School of Computing and Information Systems
COMP30026 Models of Computation
Week 3: Writing and Checking Proofs

For the homework problems, swap your answers with a friend, and critique each other's work!

Exercises

T3.1 For each of the following, determine whether it is a valid deductive argument. Justify your response.

(a) My neighbours have woken me up every night so far, and therefore they will also wake me up tonight.
(b) Suppose that all birds fly, and that Jo flies. This suggests that Jo is a bird.
(c) There is no greatest prime number.
(d) Suppose that cities, villages, and towns exist. Suppose also that some pigeons live in cities, and that some pigeons do not. Therefore, some pigeons live in towns.
(e) Suppose that $0 = 1$. Then the set of all sets exists.

T3.2 Prove by induction the following statement:

Claim: For all positive integers n,

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

T3.3 The following "proof" contains a very subtle but significant error. Can you spot it? (Reminder: for all $p, q, n \in \mathbb{Z}$, we have $p \equiv q \pmod{n}$ if and only if $p - q$ is a multiple of n.)

Claim: Let $a, b \in \mathbb{Z}$ where $a \equiv 1 \pmod 3$ and $b \equiv 2 \pmod 3$. Then $a + b \equiv 0 \pmod 3$.

"Proof:" Since $a \equiv 1 \pmod 3$ there is an integer k such that $a = 3k+1$. Since $b \equiv 2 \pmod 3$, we can write $b = 3k + 2$. Thus

$$\begin{aligned} a + b &= (3k + 1) + (3k + 2) \\ &= 6k + 3 \\ &= 3(2k + 1), \end{aligned}$$

and so $a + b \equiv 0 \pmod 3$.

T3.4 Prove by contradiction the following claim:

Claim: Let $a, b \in \mathbb{R}$. If a is rational and ab is irrational, then b is irrational.

Homework problems

P3.1 Prove or disprove the following claim:

Claim: For all sets A, B, C, if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

P3.2 Prove or disprove the following claim:

Claim: For all sets A, B, C, if $A \in B$ and $B \in C$, then $A \in C$.

P3.3 Prove that $\sqrt{3}$ is irrational.

P3.4 Write a corrected version of the proof from T3.3.

P3.5 Prove by induction the following claim:

Claim: For all positive integers n,

$$\sum_{i=1}^{n} i^2 = \frac{2n^3 + 3n^2 + n}{6}.$$

P3.6 **Definition:** An integer is even iff it is a multiple of 2.

Definition: An integer is odd iff it is not even.

Prove that the product of any two odd integers is odd.

P3.7 Prove the following claim:

Claim: Let a, b and c be odd integers. Then the polynomial $ax^2 + bx + c$ has no rational roots.