Bachelor Thesis

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2023-07-20

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Chapter 1

Introduction

Currently, some research interest has shifted from studying stationary signals to exploring the dynamics of time dependent signals. A traditional method for identifying a signals' spectral properties is the Fourier transform. Applying the Fourier transform is based on the assumption that the signal is stationary and ergodic [37]. However, this allows estimating the signal characteristics only in average but not the dynamics. One approach for analyzing non-stationary signals is to use a windowed Fourier transform. While using this method a nonstationary signal is pre-divided into segments (windows) where the signal is considered as stationary. Moreover, the Heisenberg uncertainty principle for Fourier analysis [36] makes it impossible to determine time and frequency simultaneously. We can only state that the coordinate resolution of a windowed transform is determined by the width of the window function and is inversely proportional to the frequency resolution [35]. For the Fourier and the windowed Fourier transforms these restrictions are fundamental [36].

1.1 Fourier Transformation

Fast Fourier Transform (FFT) for signal processing can be particularly problematic when the signal consists of randomly occurring transients superimposed on a more continuous signal [37]. Unfortunately, many signals, for example most of the signals resulting from biological processes are non-stationary and non-ergodic. Applying the classical spectral analysis and the Fourier transformation in the window to study the dynamics of the signals negatively affects the results. If the task is to investigate the reaction in biological systems, it is necessary to extract the information about the time and the frequency of some events in the signal.

1.2 Wavelet Transform

In this context wavelet analysis turned up as a solution to the problem of pinpointing local events. Morle and Grossman (1982-1984) [35] proposed to use wavelet transforms of nonstationary signals. Here the nonstationary signal is decomposed into certain basic functions (other than harmonic), which are obtained from a prototype function by compression and shift. The prototype function is called the mother (basic) wavelet [35]. Usually one of following functions is used as mother wavelet: Mexican Hat, Daubechies Wavelet, Haar Wavelet, Morlet Wavelet and others [35], [38]. Among the most effective wavelets we find the Hermitian wavelets - a family of continuous wavelets, which are defined as the derivative of a Gaussian distribution [39]. These wavelets are very suitable for the analysis of signals containing some peaks, for example for the detection of singularities generated by localized defects in a mechanical system [40], [41], [42], [43]. But Hermitian functions are functions of a continuous argument which leads to difficulties in their application when implementing appropriate algorithms.

1.2.1 Kratwchouk Functionals

pplication when implementing appropriate algorithms. In this context we propose to use Krawtchouk functions [2], [3] as mother wavelets. Krawtchouk functions are the discrete analogue of Hermitian functions by using a simple idea: Since polynomials are completely determined by their values at a sufficiently large number of different points, it is possible to define orthogonality (and also orthonormality) relations on certain polynomials by using only a discrete finite set of points. Their application to automated spectral analysis of arbitrary signals is free from the drawbacks which appear while using polynomials as functions of a continuous argument. Thus, our proposal to use the discrete Krawchouk functions as wavelets in signal processing is an advantage over classical approaches,but does not yet solve all problems. Also wavelets fit into the principle of uncertainty because a basic wavelet is defined on a short time interval which corresponds to high frequency. When the wavelet is stretched this time interval gets longer and the frequency decreases [35], [34], [36]. This is also true for Hermite wavelets [36], [44], [46]. Neither classical Fourier analysis nor wavelet analysis can give a satisfactory answer to the questions ”what?” and ”when?”. However, our visual system can solve the problem of finding the answer to the questions ”what?” and ”where?” in an excellent way, which for one-dimensional signals is equivalent to the question ”when?”. Therefore, we suggest to supplement the wavelet analysis by a new method which is based on a model of the visual system, a method to extract the features of the signal which are invariant to shifts.

1.2.2 Convolution Theorem

Chapter 2

Time Series Analysis

2.1 Modelling EEG and ECG

2.2 Neural Network Model

The problem of extracting a complete system of invariant features of a signal arises in signal processing (including images) as well as in automatic pattern recognition or automatic classification and diagnostics. It is necessary to separate the information about the characteristics of the signal from the information about the transformations that this signal has undergone. These transformations (e.g. shift, image rotation, scale conversion, etc.) cannot be controlled, a visual system must identify the image independent from its location, and the transformations should not affect the performance of the system. Therefore, the images that pass into each other under some specific transformations must be classified as equivalent. It should be noted that the human or mammalian visual system to some extent is capable to extract the invariant features of a signal. It took some effort to understand the functions necessary for visual form perception. Based on the knowledge about the receptive field (RF) reactions and about the visual system in [5], [14], [17], [53], [54] a theory was developed according to which the visual system performs spatialfrequency image filtering. To adapt this effect to general signal processing we have to understand how this filtering works. Let us consider in more detail the visual path from the retina to the cerebral cortex of primates. Visual information enters the retina and is transmitted to the brain through about a million nerve fibers that are united in the optic nerve [4], [6]. Most of the optic nerve fibers reach without interruption two cell nuclei that are located deep in the brain. These nuclei are called lateral geniculate nuclei (LGN). In turn, neurons of the LGN send their axons directly to the primary visual cortex [4], [56]. The RF of a retinal ganglion cell refers to the synaptic network of photoreceptors, bipolar, horizontal, and amacrine cells which come together to this one ganglion cell [6]. A concentric RF has the central zone where the receptors are stimulated to give response and a peripheral inhibitory ring (off-on), or conversely, an inhibitory central zone and a peripheral ring that gives response (on-off). Concentric fields are used to describe the image by points [6]. Mathematically, the spatiotemporal RF is defined by an impulse response function (weight function) describing the firing-rate response to a tiny spot which is on for a very short time [23]. Some methods of modelling this weight functions were developed with difference of arouse and inhibitory Gaussians [23], [9], [10], [16], [18], [20], with Gabor elements [11], [12], [13] and others [19], [20]. Retinal ganglion cells project sensory information to the lateral geniculate neurons of the thalamus. LGN neurons replicate the center-surround structure of their presynaptic partners [7]. Yet, this does not mean that the thalamic fields are direct copies of those in the retina. A single LGN neuron might receive inputs from multiple ganglion cells, hence the spatial information sent from retina is remixed [61]. One LGN neuron is overlapped with the ON (OFF) sub-region of the RF of retinal cells according to feedforward or feedback excitation [55], [51]. Hence RFs of the LGN describe spatial patterns of light and dark regions around an average illuminance level in the visual field [6], [51], [52]. The neural networks of the visual cortex do not present a contour picture of the incoming image which exists on the retina [5]. For effective processing of incoming signals it is therefore necessary to reduce the information redundancy, which, based on known studies, can be assumed to be realized in the LGN [50]. Let us look into more details of this process.

2.3 Krawtchouk

2.3.1 Fourier Decomposition

2.3.2 Coefficient of Assymetry

2.3.3 Energy Functional

2.3.4 Artifact Removal

Chapter 3

Speed Comparison

3.1 Stepwise Speed Comparison

3.2 Memory Consumption

3.3 Error of Function

Chapter 4

References