School of Computing and Information Systems
COMP30026 Models of Computation
Week 7: Regular Languages, DFAs and NFAs

Exercises

T7.1 Consider the two languages $L_1 = \{\mathtt{ab}, \mathtt{c}\}$ and $L_2 = \{\mathtt{ca}, \mathtt{c}\}$. What strings are in $(L_1 \cup L_2) \circ L_2$ What about $L_1^* \setminus L_2^*$?

T7.2 Draw DFAs recognising the following languages. Assume that the alphabet $\Sigma = \{\mathtt{0}, \mathtt{1}\}$.

(i) $\{w \mid w$ begins with a $\mathtt{1}$ and ends with a $\mathtt{0}\}$

(ii) $\{w \mid w$ contains the substring $\mathtt{0101}\}$ (so $w = x\mathtt{0101}y$ for some strings x and y)

(iii) $\{w \mid w$ is any string except $\mathtt{11}$ and $\mathtt{111}\}$

(iv) $\{\epsilon\}$

(v) The empty set

Continued in P7.1

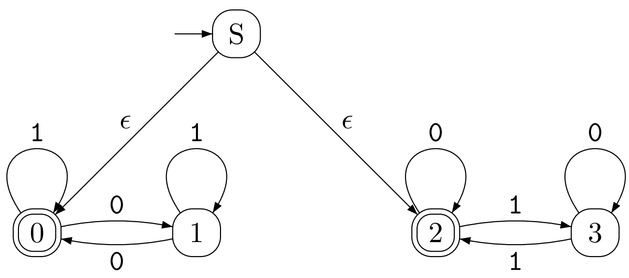
T7.3 The language

$$\{w \mid w \text{ has an even number of } \mathtt{a}\text{s and one or two } \mathtt{b}\text{s}\}$$

over the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$, can be written as the intersection of two simpler languages. First construct the DFAs for the simpler languages, then combine to get a DFA for the intersection using the following idea: If the set of states for DFA D_1 is Q_1 and the set of states for D_2 is Q_2, we let the set of states for the combined DFA D be $Q_1 \times Q_2$. We construct D so that, having consumed a string s, D will be in state (q_1, q_2) iff D_1 is in state q_1, and D_2 is in state q_2 when they have consumed s.

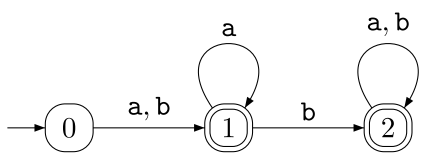
Continued in P7.2 & P7.4

T7.4 Use the subset construction method to turn the following NFA into an equivalent DFA.



Continued in P7.6 & P7.7

T7.5 Use the minimization algorithm to turn this DFA into a minimal DFA which recognises the same language.



Continued in P7.8 & P7.9

Homework problems

P7.1 Draw DFAs recognising the following languages. Assume that the alphabet $\Sigma = \{\mathtt{0}, \mathtt{1}\}$.

(i) $\{w \mid w$ is not empty and contains only 0s or only 1s$\}$
(ii) $\{w \mid w$ has length at least 3 and its third symbol is 0$\}$
(iii) $\{w \mid$ the length of w is at most 5$\}$
(iv) $\{w \mid$ the length of w is a multiple of 3$\}$
(v) $\{w \mid$ every odd position of w is a 1$\}$
(vi) $\{w \mid w$ contains at least two 0s and at most one 1$\}$
(vii) $\{w \mid$ the last symbol of w occurs at least twice in $w\}$
(viii) All strings except the empty string

P7.2 Each of the following languages is the intersection of two simpler languages. First construct the DFAs for the simpler languages, then combine them using the construction from T7.3 to create a DFA for the intersection language

(a) $\{w \mid w$ has at least three as and at least two bs$\}$
(b) $\{w \mid w$ has an odd number of as and ends with b$\}$
(c) $\{w \mid w$ has an odd number of as and has even length$\}$

P7.3 Each of the following languages is the complement of a simpler language. Again, the best way to proceed is to first construct a DFA for the simpler language, then find a DFA for the complement by transforming that DFA appropriately. Throughout this question, assume that the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

(a) $\{w \mid w$ does not contain the substring bb$\}$
(b) $\{w \mid w$ contains neither the substring ab nor ba$\}$
(c) $\{w \mid w$ is any string not in $\mathtt{A}^* \circ \mathtt{B}^*$, where $\mathtt{A} = \{\mathtt{a}\}, \mathtt{B} = \{\mathtt{b}\}\}$
(d) $\{w \mid w$ is any string not in $\mathtt{A}^* \cup \mathtt{B}^*$, where $\mathtt{A} = \{\mathtt{a}\}, \mathtt{B} = \{\mathtt{b}\}\}$
(e) $\{w \mid w$ is any string that doesn't contain exactly two as$\}$
(f) $\{w \mid w$ is any string except a and b$\}$

P7.4 The language

$$L = \{w \mid \text{the length of } w \text{ is a multiple of 2 and is not multiple of 3}\}$$

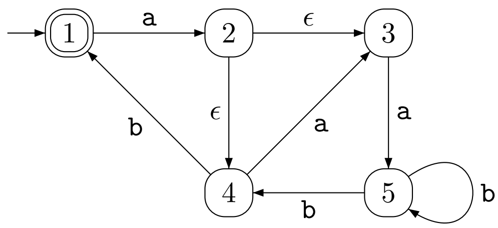
is the difference of two simpler languages. First construct DFAs for the simpler languages, then construct a DFA for L. Assume the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

P7.5 A language is regular iff it is recognised by some DFA. Prove that for any languages $L, K \subseteq \Sigma^*$

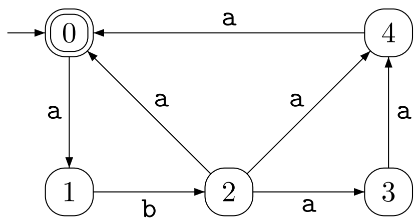
(i) If L is regular then L^c is regular
(ii) If L and K are both regular and then $L \cap K$ is regular
(iii) If L and K are both regular and then $L \setminus K$ is regular

Another way to say this is "The class of regular languages is closed under intersection, complement and difference". You have demonstrated this already for some example languages. For this exercise you should specify how this works in general, using the formal definition of a DFA. *Hint:* Your proof for (iii) can be much shorter.

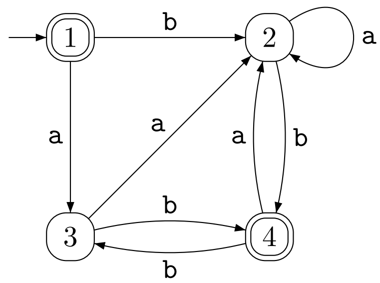
P7.6 Use the subset construction method to turn this NFA into an equivalent DFA:



P7.7 Use the subset construction method to turn this NFA into an equivalent DFA:



P7.8 Construct a minimal DFA equivalent to this one:



P7.9 Construct a minimal DFA equivalent to this one: