Spring 2018, Stat 850: Final exam

5/7/2018

Attention:

(1) This is an open notes exam. To get full credit, you need to show your work and do the actual calculations with the information provided in the questions. Simply writing down the formulas for various quantities will not get you any partial credits.

(2) Do not dwell on any individual question for too long. Answer as many questions as possible.

(3) The exam will be scored out of 40 total points. Scores for individual questions are listed by each question.

Q1. (8 pts) Researchers interested in the effects of a drug and three diets on hypertension conducted an experiment using model organism mouse. They used a total of 60 mice with high blood pressure. Each mouse was housed in its own cage and the 60 mice were randomly assigned to the following six treatment groups using a balanced and completely randomized design with 10 mice in each treatment group. In addition to their drug of interest (drug = 2 in the table below), they used a placebo (drug = 1).

Treatment	Diet	Drug
1	1	1
2	1	2
3	2	1
4	2	2
5	3	1
6	3	2

As response, the study measured the improvement in blood pressure for each mouse. Positive values of the response indicate improvement (decreasing) of blood pressure while negative values indicate an increase in blood pressure. Use the partial `R` code and results below to answer the questions (a)-(e).

```
diet <- as.factor(rep(1:3, each=20))
drug <- as.factor(rep(rep(1:2, each=10), 3))
mod1 <- lm(imp ~ diet + drug)
summary(mod1)
Coefficients:
         Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.000      1.662   0.602 0.54974
diet2       4.000      2.036   1.965 0.05440 .
diet3       6.000      2.036   2.948 0.00466 **
drug2       2.500      1.662   1.504 0.13818
```

(a) Find the best linear unbiased estimate of the mean for Treatment 6?

(b) According to the model considered by the researcher, is the mean response for Treatment 1 significantly different from zero? Provide a test statistic, its degrees of freedom, a p-value, and a conclusion for a significance level 0.05 test.

(c) A key inference question for this study is whether there are any significant differences among the three diets with regard to lowering mean blood pressure. Based on the output provided, complete the entries in the table below as much as possible, while ignoring the multiplicity issue.

Effect Difference	Estimate	SE	t-statistic	p-value $\leq 0:05$? (yes or no)
Diet 1 - Diet 2				
Diet 1 - Diet 3				
Diet 2 - Diet 3				

(d) Provide an estimate of the error variance (σ^2) based on this model that the researchers fit the data.

(e) What is the degrees of freedom associated with your estimate from part (d).

(f) The `R` output used the set-to-zero contrast in `R` to fit a treatment effects model. The researchers would like to switch to a treatment means model using sum-to-zero contrasts in `R` without refitting. Can you calculate the treatment effect estimates from such a model fit using the above `R` output? If so, provide estimates.

Q2. (8 pts) Researchers from the UW Audiology Department were interested in studying the effects of three types of surgically implanted hearing aids (labeled 1, 2, and 3) on subjects with a certain type of hearing loss in both ears. To generate preliminary data, they conducted an experiment involving a total of six subjects. This was with an understanding that many more subjects would likely be needed to obtain useful results. Two of the three hearing aid types were randomly assigned to each subject. The two hearing aid types randomly assigned to a subject were also randomly assigned to ears within subject. Prior to surgical implantation, each subject received a separate hearing test for each ear. Following surgical implantation, each ear of each subject was tested again. Thus, a pre-test and post-test score were available for each subject and each ear.

In the code below, the variable test is coded as 1 for the pre-test and 2 for the post-test, and the variable score contains the test score. The variables subject, ear, and hearingaid contain the subject ID, the ear ID, and the hearing aid type, respectively. All the variables except for score are factors in R.

The researchers consider a mixed linear model of the following standard form to analyze these data

$$y = X\beta + Zu + \epsilon.$$

```
dat <- read.table("q2.txt", header = TRUE)
show(dat)
```

```
##    subject ear hearingaid test score
## 1        1   1          1    1  54.8
## 2        1   1          1    2  66.9
## 3        1   2          2    1  56.6
## 4        1   2          2    2  73.5
## 5        2   3          1    1  43.9
## 6        2   3          1    2  56.4
## 7        2   4          2    1  44.8
## 8        2   4          2    2  61.7
## 9        3   5          1    1  40.8
## 10       3   5          1    2  51.6
## 11       3   6          3    1  38.7
## 12       3   6          3    2  74.5
## 13       4   7          1    1  52.4
## 14       4   7          1    2  63.8
## 15       4   8          3    1  48.5
## 16       4   8          3    2  81.2
## 17       5   9          2    1  32.9
## 18       5   9          2    2  49.6
## 19       5  10          3    1  39.3
## 20       5  10          3    2  73.6
```

```
## 21       6  11          2    1  69.8
## 22       6  11          2    2  84.6
## 23       6  12          3    1  60.5
## 24       6  12          3    2  96.3
```

```r
library(lme4)
```

```
## Loading required package: Matrix
```

```r
m1 <- lmer(score ~ hearingaid*test + (1|subject) + (1|ear), data=dat)
```

```r
library(lme4)
summary(m1)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: score ~ hearingaid * test + (1 | subject) + (1 | ear)
##    Data: dat
##
## REML criterion at convergence: 131.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.3856 -0.5662 -0.0011  0.5988  1.4417
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  ear      (Intercept)   9.848   3.138
##  subject  (Intercept) 104.712  10.233
##  Residual               6.826   2.613
## Number of obs: 24, groups:  ear, 12; subject, 6
##
## Fixed effects:
##                 Estimate Std. Error t value
## (Intercept)       52.405      6.719   7.800
## hearingaid       -12.356      2.451  -5.042
## test              -2.058      2.822  -0.729
## hearingaid:test   11.475      1.306   8.784
##
## Correlation of Fixed Effects:
##             (Intr) herngd test
## hearingaid  -0.730
## test        -0.630  0.740
## hearngd:tst  0.583 -0.800 -0.926
```

(a) Determine the first and last row of the design matrix X.

(b) Determine the first and last row of the matrix Z.

(c) Explicitly define the variance components of the above model.

(d) According to the model fit by the researchers, what is the variance of a single response in terms of the variance components in part (c)? Provide both the formula based on your notation in part (c) and also the numerical value.

(e) According to the model fit by the researchers, what is the correlation between the pre-test and post-test scores for the left ear of subject 1 in terms of the variance components defined in part (c)? Provide both the formula based on your notation in part (c) and also the numerical value.

(f) According to the model fit by the researchers, what is the correlation between the pre-test score for the left ear of subject 1 and the post-test score for the right ear of subject 1? Provide both the formula based on your notation in part (c) and also the numerical value.

(g) According to the model fit by the researchers (m1 above), how many different values can occur in the vector E(y)? Does this seem appropriate based on the design of the experiment? Explain why or why not.

Q3. (6 pts) Answer the following questions for each of the following experiments.

(a) Describe the design for each of the experiments. Specifically, identify treatments, experimental, obser- vational units, and, if appropriate, indicate the name of the design.

(b) Give a skeleton ANOVA (sources of variation and degrees of freedom only) for each design.

Experiment 1. A tire company is testing four steel-belt designs for one of their tire brands. From five popular car models, it randomly chooses three cars of each model. Each steel-belt design is randomly assigned one position on each car (Left Front, Right Front, Left Rear, Right Rear); the cars are driven 10,000 miles and a measure of tread wear is recorded.

Experiment 2. A study is carried out to compare three exercise programs (Program, coded 1,2,3) – each one involving a different emphasis. Of interest is how these programs may differently affect blood pressure over time. Ten subjects (Subject, coded 1,. . . ,10) were randomly assigned to each of the 3 programs (for a total of 30 subjects), and each subject exercised (using their assigned program only) each Tuesday and Thursday for seven weeks. Blood pressure measurements (BP) were made on each subject on the Friday at the end of each week (Week, coded 1,. . . ,7), for a total sample size of 210.

Experiment 3. A consulting client describes to you their study. They want to compare growth rates of pigs. The treatments are all possible combinations of 5 diets and whether or not (2 levels) the pig is injected with monensin, an antibiotic. The experiment will be conducted in a barn with 15 pens. Each pen will have 4 pigs. Pens are randomly assigned to diet (3 pens per diet) and pigs are randomly assigned to monensin or not (2 pigs per pen per trt). The study will be repeated at a total of 3 locations. In the entire study, there are 3 locations, 45 pens, and 180 pigs. The mean growth rate is expected to vary among locations. The effect of the antibiotic (monensin) is expected to be more consistent across the locations than is the effect of diet.

Q4. (6 pts) Consider a study where we investigate the effect of a type of golf ball on the distance. We have $n_T = 20$ golfers and 2 treatments (i.e. 2 types of golf balls). We are considering two types of designs:

Design I. Design I: Each golfer is randomly assigned a type of golf ball. Half of the golfers are assigned to type 1 and the remaining golfers will use the type 2 golf balls. We ask the golfers to hit a few golf balls and we compute the average distance of these hits.

Design II: Each golfer will hit each type of ball a few times and we will compute the average distance for each type of golf ball.

(a) Give the name of two experimental designs.

(b) If you were to choose between these two designs, which would you prefer? Why?

(c) Consider the design II. Denote Y_{ij} as the average distance for golfer i with golf ball type j, for $i = 1, \cdots, 20$ and $j = 1, 2$. Since the two observations Y_{i1} and Y_{i2} are from the same golfer, then we will consider them as correlated by assuming

$$\text{Cov}\{Y_{i1}, Y_{i2}\} = c, \quad \text{for} \quad i = 1 \cdots 20,$$

where c is some positive constant. Assume that $\text{Var}\{Y_{ij}\} = \sigma^2$ for all $i = 1, \cdots, 20$ and $j = 1, 2$. To compare the effect of the type of golf ball on the distance, for golfer i, we will compute $Y_{i1} - Y_{i2}$. Give an expression for the variance of $Y_{i1} - Y_{i2}$ as a function of c and σ^2.

Q5. (12 pts) Consider an experiment designed to determine which of four chocolate chip cookie recipes consumers prefer most. A total of 24 volunteer tasters were used in the experiment. Each volunteer tasted three of the four cookie recipes in an assigned order and provided a score for each recipe using a 9-point scale (1 = poor,...,9= excellent). Three batches of cookies were prepared using each of the four recipes. Each of the 12 total batches of cookies was divided into six trays. These 72 total trays were tasted by the 24 tasters (three trays per taster).

A linear mixed model with recipe and taste order as fixed effects and taster and batch as random factors was used to analyze this dataset. Consider the following partial `R` output to answer the following questions. You may or may not need all the provided output.

(a) Provide REML estimates of the model variance components.

(b) Estimate the correlation between the first and seventh observations in the data set based on the model fit.

(c) Determine the effect of recipe 1 on the taste score.

(d) Provide the standard error for the estimate in part (c).

(e) Provide an estimate of the difference between means for recipes 1 and 2.

(f) Provide the standard error for the estimate in part (e).

(g) Based on the AIC criterion, should the model include interaction between the factors recipe and taste order?

(h) Conduct a likelihood ratio test to answer the question in part (g). State the test statistic, the degrees of freedom, the p-value, and a conclusion.

```
d <- read.table("cookies.txt", header = TRUE)
d$recipe <- as.factor(d$recipe)
d$tasteorder <- as.factor(d$tasteorder)
d$batch <- as.factor(d$batch)
d$taster<- as.factor(d$taster)

 table(d$recipe)
```

```
##
##  1  2  3  4
## 18 18 18 18
```

```
 table(d$tasteorder)
```

```
##
##  1  2  3
## 24 24 24
```

```
 table(d$batch)
```

```
##
##  1  2  3  4  5  6  7  8  9 10 11 12
##  6  6  6  6  6  6  6  6  6  6  6  6
```

```
 table(d$taster)
```

```
##
##  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
##  3  3  3  3  3  3  3  3  3  3  3  3  3  3  3  3  3  3  3  3  3  3  3  3
```

```
library (lme4)
mod1 <- lmer(y ~ recipe + tasteorder + (1|batch) + (1| taster), data=d)
```

```r
# Observation 1
d[1, ]
```

```
##   recipe batch taster tasteorder y
## 1      1     1      1          1 3
```

```r
# Observation 7
d[7, ]
```

```
##   recipe batch taster tasteorder y
## 7      2     2      1          2 6
```

```r
summary(mod1)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: y ~ recipe + tasteorder + (1 | batch) + (1 | taster)
##    Data: d
##
## REML criterion at convergence: 193.2
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -2.04691 -0.45391 -0.00967  0.53562  1.98374
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  taster   (Intercept) 1.8308   1.3531
##  batch    (Intercept) 0.1292   0.3594
##  Residual             0.2587   0.5087
## Number of obs: 72, groups:  taster, 24; batch, 12
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)   3.4559     0.3812   9.065
## recipe2       1.1398     0.3560   3.202
## recipe3      -0.8139     0.3560  -2.286
## recipe4      -1.4828     0.3560  -4.166
## tasteorder2   1.5417     0.1468  10.499
## tasteorder3   1.6250     0.1468  11.066
##
## Correlation of Fixed Effects:
##             (Intr) recip2 recip3 recip4 tstrd2
## recipe2     -0.467
## recipe3     -0.467  0.500
## recipe4     -0.467  0.500  0.500
## tasteorder2 -0.193  0.000  0.000  0.000
## tasteorder3 -0.193  0.000  0.000  0.000  0.500
```

```r
ranef(mod1)
```

```
## $taster
##    (Intercept)
## 1  -0.28542847
## 2   1.94292550
## 3   0.98791665
## 4   0.03290781
```

```
## 5  -0.60376475
## 6  -1.55877360
## 7  -0.98348345
## 8   0.60819796
## 9   1.56320680
## 10 -1.62015602
## 11 -0.02847461
## 12 -0.98348345
## 13 -0.41021107
## 14  2.13647918
## 15 -2.32022876
## 16 -1.04688364
## 17  2.45481546
## 18 -0.09187479
## 19 -1.39812773
## 20  1.14856253
## 21 -1.39812773
## 22 -0.12478260
## 23  1.14856253
## 24  0.83022625
##
## $batch
##    (Intercept)
## 1  -0.32294282
## 2   0.32198034
## 3   0.03735385
## 4   0.17186041
## 5  -0.18282500
## 6  -0.09093126
## 7   0.15108241
## 8   0.13652586
## 9  -0.23666034
## 10 -0.13915535
## 11 -0.17387970
## 12  0.32759160
```

```
vcov(mod1)
```

```
## 6 x 6 Matrix of class "dpoMatrix"
##             (Intercept)       recipe2       recipe3       recipe4
## (Intercept)  0.14534652 -6.335710e-02 -6.335710e-02 -6.335710e-02
## recipe2     -0.06335710  1.267142e-01  6.335710e-02  6.335710e-02
## recipe3     -0.06335710  6.335710e-02  1.267142e-01  6.335710e-02
## recipe4     -0.06335710  6.335710e-02  6.335710e-02  1.267142e-01
## tasteorder2 -0.01078125  7.473679e-17  7.473679e-17  7.473679e-17
## tasteorder3 -0.01078125  7.473679e-17  7.473679e-17  7.473679e-17
##               tasteorder2   tasteorder3
## (Intercept) -1.078125e-02 -1.078125e-02
## recipe2      7.473679e-17  7.473679e-17
## recipe3      7.473679e-17  7.473679e-17
## recipe4      7.473679e-17  7.473679e-17
## tasteorder2  2.156250e-02  1.078125e-02
## tasteorder3  1.078125e-02  2.156250e-02
```

```r
summary(mod1)$varcor
```

```
##  Groups   Name        Std.Dev.
##  taster   (Intercept) 1.35307
##  batch    (Intercept) 0.35941
##  Residual             0.50867
```

```r
summary(mod1)$sigma^2
```

```
## [1] 0.25875
```

```r
mod1.additive.1 <- lmer(y~recipe+tasteorder+(1|batch)+(1| taster),
                REML=F, data=d)

mod1.interaction.1 <- lmer(y~recipe*tasteorder+(1|batch)+(1| taster),
                REML=F, data=d)

anova(mod1.additive.1, mod1.interaction.1)
```

```
## Data: d
## Models:
## mod1.additive.1: y ~ recipe + tasteorder + (1 | batch) + (1 | taster)
## mod1.interaction.1: y ~ recipe * tasteorder + (1 | batch) + (1 | taster)
##                    Df    AIC    BIC  logLik deviance  Chisq Chi Df
## mod1.additive.1     9 204.22 224.71 -93.112   186.22
## mod1.interaction.1 15 213.08 247.23 -91.540   183.08 3.1437      6
##                    Pr(>Chisq)
## mod1.additive.1
## mod1.interaction.1     0.7906
```