Triangle Multiple Linear Regression Simulation

Stewart Kerr

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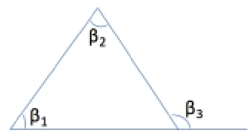
Motivation

Suppose we wish to measure the three angles β_1, β_2, and β_3 as depicted in the diagram below.



Elementary geometry shows that $\beta_1 + \beta_2 = \beta_3$. Suppose, as a check on the accuracy of the results, we decide to measure all three angles, with measurement error. Let b_j be the actual measurement for β_j, $j = 1, 2, 3$. Due to measurement error, $b_1 + b_2$ might not be equal to b_3. Assume that the measurement errors are independent and follow normal distribution with mean 0 and variance σ^2. Formulate this as a multiple linear regression model, and derive the least squares estimates $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$, with their standard deviations.

```r
b1 = c()
b2 = c()
b3 = c()
for (i in seq(1,100)){
  b1 = append(b1, 60 + rnorm(1))
  b2 = append(b2, 60 + rnorm(1))
  b3 = append(b3, 120 + rnorm(1))
}

trilm = lm(b3 ~ b1 + b2)
summary(trilm)
```

```
##
## Call:
## lm(formula = b3 ~ b1 + b2)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.8678 -0.6342 -0.1712  0.6895  2.1630
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 108.51763    6.67675  16.253   <2e-16 ***
## b1            0.03534    0.08330   0.424   0.6723
## b2            0.15852    0.08535   1.857   0.0663 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8618 on 97 degrees of freedom
## Multiple R-squared:  0.03862,    Adjusted R-squared:  0.01879
## F-statistic: 1.948 on 2 and 97 DF,  p-value: 0.1481
```