Urban Rat Simulation [Revision]

CSCI-UA 144 MATH-UA 330

Introduction to Computer Simulation

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April 2025

MATLAB Code:
`https://github.com/stevloc/matlab/blob/main/Urban%20Rats%20Simulation/project2.m`

1 Personal Note

Moving to New York City, one of the first things I noticed was how common rats were. I saw them in parks, streets, and subways. One night at the station, I saw a few run across the tracks, and I started to wonder: how do they thrive in such a crowded place? That curiosity led me to this project. I wanted to understand how rats survive in the city and what role seasonal change and predators play. This simulation let me explore that through code and data.

2 Introduction

Rats are one of the most persistent and visible species in New York City. Found in subways, streets, and parks, they thrive on food waste and shelter created by dense human infrastructure (Feng & Himsworth, 2014). Their success in urban environments is driven by high reproductive rates and adaptability to seasonal conditions. However, their population dynamics are also influenced by interactions with predators and human control efforts.

This project simulates rat population dynamics alongside two key urban predators: feral cats and red-tailed hawks. The model is formulated using a system of differential equations that describe species interactions through reproduction, death, and predation. It incorporates logistic growth for rats, soft population caps for hawks, and seasonal variability in ecological parameters. The simulation examines how these forces shape population levels over time.

Each species plays a distinct role in the urban ecosystem. Rats act as prey with high reproductive output and constant environmental support from urban food sources. Feral cats are present year-round and reproduce based on rat availability. Their growth is partially suppressed in spring and summer to reflect the effects of Trap-Neuter-Return (TNR) programs, which reduce stray cat fertility in many NYC neighborhoods (Lowe et al., 2015). Hawks are modeled as non-migratory predators with low but stable populations, consistent with red-tailed hawks observed year-round in urban parks.

Seasonal changes are built into the model by adjusting ecological parameters at each time step. Rat birth and death rates, predator efficiency, and cat reproduction vary by season to reflect environmental stress, temperature changes, and human interventions. These changes are implemented through parameter scaling rather than direct changes to population levels, allowing smoother and more biologically realistic transitions between seasons.

The simulation is implemented in MATLAB and uses the forward Euler method to approximate population changes over time. To ensure realism and numerical stability, the model includes a rat carrying capacity, a soft cap on hawks, and a minimum threshold of five individuals per species to prevent extinction. The system is tested over three durations—1 year, 10 years, and 100 years—to evaluate both short-term seasonality and long-term population trends.

The purpose of this project is to explore how seasonal variation, predator-prey interactions, and urban management strategies affect the stability and growth of rat populations. By simulating simplified but ecologically grounded relationships, this model provides insight into the long-term behavior of urban wildlife and offers a framework for studying pest control and urban ecology at multiple time scales.

3 Equations

The model describes interactions between three species—rats (R), cats (C), and hawks (H)—using a system of first-order ordinary differential equations with ecological constraints. The behavior of each species is governed by:

- Rats reproduce following logistic growth, with a carrying capacity.
- Rats die naturally and are consumed by predators (cats and hawks), with predation rates dependent on both rat and predator levels.
- Cats and hawks die at rates proportional to their own populations.
- Cats and hawks reproduce by consuming rats.
- Hawks are limited by a soft carrying capacity, and rats receive a constant environmental boost simulating food waste.

These rules yield the following system of equations:

$$
\begin{aligned}
\frac{dR}{dt} &= b_R R \left(1 - \frac{R}{K_R}\right) - d_R R - p_{RC} RC - p_{RH} RH + \beta \\
\frac{dC}{dt} &= -d_C C + b_C RC \\
\frac{dH}{dt} &= (-d_H H + b_H RH) \left(1 - \frac{H}{K_H}\right)
\end{aligned}
\tag{1}
$$

- b_R, d_R are the seasonally adjusted birth and death rates of rats.
- K_R is the rat carrying capacity (e.g., 250).
- β is a constant environmental boost to rat population (e.g., $\beta = 0.5$).
- p_{RC}, p_{RH} are predation rates of cats and hawks on rats.
- d_C, d_H are death rates of cats and hawks.
- b_C, b_H are reproduction rates of cats and hawks due to predation.
- K_H is the hawk soft cap (e.g., 30), simulating territorial limits or nest space.

This structure captures continuous population interactions with realistic ecological bounds and species persistence mechanisms.

Seasonal Modifiers

Seasonal effects are applied by adjusting model parameters—such as birth, death, and predation rates—at each timestep depending on the current season:

- **Winter:** Reduced rat birth and increased death; no predation activity; elevated predator mortality.
- **Spring:** High rat reproduction and moderate predator reproduction; feral cat birth reduced (simulating TNR).
- **Summer:** Sustained high rat growth; peak predation activity.
- **Fall:** Transition period with reduced rat reproduction and intensified predation.

These modifiers affect system dynamics by influencing interaction terms, without directly changing the population values. This approach enables a smooth and biologically grounded representation of seasonal changes in an urban ecosystem.

4 Numerical Method

The simulation advances in time using the forward Euler method, which approximates continuous-time dynamics with finite differences. At each time step of size Δt, the system updates the rat (R), cat (C), and hawk (H) populations based on the rates of change defined in the differential equations:

$$
\begin{aligned}
R_{t+\Delta t} &= \max\left(5, R_t + \left[b_R R_t \left(1 - \frac{R_t}{K_R}\right) - d_R R_t - p_{RC} R_t C_t - p_{RH} R_t H_t + \beta\right] \cdot \Delta t\right) \\
C_{t+\Delta t} &= \max\left(5, C_t + \left(-d_C C_t + b_C R_t C_t\right) \cdot \Delta t\right) \\
H_{t+\Delta t} &= \max\left(5, H_t + \left[\left(-d_H H_t + b_H R_t H_t\right) \cdot \left(1 - \frac{H_t}{K_H}\right)\right] \cdot \Delta t\right)
\end{aligned}
$$

Each term is evaluated using the current values at time t, and the result defines the new state at time $t + \Delta t$. A constant rat boost β simulates persistent food availability (e.g., trash and shelter), and logistic constraints limit uncontrolled growth of rats and hawks.

Assumptions and Stability

- A population floor of 5 individuals is enforced for each species using `max(5, ...)` to avoid artificial extinction due to numerical decline or winter stress. This models the assumption that species in NYC rarely disappear entirely.
- The rat population includes a constant positive boost $\beta = 0.5$ per time step, simulating consistent access to urban resources.
- A rat carrying capacity $K_R = 250$ and hawk soft cap $K_H = 30$ prevent runaway growth and help stabilize the simulation.
- The time step $\Delta t = 0.2$ was selected to ensure smooth behavior without sacrificing runtime efficiency. Smaller time steps produce similar outcomes, confirming stability.

Seasonal Parameter Scaling

Seasonal dynamics are implemented by modifying birth, death, and predation rates based on the current season. These modifiers are applied through a helper function and do not alter the population values directly. For example:

- Winter increases death rates and suppresses predator activity.
- Spring amplifies rat birth and predator reproduction.
- Summer maintains high interaction intensity.
- Fall begins a decline in rat birth with increased predation.

This parameter-scaling approach allows the model to simulate realistic seasonal pressures while preserving smooth and continuous dynamics throughout the simulation.

5 Validation

To ensure that the simulation is both numerically stable and ecologically plausible, the model was validated using two primary criteria: convergence across time steps and the enforcement of biologically meaningful population constraints.

5.1 Convergence Test Across Time Steps

To test numerical stability, the simulation was run over 100 years using three different time steps: $\Delta t = 0.2$, 0.1, and 0.05. All runs used the same initial populations and parameter settings. The goal was to verify that the model produces consistent dynamics across different levels of temporal resolution.

In all three cases, the population trends were qualitatively identical. Rat populations displayed seasonal oscillations with long-term damping, feral cats gradually rose to dominate the system, and hawks remained at low, stable levels. These consistent patterns confirm that the model's underlying dynamics are not sensitive to time step size.

As expected, smaller values of Δt produced smoother population curves and better resolution of seasonal transitions. The simulation with $\Delta t = 0.2$ exhibited more abrupt shifts but retained all major behavioral features. There was no divergence, instability, or change in qualitative dynamics across time steps. This confirms that the forward Euler method is numerically stable for this system and that $\Delta t = 0.2$ is a reliable choice for long-term ecological modeling.

5.2 Biological Constraints and Realism

To maintain ecological realism, the simulation enforces a minimum population threshold of 5 individuals for each species. This reflects the persistence of urban wildlife and prevents artificial extinction due to numerical decay or temporary resource scarcity. Logistic growth for rats and soft caps for hawks ensure populations remain within biologically plausible bounds. These constraints support both the interpretability and robustness of the simulation across long timeframes.

6 Interesting Aspects of the Code

One of the most interesting aspects of the simulation is how it separates biological dynamics from seasonal influences. Instead of modifying population values directly, the model uses a seasonal parameter system implemented through a helper function called `getSeasonalParameters`. This function adjusts birth, death, and predation rates based on the current season:

```
if seasonIdx == 1  % Winter
    params.ratBirth = base.ratBirth * 0.5;
    params.ratDeath = base.ratDeath * 1.4;
    params.ratPredCats = 0;
    params.ratPredHawks = 0;
    params.catBirthFromRats = 0;
    params.hawkBirthFromRats = 0;
    params.catDeath = base.catDeath * 1.2;
    params.hawkDeath = base.hawkDeath * 1.2;
end
```

This approach allows the simulation to reflect seasonal stress and ecological changes without disrupting the continuity of the population values. It also makes it easy to experiment with different seasonal strategies or urban interventions by simply modifying the parameter scaling in one place.

Another notable design feature is the enforcement of minimum population thresholds. To prevent extinction from numerical decay or sharp seasonal drops, each species population is updated using:

```
ratPop(t+1) = max(5, R + dR * timeStep);
```

This ensures biological persistence and reflects the reality that urban species like rats, feral cats, and red-tailed hawks rarely disappear completely from cities like New York.

Additionally, the hawk population is capped using a soft logistic limit:

```
dH = dH * (1 - H / hawkCarryingCapacity);
```

This prevents uncontrolled growth while still allowing dynamic fluctuations based on prey availability. The overall structure of the code enables a realistic, modular, and stable long-term simulation of species interactions in an urban ecosystem.

Results and Discussion

To better understand how seasonal pressures and predator-prey dynamics shape urban wildlife over time, the simulation was conducted for three time spans: 1 year, 10 years, and 100 years. Each scenario provides a different lens through which to examine stability, oscillations, and predator impact.

1-Year Simulation: Short-Term Fluctuations

In the 1-year simulation, population dynamics are driven almost entirely by seasonal parameters. Rats peak in spring and summer due to high birth rates and an environmental boost, while winter causes a sharp drop due to increased mortality and halted predation. Cats and hawks remain relatively stable, with hawks at a low constant level and cats fluctuating with rat availability.

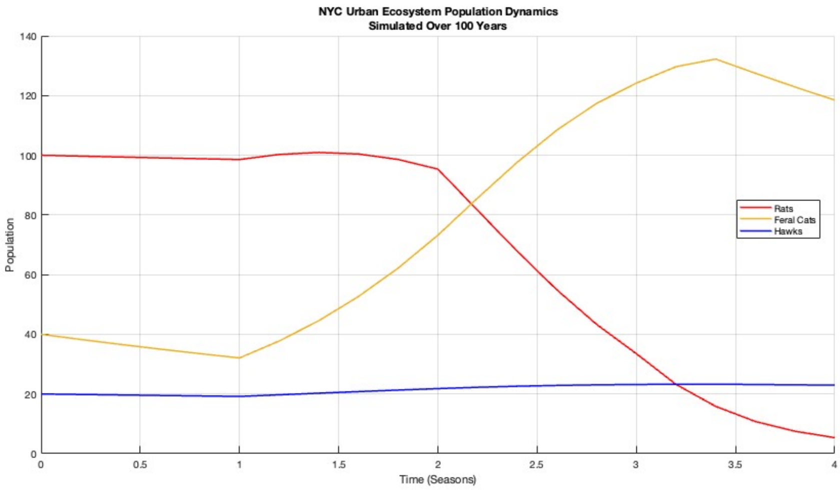

Figure 1: Population dynamics over 1 simulated year.

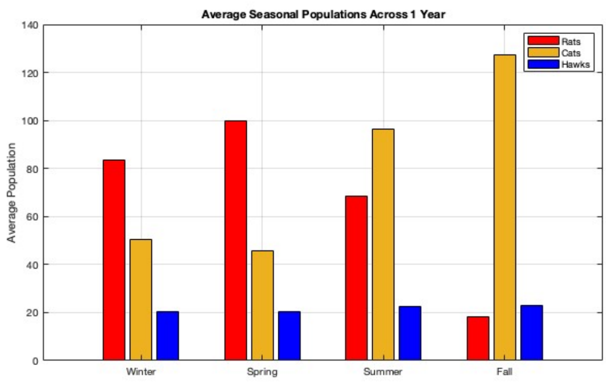

Figure 2: Average seasonal populations across 1 year. Rats peak in spring; hawks remain low.

10-Year Simulation: Medium-Term Trends

In the 10-year simulation, system dynamics become more pronounced. Initially, rats dominate due to favorable spring/summer conditions, but this leads to a spike in cat population as predators respond. Rats then decline sharply, leading to a temporary predator-prey imbalance. Over time, the system begins to stabilize into a new equilibrium with cats maintaining higher populations than rats. Hawks remain stable and low throughout.

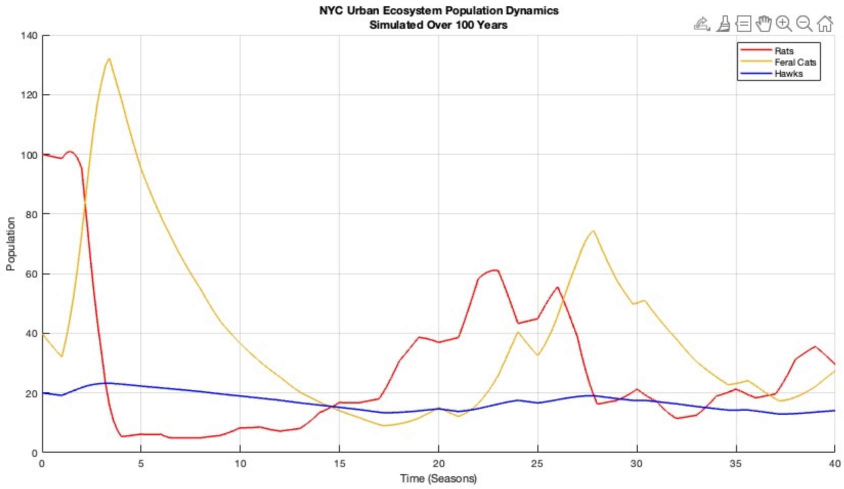

Figure 3: Population dynamics over 10 simulated years. Cats surpass rats in later years.

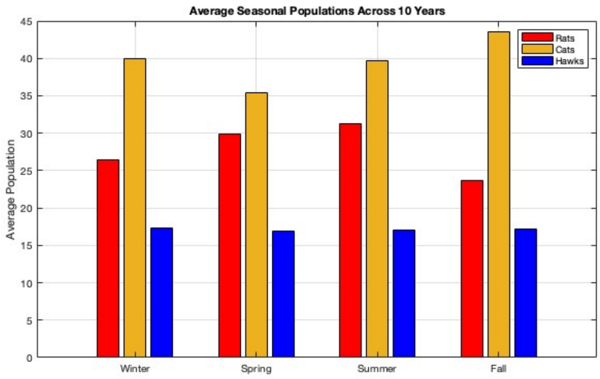

Figure 4: Average seasonal populations across 10 years. Cats dominate across all seasons.

100-Year Simulation: Long-Term Stability

The 100-year simulation reveals how the system settles into a biologically plausible steady state. After a period of large fluctuations and overshoot in early years, rat and cat populations dampen into seasonal cycles with moderate amplitudes. Rats oscillate annually in sync with seasons, while cats stabilize at slightly higher levels due to their predation advantage. Hawk populations remain capped at a low level, showing minimal seasonal variation.

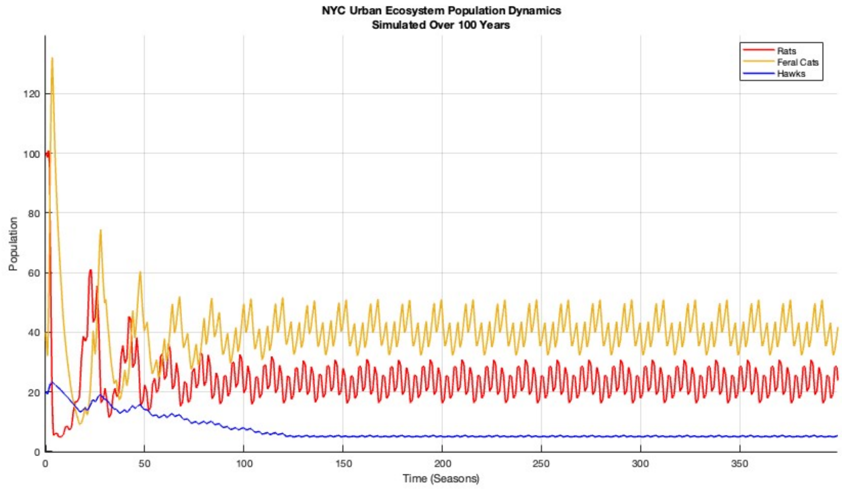

Figure 5: Population dynamics over 100 simulated years. Long-term cycles emerge.

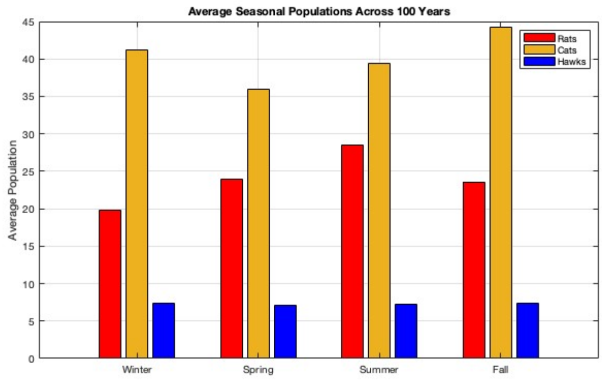

Figure 6: Average seasonal populations across 100 years. Rats remain stable but lower than cats.

Key Insights

- **Rats:** Despite high birth rates and seasonal boosts, rat populations are ultimately constrained by predation, seasonal death, and a logistic carrying capacity. They oscillate over time but do not dominate.
- **Cats:** Cats gradually become the dominant predator, especially in medium and long runs. TNR effects in spring/summer slow their growth but do not reverse it.
- **Hawks:** Hawk numbers remain low and stable across all timeframes, consistent with a soft cap and limited predation rate.

These results demonstrate that even with simple deterministic equations, the system captures complex ecological feedback over time. The interplay between seasonal modifiers, predator pressure, and carrying constraints produces cyclical and self-regulating behavior. While not spatially explicit or stochastic, this model reflects core ecological dynamics found in real urban wildlife populations.

7 Conclusion

This project examined how urban rats, feral cats, and red-tailed hawks interact within a seasonally driven ecosystem modeled after New York City. A modified Lotka-Volterra framework was used to simulate these dynamics, incorporating logistic growth for rats, soft caps for hawks, and seasonally adjusted birth, death, and predation parameters. The simulation was implemented in MATLAB using the forward Euler method and tested across three timescales: 1 year, 10 years, and 100 years.

The model produced ecologically consistent results across all durations. Rats exhibited predictable seasonal cycles, with population peaks in spring and summer followed by declines in winter. Feral cats initially responded to rat availability but eventually stabilized at higher population levels, reflecting their predation advantage and persistence. Hawks remained a stable but minor component of the system, constrained by limited reproduction and a soft carrying capacity. The simulation maintained stable, oscillating population dynamics without divergence, confirming both numerical stability and ecological plausibility.

Despite these strengths, the model includes several simplifying assumptions. It omits spatial structure, random environmental disturbances, and indirect species interactions beyond predation. Extinction is prevented by a fixed minimum population threshold, which ensures system continuity but reduces biological realism in rare-event scenarios. Seasonal effects are implemented through parameter scaling rather than environmental feedback or adaptive behaviors. Nonetheless, the project successfully demonstrated how structured rules, ecological constraints, and seasonal drivers can be integrated into a computational framework to reproduce complex urban wildlife dynamics over time.

8 References

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