Assignment 4 – Dynamic Linear Models with R (PART I).

Kalman filter for the random walk plus noise model.

Tentative: Due by April 15, 2025

For this exercise, **SEE THE LAB POSTED ON Blackboard!**
Install and load package **dlm**
For an overview: `http://core.ac.uk/download/pdf/6340213.pdf`

Exercise.
Consider the Nile data (measurements of the annual flow of the river Nile at Ashwan 1871-1970), available in `R` (`> ?Nile`).
First, plot the data. The series clearly appears non-stationary, presenting a quite evident change point. A *local level* model, i.e. a random walk plus noise, may be used to capture the main change point *and* other minor changes in the level of the Nile river. Let us consider the following random walk plus noise model

$$\begin{aligned} y_t &= \theta_t + v_t, \quad v_t \sim \mathcal{N}(0, V) \\ \theta_t &= \theta_{t-1} + w_t, \quad w_t \sim \mathcal{N}(0, W)' \end{aligned}$$

with the due assumptions. To start with, assume that the variances are known, $V = 15100$, $W = 1470$. In fact, they will have to be estimated (next assignment). As the initial distribution, let $\theta_0 \sim \mathcal{N}(1000, 1000)$.

1. FILTERING.
   Compute the filtering states estimates $m_t = E(\theta_t \mid y_{1:t})$, for $t = 1, 2, \ldots, T$.
   Compute the corresponding standard deviations
   $$\sqrt{C_t} = V(\theta_t \mid y_{1:t})^{1/2}$$
   and plot them. Comment briefly.
   Finally, plot the data together with the filtering state estimates and their 0.95 credible intervals.

2. ONLINE FORECASTING. Compute the one-step ahead forecasts $f_t = E(Y_t \mid y_{1:t-1})$, $t = 1, ..., T$.
   Plot the data, together the one-step-ahead forecasts and their 0.95 credible intervals.

3. What is the effect of the **signal-to-noise ratio** (i.e. the ratio W/V) on the forecasts? Repeat the exercise with different choices of V (observation variance) and W (evolution variance) and comment briefly.

4. SMOOTHING. So far, for computations, we pretended that the data arrived sequentially. Now consider $(y_1, \ldots, y_T)$, and provide and plot the smoothing estimate of the Nile level θ_t at time $t = 28$ together with its 95% credible interval.