BIOS 545 Week 2

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Factors

- R supports factors which are a special data type for managing categories of data
- Identifying categorical variables is usually straightforward. These are the variables by which you might want to summarize some continuous data
- Categorical variables usually take on a definite number of values
- Many times R functions will convert character labels to factors by default but not always
- Storing data as factors insures that R modeling functions will treat such data correctly

Factors

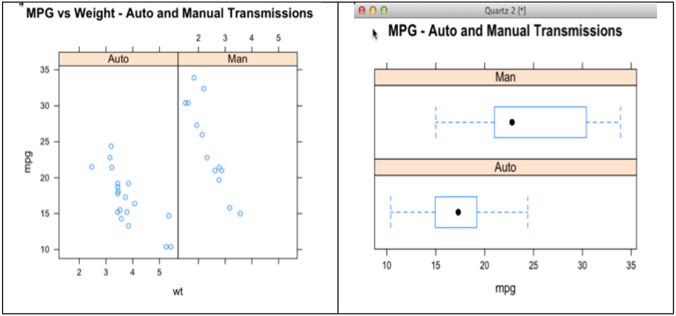
Let's say we have some automobile data that tells us if a car has an automatic transmission (0) or a manual transmission (1). We store this into a vector called transvec

```
transvec <- c(1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,0,0,0,0,0,1,1,1,1,1,1,1)

table(transvec)    #How many are Auto and Manual ? Count 'em up.
transvec
 0  1
19 13

mytransfac <- factor(transvec, levels = c(0,1), labels = c("Auto","Man") )

levels(mytransfac)
[1] "Auto" "Man"

mytransfac
[1] Man  Man  Man  Auto Auto Auto Auto Auto Auto Auto Auto Auto Auto Auto Auto
[16] Auto Auto Man  Man  Man  Auto Auto Auto Auto Auto Man  Man  Man  Man  Man
[31] Man  Man
Levels: Auto Man
```

Factors

R knows how to handle factors when doing plots. Here were get an X/Y plot and a Box Plot with very little work since R knows that mytransfac is a factor

```
library(lattice)
xyplot(mpg~wt | mytransfac, mtcars,
       main="MPG vs Weight - Auto and Manual Transmissions")

bwplot(~mpg|mytransfac, mtcars,
        main="MPG - Auto and Manual Transmissions",layout=c(1,2))
```



Factors - aggregation preview

With our knowledge of factors and vectors we can do some basic aggregation using the tapply command. We have a factor vector called mytransfac.

Let's summarize some MPG data that corresponds to the automobiles used in the mytransfac vector. So for each car we have its MPG figure and whether it has an automatic or manual transmission.

```
mympg <- c(21,21,22.8,21.4,18.7,18.1,14.3,24.4,22.8,19.2,17.8,
           16.4,17.3,15.2,10.4,10.4,14.7,32.4,30.4,33.9, 21.5,
           15.5,15.2,13.3,19.2,27.3,26,30.4,15.8,19.7,15,21.4)

# tapply( continuous_value_to_summarize,  factor_or_grouping variable,
          function_for_summary)

tapply(mympg, mytransfac, mean)
    Auto      Man
17.14737 24.39231
```

Factors - cut

It is sometimes useful to take a continuous variable and chop it up into intervals or categories for purposes of summary or grouping. R has a function to do this called "cut" to accomplish this.

Let's work through some examples to understand what is going on. Let's cut up the numbers between 0 and 10 into 4 distinct intervals

```
cut(0:10,breaks=4)

[1] (-0.01,2.5] (-0.01,2.5] (-0.01,2.5] (2.5,5]    (2.5,5]   (2.5,5]   (5,7.5]
    (5,7.5]     (7.5,10]    (7.5,10]    (7.5,10]
Levels: (-0.01,2.5] (2.5,5] (5,7.5] (7.5,10]

table(cut(0:10,breaks=4))

(-0.01,2.5]     (2.5,5]     (5,7.5]    (7.5,10]
          3           3           2           3

table(cut(0:10,breaks=4))
(-0.01,2.5]     (2.5,5]     (5,7.5]    (7.5,10]
          3           3           2           3
```

Factors - cut

Well that was cool but since we are creating 4 intervals we should probably name them

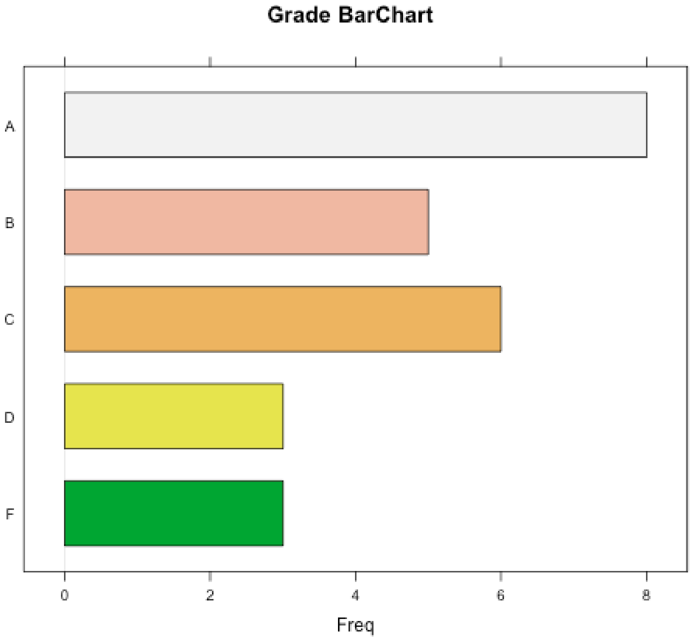
```
my.cut <- cut(0:10,breaks=4,labels=c("Q1","Q2","Q3","Q4"))
[1] Q1 Q1 Q1 Q2 Q2 Q2 Q3 Q3 Q4 Q4 Q4
Levels: Q1 Q2 Q3 Q4

table(my.cut)
my.cut
Q1 Q2 Q3 Q4
 3  3  2  3

# But you can to take finer-grained control over how the intervals are made.

quantile(0:10)
  0%  25%  50%  75% 100%
 0.0  2.5  5.0  7.5 10.0

table(cut(0:10,breaks=quantile(0:10),include.lowest=TRUE))

 [0,2.5]  (2.5,5]  (5,7.5] (7.5,10]
       3        3        2        3
```

Factors - cut

Let's summarize some exam scores according to a typical grading system.
F: $<$ 60, D: 60-70: C: 70-80: B:80-90 A:90-100

```
set.seed(123)
exam.score <- runif(25,50,100)  # Simulate some scores

cut(exam.score,breaks=c(0,60,70,80,90,100))
 [1] (60,70]  (80,90]  (70,80]  (90,100] (90,100] (0,60]   (70,80]  (90,100]
 [9] (70,80]  (70,80]  (90,100] (70,80]  (80,90]  (70,80]  (0,60]   (90,100]
[17] (60,70]  (0,60]   (60,70]  (90,100] (90,100] (80,90]  (80,90]  (90,100]
[25] (80,90]
Levels: (0,60] (60,70] (70,80] (80,90] (90,100]


cut(exam.score,breaks=c(0,60,70,80,90,100),labels=c("F","D","C","B","A"))
 [1] D B C A A F C A C C A C B C F A D F D A A B B A B
Levels: F D C B A

my.table <- table(cut(exam.score,breaks=c(0,60,70,80,90,100),
                  labels=c("F","D","C","B","A")))

F D C B A
3 3 6 5 8

barchart(my.table,main="Grade BarChart",col=terrain.colors(5))
```

Factors - cut



Factors - cut

- But the intervals don't exactly match the grading scheme
- A score of 90 will get a B when it should get an A
- Note the [and) characters show us if the interval boundary includes a score
- There is an argument that can help **right**)

```
cut(exam.score,breaks=c(0,60,70,80,90,100))
 [1] (60,70]  (80,90]  (70,80]  (90,100] (90,100] (0,60]   (70,80]  (90,100]
 [9] (70,80]  (70,80]  (90,100] (70,80]  (80,90]  (70,80]  (0,60]   (90,100]
[17] (60,70]  (0,60]   (60,70]  (90,100] (90,100] (80,90]  (80,90]  (90,100]
[25] (80,90]
Levels: (0,60] (60,70] (70,80] (80,90] (90,100]

cut(exam.score,breaks=c(0,60,70,80,90,100),right=F)
 [1] [60,70)  [80,90)  [70,80)  [90,100) [90,100) [0,60)   [70,80)  [90,100)
 [9] [70,80)  [70,80)  [90,100) [70,80)  [80,90)  [70,80)  [0,60)   [90,100)
[17] [60,70)  [0,60)   [60,70)  [90,100) [90,100) [80,90)  [80,90)  [90,100)
[25] [80,90)
Levels: [0,60) [60,70) [70,80) [80,90) [90,100)
```

Factors - cut

So if you don't think that the cut command is cool then here is how you would have had to solve the problem - ughhh !

```
set.seed(123)
exam.score <- runif(25,50,100)

acount <- 0
bcount <- 0
ccount <- 0
dcount <- 0
fcount <- 0
exam.score <- runif(25,50,100)
for (ii in 1:length(exam.score)) {

 if (exam.score[ii] < 60) {fcount = fcount + 1} else
  if ((exam.score[ii] >= 60) & (exam.score[ii] < 70)) {dcount = dcount + 1} else
   if ((exam.score[ii] >= 70) & (exam.score[ii] < 80)) {ccount = ccount +1} else
    if ((exam.score[ii] >= 80) & (exam.score[ii] < 90)) {bcount = bcount +1} else
     if ((exam.score[ii] >= 90) & (exam.score[ii] <= 100)) {acount = acount +1}
}
cat("acount bcount ccount dcount fcount")
cat(acount,bcount,ccount,dcount,fcount)
acount bcount ccount dcount fcount
8 5 7 3 2
```

Factors - cut

Sometimes we want our factors to be ordered. For example, we intuitively know that January comes before February and so on. Can we get R to create ordered factors ?

```
mons <- c("Jan","Feb","Mar","Apr","May","Jun","Jan",
          "Feb","May","Jun", "Apr","Mar")

my.fact.mons <- factor(mons)
 [1] Jan Feb Mar Apr May Jun Jan Feb May Jun Apr Mar
Levels: Apr Feb Jan Jun Mar May

my.fact.mons[1] < my.fact.mons[2]

Warning message:
In Ops.factor(my.fact.mons[1], my.fact.mons[2]) :
  < not meaningful for factors

levels(my.fact.mons)
[1] "Apr" "Feb" "Jan" "Jun" "Mar" "May"
```

http://www.stat.berkeley.edu/classes/s133/factors

Factors - cut

Sometimes we want our factors to be ordered. For example, we intuitively know that January comes before February and so on. Can we get R to create ordered factors ?

```
my.fact.mons <- factor(mons, labels=c("Jan","Feb","Mar","Apr","May","Jun"),
                        ordered=TRUE)
my.fact.mons
 [1] Mar Feb May Jan Jun Apr Mar Feb Jun Apr Jan May
Levels: Jan < Feb < Mar < Apr < May < Jun

my.fact.mons[1] < my.fact.mons[2]
[1] FALSE

table(my.fact.mons)
my.fact.mons
Jan Feb Mar Apr May Jun
  2   2   2   2   2   2

levels(my.fact.mons)                               # This is what we want !
[1] "Jan" "Feb" "Mar" "Apr" "May" "Jun"
```

http://www.stat.berkeley.edu/classes/s133/factors

Factors - cut - Supplemental

Let's do an AOV on the mtcars data set variables MPG and number of gears the latter of which takes on the values 3,4,5. So it is well suited to be a factor.

```
mtcars$gear <- factor(mtcars$gear)  # Turn gear into a factor
aov.ex1 = aov(mpg ~ gear,mtcars)
summary(aov.ex1)
             Df Sum Sq Mean Sq F value    Pr(>F)
factor(gear)  2 483.24 241.622  10.901 0.0002948 ***
Residuals    29 642.80  22.166
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
print(model.tables(aov.ex1,"means"))
Tables of means
Grand mean

20.09062

 gear
        3     4     5
    16.11 24.53 21.38
rep 15.00 12.00  5.00
```

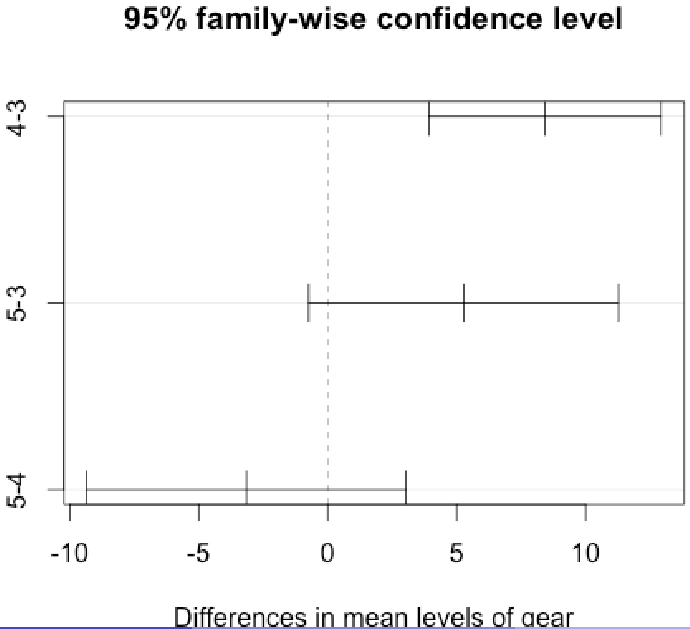
Factors - cut - Supplemental

Let's do an AOV on the mtcars data set variables MPG and number of gears the latter of which takes on the values 3,4,5. So it is well suited to be a factor.

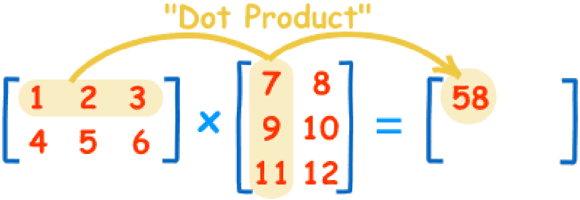
```
my.tukey <- TukeyHSD(aov.ex1,"gear")  # Tukey Multiple Comparisons
my.tukey
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = mpg ~ gear, data = mtcars)

$gear
         diff        lwr       upr     p adj
4-3  8.426667  3.9234704 12.929863 0.0002088
5-3  5.273333 -0.7309284 11.277595 0.0937176
5-4 -3.153333 -9.3423846  3.035718 0.4295874

plot(my.tukey)
```

Differences between Gears are significant at the five percent level if the confidence interval around the estimation of the difference does not contain zero

Factors - cut - Supplemental



Matrices



Matrices

- Matrices are important mathematical structures for which R has excellent support
- Matrices are ideal for storing information on scientific data
- Think of a matrix as being a vector with dimnensions
- There are two common ways to create a matrix

Matrices - Creating

1) Using the **dim()** function. You can think of the following vector as being a matrix with one row and twelve columns.

```
myvec <- c(1:12)
```

To create, for example, a 3x4 matrix use the **dim()** function to adjust the dimensions of the vector

```
dim(myvec) <- c(3,4)
myvec
     [,1] [,2] [,3] [,4]
[1,]    1    4    7   10
[2,]    2    5    8   11
[3,]    3    6    9   12
```

Note that columns are "filled" before rows. Note also that the requested dimension must make sense with the available number of elements

```
dim(myvec) <- c(5,4)
Error in dim(myvec) <- c(5, 4) :
  dims [product 20] do not match the length of object [12]
```

Matrices - Creating

2) Using the **matrix()** function.

```
mymat <- matrix( c(7, 4, 2, 4, 7, 2), nrow=3, ncol=2)
     [,1] [,2]
[1,]    7    4
[2,]    4    7
[3,]    2    2
```

You can use the **nrow** and **ncol** arguments to explicity specify the desired number of rows and columns. You can also request that the rows get filled first as opposed to the columns:

```
mymat <- matrix( c(7, 4, 2, 4, 7, 2), nrow=3, ncol=2, byrow=TRUE)
mymat
     [,1] [,2]
[1,]    7    4
[2,]    2    4
[3,]    7    2
```

Matrices - Naming Rows and Columns

It is useful to name the rows and columns of matrices

```
set.seed(123)
X <- matrix(rpois(20,1.5),nrow=4)
X
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    4    1    2    1
[2,]    2    0    1    2    0
[3,]    1    1    4    0    1
[4,]    3    3    1    3    4
```

Let's say that these refer to four trials. We want to label the rows "Trial.1", "Trial.2", etc.

```
rownames(X) <- paste("Trial",1:nrow(X),sep=".")
X
        [,1] [,2] [,3] [,4] [,5]
Trial.1    1    4    1    2    1
Trial.2    2    0    1    2    0
Trial.3    1    1    4    0    1
Trial.4    3    3    1    3    4
```

Matrices - Naming Rows and Columns

We can something similar with the columns

```
colnames(X) <- paste("P",1:ncol(X),sep=".")
```

```
X
        P.1 P.2 P.3 P.4 P.5
Trial.1   1   4   1   2   1
Trial.2   2   0   1   2   0
Trial.3   1   1   4   0   1
Trial.4   3   3   1   3   4
```

Example taken from “The R Book - Michael J. Crawley

Matrices - Naming Rows and Columns

We aren't restricted to naming things according to a pattern

```
drug.names <- c("aspirin","paracetamol","nurofen","hedex","placebo")

colnames(X) <- drug.names

X
        aspirin paracetamol nurofen hedex placebo
Trial.1       1           4       1     2       1
Trial.2       2           0       1     2       0
Trial.3       1           1       4     0       1
Trial.4       3           3       1     3       4
```

Example taken from "The R Book - Michael J. Crawley

Matrices - Indexing

Names provide an intutive way to index into a matrix structure

```
X
        aspirin paracetamol nurofen hedex placebo
Trial.1       1           4       1     2       1
Trial.2       2           0       1     2       0
Trial.3       1           1       4     0       1
Trial.4       3           3       1     3       4

X['Trial.1',]   # Gets all columns for Trial #1

    aspirin paracetamol     nurofen       hedex     placebo
          1           4           1           2           1

# Get's the nurofen column for Trial.1

X['Trial.1','nurofen']
[1] 1
```

Example taken from "The R Book - Michael J. Crawley

Matrices - Indexing

Names provide an intutive way to index into a matrix structure

```
X
        aspirin paracetamol nurofen hedex placebo
Trial.1       1           4       1     2       1
Trial.2       2           0       1     2       0
Trial.3       1           1       4     0       1
Trial.4       3           3       1     3       4


X[,'nurofen']                    # Get all Trials for nurofen
Trial.1 Trial.2 Trial.3 Trial.4
      1       1       4       1

X[,'nurofen',drop=FALSE]   # Preserves matrix structure if desired
  nurofen
Trial.1       1
Trial.2       1
Trial.3       4
Trial.4       1
```

Matrices - Indexing

It is more common to use numeric indexing especially if you are accessing parts of a matrix from a program

```
set.seed(123)
X <- matrix(rpois(9,1.5),nrow=3)
X
     [,1] [,2] [,3]
[1,]    1    3    1
[2,]    2    4    3
[3,]    1    0    1

X[1,1]     # First row, First Column
[1] 1

X[2,2]     # Second row, Second Column
[1] 4

X[3,3]     # Third row, Third column
1] 1

diag(X)     # Ah, there is a function that gets the diagonal.
[1] 1 4 1   # Always check to see if there is already a function
            # to do what you want
```

Example taken from “The R Book - Michael J. Crawley

Matrices - Indexing

Extracting information from a matrix is an important skill. Practice makes perfect

```
X
     [,1] [,2] [,3]
[1,]    1    3    1
[2,]    2    4    3
[3,]    1    0    1


X[1:2,1]     # Gets First and second rows and the first column
[1] 1 2

X[1:2,2]     # Gets First and second rows and the second column
[1] 3 4

X[1:2,]      # Gets First and second rows and ALL columns
    [,1] [,2] [,3]
[1,]    1    3    1
[2,]    2    4    3
```

Example taken from “The R Book - Michael J. Crawley

Matrices - Indexing

Extracting information from a matrix is an important skill. Practice makes perfect

```
X
     [,1] [,2] [,3]
[1,]    1    3    1
[2,]    2    4    3
[3,]    1    0    1

X[,c(1,3)]     # Get all rows but only columns 1 and 3
      [,1] [,2]
[1,]    1    1
[2,]    2    3
[3,]    1    1


X[,-2]   # Same effect as above. Get all rows and columns except 2
     [,1] [,2]
[1,]    1    1
[2,]    2    3
[3,]    1    1
```

Example taken from "The R Book - Michael J. Crawley

Matrices - Indexing

Remember that a matrix is just a vector with dimensions so you could index into a matrix using single bracket notation.

```
X
     [,1] [,2] [,3]
[1,]    1    3    1
[2,]    2    4    3
[3,]    1    0    1

X[1:4]
[1] 1 2 1 3

X >= 2
      [,1]  [,2]  [,3]
[1,] FALSE  TRUE FALSE
[2,]  TRUE  TRUE  TRUE
[3,] FALSE FALSE FALSE


X[X >= 2]   # Returns which values are greater or equal to 2
[1] 2 3 4 3

which(X >= 2)   # Returns which elements are greater or equal to 2
[1] 2 4 5 8
```

Matrices - Indexing

Remember that a matrix is just a vector with dimensions so you could index into a matrix using single bracket notation.

```
     [,1] [,2] [,3]
[1,]    1    3    1
[2,]    2    4    3
[3,]    1    0    1

X %% 2 == 0          # Returns a logical matrix
      [,1]  [,2]  [,3]
[1,] FALSE FALSE FALSE
[2,]  TRUE  TRUE FALSE
[3,] FALSE  TRUE FALSE


X[ X %% 2 == 0 ]   # Finds the actual element values
[1] 2 4 0
```

Matrices - Indexing

Remember that a matrix is just a vector with dimensions so you could index into a matrix using single bracket notation.

```
X
     [,1] [,2] [,3]
[1,]    1    3    1
[2,]    2    4    3
[3,]    1    0    1


X[X %%  2 == 0] <- 99

X
     [,1] [,2] [,3]
[1,]    1    3    1
[2,]   99   99    3
[3,]    1   99    1
```

Matrices - Indexing

There are two functions called row and col that return the numeric row and column, respectively of the matrix.

```
X
     [,1] [,2] [,3]
[1,]    1    3    1
[2,]    2    4    3
[3,]    1    0    1

row(X)
     [,1] [,2] [,3]
[1,]    1    1    1    # The values correspond to the actual row number
[2,]    2    2    2
[3,]    3    3    3

col(X)                   # The values correspond to the actual col number
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    1    2    3
[3,]    1    2    3
```

Matrices - Indexing

There are two functions called row and col that return the numeric row and column, respectively of the matrix.

```
row(X) == col(X)

      [,1]  [,2]  [,3]
[1,]  TRUE FALSE FALSE
[2,] FALSE  TRUE FALSE
[3,] FALSE FALSE  TRUE


X[row(X) == col(X)]
[1] 1 4 1

X[row(X) == col(X)] <- 0  # Put zeroes in the diagonal

     [,1] [,2] [,3]
[1,]    0    3    1
[2,]    2    0    3
[3,]    1    0    0
```

Matrices - Calculations

Let's look at some examples involving calculations on matricies:

```
set.seed(123)

X <- matrix(rpois(9,1.5),nrow=3)

colnames(X) <- c("aspirin","paracetamol","nurofen")

rownames(X) <- paste("Trial",1:3,sep=".")

X
        aspirin paracetamol nurofen
Trial.1       1           3       1
Trial.2       2           4       3
Trial.3       1           0       1

mean(X[,3])  # Mean of the 3rd column
[1] 1.666667

var(X[3,]) # Variance of the 3rd row
[1] 0.3333333
```

Matrices - Calculations

There are functions optimized specifcally for use with matrices. They are very fast and work well on large matrices

```
X
        aspirin paracetamol nurofen
Trial.1       1           3       1
Trial.2       2           4       3
Trial.3       1           0       1

rowSums(X)
Trial.1 Trial.2 Trial.3
      5       9       2

colSums(X)
aspirin paracetamol     nurofen
   4           7           5
```

Matrices - Calculations

There are functions optimized specfcally for use with matrices. They are very fast and work well on large matrices.

However be careful if the matrix has missing data in wich case you will need to include the **na.rm=TRUE** arugement.

```
rowMeans(X)
  Trial.1   Trial.2   Trial.3
1.6666667 3.0000000 0.6666667

colMeans(X,na.rm=TRUE)
    aspirin paracetamol     nurofen
   1.333333    2.333333    1.666667

colMeans(X)[3]
 nurofen
1.666667
```

Matrices - Calculations

There are functions optimized specfcally for use with matrices. They are very fast and work well on large matrices.

However be careful if the matrix has missing data in wich case you will need to include the **na.rm=TRUE** arugement.

```
X[1,2] <- NA
X
        aspirin paracetamol nurofen
Trial.1       1          NA       1
Trial.2       2           4       3
Trial.3       1           0       1



colMeans(X)
    aspirin paracetamol     nurofen
   1.333333          NA    1.666667

colMeans(X, na.rm=TRUE)
    aspirin paracetamol     nurofen
   1.333333    2.000000    1.666667
```

Matrices - apply

There is a function called **apply** which simplifies looping over the rows or columns of a matrix. The "apply family" of functions is important in R.

```
X
        aspirin paracetamol nurofen
Trial.1       1           3       1
Trial.2       2           4       3
Trial.3       1           0       1

apply(X,1,range)            # 1 is for rows
     Trial.1 Trial.2 Trial.3
[1,]       1       2       0
[2,]       3       4       1

apply(X,2,range)            # 2 is for columns
     aspirin paracetamol nurofen
[1,]       1           0       1
[2,]       2           4       3
```

Matrices - Linear Algebra

R supports common linear algebra operations

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 8 \\ 2 & 9 \end{bmatrix} t(A) = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 8 & 9 \end{bmatrix} \tag{1}$$

```
A <- matrix(c(1,3,2,2,8,9),3,2)
A
     [,1] [,2]
[1,]    1    2
[2,]    3    8
[3,]    2    9
```

```
t(A)
     [,1] [,2] [,3]
[1,]    1    3    2
[2,]    2    8    9
```

Matrices - Linear Algebra

```
B
     [,1] [,2]
[1,]    5    4
[2,]    8    2

diag(B)              # Fetches the diagonal
[1] 5 2

diag(c(1,2,3))       # Creates a matrix with 1,2,3 on the diagonal
     [,1] [,2] [,3]
[1,]    1    0    0
[2,]    0    2    0
[3,]    0    0    3

diag(1,4)            # Creates a 4 x 4 Indentity matrix
     [,1] [,2] [,3] [,4]
[1,]    1    0    0    0
[2,]    0    1    0    0
[3,]    0    0    1    0
[4,]    0    0    0    1
```

Matrices - Linear Algebra

The inverse of a n x n matrix A is the matrix B (which is also n x n) that when multiplied by A gives the identity matrix

```
A <- matrix(1:4,2,2)
A
     [,1] [,2]
[1,]    1    3
[2,]    2    4

B <- solve(A)


B
     [,1] [,2]
[1,]   -2  1.5
[2,]    1 -0.5

A %*% B             # We get the identity matrix
     [,1] [,2]
[1,]    1    0
[2,]    0    1
```

http://bendixcarstensen.com/APC/linalg-notes-BxC.pdf

Matrices - Linear Algebra

Suppose we have the following system of equations

$$\begin{aligned} x_1 + 3x_2 &= 7 \\ 2x_1 + 4x_2 &= 10 \end{aligned}$$	$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 10 \end{bmatrix} \text{ i.e. } Ax = b$$

```
A
     [,1] [,2]
[1,]    1    3
[2,]    2    4

b <- c(7,10)
x <- solve(A) %*% b
x
     [,1]
[1,]    1
[2,]    2
```

http://bendixcarstensen.com/APC/linalg-notes-BxC.pdf

Matrices - replicate

- We can create matrices using the replicate command
- Useful when doing repeated sampling activity like bootstrapping

```
replicate(4,rnorm(5))
             [,1]       [,2]       [,3]        [,4]
[1,] -1.181720384  0.2717525 -1.4716542  2.26654104
[2,]  0.268970133  0.3423814  0.9553185  0.07749788
[3,]  0.007413904 -1.2102476  0.2273662 -0.46742459
[4,]  1.726961040  0.9977138  2.0491924  0.77174367
[5,]  0.950821481 -1.8599874 -0.8587209  0.95906263

some.population <- rnorm(1000)

replicate(4,sample(some.population, 5, replace=TRUE))
           [,1]        [,2]       [,3]       [,4]
[1,] -0.4680211  0.27727612 -0.5346220 0.94161600
[2,]  0.3138391  0.36105532  0.1108916 0.35186402
[3,] -1.8416441 -0.05812402  1.3535505 0.05288187
[4,] -0.9483933 -0.24572418  1.6950778 1.30636068
[5,]  1.0369327 -0.66983941  0.3055545 1.57318148
```

Matrices - Functions

Operation	Description
A * B	Element-wise multiplication
A t(A)	Transpose
diag(x)	Creates diagonal matrix with elements of x in diagonal
diag(A)	Returns a vector with the elements of the diagonal
diag(k)	Creates a k x k identity matrix
solve(A,b)	Returns vector x in the equation b = Ax
solve(A)	Inverse of A where A is a square matrix
y <- eigen(A)	Gets eigenvalues and eigenvectors of A
y <- svd(A)	Single value decomposition of A
y <- qr(A)	QR decomposition of A
cbind(A,B,...)	Combine matrices/vectors horizontally
rbind(A,B,...)	Combine matrices/vectors vertically
rowMeans(A)	Retruns vector of row means
rowSums(A)	Returns vector of row sums
colMeans(A)	Returns vector of column means
colSums(A)	Returns vector of column means

Matrices - Eigen values - Supplemental

Eigen values/vectors show up a lot in math - like with Principal Components

```
my.wines <- read.csv("http://steviep42.bitbucket.org/YOUTUBE.DIR/wines.csv",
                     header=T)
my.scaled.wines <- scale(my.wines[,-1])   # Scale the data
my.cov <- cor(my.wines[,-1])        # Get the covariance matrix
my.eigen <- eigen(my.cov)             # Now find the eigen values/vectors

options(digits=3)
my.eigen                             # Check out the Eigen values and vectors
$values
[1]  4.76e+00  1.81e+00  3.53e-01  7.44e-02  3.73e-16 -2.61e-16 -2.99e-16

$vectors
        [,1]    [,2]     [,3]    [,4]      [,5]      [,6]      [,7]
[1,] -0.3965  0.1149  0.80247  0.0519 -1.46e-01  0.00e+00 -4.02e-01
[2,] -0.4454 -0.1090 -0.28106 -0.2745  4.84e-01 -5.18e-01 -3.64e-01
[3,] -0.2646 -0.5854 -0.09607  0.7603  5.41e-16  3.75e-16 -1.16e-15
[4,]  0.4160 -0.3111  0.00734 -0.0939  3.24e-01  4.88e-01 -6.15e-01
[5,] -0.0485 -0.7245  0.21611 -0.5474 -2.16e-01 -3.23e-02  2.80e-01
[6,] -0.4385  0.0555 -0.46576 -0.1687 -5.67e-01  3.86e-01 -2.97e-01
[7,] -0.4547  0.0865  0.06430 -0.0835  5.20e-01  5.85e-01  4.01e-01

$loadings <- my.eigen$vectors
```

Matrices - Eigen values Supplemental

Eigen values/vectors show up a lot in math - like with Principal Components. The loadings dervied in the previous slide are the principal componenents

```
loadings <- my.eigen$vectors
```

The scores are the product of the matrix multiplication between the scaled.wines and the loadings. This takes the original wine data and re-expresses it in terms of the "principal components".

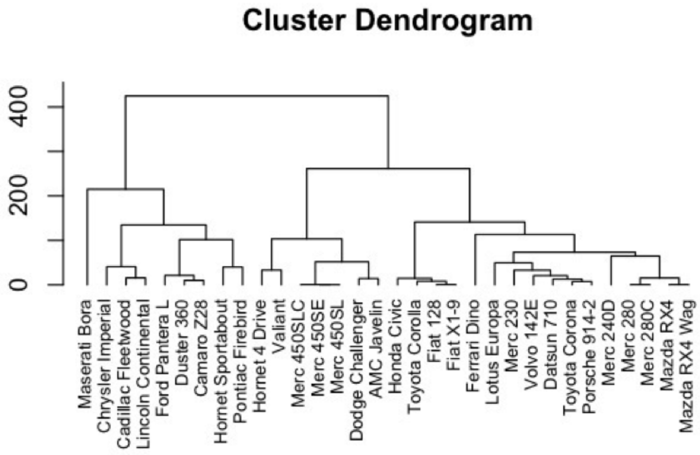
```
scores <- my.scaled.wines %*% loadings
```

Matrices - Clustering - Supplemental

Matrices are also used a lot in cluster analysis. Let's look at a matrix of 32 cars and attempt to cluster them according to their various attributes such as MPG, Number of Cylinders, Gears, Weight, etc. This data set (mtcars) is internal to R so you can refer to it easily.

```
                    mpg cyl  disp  hp drat   wt qsec vs am gear carb
Mazda RX4           21.0   6 160.0 110 3.90 2.62 16.5  0  1    4    4
Mazda RX4 Wag       21.0   6 160.0 110 3.90 2.88 17.0  0  1    4    4
Datsun 710          22.8   4 108.0  93 3.85 2.32 18.6  1  1    4    1
Hornet 4 Drive      21.4   6 258.0 110 3.08 3.21 19.4  1  0    3    1
Hornet Sportabout   18.7   8 360.0 175 3.15 3.44 17.0  0  0    3    2
..
..

# We first compute a distance between the rows and then cluster them.

hc <- hclust(dist(mtcars[,2:11]))  # The first column is a label.
plot(hc, hang = -1,cex=0.7)
```

Matrices - Clustering - Supplemental



Lists

Lists

- Lists provide a way to store information of different types within a single data structure
- Remember that vectors and matrices restrict us to only one data type at a time.
- That is we cannot mix, for example, characters and numbers within a vector or matrix.
- Many functions in R return information stored in lists
- Consider the following example wherein we store information about a family. Not all this information is of the same type

Lists

```
family1 <- list(husband="Fred", wife="Wilma", numofchildren=3,
                agesofkids=c(8,11,14))

length(family1)  # Has 4 elements
[1] 4

family1
$husband
[1] "Fred"

$wife
[1] "Wilma"

$numofchildren
[1] 3

$agesofkids
[1]  8 11 14

str(family1)
List of 4
 $ husband      : chr "Fred"
 $ wife         : chr "Wilma"
 $ numofchildren: num 3
 $ agesofkids   : num [1:3] 8 11 14
```

Lists - Creating

If possible, always create named elements. It is easier for humans to index into a named list

```
family1 <- list(husband="Fred", wife="Wilma", numofchildren=3,
                agesofkids=c(8,11,14))

# If the list elements have names then use "$" to access the element

family1$agesofkids
[1]  8 11 14

family1$agesofkids[1:2]
[1]  8 11

family1[1]
$husband
[1] "Fred"

family1[[1]]
[1] "Fred"
```

Lists - Creating

If the list elements have no names then you have to use numeric indexing

```
family2 <- list("Barney","Betty",2,c(4,6))
[[1]]
[1] "Barney"

[[2]]
[1] "Betty"

[[3]]
[1] 2

[[4]]
[1] 4 6

str(family2)
List of 4
 $ : chr "Barney"
 $ : chr "Betty"
 $ : num 2
 $ : num [1:2] 4 6
```

Lists - Creating

If the list elements have no names then you have to use numeric indexing

```
family2 <- list("Barney","Betty",2,c(4,6))

family2[4]    # Accesses the 4th index and associated element
[[1]]
[1] 4 6

family2[[4]]   # Accesses the 4th element value only - more direct
[1] 4 6

family2[3:4]   # Get 3rd and 4th indices and associate values
[[1]]
[1] 2

[[2]]
[1] 4 6
```

Lists - Uses

As newcomers to R we usually doesn't create lists except in two major cases:

- As a precursor to creating a a data frame, which represents a hybrid object with characteristics of a list and a matrix
- We are writing a function that does some interesting stuff and we want to return to the user a structure that has information of varying types. R does this all of the time by returning list structures from statistical modeling functions.

Lists - Functions

R has lots of statistical functions that return lists of information.

```
data(mtcars)   # Load mtcars into the environment

mylm <- lm(mpg ~ wt, data = mtcars)

print(mylm)

Call:
lm(formula = mpg ~ wt, data = mtcars)

Coefficients:
(Intercept)           wt
     37.285       -5.344

# But there is a lot more information

typeof(mylm)
[1] "list"
```

Lists - Functions

```
str(mylm,give.attr=F)  # Lots of stuff here

List of 12
 $ coefficients : Named num [1:2] 37.29 -5.34
 $ residuals    : Named num [1:32] -2.28 -0.92 -2.09 1.3 -0.2 ...
 $ effects      : Named num [1:32] -113.65 -29.116 -1.661 1.631 0.111 ...
 $ rank         : int 2
 $ fitted.values: Named num [1:32] 23.3 21.9 24.9 20.1 18.9 ...
 $ assign       : int [1:2] 0 1
 $ qr           :List of 5
  ..$ qr   : num [1:32, 1:2] -5.657 0.177 0.177 0.177 0.177 ...
  ..$ qraux: num [1:2] 1.18 1.05
  ..$ pivot: int [1:2] 1 2
  ..$ tol  : num 1e-07
  ..$ rank : int 2
 $ df.residual  : int 30
 $ xlevels      : Named list()
 $ call         : language lm(formula = mpg ~ wt, data = mtcars)
 $ terms        :Classes 'terms', 'formula' length 3 mpg ~ wt
 $ model        :'data.frame':  32 obs. of  2 variables:
  ..$ mpg: num [1:32] 21 21 22.8 21.4 18.7 18.1 14.3 24.4 22.8 19.2 ...
  ..$ wt : num [1:32] 2.62 2.88 2.32 3.21 3.44 ...
```

Lists - Functions

```
names(mylm)
 [1] "coefficients"  "residuals"     "effects"       "rank"
 [5] "fitted.values" "assign"        "qr"            "df.residual"
 [9] "xlevels"       "call"          "terms"         "model"

mylm$effects
 (Intercept)          wt
-113.6497374  -29.1157217   -1.6613339    1.6313943    0.1111305   -0.3840041
  -3.6072442    4.5003125    2.6905817    0.6111305   -0.7888695    1.1143917
   0.2316793   -1.6061571    1.3014525    2.2137818    6.0995633    7.3094734
   2.2421594    6.8956792   -2.2010595   -2.6694078   -3.4150859   -3.1915608
   2.7346556    0.8200064    0.5948771    1.7073457   -4.2045529   -2.4018616
  -2.9072442   -0.6494289

# Some use the $ notation to extract desired information they want straight
# from the function call

lm(mpg ~ wt, data = mtcars)$coefficients
(Intercept)          wt
  37.285126   -5.344472
```

Lists - Functions

Some other basic R functions will return a list - such as some of the character functions:

```
mystring <- "This is a test"

mys <- strsplit(mystring, " ")

str(mys)
List of 1
 $ : chr [1:4] "This" "is" "a" "test"

mys
[[1]]
[1] "This" "is"   "a"    "test"

mys[[1]][1]
[1] "This"

mys[[1]][1:2]
[1] "This" "is"

unlist(mys)
[1] "This" "is"   "a"    "test"
```

Lists - Functions

When we create our own functions we can return a list

```
my.summary <- function(x) {
  return.list <- list()
  return.list$mean <- mean(x)
  return.list$sd <- sd(x)
  return.list$var <- var(x)
  return(return.list)
}

my.summary(1:10)
$mean
[1] 5.5

$sd
[1] 3.02765

$var
[1] 9.166667

names(my.summary(1:10))
[1] "mean" "sd"   "var"

my.summary(1:10)$var
[1] 9.166667
```

Lists - sapply/lapply

As with the apply command for matrices, there is a command(s) that will allow us to process each element of a list. This helps us avoid having to write a "for-loop" every time we want to process a list.

```
# sapply( vector_or_list, function_to_apply_to_each element)

family1 <- list(husband="Fred", wife="Wilma", numofchildren=3,
                agesofkids=c(8,11,14))

sapply(family1,class)
      husband          wife numofchildren    agesofkids
  "character"   "character"     "numeric"     "numeric"

sapply(family1,length)
      husband          wife numofchildren    agesofkids
            1             1             1             3
```

Lists - sapply/lapply

sapply tries to return a "simplified" version of the output (either a vector, list, or a matrix), hence the "s" in the "sapply". If you don't use something like sapply then the example on the previous slide would look this:

```
# sapply( vector_or_list, function_to_apply_to_each element)

family1 <- list(husband="Fred", wife="Wilma", numofchildren=3,
                agesofkids=c(8,11,14))

for (ii in 1:length(family1)) {
   cat(names(family1)[ii]," : ",class(family1[[ii]]),"\n")
}

# More involved than just doing

sapply(family1,class)
      husband          wife numofchildren    agesofkids
  "character"   "character"     "numeric"     "numeric"
```

Lists - sapply/lapply

- Similar to **sapply**, the **lapply** function let's you "apply" some function over each element of a list or vector.
- It will return a list version of the output hence the "l" in the "lapply".
- So deciding between sapply and lapply simply is a question of format. What do you want back ? A vector or list ? Most of the time I use sapply.

Lists - sapply/lapply

```
sapply(family1,mean)
$husband
NULL

$wife
NULL

$numofchildren
[1] 3

$agesofkids
[1] 11

Warning messages:
1: In mean.default(X[[1L]], ...) :
  argument is not numeric or logical: returning NA
2: In mean.default(X[[2L]], ...) :
  argument is not numeric or logical: returning NA
```

Lists - sapply/lapply

```
my.func <- function(x) {
  if(class(x)=="numeric") {
    return(mean(x))
  }
}

sapply(family1, my.fun)
$husband
NULL

$wife
NULL

$num.of.children
[1] 3

$child.ages
[1] 6.67
```

You may wish to view the following YouTube playlist wherein I have three videos that describe lists and lapply in detail:

`https://www.youtube.com/playlist?list=PL905DXZOAgwwj16m6C3ioh6aVKDDrEiiO`

Also see my blog post on lapply at:

`https://rollingyours.wordpress.com/2014/10/20/the-lapply-command-101/`

And see this one on apply for matricies at:

`https://rollingyours.wordpress.com/2014/05/15/the-apply-command-101/`

Lists - twitteR - Supplemental

Example from the twitteR package

```
delta.tweets <- searchTwitter('@delta', n = 100)
class(delta.tweets)
[1] "list"

delta.tweets
[[1]]
[1] "sotsoy: Apparently if you use your frequent flier miles on @delta
they stick you at the back of the plane on every flight next to the bathroom"

[[2]]
[1] "ImTooNonFiction: My @Delta flight has been delayed for the last 2 hrs.
We've been on plane at gate for 2+ hours and no mention of a voucher
or compensation"

[[3]]
[1] "ShaneNHara: @Delta and @DeltaAssist, thank you for a swift boarding
process here at SEA en route to LAX. Taking care of your loyal flyers
= appreciated."

[[4]]
[1] "NaiiOLLG: RT @TheRealNickMara: ThankYou @Delta for a great flight!! #Work!!!"
```