School of Computing and Information Systems
COMP30026 Models of Computation
Week 9: Pushdown Automata and Context-Free Languages

Exercises

T9.1 Construct pushdown automata (PDAs) for the following languages.

(i) $\{\mathtt{a}^i\mathtt{b}\mathtt{a}^j \mid i > j \geq 0\}$

(ii) $\{w \in \{\mathtt{a},\mathtt{b}\}^* \mid w \text{ is a palindrome}\}$

Continued in P9.1 & P9.2

T9.2 A PDA $(Q, \Sigma, \Gamma, \delta, q_0, F)$ is *deterministic* if, from any configuration, it has at most one available move. That is,

$$\forall q \in Q\, \forall v \in \Sigma\, \forall a \in \Gamma \big(|\delta(q, v, a)| + |\delta(q, v, \epsilon)| + |\delta(q, \epsilon, a)| + |\delta(q, \epsilon, \epsilon)| \leq 1\big)$$

Which of your PDAs from T9.1 are deterministic?

T9.3 (a) Consider the language $A = \{\mathtt{a}^i\mathtt{b}^j\mathtt{a}^i\mathtt{b}^j \mid i, j \geq 0\}$. Use the pumping lemma for context-free languages to show that A is not context-free.

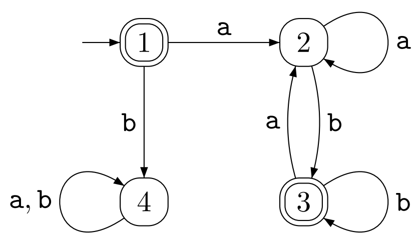
(b) Now consider $B = \{\mathtt{a}^i\mathtt{b}^j\mathtt{a}^j\mathtt{b}^i \mid i, j \geq 0\}$. Give a context-free grammar for B.

(c) A and B look very similar. We might try to prove that B is not context-free by doing what we did to prove that A is not context-free. Where does the attempted proof fail?

T9.4 Let A and B be arbitrary context-free languages. Show how context-free grammars for A and B can be manipulated to construct context-free grammars for $A \cup B$, $A \circ B$, and A^*.

Hint: Try to construct a new grammar using grammars for A and B that can derive strings from A *or* B, *etc.* **Continued in P9.3**

T9.5 Give a context-free grammar for the language recognised by this DFA:



Homework problems

P9.1 Construct a pushdown automaton for

$$\{w \in \{\texttt{0}, \texttt{1}\}^* \mid \text{the length of } w \text{ is odd and its middle symbol is } \texttt{0}\}.$$

What's the minimum number of states you can achieve?

P9.2 Take any context-free grammar in any problem set or tutorial, and construct a pushdown automata which recognises the language of that grammar. Try using the CFG to PDA conversion from the lectures, otherwise you can try to understand what the language is, and construct a PDA from scratch. Repeat this a few times for practice.

P9.3 We have seen that the set of context-free languages is not closed under intersection. However, it *is* closed under intersection with regular languages. That is, if L is context-free and R is regular then $L \cap R$ is context-free.

We can show this if we can show how to construct a push-down automaton P' for $L \cap R$ from a push-down automaton P for L and a DFA D for R. The idea is that we can do something similar to what we did in T8.3 when we built "product automata", that is, DFAs for languages $R_1 \cap R_2$ where R_1 and R_2 were regular languages. If P has state set Q_P and D has state set Q_D, then P' will have state set $Q_P \times Q_D$.

More precisely, let $P = (Q_P, \Sigma, \Gamma, \delta_P, q_P, F_P)$ and let $D = (Q_D, \Sigma, \delta_D, q_D, F_D)$. Recall the types of the transition functions:

$$\begin{aligned} &\delta_P : (Q_P \times \Sigma_\epsilon \times \Gamma_\epsilon) \to \mathcal{P}(Q_P \times \Gamma_\epsilon) \\ &\delta_D : (Q_D \times \Sigma) \to Q_D \end{aligned}$$

We construct P' with the following components: $P' = (Q_P \times Q_D, \Sigma, \Gamma, \delta, (q_P, q_D), F_P \times F_D)$. Discuss how P' can be constructed from P and D. Then give a formal definition of δ, the transition function for P'.

P9.4 Give a context-free grammar for $\{\texttt{a}^i\texttt{b}^j\texttt{c}^k \mid i = j \vee j = k \text{ where } i, j, k \geq 0\}$. Is your grammar ambiguous? Why or why not?

P9.5 Consider the context-free grammar $G = (\{S, A, B\}, \{\texttt{a}, \texttt{b}\}, R, S)$ with rules R:

$$\begin{array}{rcl} S & \to & A\ B\ A \\ A & \to & \texttt{a}\ A \mid \epsilon \\ B & \to & \texttt{b}\ B \mid \epsilon \end{array}$$

(a) Show that G is ambiguous.

(b) The language generated by G is regular; give a regular expression for $L(G)$.

(c) Give an unambiguous context-free grammar, equivalent to G. *Hint:* As an intermediate step, you may want to build a DFA for $L(G)$.