School of Computing and Information Systems
COMP30026 Models of Computation
Week 3: Writing and Checking Proofs

For the homework problems, swap your answers with a friend, and critique each other's work!

Homework problems

P3.1 Prove or disprove the following claim:

Claim: For all sets A, B, C, if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

Solution: Prove from the definition of subset. (Given sets X and Y, we have $X \subseteq Y$ if and only if every member of X is a member of Y.)

P3.2 Prove or disprove the following claim:

Claim: For all sets A, B, C, if $A \in B$ and $B \in C$, then $A \in C$.

Solution: Give a counterexample, and argue that the premises are satisfied but that the conclusion is not.

P3.3 Prove that $\sqrt{3}$ is irrational.

Solution: Use the same strategy we used when proving the $\sqrt{2}$ is irrational.

P3.4 Write a corrected version of the proof from **??**.

P3.5 Prove by induction the following claim:

Claim: For all positive integers n,

$$\sum_{i=1}^{n} i^2 = \frac{2n^3 + 3n^2 + n}{6}.$$

Solution: Use the same strategy we used to show that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

P3.6 **Definition:** An integer is even iff it is a multiple of 2.

Definition: An integer is odd iff it is not even.

Prove that the product of any two odd integers is odd.

P3.7 Prove the following claim:

Claim: Let a, b and c be odd integers. Then the polynomial $ax^2 + bx + c$ has no rational roots.

Solution: If you want to use the quadratic formula, remember to handle the case when $a = 0$.

A simpler strategy is to proceed by contradiction, and consider an arbitary rational root. Substitute it into the polynomial, then reason by cases. Which terms are even or odd? Show that this leads to a contradiction in all cases.