School of Computing and Information Systems
COMP30026 Models of Computation Tutorial Week 12

Content: reducibility, mapping reducibility

Exercises

T12.1 Show that each the following reduciblity relations hold by giving a computable mapping reduction: Show that each the following reduciblity relations hold by giving a computable mapping reduction:

(a) $A_{NFA} \leq_m A_{DFA}$

(b) $A_{DFA} \leq_m A_{NFA}$

(c) $A_{REX} \leq_m A_{DFA}$

(d) $EQ_{DFA} \leq_m E_{DFA}$

Remember to justify why your mapping reduction is correct and why your mapping reduction is computable. You can do the latter by referencing a procedure from slides/tutes.

T12.2 Consider the languages $ALL_{CFG} = \{\langle G \rangle \mid G$ is a grammar and $L(G) = \Sigma^*\}$ and $EQ_{CFG} = \{\langle G, H \rangle \mid G$ and H are grammars and $L(G) = L(H)\}$.

(a) Show that $ALL_{CFG} \leq_m EQ_{CFG}$.

(b) It is a fact that ALL_{CFG} is undecidable. What can we conclude from this fact and the above?

T12.3 Show that $E_{TM} \leq_m EQ_{TM}$. Using only the facts that $E_{TM} \leq_m EQ_{TM}$ and that E_{TM} is undecidable, can we conclude that EQ_{TM} is also undecidable?

T12.4 Consider the problem of whether a given context-free grammar with alphabet $\{\mathtt{0}, \mathtt{1}\}$ is able to generate a string in $L(\mathtt{1}^*)$. Is that decidable? In other words, is the language

$$\{\langle G \rangle \mid G \text{ is a context-free grammar over } \{\mathtt{0}, \mathtt{1}\} \text{ and } L(G) \cap L(\mathtt{1}^*) \neq \emptyset\}$$

decidable? Show that it is decidable, using a decider E_{CFG} for the decidable language

$$\{\langle G \rangle \mid G \text{ is a context-free grammar over } \{\mathtt{0}, \mathtt{1}\} \text{ and } L(G) = \emptyset\}$$

Hint: consider known closure properties for context-free languages.

T12.5 **Now continue with any other practice problems you may have missed.**

Practice Problems

P12.1 Show that $A_{DFA} \leq_m A_{CFG} \leq_m A_{TM}$.

P12.2 Earlier, we gave a mapping reduction from an undecidable problem ALL_{CFG} to EQ_{CFG} and concluded that EQ_{CFG} is undecidable. Show that the language

$$\overline{EQ_{CFG}} = \{\langle G, H\rangle \mid G \text{ and } H \text{ are grammars and } L(G) \neq L(H)\}$$

is Turing-recognisable. Conclude that EQ_{CFG} is in fact not Turing-recognisable.

P12.3 Let $T = \{\langle M\rangle \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w\}$. Show that T is undecidable.

P12.4 Show that if A is Turing-recognisable and $A \leq_m \overline{A}$, then A is decidable.