1 Calculating the real heat conduction

Here, we test various assumptions made by [1]. First, we see, whether the assumption of an effective temperature difference holds:

$$\Delta_{eff} = 2(T_b - T_c),$$

For this test, we calculate the real heat conduction q_{cond} based on the heat conduction at every point $\lambda(z)$ and integrating the:

$$q_{cond} = \lambda(z)\frac{\partial T}{\partial z}.$$

The integration is done via a shooting method starting from the top plate temperature T_t with a varying parameter q_{cond} to hit T_b at z=L (L being the height).

From the real temperature profile T(z) one calculates the density ϱ at each z and subsequently the hydrostatic pressure p(z) $\partial p/\partial z = g\varrho z$. In this integration, there is a constant p_0 that is chosen such that $p(L/2) = P_m$ being the pressure in the U-Boot. From p(z) and T(z) we recalculate the fluid properties $\varrho(z)$ and $\lambda(z)$, that we plug back into Ficks law. In an iterative way, we get the real profile $T(z)$, p(z), $\varrho(z)$, and $\alpha(z)$.

For test purposes, I assume: L=2.2m;$T_b = 40$C; $T_t = 13$C and P=19 bar. We note, that T_t at this pressure is already in the liquid regime. However, the SF6 program (by Guenter) extrapolate the gas values into this regime. For test purposes this should be fine.

Integration is done with 10000 intervals (in z-direction) using a simple Euler scheme.

Already after 2 iterations, q has reached its final value: Iteration:

- 0: q = 0.189488 [W/m^2] (linear profile)
- 1: q = 0.190150 [W/m^2]
- :
- 6: q = 0.190202 [W/m^2]
- Heat conduction assuming linear profile and $\lambda(T_m)$: $q_{cond} = 0.1895 W/m^2$

The result shows, that in this regime the conductive heat transport q_{cond} can well be approximated by $q_{cond} \approx \lambda(T_m)\Delta T/L$. The reason becomes clear from fig. 1. The variation of the temperature from a linear profile is small in comparision to ΔT. The hydrostatic pressure is rather small and thus also λ doesn't change significant with z. Only ϱ changes remarkable due to the temperature gradients.

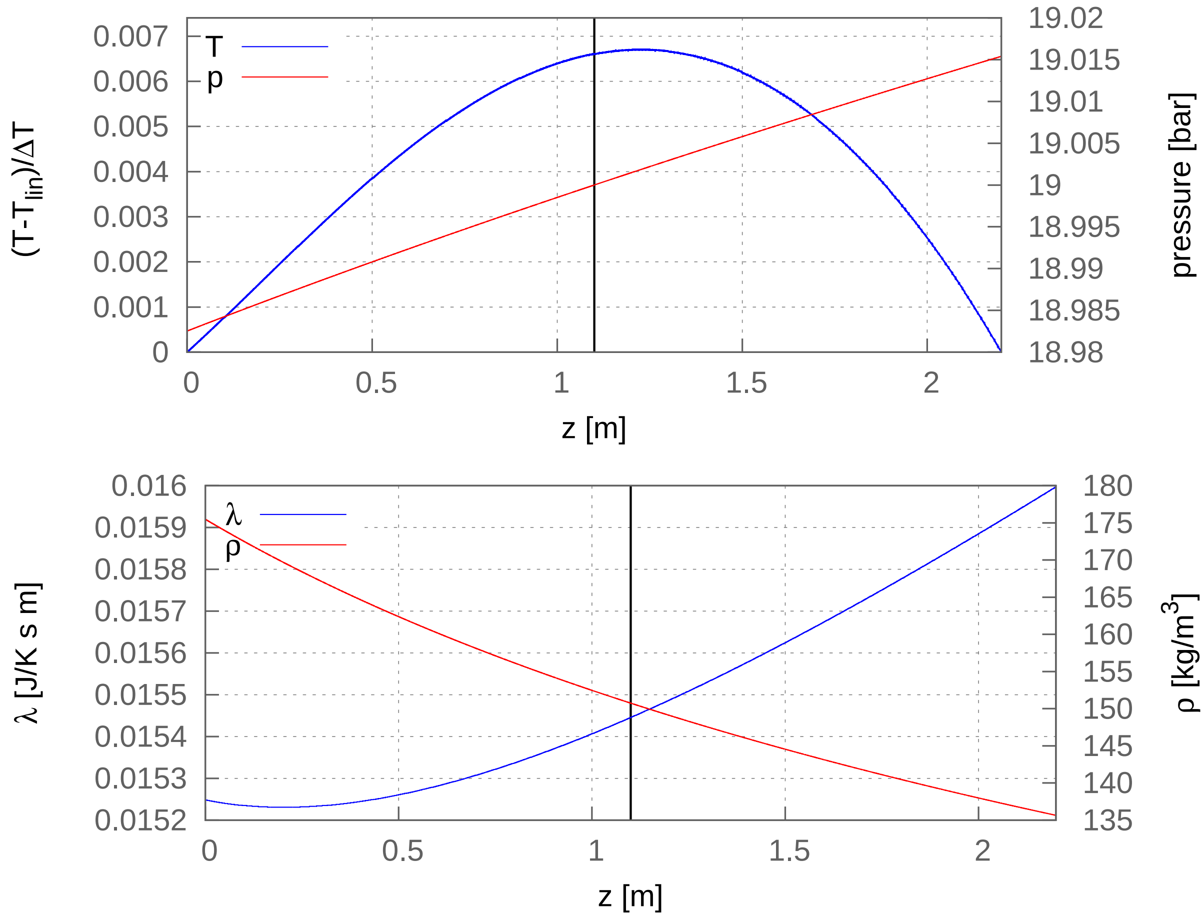

Figure 1: Top: Variation from the linear Temperature profile and pressure profile p(z). Bottom: Density $\varrho(z)$ and heat heat conductivity $\lambda(z)$. **Note, that the z- axis shows in downward direction (against usual convention).** The vertical black line shows the midheight of the cell.

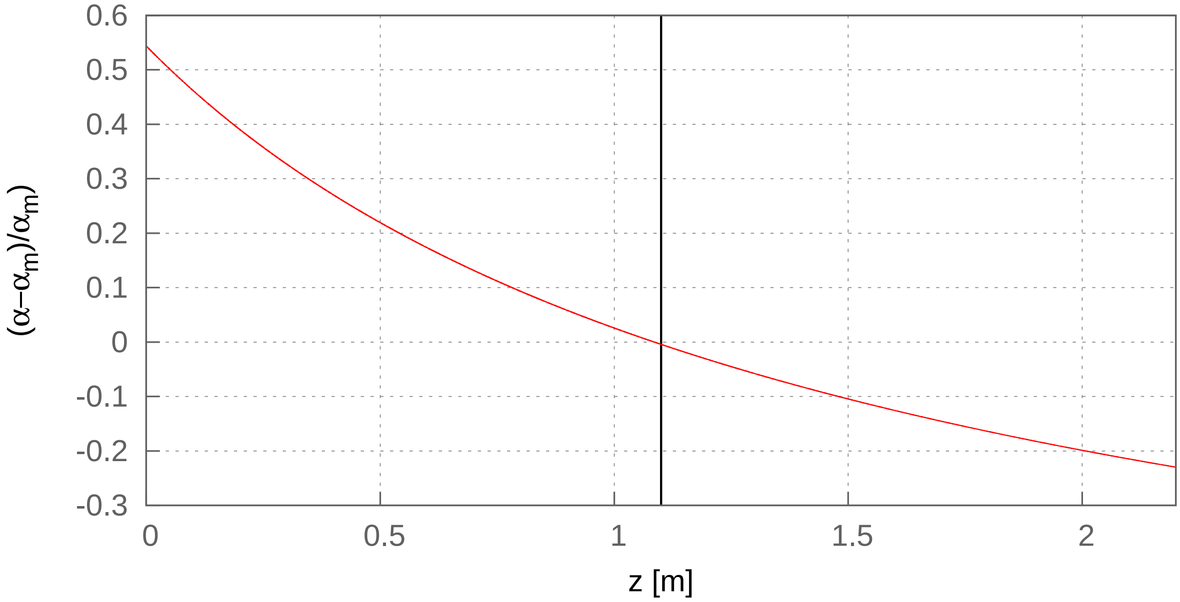

Figure 2: The normalised heat expansion coefficient as a function of z, assuming the T(z) and p(z) as shown in fig. 1. The vertical black line shows the midheight of the cell.

2 Analysing $\Gamma = 1$-Data

In the following, we analyse the $\Gamma = 1$- data, provided by Xiaozhou. As we have seen above, q_{cond} doesn't change much when numerically calculated versus via $\lambda(T_m)\Delta T/L$.

We use in the following: L=1.12 m. The result of an reanalysis is shown in fig. 3. One clearly sees that whatever q_{cond} we choose, the differences in $Nu/Ra^{1/3}$ is small. In particular, the transition at around $Ra = 10^{14}$ is clearly visible. However, any corrections reduces $Nu/Ra^{1/3}$ slightly. The blue bullets show the original analysis with fluid properties evaluated at $T_m = (T_t + T_b)/2$. Using the numerically calculated conduction profile for calculating Nu (denoted as Nu_r) decreases Nu very little (less than 0.2%). The largest degrease is visible in the open pink squares, that show the numerically calculated Nu, normalised by Ra_c which is Ra with the fluid parameters calculated at the midheight temperature T_c.

References

[1] L. Skrbek and P. Urban. Has the ultimate state of turbulent thermal convection been observed? *Journal of Fluid Mechanics*, 785:270–282, 12 2015.

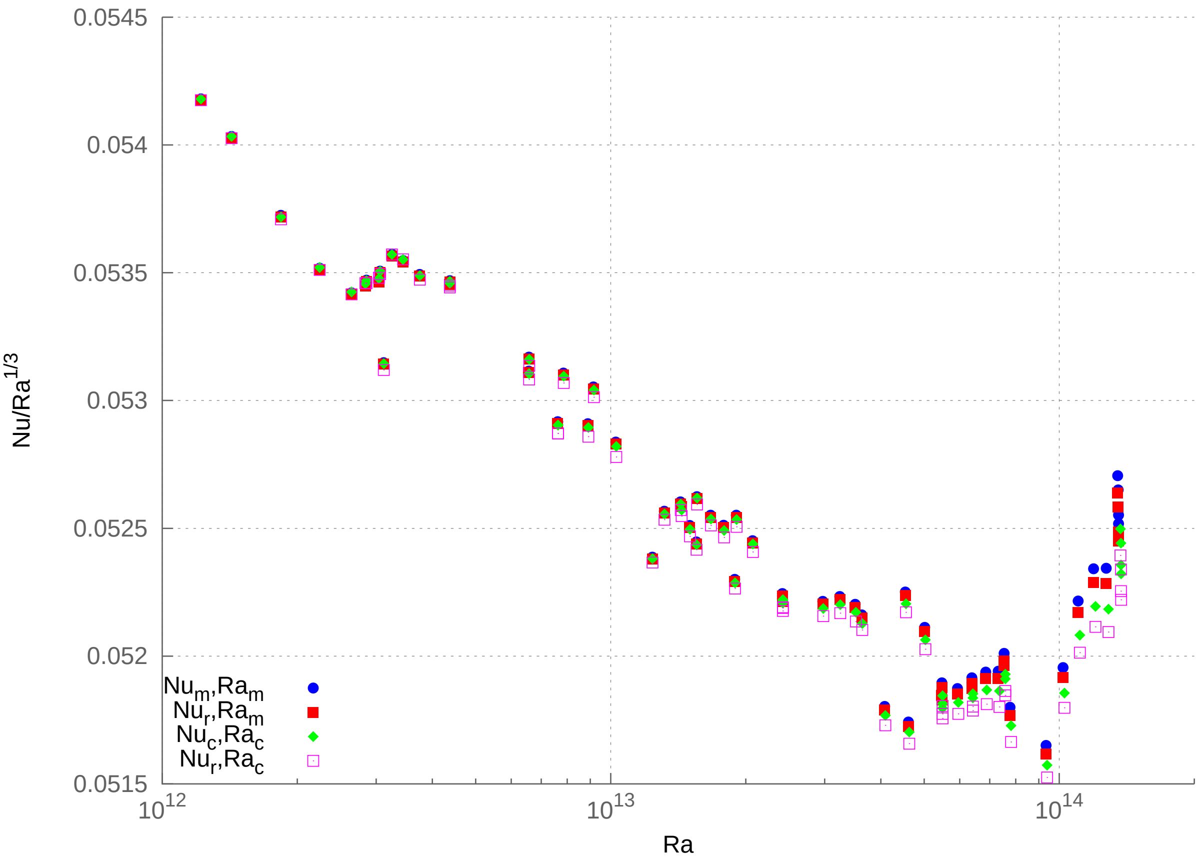

Figure 3: Various ways to calculate Nu and Ra compared with each other. Data are from the $\Gamma = 1$-experiment. Nu_r denote Nu based on the real numerically calculated q_{cond}. Nu_m stands for Nu with q_{cond} calculated $\lambda(T_m)$, and Nu_c uses $\lambda(T_c)$. Similarly for Ra.