Assignment 2

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```r
# To solve conflicts
library(conflicted)
conflicts_prefer(dplyr::filter)
```

```
[conflicted] Will prefer dplyr::filter over any other package.
```

```r
# Time series packages
library(dlm)
library(TSstudio)
library(feasts)
```

```
Loading required package: fabletools
Registered S3 method overwritten by 'tsibble':
  method               from
  as_tibble.grouped_df dplyr
```

```r
library(tseries)
```

```
Registered S3 method overwritten by 'quantmod':
  method            from
  as.zoo.data.frame zoo
```

```r
  # Necessary packages for quantmod
  library(zoo)
  library(xts)
library(quantmod)
```

```
Loading required package: TTR
```

```r
#Specifically for Assignment 2
library(depmixS4)
```

```
Loading required package: nnet
Loading required package: MASS
Loading required package: Rsolnp
Loading required package: nlme

Attaching package: 'nlme'

The following object is masked from 'package:feasts':

    ACF
```

```r
library(HiddenMarkov)

# Datasets
library(readr)
library(fpp3)
```

```
-- Attaching packages -------------------------------------------- fpp3 1.0.1 --
v tibble      3.2.1     v ggplot2     3.5.1
v dplyr       1.1.4     v tsibble     1.1.6
v tidyr       1.3.1     v tsibbledata 0.4.1
v lubridate   1.9.4     v fable       0.4.1
```

```r
# For fancy plots
library(ggthemes)
  # Necessary packages for viridis
  library(viridisLite)
library(viridis)
library(gridExtra)
library(magrittr)
library(textab)

# Packages related to tidyverse, for data manipulation
library(tidyverse) # includes (lubridate), (dplyr), (ggplot2), (tidyr), (tidyselect)
```

```
-- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
v forcats 1.0.0     v stringr 1.5.1
v purrr   1.0.2
```

```r
library(tinytex)

# To handle time changes
library(timechange)
```

```r
# Importing the data
urlfile = "https://raw.githubusercontent.com/stfgrz/20236-timeseries-ps/97a64d30e6a67339343f3
data<-read_csv(url(urlfile), show_col_types = FALSE)

# Getting the nominal data
nom_int_data <- data[, 2]
ts_data_nom <- ts(as.vector(t(nom_int_data)), start = c(1997, 1), frequency = 12)
```

```r
? depmixS4
```

```r
? lm
```

```r
? glm
```

```r
? depmix
```

```r
? posterior
```

2. Hidden Markov Models

The dataset provided in the file data_assHMM.cvs (posted on BBoard) provides monthly data including 10 years Italian government bond's interest rate, inflation represented by the Harmonised Index of Consumer Prices (HICP) and default ratings assigned by the agencies Moody's and Fitch, in the investment grade range, i.e. from Aaa/AAA to Baa3-/BBB-. The data set collects data for the period January 1997 to July 2019, and it has been built mainly using OECD data.

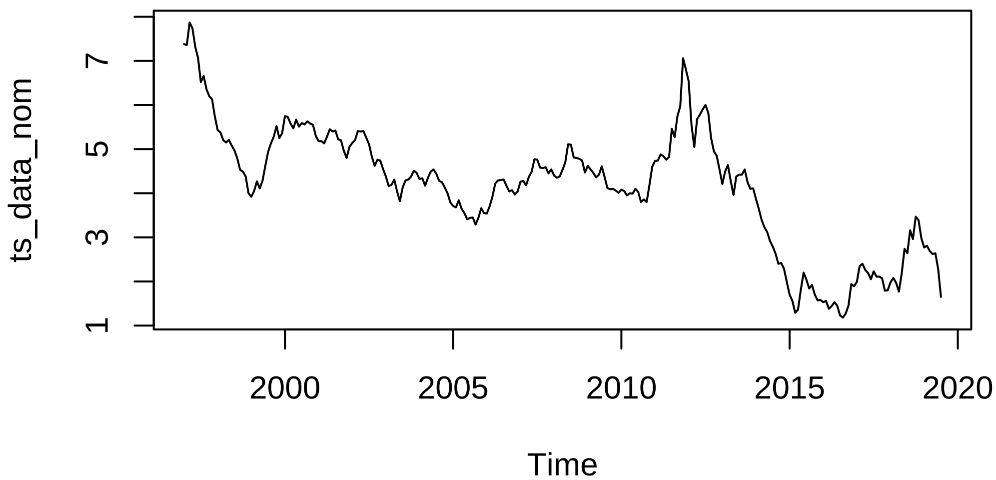
2.1 Part (a)

Question 1

Let us focus on the nominal interest rate for the 10 years Italian government bond. In fact, you may want to consider the real interest rate, calculated from the HICP.

Plot the data and comment briefly if and why a HMM could be a reasonable model.

```r
#plot the dataset for nominal interest rates
plot.ts(ts_data_nom)
```



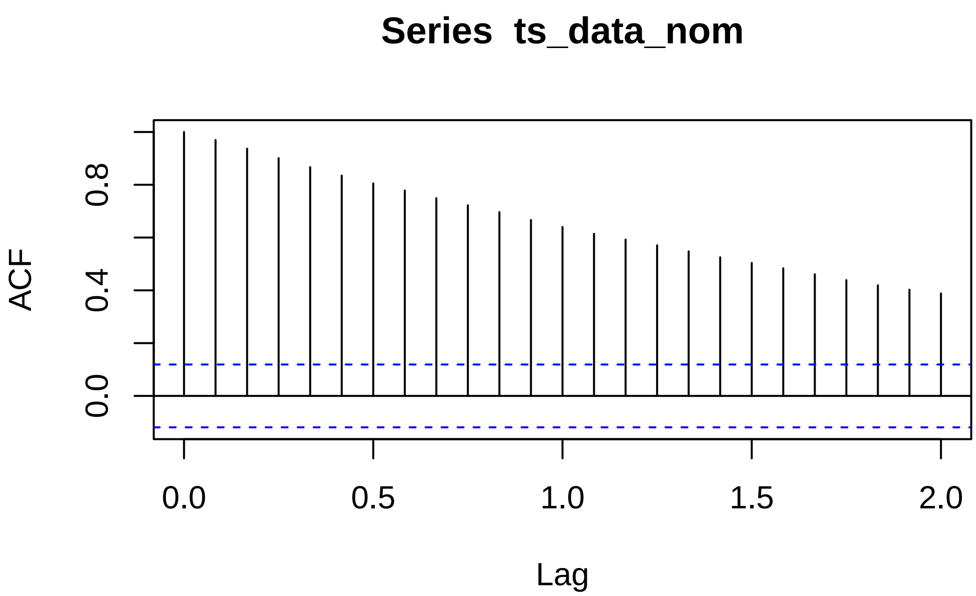

```r
#nominal
y1 <- as.numeric(ts_data_nom)
model1 <- depmix(y1 ~ 1, data=data.frame(ts_data_nom), nstates=3)
model1
```

```
Initial state probabilities model
  pr1   pr2   pr3
0.333 0.333 0.333

Transition matrix
        toS1  toS2  toS3
fromS1 0.333 0.333 0.333
fromS2 0.333 0.333 0.333
fromS3 0.333 0.333 0.333

Response parameters
Resp 1 : gaussian
    Re1.(Intercept) Re1.sd
St1               0      1
St2               0      1
St3               0      1
```

```r
#test for stationarity
acf(ts_data_nom)
```




```r
kpss.test(ts_data_nom)
```

```
Warning in kpss.test(ts_data_nom): p-value smaller than printed p-value
```

```
    KPSS Test for Level Stationarity

data:  ts_data_nom
KPSS Level = 2.4812, Truncation lag parameter = 5, p-value = 0.01
```

```r
adf.test(ts_data_nom)
```

```
    Augmented Dickey-Fuller Test

data:  ts_data_nom
Dickey-Fuller = -2.4709, Lag order = 6, p-value = 0.3776
alternative hypothesis: stationary
```

A HMM model is better suitable for non-stationary time series. At first visual inspection the series seems to show different means and likely different dispersion over time. We also propose various indirect formal evidence against stationarity. The autocorrelation function does not decrease over time very quickly, suggesting non-stationarity. Both the KPSS and the ADF tests, two common test for assessing stationarity, provide support for non-stationarity.

Given the data it seems likely to observe three different latent states, where the peaks before the 2000 and in the early 2010's seem to represent a recession, the years after 2015 the boom, and stable states the remaining years.

A HMM model might be appropriate for the phenomenon at stake, as financial markets are generally very quick in converging to new equilibria mainly depending on exogenous shocks. Hence, postulating a hidden process guiding such shifts is reasonable.

Question 2

Let us indeed use a Hidden Markov Model, with 3 states (representing, say, boom (i.e. less risky, lower interest rates), recession (high risk, high interest rates) and a stable path), and Gaussian emission ditributions, with state-dependent mean and variance.

> Provide the MLEs of the unknown paramters of the model (and their standard errors). Comment briefly.

```
#nominal interest rates
fmodel1 <- fit(model1)
```

```
converged at iteration 28 with logLik: -216.452
```

```
fmodel1
```

```
Convergence info: Log likelihood converged to within tol. (relative change)
'log Lik.' -216.452 (df=14)
AIC:  460.9041
BIC:  511.3337
```

```
summary(fmodel1)
```

```
Initial state probabilities model
pr1 pr2 pr3
  0   0   1

Transition matrix
        toS1  toS2  toS3
fromS1 1.000 0.000 0.000
fromS2 0.008 0.976 0.016
fromS3 0.000 0.041 0.959

Response parameters
Resp 1 : gaussian
    Re1.(Intercept) Re1.sd
St1           2.137  0.581
St2           4.251  0.377
St3           5.639  0.713
```

```r
MLEse1=standardError(fmodel1)
MLEse1
```

```
            par constr          se
1  0.000000e+00    bnd          NA
2  0.000000e+00    bnd          NA
3  1.000000e+00    bnd          NA
4  1.000000e+00    bnd          NA
5  6.959899e-34    bnd          NA
6  1.137589e-58    bnd          NA
7  7.625438e-03    inc 0.007606343
8  9.762212e-01    inc 0.013669112
9  1.615333e-02    inc 0.011405695
10 9.927922e-33    bnd          NA
11 4.128159e-02    inc 0.023266241
12 9.587184e-01    inc 0.023266241
13 2.137116e+00    inc 0.074046873
14 5.810055e-01    inc 0.052838729
15 4.250948e+00    inc 0.037628730
16 3.766686e-01    inc 0.025714756
17 5.639089e+00    inc 0.092323809
18 7.134020e-01    inc 0.058592423
```

```r
posterior(fmodel1)$state
```

```
Warning in .local(object, ...): Argument 'type' not specified and will default
to 'viterbi'. This default may change in future releases of depmixS4. Please
see ?posterior for alternative options.
```

```
  [1] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3
 [38] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 2 2 2 2 2 2 2
 [75] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
[112] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
[149] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3
[186] 3 3 3 3 3 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[223] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[260] 1 1 1 1 1 1 1 1 1 1 1 1
```

Estimated initial probabilities suggest an initial stage at state 3 (recession).

Overall the transition matrix suggests a persistence in state, where the probabilities of remaining in current states are over 95% for both state 1 and state 3. By plotting the data and interpreting state 1 as stability, state 2 as boom and state 3 as recession, the transitions probabilities are then reasonable. It is impossible ($p_{3,2} = p_{2,3} = 0$) to directly shift from recession to boom (and viceversa). Starting from a stable state, it is more likely to end up in a recession than a boom ($p_{1,3} > p_{1,2}$). FInally, the probability 1 for $p_{2,2}$ is justified as the "classification" of state 2 is limited to the last periods of the dataset and no different state follows a state 2 observation, hence the probability 1.

Emission distributions show intermediate nominal interest rates for stable periods (state 1) and the lowest variance. The highest mean is for recession periods, as expected, as well as the highest variance. Booms, reflecting excitement, show lower interest rates but also higher variability compared to stable periods, yet lower than recessions.

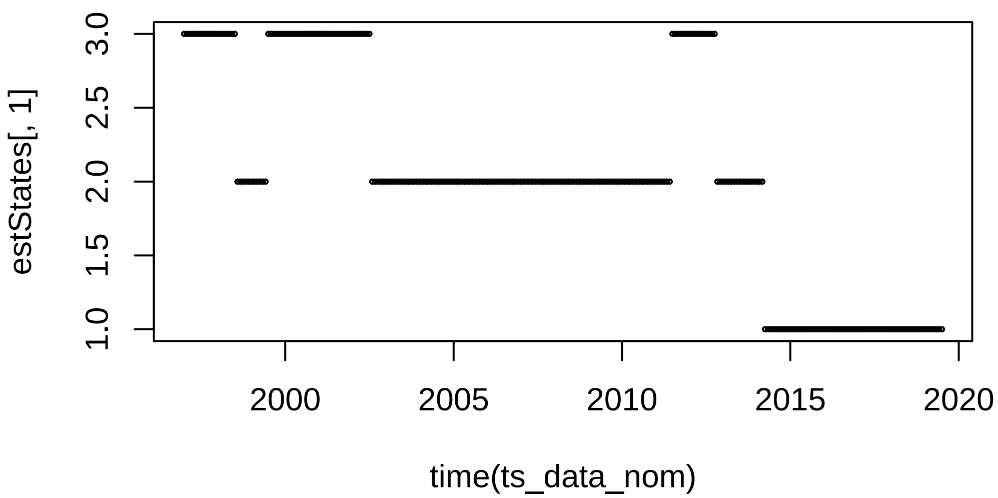
Question 3

> Find the optimal state sequence ("deconding") and plot it, comparing it with the data.

```r
estStates <- posterior(fmodel1)
```

```
Warning in .local(object, ...): Argument 'type' not specified and will default
to 'viterbi'. This default may change in future releases of depmixS4. Please
see ?posterior for alternative options.
```

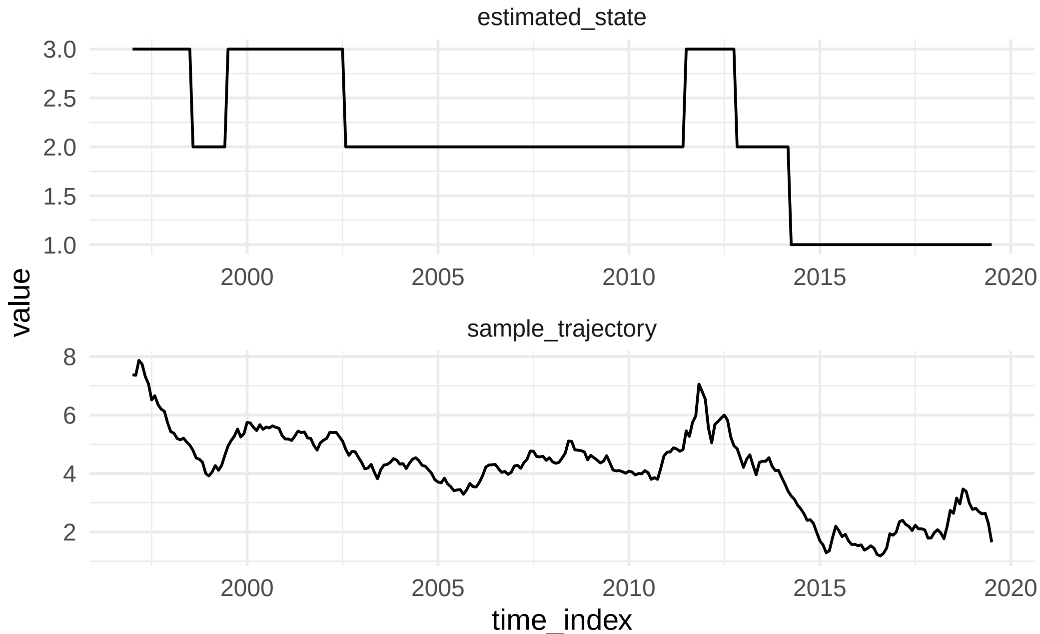
```
plot(time(ts_data_nom), estStates[,1], cex=.3)
```




```
#show it with ggplot
results_df <- data.frame(time_index=time(ts_data_nom) %>% as.numeric(),
sample_trajectory=ts_data_nom %>% as.numeric(),
estimated_state=posterior(fmodel1)$state) %>%
gather("variable", "value", -time_index)
```

```
Warning in .local(object, ...): Argument 'type' not specified and will default
to 'viterbi'. This default may change in future releases of depmixS4. Please
see ?posterior for alternative options.
```

```
ggplot(results_df, aes(time_index, value)) + geom_line() +
facet_wrap(variable ~ ., scales="free", ncol=1) + theme_minimal()
```



2.2 Part (b)

HMMs are particularly useful for time series that present change points. However, one may want to go further, trying to improve prediction of a possible change point through avialble covariates. To this aim, one may use non-homogeneous HMMs, allowing the transition matrix to depend on covariates. A reference is Zucchini, W., MacDonald, I-L. and Langrock, R. (2016) Hidden Markov Models for Time Series: an introduction using R. Chapman and Hall/CRC;and the R package 'depmixS4' allows this extension, see Visser and Speekenbrink (2010), Journal of Statistical Software– both references are posted on BBoard.

You may want to explore this more general class of HMMs for the data under study here.

```
data.b <- as.data.frame(data[, 2:7] )
y1 <- as.numeric(ts_data_nom)
```

Model 1: baseline model

```
#model 1b (controlling for inflation)
mod <- depmix(response = y1 ~ 1, data = data.b, nstates = 3, family = gaussian(),
              transition = ~ inflation)
fmodel2 <- fit(mod)
```

```
converged at iteration 58 with logLik: -214.1302
```

```r
fmodel2
```

```
Convergence info: Log likelihood converged to within tol. (relative change)
'log Lik.' -214.1302 (df=20)
AIC:  468.2603
BIC:  540.3027
```

```r
summary(fmodel2)
```

```
Initial state probabilities model
pr1 pr2 pr3
  1   0   0

Transition model for state (component) 1
Model of type multinomial (mlogit), formula: ~inflation
Coefficients:
            St1       St2        St3
(Intercept)   0 -4.534609   7.726854
inflation     0  0.590847 -11.751170
Probalities at zero values of the covariates.
0.0004406325 4.728471e-06 0.9995546

Transition model for state (component) 2
Model of type multinomial (mlogit), formula: ~inflation
Coefficients:
            St1       St2       St3
(Intercept)   0  6.376119  4.985542
inflation     0 -1.122777 -4.296848
Probalities at zero values of the covariates.
0.001360683 0.7995949 0.1990444

Transition model for state (component) 3
Model of type multinomial (mlogit), formula: ~inflation
Coefficients:
            St1      St2       St3
(Intercept)   0 -55.2404  79.02395
inflation     0  31.0821 -30.32285
Probalities at zero values of the covariates.
4.790007e-35 4.894807e-59 1
```

```
Response parameters
Resp 1 : gaussian
    Re1.(Intercept) Re1.sd
St1           5.612  0.714
St2           4.236  0.365
St3           2.137  0.581
```

```
MLEse2=standardError(fmodel2)
```

```
Warning in sqrt(diag(vc$vcov)): NaNs produced
```

```
MLEse2
```

```
          par constr           se
1    1.0000000    bnd           NA
2    0.0000000    bnd           NA
3    0.0000000    bnd           NA
4    0.0000000    fix           NA
5   -4.5346094    inc   2.46774574
6    7.7268538    inc 144.57410112
7    0.0000000    fix           NA
8    0.5908470    inc   0.88195186
9  -11.7511697    inc 114.77835038
10   0.0000000    fix           NA
11   6.3761187    inc   3.36041999
12   4.9855415    inc   3.65161502
13   0.0000000    fix           NA
14  -1.1227766    inc   1.40890044
15  -4.2968477    inc   2.63757671
16   0.0000000    fix           NA
17 -55.2403992    inc          NaN
18  79.0239465    inc          NaN
19   0.0000000    fix           NA
20  31.0821041    inc 287.65161045
21 -30.3228531    inc 199.74585103
22   5.6118446    inc   0.10051847
23   0.7140541    inc   0.05757647
24   4.2355173    inc   0.04406757
25   0.3654813    inc   0.03003733
```

```
26    2.1369221     inc    0.07362210
27    0.5806107     inc    0.05243830
```

```r
posterior(fmodel2)$state
```

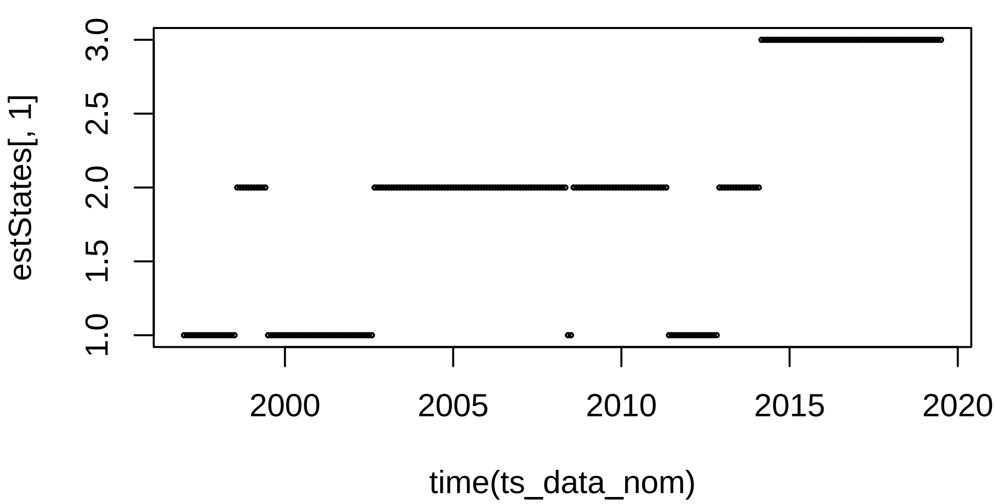
```
Warning in .local(object, ...): Argument 'type' not specified and will default
to 'viterbi'. This default may change in future releases of depmixS4. Please
see ?posterior for alternative options.
```

```
  [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1
 [38] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2
 [75] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
[112] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 2 2 2 2 2 2 2 2 2
[149] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1
[186] 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
[223] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
[260] 3 3 3 3 3 3 3 3 3 3 3 3
```

```r
estStates <- posterior(fmodel2)
```

```
Warning in .local(object, ...): Argument 'type' not specified and will default
to 'viterbi'. This default may change in future releases of depmixS4. Please
see ?posterior for alternative options.
```

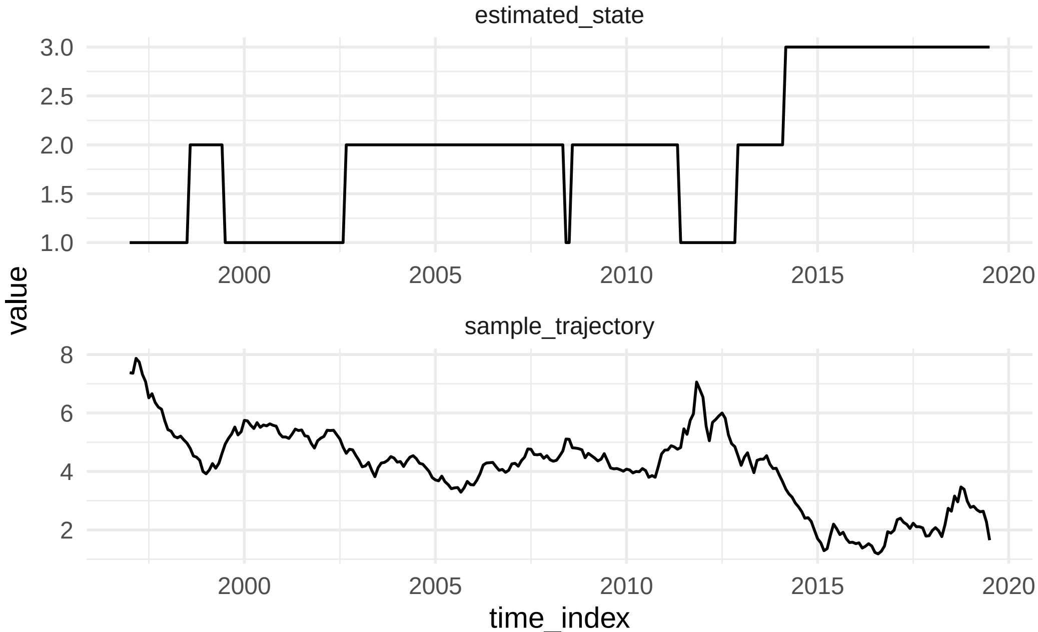
```r
plot(time(ts_data_nom), estStates[,1], cex=.3)
```



```
#ggplot
results_df <- data.frame(time_index=time(ts_data_nom) %>% as.numeric(),
sample_trajectory=ts_data_nom %>% as.numeric(),
estimated_state=posterior(fmodel2)$state) %>%
gather("variable", "value", -time_index)
```

```
Warning in .local(object, ...): Argument 'type' not specified and will default
to 'viterbi'. This default may change in future releases of depmixS4. Please
see ?posterior for alternative options.
```

```
ggplot(results_df, aes(time_index, value)) + geom_line() +
facet_wrap(variable ~ ., scales="free", ncol=1) + theme_minimal()
```



Model 2: controlling for gdp

```
mod2 <- depmix(response = y1 ~ 1, data = data.b, nstates = 3, family = gaussian(),
               transition = ~ gdp)
fmodel3 <- fit(mod2)
```

```
converged at iteration 33 with logLik: -215.7819
```

```r
fmodel3
```

```
Convergence info: Log likelihood converged to within tol. (relative change)
'log Lik.' -215.7819 (df=20)
AIC:  471.5637
BIC:  543.6061
```

```r
summary(fmodel3)
```

```
Initial state probabilities model
pr1 pr2 pr3
  0   1   0

Transition model for state (component) 1
Model of type multinomial (mlogit), formula: ~gdp
Coefficients:
            St1        St2        St3
(Intercept)   0 -4.2810454 -4.8542469
gdp           0  0.8807132  0.1042325
Probalities at zero values of the covariates.
0.9788343 0.01353551 0.007630211

Transition model for state (component) 2
Model of type multinomial (mlogit), formula: ~gdp
Coefficients:
            St1       St2        St3
(Intercept)   0 3.1137101 -9.9505399
gdp           0 0.7601739  0.4167766
Probalities at zero values of the covariates.
0.04254517 0.9574528 2.029484e-06

Transition model for state (component) 3
Model of type multinomial (mlogit), formula: ~gdp
Coefficients:
            St1        St2       St3
(Intercept)   0  0.9297441 14.304771
gdp           0 -0.9181655  2.768894
Probalities at zero values of the covariates.
6.130784e-07 1.553455e-06 0.9999978
```

```
Response parameters
Resp 1 : gaussian
    Re1.(Intercept) Re1.sd
St1           4.248  0.375
St2           5.632  0.714
St3           2.137  0.581
```

```r
MLEse3=standardError(fmodel3)
```

```
Warning in sqrt(diag(vc$vcov)): NaNs produced
```

```r
MLEse3
```

```
          par constr           se
1   0.0000000    bnd           NA
2   1.0000000    bnd           NA
3   0.0000000    bnd           NA
4   0.0000000    fix           NA
5  -4.2810454    inc   1.12445396
6  -4.8542469    inc   1.01198347
7   0.0000000    fix           NA
8   0.8807132    inc   2.09071863
9   0.1042325    inc   1.41291508
10  0.0000000    fix           NA
11  3.1137101    inc   0.60048919
12 -9.9505399    inc  98.14191235
13  0.0000000    fix           NA
14  0.7601739    inc   0.88181174
15  0.4167766    inc          NaN
16  0.0000000    fix           NA
17  0.9297441    inc  17.21967904
18 14.3047709    inc 116.49875319
19  0.0000000    fix           NA
20 -0.9181655    inc  19.72050056
21  2.7688941    inc  28.21562791
22  4.2482162    inc   0.03942895
23  0.3753438    inc   0.02665222
24  5.6323303    inc   0.09560580
25  0.7144388    inc   0.05852701
26  2.1371237    inc   0.07404068
27  0.5810065    inc   0.05283736
```

```r
posterior(fmodel3)$state
```

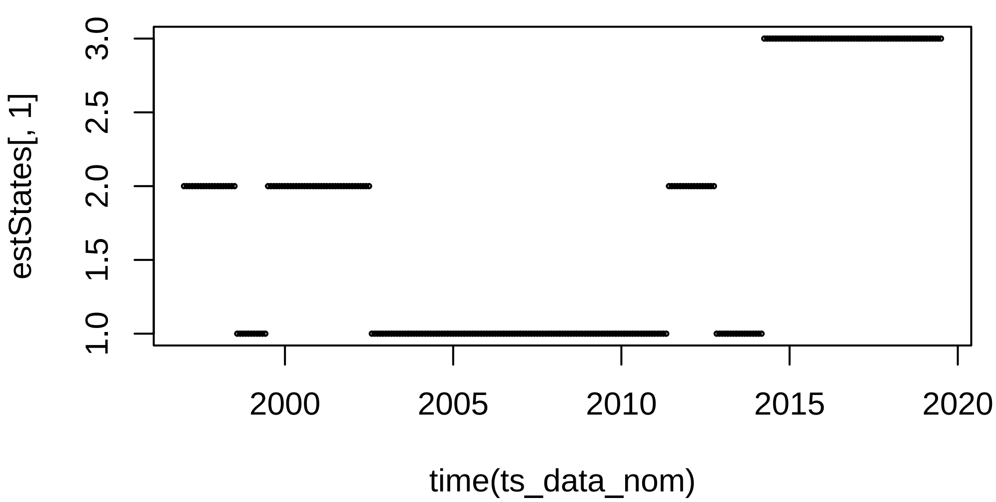
```
Warning in .local(object, ...): Argument 'type' not specified and will default
to 'viterbi'. This default may change in future releases of depmixS4. Please
see ?posterior for alternative options.
```

```
  [1] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2
 [38] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1
 [75] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[112] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[149] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2
[186] 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 3 3 3 3 3 3 3 3 3 3 3 3 3 3
[223] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
[260] 3 3 3 3 3 3 3 3 3 3 3 3
```

```r
estStates <- posterior(fmodel3)
```

```
Warning in .local(object, ...): Argument 'type' not specified and will default
to 'viterbi'. This default may change in future releases of depmixS4. Please
see ?posterior for alternative options.
```

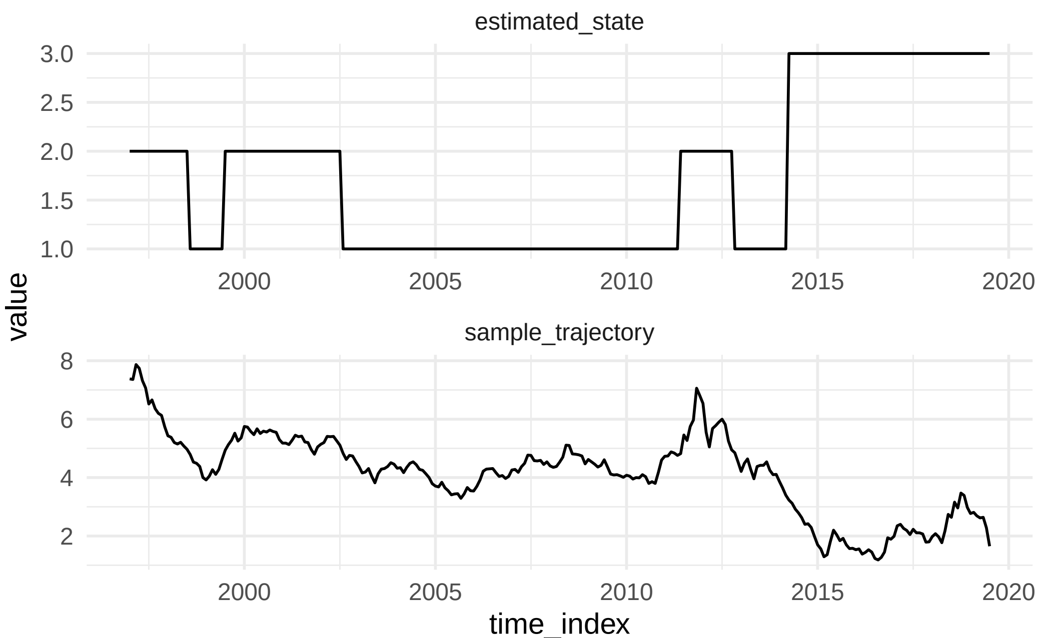
```r
plot(time(ts_data_nom), estStates[,1], cex=.3)
```



```
#ggplot
results_df <- data.frame(time_index=time(ts_data_nom) %>% as.numeric(),
sample_trajectory=ts_data_nom %>% as.numeric(),
estimated_state=posterior(fmodel3)$state) %>%
gather("variable", "value", -time_index)
```

```
Warning in .local(object, ...): Argument 'type' not specified and will default
to 'viterbi'. This default may change in future releases of depmixS4. Please
see ?posterior for alternative options.
```

```
ggplot(results_df, aes(time_index, value)) + geom_line() +
facet_wrap(variable ~ ., scales="free", ncol=1) + theme_minimal()
```



Model 3: controlling for gdp and inflation

```
mod3 <- depmix(response = y1 ~ 1, data = data.b, nstates = 3, family = gaussian(),
               transition = ~ gdp + inflation)
fmodel4 <- fit(mod3)
```

```
converged at iteration 26 with logLik: -212.1899
```

```r
fmodel4
```

```
Convergence info: Log likelihood converged to within tol. (relative change)
'log Lik.' -212.1899 (df=26)
AIC:  476.3797
BIC:  570.0348
```

```r
summary(fmodel4)
```

```
Initial state probabilities model
pr1 pr2 pr3
  1   0   0

Transition model for state (component) 1
Model of type multinomial (mlogit), formula: ~gdp + inflation
Coefficients:
            St1         St2        St3
(Intercept)   0 -0.04957338   9.675829
gdp           0 -1.85466145   3.058293
inflation     0 -1.26127930 -16.830633
Probalities at zero values of the covariates.
6.277516e-05 5.973906e-05 0.9998775

Transition model for state (component) 2
Model of type multinomial (mlogit), formula: ~gdp + inflation
Coefficients:
            St1        St2        St3
(Intercept)   0  7.2522619  6.1242106
gdp           0  0.2559071 -0.3583218
inflation     0 -1.5504992 -5.1229882
Probalities at zero values of the covariates.
0.0005350233 0.7550749 0.2443901

Transition model for state (component) 3
Model of type multinomial (mlogit), formula: ~gdp + inflation
Coefficients:
            St1        St2         St3
(Intercept)   0 -45.469617  70.7524128
gdp           0  -1.331988  -0.5211976
inflation     0  26.831880 -26.5794866
Probalities at zero values of the covariates.
```

```
1.873344e-31 3.352847e-51 1
```

```
Response parameters
Resp 1 : gaussian
    Re1.(Intercept) Re1.sd
St1           5.575  0.717
St2           4.217  0.355
St3           2.137  0.580
```

```
MLEse4=standardError(fmodel4)
```

```
Warning in sqrt(diag(vc$vcov)): NaNs produced
```

```
MLEse4
```

```
            par constr           se
1    1.00000000    bnd           NA
2    0.00000000    bnd           NA
3    0.00000000    bnd           NA
4    0.00000000    fix           NA
5   -0.04957338    inc          NaN
6    9.67582852    inc  82.51264009
7    0.00000000    fix           NA
8   -1.85466145    inc   0.84067931
9    3.05829292    inc          NaN
10   0.00000000    fix           NA
11  -1.26127930    inc   0.03133484
12 -16.83063310    inc 145.82307930
13   0.00000000    fix           NA
14   7.25226188    inc   2.95591615
15   6.12421064    inc   3.32465124
16   0.00000000    fix           NA
17   0.25590707    inc   0.58371300
18  -0.35832183    inc   0.64397243
19   0.00000000    fix           NA
20  -1.55049921    inc   1.10714580
21  -5.12298824    inc   2.85885605
22   0.00000000    fix           NA
23 -45.46961717    inc 397.63248689
24  70.75241280    inc 665.23030451
```

```
25   0.00000000    fix           NA
26  -1.33198815    inc  13.31298880
27  -0.52119761    inc          NaN
28   0.00000000    fix           NA
29  26.83188024    inc          NaN
30 -26.57948659    inc          NaN
31   5.57473278    inc   0.08878003
32   0.71749756    inc   0.05641503
33   4.21716952    inc   0.03699022
34   0.35495864    inc   0.02510926
35   2.13677895    inc   0.07355786
36   0.58046696    inc   0.05235337
```

```
posterior(fmodel4)$state
```

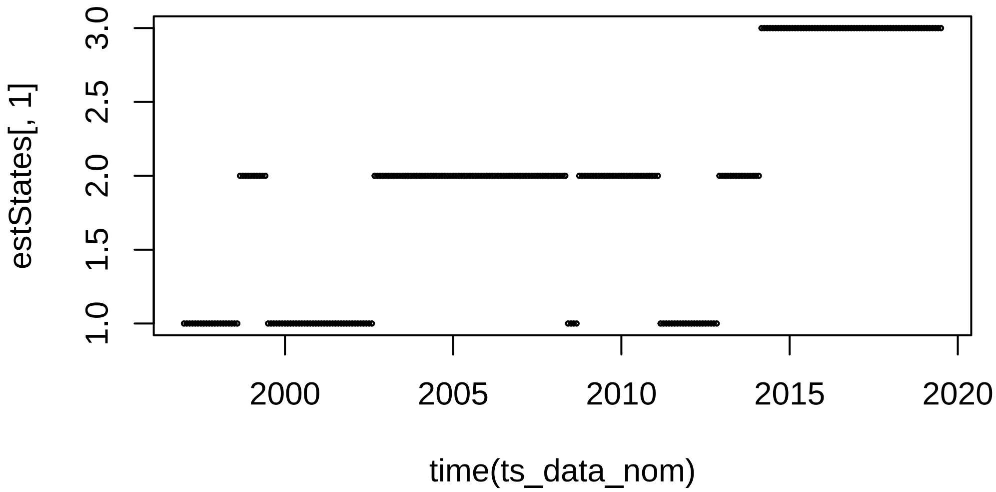
```
Warning in .local(object, ...): Argument 'type' not specified and will default
to 'viterbi'. This default may change in future releases of depmixS4. Please
see ?posterior for alternative options.
```

```
  [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1
 [38] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2
 [75] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
[112] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1 2 2 2 2 2 2 2
[149] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[186] 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
[223] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
[260] 3 3 3 3 3 3 3 3 3 3 3 3
```

```
estStates <- posterior(fmodel4)
```

```
Warning in .local(object, ...): Argument 'type' not specified and will default
to 'viterbi'. This default may change in future releases of depmixS4. Please
see ?posterior for alternative options.
```

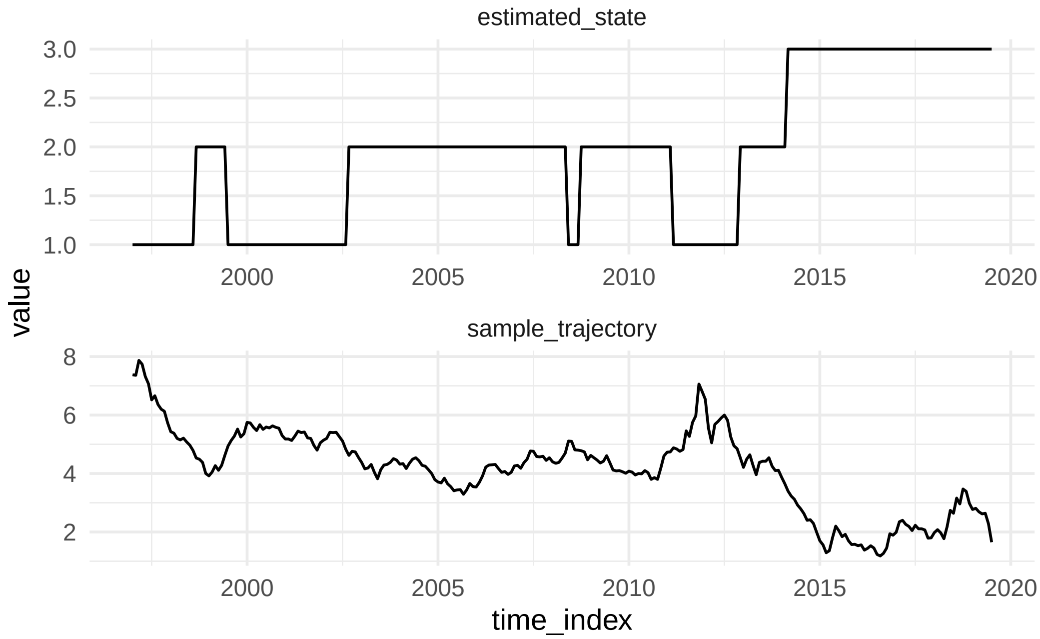
```
plot(time(ts_data_nom), estStates[,1], cex=.3)
```




```
#ggplot
results_df <- data.frame(time_index=time(ts_data_nom) %>% as.numeric(),
sample_trajectory=ts_data_nom %>% as.numeric(),
estimated_state=posterior(fmodel4)$state) %>%
gather("variable", "value", -time_index)
```

```
Warning in .local(object, ...): Argument 'type' not specified and will default
to 'viterbi'. This default may change in future releases of depmixS4. Please
see ?posterior for alternative options.
```

```
ggplot(results_df, aes(time_index, value)) + geom_line() +
facet_wrap(variable ~ ., scales="free", ncol=1) + theme_minimal()
```



Results

```
results_df <- data.frame(
  time_index = as.numeric(time(ts_data_nom)),
  sample_trajectory = as.numeric(ts_data_nom),
  estimated_state_fmodel4 = posterior(fmodel4)$state,
  estimated_state_fmodel3 = posterior(fmodel3)$state,
  estimated_state_fmodel2 = posterior(fmodel2)$state
) %>%
  gather("variable", "value", -time_index)
```

```
Warning in .local(object, ...): Argument 'type' not specified and will default
to 'viterbi'. This default may change in future releases of depmixS4. Please
see ?posterior for alternative options.
Warning in .local(object, ...): Argument 'type' not specified and will default
to 'viterbi'. This default may change in future releases of depmixS4. Please
see ?posterior for alternative options.
Warning in .local(object, ...): Argument 'type' not specified and will default
to 'viterbi'. This default may change in future releases of depmixS4. Please
see ?posterior for alternative options.
```

```
# Plot
ggplot(results_df, aes(time_index, value, color = variable)) +
  geom_line() +
  facet_wrap(~ variable, scales = "free", ncol = 1) +
  theme_minimal() +
  labs(title = "Comparison of Estimated States from fmodel4 and fmodel2",
       x = "Time Index",
       y = "Value") +
  theme(legend.position = "bottom")
```