**School of Computing and Information Systems**
**COMP30026 Models of Computation**
**Week 6: Predicate Logic — Semantics and Resolution**

## Homework problems

P6.1 Consider the following predicates:

- $C(x)$, which stands for "$x$ is a cat"
- $M(x)$, which stands for "$x$ is a mouse"
- $L(x, y)$, which stands for "$x$ likes $y$"

Express the statement "No mouse likes a cat who likes all mice" as a formula in first-order predicate logic. Once you have, convert that formula into clausal form.

**Solution:** Here is how we might capture "$z$ is a mouse": $M(z)$, and here is how we can say that "$x$ is a cat who likes mice": $C(x) \wedge \forall y(M(y) \rightarrow L(x, y))$. Now we want to say that if both of those two statements are true then $z$ does not like $x$, and that's that case no matter which $z$ and which $x$ we are talking about:

$$\forall x \forall z \Big( M(z) \wedge C(x) \wedge \forall y(M(y) \rightarrow L(x, y)) \rightarrow \neg L(z, x) \Big)$$

This might not follow with a natural reading of the statement in English. However, $\forall x, P(x)$ will hold iff $\neg\exists x\, \neg P(x))$. With a natural reading of the sentence in English, for some definition of "natural", one may read the following:

$$\neg\exists z \Big( M(z) \wedge \exists x \big( C(x) \wedge \forall y(M(y) \rightarrow L(x, y)) \wedge L(z, x) \big) \Big)$$

P6.2 For this question use the following predicates:

- $G(x)$ for "$x$ is a green dragon"
- $R(x)$ for "$x$ is a red dragon"
- $H(x)$ for "$x$ is a happy dragon"
- $S(x)$ for "$x$ can spit fire"
- $P(x, y)$ for "$x$ is a parent of $y$"
- $C(x, y)$ for "$x$ is a child of $y$"

(a) Express the following statements as formulas in first-order predicate logic:

i. $x$ is a parent of $y$ if and only if $y$ is a child of $x$.
ii. A dragon is either green or red; not both.
iii. A dragon is green if and only if at least one of its parents is green.
iv. Green dragons can spit fire.
v. A dragon is happy if all of its children can spit fire.

(b) Translate each of the five formulas to clausal form.

(c) Prove, using resolution, that all green dragons are happy.

P6.3 Consider the following formulas:

(a) $\forall x\, \neg L(x, x)$

(b) $\forall x \exists y\, L(x, y)$

(c) $\forall x \forall z(L(x, z) \to \exists y(L(x, y) \wedge L(y, z)))$

Give a model which satisfies all three formulas. Can this be done with a finite universe? If so, how many elements does the smallest such universe have?

**Solution:** These formulas are true in the model where the universe is $\mathbb{Q}$ and $L$ stands for the usual "less than" predicate.

We can also do this with a finite universe. We can find a finite model $M$ by thinking about what the directed graph of the relation $I(L)$ has to look like, where $I$ is the interpretation function of $M$. In terms of directed graphs, the formulas say the following things, in order:

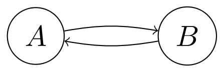
(a) There are no self-loops.

(b) Every node has an edge going out of it.

(c) If there is an edge from a node $x$ to a node $z$, then there is also a node $y$ such that there is an edge from $x$ to $y$ and from $y$ to $z$.

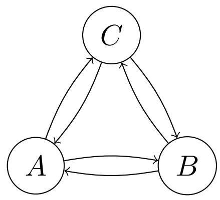
Can a graph with just one node satisfy these conditions? No, because there has to be an edge going out of the node, but if there is only one node, then that edge must a self-loop.

How about a two-node graph? Well, we can satisfy the first two conditions with this graph:




This is also the only two-node graph satisfying the first two conditions: we cannot delete any edge without violating the second condition, and we cannot add any edge without violating the first condition. But the third condition is not satisfied: there is an edge from $A$ to $B$, but there is no node $y$ such that there is an edge from $A$ to $y$ and from $y$ to $B$.

So a two-node graph won't work, but we seem to be close. All we need to do is add one extra node to satisfy the third condition:




This one works! No self-loops, every node has an edge coming out of it, and you can always find a "detour" to satisfy the third condition. So a three-node graph is the smallest solution you can get. This immediately gives us a model where the universe is the set of vertices $V = \{A, B, C\}$, and $I(L)$ is the set of edges $E = \{(A, B), (B, A), (A, C), (C, A), (B, C), (C, B)\}$. This model is also the only 3-element model, up to renaming. Larger (but still finite) models exist too. Can you figure out how to construct a model of any given size greater than 3?

P6.4 Using equivalences, show that $\neg \forall x \exists y\ (\neg P(x) \wedge P(y))$ is valid. Then convert the formula to clausal form. **Solution:** It is easy to see that $\neg \forall x \exists y\ (\neg P(x) \wedge P(y))$ is valid. For example, first push negation in, to get $\exists x \forall y\ (P(x) \vee \neg P(y))$. Both quantifiers can be pushed in, since there is no $y$ in the left conjunct. So this formula is equivalent: $\exists x\ (P(x)) \vee \forall y\ (\neg P(y))$. This is clearly valid. It is also straight-forward to turn into clausal form; we get just one clause: $P(a) \vee \neg P(y)$.

P6.5 Prove the theorem given in **??**.

P6.6 Turn $\neg\forall x\ \exists y\ \left[\forall z\ \Big(Q(x,z) \wedge P(y)\Big) \wedge \forall u\ \Big(\neg Q(u,x)\Big)\right]$ into clausal form.

**Solution:** Let's start by pushing negation in. The formula becomes

$$\exists x\ \forall y \Big(\exists z\ (\neg Q(x,z) \vee \neg P(y)) \vee \exists u\ Q(u,x)\Big)$$

Now we can Skolemize and remove universal quantifiers. The result is a single clause:

$$\neg Q(a, f(y)) \vee \neg P(y) \vee Q(g(y), a)$$

P6.7 Turn $\forall x \forall y \exists z \Big( P(x) \rightarrow \forall y \forall z (Q(y,z))\Big)$ into a simpler, equivalent formula of the form $\varphi \rightarrow \psi$.

**Solution:** We can use the rules of passage for the quantifiers. First, note that the sub-formula

$$\Big(P(x) \rightarrow \forall y \forall z (Q(y,z))\Big)$$

has no free occurrence of $y$ or $z$. Hence, we can simply drop the quantifiers $\forall y \exists z$ that we find in front of that sub-formula, which leaves us with

$$\forall x \Big( P(x) \rightarrow \forall y \forall z (Q(y,z))\Big).$$

Since the right-hand side of the arrow has no free occurrence of $x$, we can push the remaining universal quantifier in, which yields $\exists x(P(x)) \rightarrow \forall y \forall z(Q(y,z))$. This is of the required form. If you wonder about the universal quantifier turning into an existential quantifier, complete this exercise by filling in the details (start by rewriting the implication to a disjunction).

P6.8 For each of the following pairs of terms, determine whether the pair is unifiable. If it is, give the most general unifier. (Don't forget our agreed convention: for constants we use letters from the beginning of the alphabet, here $a$ and $b$, whereas for variables we use letters from the end of the alphabet.)

(i) $h(f(x), g(y, f(x)), y)$ and $h(f(u), g(v, v), u)$
(ii) $h(f(g(x, y)), y, g(y, y))$ and $h(f(u), g(a, v), u)$
(iii) $h(g(x, x), g(y, z), g(y, f(z)))$ and $h(g(u, v), g(v, u), v)$
(iv) $h(v, g(v), f(u, a))$ and $h(g(x), y, x)$
(v) $h(f(x, x), y, y, x)$ and $h(v, v, f(a, b), a)$

P6.9 Consider the following predicates:

- $E(x, y)$, which stands for "$x$ envies $y$"
- $F(x, y)$, which stands for "$x$ is more fortunate than $y$"

(a) Using '$a$' for Adam, express, in first-order predicate logic, the sentence "Adam envies everyone more fortunate than him."
(b) Using '$e$' for Eve, express, in first-order predicate logic, the sentence "Eve is no more fortunate than any who envy her."
(c) Formalise an argument for the conclusion that "Eve is no more fortunate than Adam." That is, express this statement in first-order predicate logic and show that it is a logical consequence of the other two.

**Solution:**

(a) $\forall x\ (F(x,a) \to E(a,x))$

(b) $\forall x\ (E(x,e) \to \neg F(e,x))$

(c) We capture "Eve is no more fortunate than Adam" as $\neg F(e,a)$. To show that this is a logical consequence of the other two statements, we need to show that every model of $\forall x\ (F(x,a) \to E(a,x)) \land \forall x\ (E(x,e) \to \neg F(e,x))$ makes $F(e,a)$ false. Assume (for contradiction) that there is a model in which $F(e,a)$ is true. Then, by the left conjunct, $E(a,e)$ is also true in this model. But then, by the right conjunct, $\neg F(e,a)$ is also true, that is, $F(e,a)$ is false. But this is a contradiction, so $F(e,a)$ must be false. Indeed a proof by resolution is easy:

$$\begin{array}{ccccc} \neg F(x,a), E(a,x) & & \neg E(x',e), \neg F(e,x') & & F(e,a) \\ & \neg F(e,a) & & & \\ & & & \bot & \end{array}$$

P6.10 Using the unification algorithm, determine whether $Q(f(g(x),y,f(y,z,z)),g(f(a,y,z)))$ and $Q(f(u,g(a),v),u)$ are unifiable. If they are, give a most general unifier. (As usual, we use letters from the end of the alphabet for variables, and letters from the beginning of the alphabet for constants.)

P6.11 Determine whether $P(f(g(x),f(g(x),g(a))),x)$ and $P(f(u,f(v,v)),u)$ are unifiable. If they are, give a most general unifier.

**Solution:**

(i) The pair of terms $(h(f(x),g(y,f(x)),y),h(f(u),g(v,v),u))$ is not unifiable. Applying rule 1 (decomposition) to $\{h(f(x),g(y,f(x)),y) = h(f(u),g(v,v),u)\}$, we get

$$\left\{ \begin{array}{rcl} f(x) & = & f(u) \\ g(y,f(x)) & = & g(v,v) \\ y & = & u \end{array} \right\}$$

Applying rule 1 (decomposition) again, to each of the first two equations, yields

$$\left\{ \begin{array}{rcl} x & = & u \\ y & = & v \\ f(x) & = & v \\ y & = & u \end{array} \right\}$$

Applying rule 6 (substitution) with the first equation, we get

$$\left\{ \begin{array}{rcl} x & = & u \\ y & = & v \\ f(u) & = & v \\ y & = & u \end{array} \right\}$$

Applying rule 4 (reorientation) to the third equation, followed by rule 6 to the result yields

$$\left\{ \begin{array}{rcl} x & = & u \\ y & = & f(u) \\ v & = & f(u) \\ y & = & u \end{array} \right\}$$

Applying rule 6 (substitution) with the last equation yields

$$\left\{\begin{array}{lcl} x & = & u \\ u & = & f(u) \\ v & = & f(u) \\ y & = & u \end{array}\right\}$$

Now the occur check applied to the second equation yields failure.

(ii) The pair of terms $(h(f(g(x,y)),y,g(y,y)), h(f(u),g(a,v),u))$ is unifiable. Applying rule 1 (decomposition) to $\{h(f(g(x,y)),y,g(y,y)) = h(f(u),g(a,v),u)\}$, we get

$$\left\{\begin{array}{rcl} f(g(x,y)) & = & f(u) \\ y & = & g(a,v) \\ g(y,y) & = & u \end{array}\right\}$$

and a second application yields

$$\left\{\begin{array}{rcl} g(x,y) & = & u \\ y & = & g(a,v) \\ g(y,y) & = & u \end{array}\right\}$$

Applying rule 4 (reorientation) to the first and the third equation, we have

$$\left\{\begin{array}{lcl} u & = & g(x,y) \\ y & = & g(a,v) \\ u & = & g(y,y) \end{array}\right\}$$

Applying rule 6 (to the first equation) we then get

$$\left\{\begin{array}{rcl} u & = & g(x,y) \\ y & = & g(a,v) \\ g(x,y) & = & g(y,y) \end{array}\right\}$$

which, after an application of rule 1 gives

$$\left\{\begin{array}{lcl} u & = & g(x,y) \\ y & = & g(a,v) \\ x & = & y \\ y & = & y \end{array}\right\}$$

The last equation is dropped, by rule 3, and then rule 6 applied to the third equation gives

$$\left\{\begin{array}{lcl} u & = & g(y,y) \\ y & = & g(a,v) \\ x & = & y \end{array}\right\}$$

Finally, rule 6 applied to the second equation gives

$$\left\{\begin{array}{lcl} u & = & g(g(a,v),g(a,v)) \\ y & = & g(a,v) \\ x & = & g(a,v) \end{array}\right\}$$

This is a normal form and $\{u \mapsto g(g(a,v),g(a,v)), y \mapsto g(a,v), x \mapsto g(a,v)\}$ is the most general unifier.

(iii) The pair of terms $(h(g(x,x),g(y,z),g(y,f(z))),h(g(u,v),g(v,u),v))$ is not unifiable. Applying rule 1 (decomposition) to $\{h(g(x,x),g(y,z),g(y,f(z))) = h(g(u,v),g(v,u),v)\}$, we get

$$\left\{\begin{array}{rcl} g(x,x) & = & g(u,v) \\ g(y,z) & = & g(v,u) \\ g(y,f(z)) & = & v \end{array}\right\}$$

Applying rule 1 (decomposition) again, to each of the first two equations, yields

$$\left\{\begin{array}{rcl} x & = & u \\ x & = & v \\ y & = & v \\ z & = & u \\ g(y,f(z)) & = & v \end{array}\right\}$$

Applying rule 4 (reorientation) to the last equation, followed by rule 6 applied to $v$ yields

$$\left\{\begin{array}{rcl} x & = & u \\ x & = & g(y,f(z)) \\ y & = & g(y,f(z)) \\ z & = & u \\ v & = & g(y,f(z)) \end{array}\right\}$$

Now the occur check (rule 5) applied to the third equation yields failure.

(iv) The pair of terms $(h(v,g(v),f(u,a)),h(g(x),y,x))$ is unifiable. Applying rule 1 (decomposition) to $\{h(v,g(v),f(u,a)) = h(g(x),y,x)\}$, we get

$$\left\{\begin{array}{rcl} v & = & g(x) \\ g(v) & = & y \\ f(u,a)) & = & x \end{array}\right\}$$

Reorienting the last two equations:

$$\left\{\begin{array}{rcl} v & = & g(x) \\ y & = & g(v) \\ x & = & f(u,a)) \end{array}\right\}$$

Now replacing $x$ (rule 6):

$$\left\{\begin{array}{rcl} v & = & g(f(u,a)) \\ y & = & g(v) \\ x & = & f(u,a)) \end{array}\right\}$$

Finally replacing $v$ (rule 6):

$$\left\{\begin{array}{rcl} v & = & g(f(u,a)) \\ y & = & g(g(f(u,a))) \\ x & = & f(u,a)) \end{array}\right\}$$

we have a normal form and $\{v \mapsto g(f(u,a)), x \mapsto f(u,a), y \mapsto g(g(f(u,a)))\}$ is the most general unifier.

(v) The pair of terms $(h(f(x,x),y,y,x),h(v,v,f(a,b),a))$ is not unifiable. Applying rule 1 (decomposition) to $\{h(f(x,x),y,y,x) = h(v,v,f(a,b),a)\}$, we get

$$\left\{\begin{array}{rcl} f(x,x) & = & v \\ y & = & v \\ y & = & f(a,b) \\ x & = & a \end{array}\right\}$$

Reorienting the first equation yields

$$\left\{\begin{array}{rcl} v & = & f(x,x) \\ y & = & v \\ y & = & f(a,b) \\ x & = & a \end{array}\right\}$$

Now applying rule 6 to $x$ and then to $v$, we get

$$\left\{\begin{array}{rcl} v & = & f(a,a) \\ y & = & f(a,a) \\ y & = & f(a,b) \\ x & = & a \end{array}\right\}$$

Now apply rule 6 to, say, the second equation and get

$$\left\{\begin{array}{rcl} v & = & f(a,a) \\ y & = & f(a,a) \\ f(a,a) & = & f(a,b) \\ x & = & a \end{array}\right\}$$

Decomposition (rule 1) then yields

$$\left\{\begin{array}{rcl} v & = & f(a,a) \\ y & = & f(a,a) \\ a & = & a \\ a & = & b \\ x & = & a \end{array}\right\}$$

Now the second-last equation gives match failure (rule 2 applies), and so the original pair of terms were not unifiable.

**Solution:**

(a) i. $\forall x \forall y (P(x,y) \leftrightarrow C(y,x))$

ii. $\forall x (G(x) \oplus R(x))$

iii. $\forall x (G(x) \leftrightarrow \exists y (P(y,x) \land G(y)))$

iv. $\forall x (G(x) \rightarrow S(x))$

v. $\forall x (\forall y [C(y,x) \rightarrow S(y)] \rightarrow H(x))$

(b) Before we generate clauses, let us simplify the third formula. Replacing $\leftrightarrow$, we get

$$\forall x (\neg G(x) \lor \exists y (P(y,x) \land G(y)) \land (G(x) \lor \neg \exists y (P(y,x) \land G(y))))$$

Pushing negation in:

$$\forall x (\neg G(x) \lor \exists y (P(y,x) \land G(y)) \land (G(x) \lor \forall y (\neg P(y,x) \lor \neg G(y))))$$

We see that the existentially quantified $y$ needs to be Skolemized. Let us use the function symbol $p$, so that $p(x)$ reads "parent of $x$".

Similarly, let us simplify the fifth formula. Replacing the implication symbols, we get $\forall x [\neg \forall y [\neg C(y,x) \lor S(y)] \lor H(x)]$. Pushing the negations in, we then get

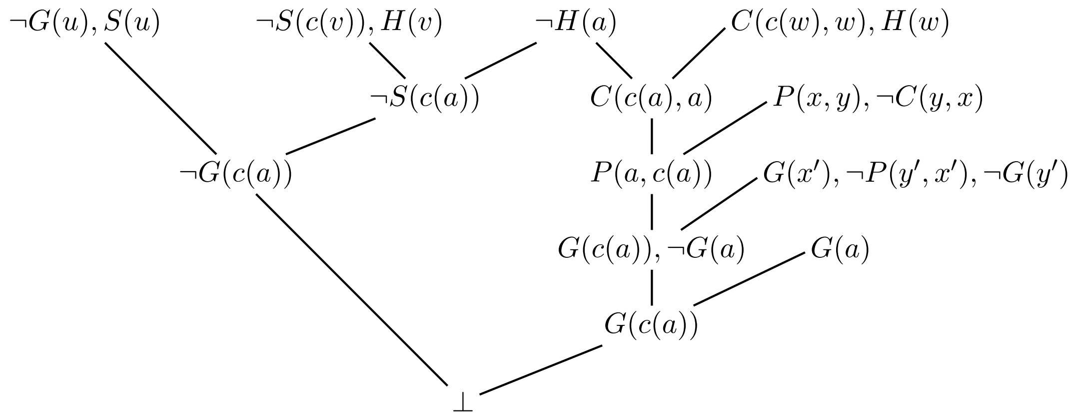
$$\forall x [\exists y [C(y,x) \land \neg S(y)] \lor H(x)]$$

Again, the existentially quantified $y$ needs to be Skolemized, and we must use a fresh function symbol—let us choose $c$, so that $c(x)$ reads "child of $x$".

We can now list the clauses:

i. Two clauses: $\{\neg P(x,y), C(y,x)\}$ and $\{P(x,y), \neg C(y,x)\}$
ii. Two clauses: $\{G(x), R(x)\}$ and $\{\neg G(x), \neg R(x)\}$
iii. Three clauses: $\{\neg G(x), P(p(x),x)\}$, $\{\neg G(x), G(p(x))\}$, and $\{G(x), \neg P(y,x), \neg G(y)\}$
iv. One clause: $\{\neg G(x), S(x)\}$
v. Two clauses: $\{C(c(x),x), H(x)\}$ and $\{\neg S(c(x)), H(x)\}$

(c) The statement to prove is $\forall x(G(x) \to H(x))$. Negating this statement, we have $\exists x(G(x) \land \neg H(x))$. In clausal form this is $G(a)$ and $\neg H(a)$ (two clauses). Altogether we now have 12 clauses, but fortunately a refutation can be found that uses just seven:




P6.12 The barber paradox is a variant of Russell's paradox: say there is a barber who shaves those and *only* those who do not shave themselves. Does the barber shave themselves? The question has no answer: both "yes" and "no" immediately lead to a contradiction.

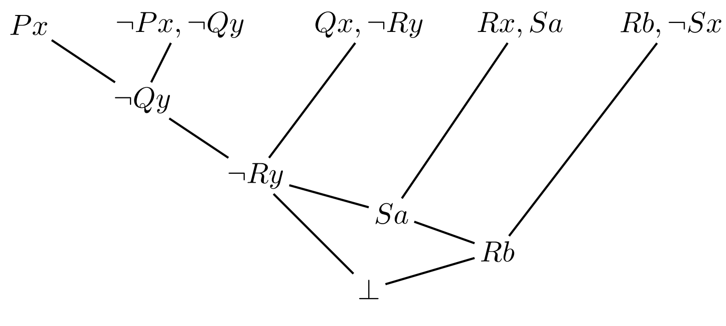
Using $B(x)$ to mean "$x$ is a barber" and $S(u,v)$ to mean "$u$ shaves $v$", translate the premise of the paradox into predicate logic. Then, using resolution (with factoring!), show that it is unsatisfiable.

P6.13 Consider the following unsatisfiable set of clauses:

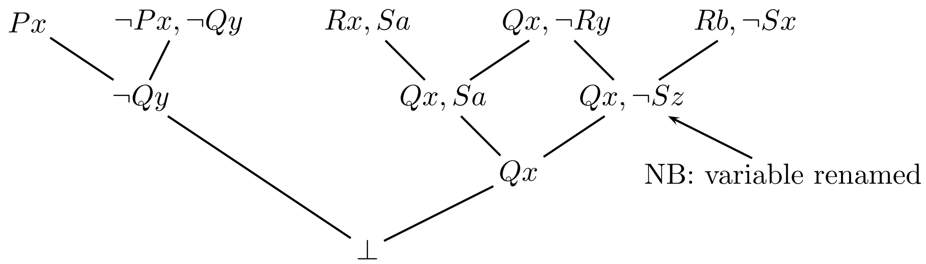
$$\{\{P(x)\}, \{\neg P(x), \neg Q(y)\}, \{Q(x), \neg R(y)\}, \{R(x), S(a)\}, \{R(b), \neg S(x)\}\}$$

What is the simplest refutation proof, if "simplest" means "the refutation tree has minimal depth"? What is the simplest refutation proof, if "simplest" means "the refutation tree has fewest nodes"?

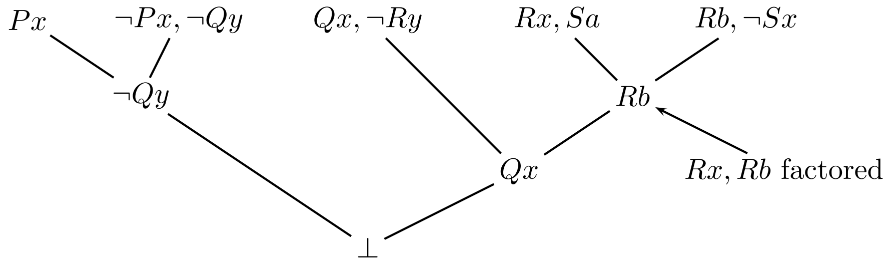
**Solution:** (We should rename clauses apart, but in this case, no confusion arises, so we omit that.) We can construct the refutation in 5 resolution steps, that is, the refutation tree has only 5 internal nodes:



Here is another way (5 steps), in which the depth of the refutation tree is somewhat smaller:




With factoring, we can do it in 4 resolution steps, plus one factoring step:




P6.14 Consider these statements:

| | |
|---|---|
| $S_1$: | "No politician is honest." |
| $S_2$: | "Some politicians are not honest." |
| $S_3$: | "No Australian politician is honest." |
| $S_4$: | "All honest politicians are Australian." |

(a) Using the predicate symbols $P$ and $H$ for being a politician and being honest, respectively, express $S_1$ and $S_2$ as formulas of predicate logic $F_1$ and $F_2$.

(b) Is $F_1 \to F_2$ satisfiable?

(c) Is $F_1 \to F_2$ valid?

(d) Using the predicate symbol $A$ for "is Australian", express $S_3$ and $S_4$ in clausal form.

(e) Using resolution, show that $S_1$ is a logical consequence of $S_3$ and $S_4$.

(f) Prove or disprove the statement "$S_2$ is a logical consequence of $S_3$ and $S_4$."

**Solution:**

(a) The two statements

| | | | |
|---|---|---|---|
| $S_1$: | "No politician is honest." | become | $F_1 \; : \; \forall x\ (\neg P(x) \vee \neg H(x))$ |
| $S_2$: | "Some politicians are not honest." | | $F_2 \; : \; \exists x\ (P(x) \wedge \neg H(x))$ |

(b) $F_1 \to F_2$ is satisfiable. First let us simplify the formula. Normally it would be a good idea to rename the bound variables, but in this case, it will be preferable to keep the $x$.

$$
\begin{array}{lll}
 & F_1 \to F_2 & \\
\equiv & \forall x\ (\neg P(x) \vee \neg H(x)) \to \exists x\ (P(x) \wedge \neg H(x)) & \text{spell out} \\
\equiv & \neg \forall x\ (\neg P(x) \vee \neg H(x)) \vee \exists x\ (P(x) \wedge \neg H(x)) & \text{eliminate implication} \\
\equiv & \exists x\ (P(x) \wedge H(x)) \vee \exists x\ (P(x) \wedge \neg H(x)) & \text{push negation in} \\
\equiv & \exists x\ ((P(x) \wedge H(x)) \vee (P(x) \wedge \neg H(x))) & \exists \text{ distributes over } \vee \\
\equiv & \exists x\ (P(x) \wedge (H(x) \vee \neg H(x))) & \text{factor out } P(x) \\
\equiv & \exists x\ P(x) & \text{eliminate trivially true conjunct}
\end{array}
$$

For this formula we can clearly find a model that makes it true. For example, take the universe $\{alf, bill, charlie\}$ and let $P$ and $H$ hold for all elements. Or, take the universe $\mathbb{Z}$, let $P$ stand for "is a prime" and let $H$ stand for "is zero".

(c) $F_1 \to F_2$ is not valid. It is easy to find a model that makes $\exists x\ P(x)$ false. For example, take the universe $\{alf, bill, charlie\}$ and let $P$ hold for none of the elements ($H$ can be given any interpretation). Or, take the universe $\mathbb{Z}$, let $P$ stand for "is an even prime greater than 2" and let $H$ stand for "is zero".

(d) The statements

$S_3$: "No Australian politician is honest."
$S_4$: "All honest politicians are Australian."

can be expressed

$$S_3:\quad \forall x\ ((A(x) \land P(x)) \to \neg H(x))$$
$$S_4:\quad \forall y\ ((P(y) \land H(y)) \to A(y))$$

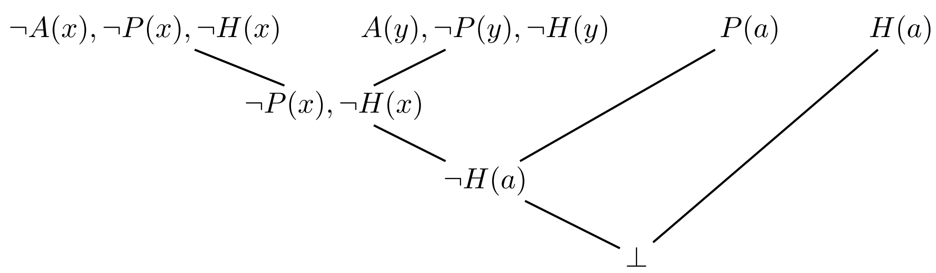
Each of these formulas corresponds to exactly one clause. The clausal forms are:

$$\{\{\neg A(x), \neg H(x), \neg P(x)\}\}$$
$$\{\{A(y), \neg H(y), \neg P(y)\}\}$$

(e) We can show that $S_1$ is a logical consequence of $S_3$ and $S_4$ by refuting $S_3 \land S_4 \land \neg S_1$. So let us write $\neg S_1$ in clausal form (note that we *must* apply the negation *before* "clausifying"; the other way round generally gives an incorrect result):

| | |
|---|---|
| $\neg\forall x\ (\neg P(x) \lor \neg H(x))$ | |
| $\exists x\ (P(x) \land H(x))$ | push negation in |
| $P(a) \land H(a)$ | Skolemize |

Or, written as a set of sets: $\{\{P(a)\}, \{H(a)\}\}$. Added to the other clauses, these allow us to complete the proof by resolution:




(f) The statement "$S_2$ is a logical consequence of $S_3$ and $S_4$" is false. We can show this by constructing a model which makes $S_3$ and $S_4$ true, while making $S_2$ false. Any model with universe $D$, in which $P$ is false for all elements of $D$, will do.

P6.15 Consider a model $M$ with universe $U = \{\text{Jemima}, \text{Thelma}, \text{Louise}\}$ and interpretation function $I$ such that

$$I(a) = \text{Jemima}, \quad I(b) = \text{Thelma}, \quad I(c) = \text{Louise},$$
$$I(F) = \{(\text{Jemima}, \text{Louise}), (\text{Thelma}, \text{Jemima}), (\text{Thelma}, \text{Thelma}), (\text{Louise}, \text{Thelma})\},$$
$$I(M) = \{\text{Jemima}, \text{Louise}\}.$$

Let $v$ be a variable assignment such that $v(x) = \text{Louise}, v(y) = \text{Thelma}, v(z) = \text{Jemima}$.

For each of the following formulas, determine whether it is true or false in the model $M$ under the variable assignment $v$. In each instance, prove your claim from the formal semantics.

(i) $F(x, a)$

(ii) $\exists y\, F(x, y)$

(iii) $\forall x \exists y\, F(x, y)$

(iv) $\forall y\, F(b, y)$

(v) $\exists x \forall y\, F(x, y)$

P6.16 Show that following "equivalences" are incorrect, by specifying a model which makes one formula true and the other false.

(i) $\exists x(F(x) \wedge G(x)) \overset{?}{\equiv} \exists x F(x) \wedge \exists x G(x)$

(ii) $\forall x(F(x) \vee G(x)) \overset{?}{\equiv} \forall x F(x) \vee \forall x G(x)$

(iii) $\forall x \exists y R(x, y) \overset{?}{\equiv} \exists x \forall y R(x, y)$