Assignment 9 — due April 15, 2019

1. Consider the random regression model seen in lecture, with subjects $i = 1, \dots, n$ observed at ages x_{ij}, $j = 1, \dots, 4$. Specifically, consider the model with uncorrelated random effects:

$$Y_{ij} = \beta_0 + b_{0i} + \beta_1 x_{ij} + b_{1i} x_{ij} + \varepsilon_{ij} \tag{1}$$

with

$$\begin{pmatrix} b_{0i} \\ b_{1i} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_0^2 & 0 \\ 0 & \sigma_1^2 \end{pmatrix} \right) \tag{2}$$

independent from subject to subject. Also $\varepsilon_{ij} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ are assumed independent between each other, and independent of the b_{0i}'s and b_{1i}'s.

   (a) Consider reparametrizing the model using $\tilde{x}_{ij} = x_{ij} - c$ where c is a constant. (Often times, this constant is chosen in the center of the distribution of the x values). Re-express model (1) in terms of $\tilde{x}_{ij}$ instead of x_{ij}. Prove that the new parametrization uses:

$$\tilde{\varepsilon}_{ij} = \varepsilon_{ij}, \quad \tilde{\beta}_1 = \beta_1, \quad \tilde{\beta}_0 = \beta_0 + \beta_1 c, \quad \tilde{b}_{1i} = b_{1i}, \quad \tilde{b}_{0i} = b_{0i} + b_{1i} c \, .$$

   (b) Prove that

$$\begin{pmatrix} \tilde{b}_{0i} \\ \tilde{b}_{1i} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_0^2 + c^2\sigma_1^2 & c\sigma_1^2 \\ c\sigma_1^2 & \sigma_1^2 \end{pmatrix} \right)$$

   Hint: start by expressing

$$\begin{pmatrix} \tilde{b}_{0i} \\ \tilde{b}_{1i} \end{pmatrix} = \mathbf{M} \begin{pmatrix} b_{0i} \\ b_{1i} \end{pmatrix}$$

   for some matrix $\mathbf{M}$.

   Is the class of models that can be written in form (1)–(2) invariant to the reparametrization $x \mapsto x - c$?

   (c) Consider the linear model (1) with the more complex covariance below:

$$\begin{pmatrix} b_{0i} \\ b_{1i} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_0^2 & \rho\sigma_0\sigma_1 \\ \rho\sigma_0\sigma_1 & \sigma_1^2 \end{pmatrix} \right) . \tag{3}$$

   For model (1) with covariance (3), re-do (a) and (b). Hint: you should find that the results in (a) still hold, but you should find a new covariance matrix in (b).

   Is the class of models that can be written in form (1)–(3) invariant to the reparametrization $x \mapsto x - c$? Conclude about model choice, between covariance model (2) versus (3).

2. Read the 2019 comment "Scientists rise up against statistical significance" (Amrhein, Greenland & McShane, Nature, `https://doi.org/10.1038/d41586-019-00857-9`, available on Canvas) and read one of the 43 papers from *The American Statistician*'s special issue on "A World Beyond p $<$ 0.05", available open-access at `https://www.tandfonline.com/toc/utas20/73/sup1?nav=tocList`.

   Then post a comment to Canvas, in the "graded discussion" set up for this assignment. Your comment should be at least one paragraph. It should include the title of the paper that you read from *The American Statistician*, and your thoughts on both papers. For your own interest, make your comment meaningful and make it your own. Do not plagiarize other comments that you might read on the web. Your comment may be a response to someone else's comment, so long as you bring new meaningful contributions. Discussions are encouraged (you may post follow-up notes). Your comments must be respectful of everyone else's.