Assignment 4 – Dynamic Linear Models with R (PART I).

Kalman filter for the random walk plus noise model.

Due by April 30, 2025

For this exercises, **SEE THE LAB POSTED ON Blackboard!**
Install and load package **dlm**
For an overview: `http://core.ac.uk/download/pdf/6340213.pdf`

Exercise.
Consider the Nile data (measurements of the annual flow of the river Nile at Ashwan 1871-1970), available in `R` (`> ?Nile`).
First, plot the data. The series clearly appears non-stationary, presenting a quite evident change point. A *local level* model, i.e. a random walk plus noise, may be used to capture the main change point *and* other minor changes in the level of the Nile river. Let us consider the following random walk plus noise model

$$y_t = \theta_t + v_t, \quad v_t \sim \mathcal{N}(0, \sigma_v^2)$$
$$\theta_t = \theta_{t-1} + w_t, \quad w_t \sim \mathcal{N}(0, \sigma_w^2)'$$

with the due assumptions. To start with, assume that the variances are known, $\sigma_v^2 = 15100$, $\sigma_w^2 = 1470$. (In fact, they are estimated by MLE; see the last exercise below). As the initial distribution, let $\theta_0 \sim \mathcal{N}(1000, 1000)$.

1. FILTERING.
   Compute the filtering states estimates $m_t = E(\theta_t \mid y_{1:t})$, for $t = 1, 2, \ldots, T$.
   Compute the corresponding standard deviations
   $$\sqrt{C_t} = V(\theta_t \mid y_{1:t})^{1/2}$$
   and plot them. Comment briefly.
   Finally, plot the data together with the filtering state estimates and their 0.95 credible intervals.

2. ONLINE FORECASTING. Compute the one-step ahead forecasts $f_t = E(Y_t \mid y_{1:t-1})$, $t = 1, ..., T$.
   Plot the data, together the one-step-ahead forecasts and their 0.95 credible intervals.

3. What is the effect of the **signal-to-noise ratio** (i.e. the ratio W/V) on the forecasts? Repeat the exercise with different choices of σ_v^2 (observation variance) and σ_w^2 (evolution variance) and comment briefly.

4. MODEL CHECKING. Finally, check the model, using the properties of the forecast errors.

5. SMOOTHING. So far, for computations, we pretended that the data arrived sequentially. Now consider $(y_1, \ldots, y_T)$, and provide and plot the smoothing estimate of the Nile level θ_t at time $t = 28$ together with its 95% credible interval.

6. MAXIMUM LIKELIHOOD ESTIMATES. So far, you have considered the variances σ_v^2 and σ_w^2 as known. In fact, they are unknown and have to be estimated.
   Following the **LAB on DLMs with R: MLEs** (posted on BBoard), compute the maximum likelihood estimates (MLEs) of the unknown variances σ_v^2 and σ_w^2.

   you might use the R function **StructTS**

   Also provide their standard errors.

   in the LAB-DLM-MLE *on BBoard, this is dome through the Rpackage* **dlm**.