Evaluating Traveling Efficiency in Different Urban Layouts

CSCI-UA 144 MATH-UA 330

Introduction to Computer Simulation

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MATLAB Code:
`https://github.com/stevloc/matlab/tree/main/Evaluating%20Traveling%20Efficiency%20in%20Different%20Urban%20Layouts`

1 Introduction

The efficiency of travel within urban environments is significantly influenced by the geometric structure of street networks. The configuration of roads dictates how directly individuals can move between two points, impacting accessibility, commuting time, and energy consumption. This study investigates how two contrasting urban designs—a grid layout and a radial-ring system—affect travel efficiency, utilizing simulated movement data.

In grid-based cities like Manhattan, streets intersect at right angles, forming a uniform rectangular grid. This structure allows for multiple paths of equal length between most points but restricts movement to horizontal and vertical directions. The distance between two locations in such a setting is typically calculated using the Manhattan distance, which sums the absolute differences in the horizontal and vertical positions. This metric effectively models movement along a grid-like street network, making it useful for urban planning and logistics (DataCamp, 2023).

Conversely, cities like Amsterdam often employ radial-ring systems, where streets form a series of concentric circles connected by radial spokes. This design facilitates both circumferential and radial travel, allowing some trips to pass directly through the city center. However, the uneven spacing of connections may lead to longer paths between certain points. Recent studies have explored how ring-radial layouts in cities like Dalian and Paris shape block morphology and performance, highlighting the impact of urban form on travel efficiency (Zhang & Li, 2025).

To compare these layouts, this study defines efficiency as the ratio between the Euclidean distance (the straight-line distance between two points) and the actual path length traveled. An efficiency closer to 1 indicates a route that closely follows the optimal straight line, while lower values suggest greater deviation. Using randomized start and end points, a fixed movement speed, and consistent block lengths, this project simulates travel on both network types to evaluate which urban geometry promotes more direct and efficient movement patterns.

2 Equations

In this project, we simulate travel efficiency between two random points in two types of urban layouts: a grid-based layout (Manhattan) and a radial-ring layout (Amsterdam). Each system has its own governing equations based on geometric rules and travel assumptions. The unknowns are the start and goal positions. The known quantities, such as block length and travel speed, are fixed parameters for the simulation.

2.1 Manhattan Model (Grid-Based Layout)

The following values are known and used to define the simulation setup:

- $x_{\text{start}}, y_{\text{start}}$: Start location (integer grid points)
- $x_{\text{goal}}, y_{\text{goal}}$: Goal location (integer grid points)

- L: Block length (meters)
- v: Travel speed (meters per second)

The travel distance along the grid is:

$$d_{\text{travel}} = L \times (|x_{\text{goal}} - x_{\text{start}}| + |y_{\text{goal}} - y_{\text{start}}|) \tag{1}$$

This equation calculates the total distance traveled along the Manhattan grid, where movement is restricted to horizontal and vertical streets. The Euclidean (straight-line) distance between start and goal is:

$$d_{\text{euclid}} = L \times \sqrt{(x_{\text{goal}} - x_{\text{start}})^2 + (y_{\text{goal}} - y_{\text{start}})^2} \tag{2}$$

This represents the straight-line distance, ignoring the grid structure. The travel time required to complete the trip is:

$$t = \frac{d_{\text{travel}}}{v} \tag{3}$$

This assumes constant travel speed v.

2.2 Amsterdam Model (Radial-Ring Layout)

The following values are known and used to define the simulation setup:

- $r_{\text{start}}, \theta_{\text{start}}$: Start position (radius and angle in polar coordinates)
- $r_{\text{goal}}, \theta_{\text{goal}}$: Goal position (radius and angle in polar coordinates)
- L: Radial block length (meters)
- v: Travel speed (meters per second)
- S: Total number of spokes in the layout

Three possible travel paths are considered:
1. Radial first, then arc at goal radius:

$$d_A = |r_{\text{goal}} - r_{\text{start}}| + r_{\text{goal}} \times |\theta_{\text{goal}} - \theta_{\text{start}}| \tag{4}$$

This path first moves radially to match the goal radius, then along the arc to reach the final angular position.
2. Arc first at start radius, then radial:

$$d_B = r_{\text{start}} \times |\theta_{\text{goal}} - \theta_{\text{start}}| + |r_{\text{goal}} - r_{\text{start}}| \tag{5}$$

This path first moves along the arc at the start radius, then radially to the goal.
3. Through the center:

$$d_C = r_{\text{start}} + r_{\text{goal}} \tag{6}$$

This path travels inward to the city center (radius zero) and outward again to the goal.
The travel distance is selected as the minimum of these three paths:

$$d_{\text{travel}} = \min(d_A, d_B, d_C) \tag{7}$$

The start and goal points are converted from polar to Cartesian coordinates:

$$x_{\text{start}} = r_{\text{start}} \cos(\theta_{\text{start}}) \tag{8}$$
$$y_{\text{start}} = r_{\text{start}} \sin(\theta_{\text{start}}) \tag{9}$$
$$x_{\text{goal}} = r_{\text{goal}} \cos(\theta_{\text{goal}}) \tag{10}$$
$$y_{\text{goal}} = r_{\text{goal}} \sin(\theta_{\text{goal}}) \tag{11}$$

The Euclidean distance is then:

$$d_{\text{euclid}} = \sqrt{(x_{\text{goal}} - x_{\text{start}})^2 + (y_{\text{goal}} - y_{\text{start}})^2} \tag{12}$$

This measures the straight-line distance between start and goal.
The travel time is:

$$t = \frac{d_{\text{travel}}}{v} \tag{13}$$

This assumes movement at a constant speed v.

2.3 Efficiency Calculation (Both Models)

For both models, the efficiency of the trip is defined as:

$$\text{Efficiency} = \frac{d_{\text{euclid}}}{d_{\text{travel}}} \tag{14}$$

Efficiency quantifies how direct the trip is relative to the straight-line distance. Values closer to 1 indicate more direct paths.

2.4 Setup Equations

Random initial and goal positions are selected in both models. In the Amsterdam model, the angular locations are determined by evenly spacing spokes across a half-circle:

$$\theta = \text{linspace}\left(-\frac{\pi}{2}, \frac{\pi}{2}, \frac{S}{2}\right) \tag{15}$$

This ensures that the available directions are evenly distributed between $-90°$ and $90°$.

3 Numerical Method

The governing equations for this project are algebraic and are used directly for the numerical solution. No differential equations or time-stepping methods are required.

Manhattan Model Coordinate System

The Manhattan model is based on a Cartesian grid. Coordinates are represented as integer grid indices (x, y), where:

- x increases from left to right (columns),
- y increases from bottom to top (rows),
- Each grid block has uniform length L.

The numerical method consists of the following steps:

1. Randomly generate start and goal coordinates $(x_{\text{start}}, y_{\text{start}})$ and $(x_{\text{goal}}, y_{\text{goal}})$ within the grid bounds.
2. Compute the travel distance using
$$d_{\text{travel}} = L \times (|x_{\text{goal}} - x_{\text{start}}| + |y_{\text{goal}} - y_{\text{start}}|). \tag{16}$$
3. Compute the Euclidean distance using
$$d_{\text{euclid}} = L \times \sqrt{(x_{\text{goal}} - x_{\text{start}})^2 + (y_{\text{goal}} - y_{\text{start}})^2}. \tag{17}$$
4. Compute the travel time
$$t = \frac{d_{\text{travel}}}{v}, \tag{18}$$
and the efficiency
$$\text{Efficiency} = \frac{d_{\text{euclid}}}{d_{\text{travel}}}. \tag{19}$$

Amsterdam Model Coordinate System

The Amsterdam model is structured as a half-circle composed of concentric rings and radial spokes. Each location is described using polar coordinates (r, θ), where:

- r is the radial index from the center (e.g., ring number), scaled by block length L so that $r = \text{ring} \times L$,
- θ is the angular position, determined by the spoke index,
- Angular positions are evenly spaced between $-\pi/2$ (leftmost spoke) and $+\pi/2$ (rightmost spoke).

The numerical method proceeds as follows:

1. Randomly generate start and goal positions $(r_{\text{start}}, \theta_{\text{start}})$ and $(r_{\text{goal}}, \theta_{\text{goal}})$ based on uniformly sampled ring and spoke indices.
2. Compute the three candidate path distances:
$$d_A = |r_{\text{goal}} - r_{\text{start}}| + r_{\text{goal}} \times |\theta_{\text{goal}} - \theta_{\text{start}}|, \tag{20}$$
$$d_B = r_{\text{start}} \times |\theta_{\text{goal}} - \theta_{\text{start}}| + |r_{\text{goal}} - r_{\text{start}}|, \tag{21}$$
$$d_C = r_{\text{start}} + r_{\text{goal}}. \tag{22}$$
3. Select the minimum of these three values:
$$d_{\text{travel}} = \min(d_A, d_B, d_C). \tag{23}$$

4. Convert polar coordinates to Cartesian coordinates using:

$$x_{\text{start}} = r_{\text{start}} \cos(\theta_{\text{start}}), \tag{24}$$
$$y_{\text{start}} = r_{\text{start}} \sin(\theta_{\text{start}}), \tag{25}$$
$$x_{\text{goal}} = r_{\text{goal}} \cos(\theta_{\text{goal}}), \tag{26}$$
$$y_{\text{goal}} = r_{\text{goal}} \sin(\theta_{\text{goal}}). \tag{27}$$

5. Compute the Euclidean distance:

$$d_{\text{euclid}} = \sqrt{(x_{\text{goal}} - x_{\text{start}})^2 + (y_{\text{goal}} - y_{\text{start}})^2}. \tag{28}$$

6. Compute the travel time

$$t = \frac{d_{\text{travel}}}{v}, \tag{29}$$

and the efficiency

$$\text{Efficiency} = \frac{d_{\text{euclid}}}{d_{\text{travel}}}. \tag{30}$$

4 Validation

To ensure the correctness of both the Manhattan and Amsterdam simulation models, we validated the code using a combination of known-case testing, path selection verification, and geometric logic checks.

4.1 Manhattan Model

In the Manhattan simulation, we verified that when the start and goal points lie on the same row or column, the resulting path is a straight line and the computed distance matches expected Manhattan logic.

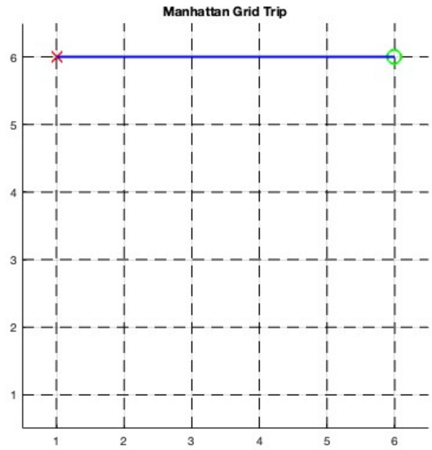


Figure 1: Manhattan simulation with start/goal on the same row-computed path is a straight horizontal line

For more general positions, we confirmed that the path follows an L-shape and the total distance equals the sum of horizontal and vertical displacements, scaled by block length.

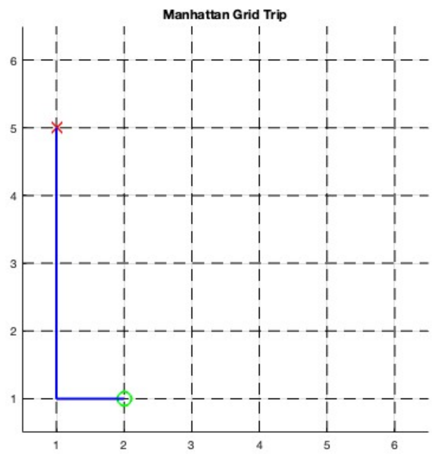
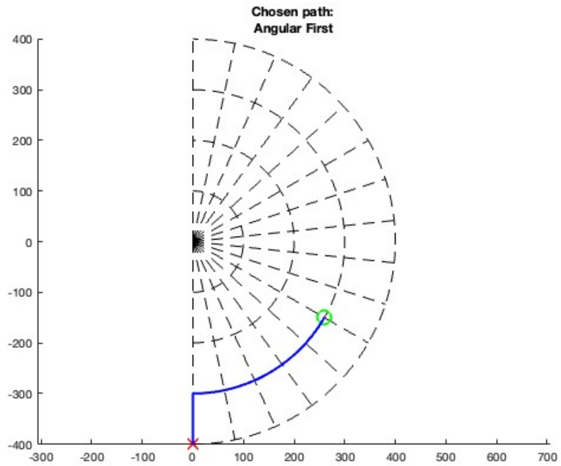


Figure 2: L-shaped Manhattan path between arbitrary start and goal points. The distance matches expected block-based travel.

4.2 Amsterdam Model

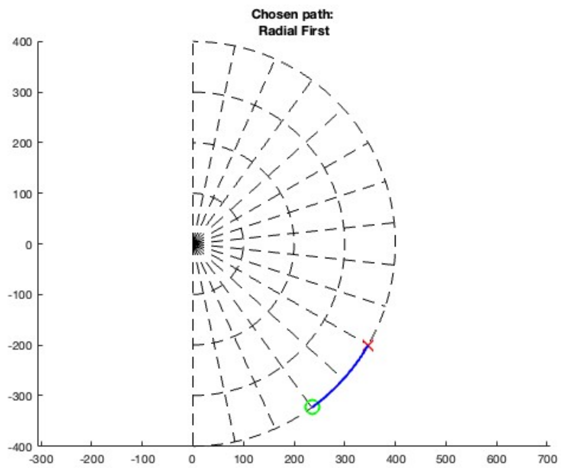
In the Amsterdam simulation, the code evaluates three routing strategies: radial-first, angular-first, and through-center. In each trial, we confirmed that the algorithm selects the option with the shortest total distance.




```
=== Path Options ===
1. Radial then Arc     : 518.88 meters
2. Arc then Radial     : 414.16 meters
3. Through Center      : 700.00 meters
→ Chosen Path: Angular First (414.16 meters)
```

Figure 3: Amsterdam simulation showing correct selection of the shortest path among three options.

We also validated special cases such as start and goal points located on the same radial or ring. The pathfinding logic correctly simplifies to pure radial or arc travel in these cases.




```
=== Path Options ===
1. Radial then Arc     : 167.55 meters
2. Arc then Radial     : 167.55 meters
3. Through Center      : 800.00 meters
→ Chosen Path: Radial First (167.55 meters)
```

Figure 4: Amsterdam simulation with start and goal on the same ring. The algorithm correctly chooses a direct arc path.

4.3 Efficiency Constraint Check

Efficiency is defined as the ratio of Euclidean distance to path distance. All values should remain strictly within the range $(0, 1]$.

5 Interesting Aspects of the Code

In the Amsterdam simulation, one key feature is the logic that evaluates three different routing options based on the circular city layout. These include a radial-first route, an angular-first route, and a route that passes through the center. The code computes each path using basic geometric formulas: radial distances are treated as straight-line segments, and arc distances are computed using the formula for arc length, $r \cdot \Delta\theta$, where r is the radius and $\Delta\theta$ is the angular difference in radians. The following lines calculate the distances:

```
dist_A = abs(r2 - r1) + abs(theta2 - theta1) * r2;
dist_B = abs(theta2 - theta1) * r1 + abs(r2 - r1);
dist_C = r1 + r2;
```

The code then selects the shortest of the three paths using MATLAB's `min` function:

```
[distance, option] = min([dist_A, dist_B, dist_C]);
```

This logic ensures the algorithm always chooses the minimal-distance option without hardcoding any specific routing logic.

The Amsterdam model uses polar coordinates to convert each start and goal position to Cartesian coordinates. This is done using MATLAB's `pol2cart` function:

```
[x1, y1] = pol2cart(theta1, r1);
[x2, y2] = pol2cart(theta2, r2);
```

This allows the simulation to accurately plot the trip and also calculate the Euclidean (straight-line) distance between the two points using:

```
euclid = norm([x2 - x1, y2 - y1]);
```

In both the Manhattan and Amsterdam models, efficiency is calculated as the ratio of Euclidean distance to the total path distance. This gives a value between 0 and 1 and serves as a measure of how direct the chosen route is:

```
efficiency = euclid / distance;
```

The Manhattan model is based on a Cartesian grid. The path distance is computed using:

```
dx = abs(goal(1) - start(1));
dy = abs(goal(2) - start(2));
distance = (dx + dy) * block_length;
```

The path is drawn as a simple L-shape, first moving horizontally and then vertically:

```
intermediate = [goal(1), start(2)];
path = [start; intermediate; goal];
plot(path(:,1), path(:,2), 'b-', 'LineWidth', 2);
```

In both models, randomized inputs are generated using `randi`, and a `while` loop ensures that the start and goal points are not identical:

```
start = [randi(R), randi(S_half)];
goal  = [randi(R), randi(S_half)];
while isequal(start, goal)
    goal = [randi(R), randi(S_half)];
end
```

Both simulations include visual outputs showing the structure of the model (rings or grid), the start and goal positions, and the computed path. These plots are useful for checking that the routing logic behaves correctly under a variety of random configurations.

6 Results and Discussion

All simulations were conducted using consistent parameter values for both the Manhattan and Amsterdam models. The purpose of these trials was to evaluate travel efficiency under random start and goal conditions while holding environmental factors constant.

6.1 Simulation Parameters

- **Speed:** 10 meters per second (constant for all simulations)
- **Block/Step Length (L):** 100 meters

Manhattan Grid:

- Grid Dimensions: 6×6
- Coordinates (x, y) range from $(1, 1)$ (bottom-left) to $(6, 6)$ (top-right)
- Start and goal positions are selected randomly from within the grid

Amsterdam Layout:

- Number of Rings: 4
- Number of Spokes: 16 (evenly spaced from $-\pi/2$ to $+\pi/2$)
- Radial positions are integer multiples of the step length (100 m)
- Start and goal positions are randomly selected ring–spoke pairs

6.2 Manhattan

In the Manhattan model, travel is restricted to axis-aligned grid movement. This means travelers can only move horizontally or vertically along the city's grid lines. Diagonal movement is not allowed, which reflects real-world cities like New York where streets form rectangular blocks.

According to the simulation, the shortest path between two points is determined by the total number of blocks traveled, regardless of the order. If the start and goal lie on the same row or column, the path is a straight line. Otherwise, multiple equally short L-shaped paths exist (e.g., moving right then up, or up then right).

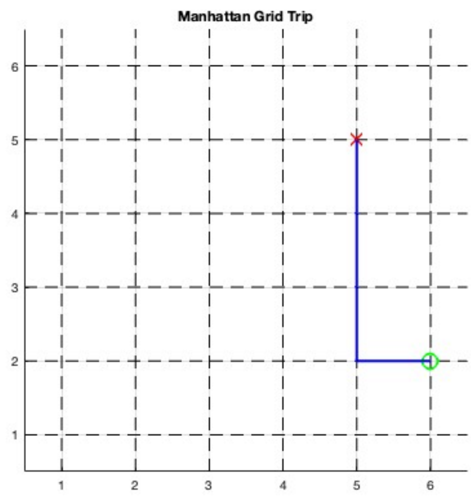


Figure 5: Manhattan Grid Trip

Figure 5 shows a simulated trip starting at coordinate $(6, 2)$ and ending at $(5, 5)$. The traveler moves one unit to the left and three units upward. According to the simulation:

- Path Distance: 400.00 meters
- Euclidean Distance: 316.23 meters

This difference highlights how grid layouts increase path length even when the destination is nearby in a straight line.

6.3 Amsterdam

The Amsterdam model is based on a semicircular layout with concentric rings (like canals) and radial spokes. According to the simulation, three path strategies are evaluated:

- Radial First: move radially first, then along a curved arc
- Angular First: move along the arc first, then radially
- Through Center: pass directly through the city center

The algorithm computes the total distance for each option and selects the shortest.

6.3.1 Angular First

In this scenario, the simulation selected the angular-first strategy, where the traveler begins by moving along the curve of a ring before heading radially outward. The program used the following start and goal points:

- Start: ring 2, spoke 5
- Goal: ring 4, spoke 14

According to the simulation, the following distances were computed:

- Radial then Arc: 953.98 meters
- Arc then Radial: 576.99 meters
- Through Center: 600.00 meters

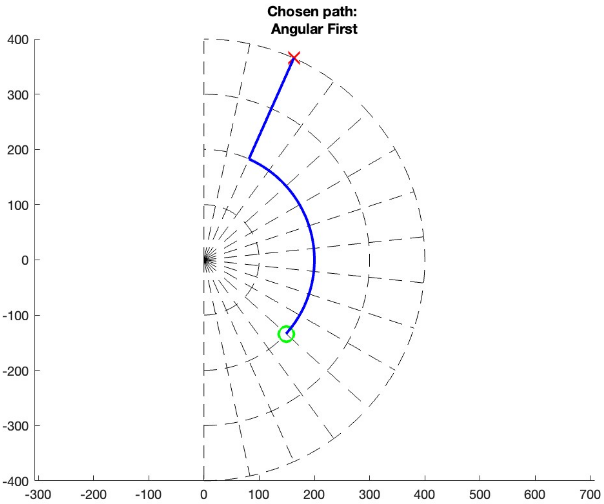


Figure 6: Amsterdam (Angular First)

According to the results, the shortest path is arc-then-radial, with a distance of 576.99 meters.

6.3.2 Radial First

In this configuration, the simulation found it shortest to move radially first before turning along the arc. This happens when the angular separation is relatively small but the radial difference is significant. The program used the following start and goal points:

- Start: ring 3, spoke 11
- Goal: ring 2, spoke 5

According to the simulation, the following distances were computed:

- Radial then Arc: 351.33 meters
- Arc then Radial: 476.99 meters
- Through Center: 500.00 meters

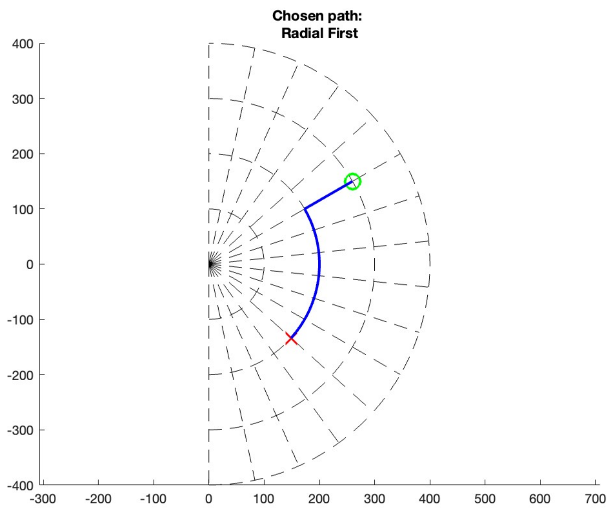


Figure 7: Amsterdam (Radial First)

According to the simulation, this configuration resulted in the radial-first path being the shortest.

6.3.3 Through Center

When the angular separation is very large, the simulation shows that it can be shorter to go through the center — essentially forming two straight-line paths that meet at the origin. The program used the following start and goal points:

- Start: ring 4, spoke 3
- Goal: ring 4, spoke 14

According to the simulation, the following distances were computed:

- Radial then Arc: 837.76 meters
- Arc then Radial: 837.76 meters
- Through Center: 800.00 meters

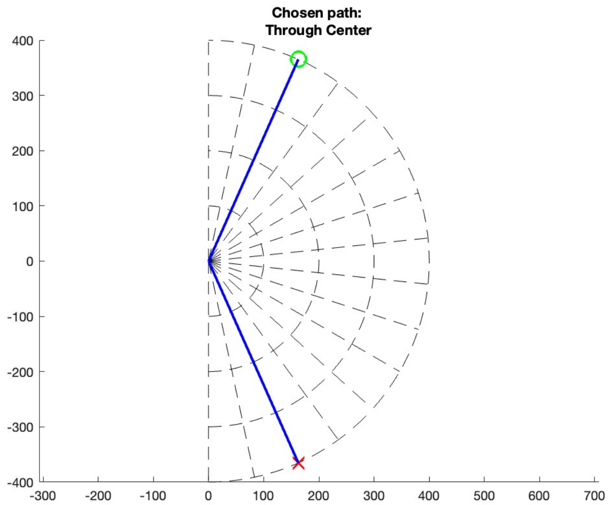


Figure 8: Amsterdam (Through Center)

As shown in Figure 8, the center path was selected since it had the shortest distance overall, even though it required passing through the origin.

6.4 Manhattan vs Amsterdam

To evaluate which layout is more efficient overall, we ran two sets of randomized trials: one with 100 simulations and another with 1000. In each trial, the start and goal points were chosen randomly, and the efficiency of the route was calculated as the ratio of the Euclidean (straight-line) distance to the actual path distance. A value of 1.0 would represent a perfectly direct path.

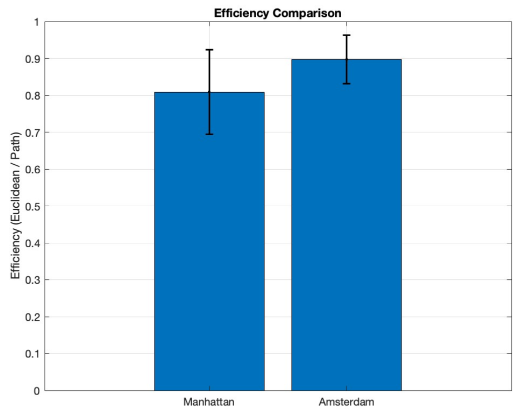


Figure 9: Manhattan vs Amsterdam (100 Trials)

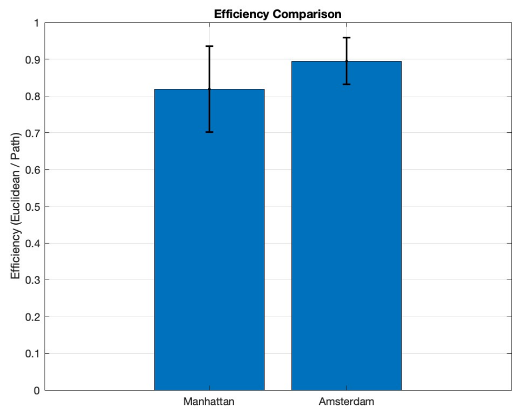


Figure 10: Manhattan vs Amsterdam (1000 Trials)

According to the results of the 100-trial simulation:

- Manhattan Mean Efficiency: 0.81 ± 0.12
- Amsterdam Mean Efficiency: 0.91 ± 0.07

From the 1000-trial simulation:

- Manhattan Mean Efficiency: 0.82 ± 0.12
- Amsterdam Mean Efficiency: 0.90 ± 0.07

These results show that the Amsterdam model consistently allows for more direct travel than the Manhattan model. The reason is that the Amsterdam layout provides more flexibility in movement, including options to cut across the grid through the center or follow curved paths. In contrast, the Manhattan model forces all movement to follow a rigid block pattern, which can lead to longer routes even when the start and goal are geographically close.

The efficiency gap, while not extreme, is stable and statistically significant across trials. This supports the hypothesis that more flexible or radial layouts lead to improved travel efficiency in randomly distributed travel scenarios.

7 Conclusion

This project investigated how different city layouts affect travel efficiency by simulating random trips in two contrasting urban models: the Manhattan grid and the Amsterdam radial network. Both simulations used a consistent framework for evaluating each route: calculating the total distance traveled, the direct (Euclidean) distance, and the resulting efficiency, defined as the ratio of directness to total path length.

In the Manhattan model, movement is strictly limited to vertical and horizontal segments, which often forces travelers to take longer routes even when the destination is close in straight-line terms. The Amsterdam model, on the other hand, provides multiple routing options—along arcs, radials, or directly through the center—and selects the shortest of these in each case. As a result, the Amsterdam simulation demonstrated higher average efficiency across both 100 and 1000 randomized trials.

The results support the idea that urban layouts with greater geometric flexibility can yield more efficient travel paths. The simulation also showed how seemingly small structural differences, such as the option to move diagonally or pass through a center, can have measurable effects on route efficiency. Overall, this project demonstrates that even simple mathematical models can provide meaningful insights into how the design of a city influences mobility and accessibility.

8 References

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