofchildren=3,
               agesofkids=c(8,11,14))

for (ii in 1:length(family1)) {
  cat(names(family1)[ii], " : ", class(family1[[ii]]), "\n")
}
```

More involved than just doing

```
sapply(family1, class)
      husband      wife numofchildren  agesofkids
"character"  "character"    "numeric"    "numeric"
```

Lists - sapply/lapply

- Similar to **sapply**, the **lapply** function let's you “apply” some function over each element of a list or vector.
- It will return a list version of the output hence the “l” in the “lapply”.
- So deciding between sapply and lapply simply is a question of format. What do you want back ? A vector or list ? Most of the time I use sapply.

Lists - sapply/lapply

```
# lapply( vector_or_list, function_to_apply_to_each element)

family1 <- list(husband="Fred", wife="Wilma", numofchildren=3,
               agesofkids=c(8,11,14))

lapply(family1,class)
      $husband
[1] "character"

      $wife
[1] "character"

      $numofchildren
[1] "numeric"

      $agesofkids
[1] "numeric"
```

Lists - sapply/lapply

```
lapply(family1,mean)
```

```
$husband
```

```
[1] NA
```

```
$wife
```

```
[1] NA
```

```
$numofchildren
```

```
[1] 3
```

```
$agesofkids
```

```
[1] 11
```

Warning messages:

```
1: In mean.default(X[[1L]], ...) :
```

```
argument is not numeric or logical: returning NA
```

```
2: In mean.default(X[[2L]], ...) :
```

```
argument is not numeric or logical: returning NA
```

Lists - sapply/lapply

```
my.func <- function(x) {  
  if(class(x)=="numeric") {  
    return(mean(x))  
  }  
}
```

```
lapply(Family, my.func)  
$husband  
NULL
```

```
$wife  
NULL
```

```
$num.of.children  
[1] 3
```

```
$child.ages  
[1] 6.67
```

Lists - twitterR

Example from the twitterR package

```
delta.tweets <- searchTwitter('@delta', n = 100)
class(delta.tweets)
[1] "list"
```

```
delta.tweets
[[1]]
[1] "sotsoy: Apparently if you use your frequent flier miles on @delta
they stick you at the back of the plane on every flight next to the bathroom"
```

```
[[2]]
[1] "ImTooNonFiction: My @Delta flight has been delayed for the last 2 hrs.
We've been on plane at gate for 2+ hours and no mention of a voucher
or compensation"
```

```
[[3]]
[1] "ShaneNHara: @Delta and @DeltaAssist, thank you for a swift boarding
process here at SEA en route to LAX. Taking care of your loyal flyers
= appreciated."
```

```
[[4]]
[1] "NaiiOLLG: RT @TheRealNickMara: ThankYou @Delta for a great flight!! #Work!!!"
```