School of Computing and Information Systems
COMP30026 Models of Computation
Week 4: Translation and Resolution

Exercises

T4.1 We say that a formula is in RCNF (reduced conjunctive normal form) if it is in CNF and none of the clauses are tautologies, have duplicate literals, or subsume any other clause.

Put the following formulas into RCNF:

(a) $\neg(A \land \neg(B \land C))$

(b) $A \lor (\neg B \land (C \lor (\neg D \land \neg A)))$

(c) $(A \lor B) \to (C \land D)$

(d) $A \land (B \to (A \to B))$

Continued in P4.2 & P4.4

T4.2 Resolution works by producing resolvents of a set of disjunctive clauses. These resolvents are logical consequences of the original set, so if a resolvent is unsatisfiable, so is the original set of clauses.

Using resolution, show that the following set of clauses is unsatisfiable:

$$\{\{A, B\}, \{A, \neg B\}, \{C, \neg A, \neg D\}, \{\neg C, \neg D\}, \{D\}\}.$$

Continued in P4.1

T4.3 Consider the following premises:

(a) If Jemima eats duck food, then she is healthy.

(b) If Jemima is healthy, she will lay an egg.

(c) Jemima eats duck food.

Translate them into formulas of propositional logic. Then, draw an appropriate resolution **refutation**, and use it to prove that Jemima will lay an egg.

Continued in P4.5 & P4.6

T4.4 We now have 3 different tools to reason about propositional formulas: truth tables/assignments, equivalences, and resolution. Discuss how we can use these tools to do the following. Which is best in each situation?

(i) Prove $F \equiv G$.

(ii) Disprove $F \equiv G$.

(iii) Prove F is unsatisfiable.

(iv) Prove F is satisfiable.

(v) Prove F is valid.

(vi) Prove $F \models G$.

(vii) Prove that F must hold under the premises A, B, and C. (That is, prove that $A \land B \land C \models F$.)

(viii) Disprove $A \land B \land C \models F$.

Homework problems

P4.1 Using resolution, show that the following set of conjunctive clauses is unsatisfiable:

$$\{\{P, R, \neg S\}, \{P, S\}, \{\neg Q\}, \{Q, \neg R, \neg S\}, \{\neg P, Q\}\}.$$

P4.2 Find the reduced CNF of $\neg((\neg B \rightarrow \neg A) \rightarrow ((\neg B \rightarrow A) \rightarrow B))$ and express the result in clausal form. Then determine whether a refutation of the resulting set is possible.

P4.3 Use resolution to show that each of these formulas is a tautology:

(a) $(P \vee Q) \rightarrow (Q \vee P)$
(b) $(\neg P \rightarrow P) \rightarrow P$
(c) $((P \rightarrow Q) \rightarrow P) \rightarrow P$
(d) $P \leftrightarrow ((P \rightarrow Q) \rightarrow P)$

P4.4 For each of the following clause sets, write down a propositional formula in CNF to which it corresponds. Which of the resulting formulas are satisfiable? Give models of those that are.

(a) $\{\{A, B\}, \{\neg A, \neg B\}, \{\neg A, B\}\}$
(b) $\{\{A, \neg B\}, \{\neg A\}, \{B\}\}$
(c) $\{\{A\}, \emptyset\}$
(d) $\{\{A, B\}, \{\neg A, \neg B\}, \{B, C\}, \{\neg B, \neg C\}, \{A, C\}, \{\neg A, \neg C\}\}$

P4.5 Consider these assumptions:

(a) If Ann can clear 2 meters, she will be selected.
(b) If Ann trains hard then, if she gets the flu, the selectors will be sympathetic.
(c) If Ann trains hard and does not get the flu, she can clear 2 meters.
(d) If the selectors are sympathetic, Ann will be selected.

Does it follow that Ann will be selected? Does she get selected if she trains hard? Use any of the propositional logic techniques we have discussed, to answer these questions.

P4.6 Consider the following four statements:

(a) The commissioner cannot attend the function unless he resigns and apologises.
(b) The commissioner can attend the function if he resigns and apologises.
(c) The commissioner can attend the function if he resigns.
(d) The commissioner can attend the function only if he apologises.

Identify the basic propositions involved and translate the statements into propositional logic. In particular, what is the translation of a statement of the form "X does not happen unless Y happens"? Identify cases where one of the statements entails some other statement in the list.

P4.7 Letting F and G be two different formulas from the set

$$\{(P \wedge Q) \vee R, \quad (P \vee Q) \wedge R, \quad P \wedge (Q \vee R), \quad P \vee (Q \wedge R)\}$$

list all combinations that satisfy $F \models G$.

P4.8 A formula is in *disjunctive* normal form (DNF) iff it is a disjunction of conjunctions of literals. We say a formula is in RDNF (reduced disjunctive normal form) iff it is in DNF and no conjunctive clause has duplicate literals, is a contradiction, or subsumes another conjunctive clause. (If F and G are *conjunctive* clauses, we say F subsumes G iff $F \vee G \equiv F$.)

We claim that the formula

$$(P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge R) \vee (Q \wedge \neg R)$$

is logically equivalent to the simpler

$$\neg P \vee Q$$

with both being in reduced disjunctive normal form (RDNF). Show that the claim is correct.

P4.9 In P4.8 we saw that truth of this formula does not depend on the truth of R:

$$(P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge R) \vee (Q \wedge \neg R)$$

Find a shortest logically equivalent CNF formula which includes R.