Syntax

$$p, q \in \mathit{Path} ::= x \mid p.f$$
$$a, b, c \in \mathit{Constr} ::= p \equiv q \mid p :: C \mid p.\mathbf{cls} \equiv C$$
$$t \in \mathit{Type} ::= [x.\ \overline{a}]$$

$\mathtt{Zero}(x.\ \epsilon)$

$\forall x.\ x :: \mathtt{Zero} \Rightarrow x :: \mathtt{Nat}$

$\mathtt{Succ}(x.\ x.p :: \mathtt{Nat})$

$\forall x.\ x :: \mathtt{Succ}, x.p :: \mathtt{Nat} \Rightarrow x :: \mathtt{Nat}$

$\mathtt{prev}(x.\ x :: \mathtt{Nat}) : [y.\ y :: \mathtt{Nat}]$

$\mathtt{prev}(x.\ x :: \mathtt{Zero}) : [y.\ y :: \mathtt{Nat}] := \textbf{new}\ \mathtt{Zero}()$

$\mathtt{prev}(x.\ x :: \mathtt{Succ}, x.p :: \mathtt{Nat}) : [y.\ y :: \mathtt{Nat}] := x.p$

Constraint System

$$\frac{}{a \vdash a}\ (\text{C-Ident}) \qquad \frac{}{\epsilon \vdash p \equiv p}\ (\text{C-Refl})$$

$$\frac{\overline{a} \vdash p.\mathbf{cls} \equiv C}{\overline{a} \vdash p :: C}\ (\text{C-Class}) \qquad \frac{\overline{a} \vdash c \qquad \overline{a'}, c \vdash b}{\overline{a}, \overline{a'} \vdash b}\ (\text{C-Cut})$$

$$\frac{\overline{a} \vdash a_{\{\!\{x \mapsto p\}\!\}} \qquad \overline{a} \vdash p' \equiv p}{\overline{a} \vdash a_{\{\!\{x \mapsto p'\}\!\}}}\ (\text{C-Subst})$$

$$\frac{(\forall x.\ \overline{a} \Rightarrow a) \in P \qquad \overline{b} \vdash \overline{a}_{\{\!\{x \mapsto p\}\!\}}}{\overline{b} \vdash a_{\{\!\{x \mapsto p\}\!\}}}\ (\text{C-Prog})$$

$$\overline{a} \vdash \overline{b} := \bigwedge_{b \in \overline{b}} \overline{a} \vdash b$$

Substitution and Generalization

Substitution: $a_{\{\!\{x \mapsto p\}\!\}}$
Generalization: $a_{\{\!\{p \mapsto x\}\!\}}$

$$(a_{\{\!\{x \mapsto p\}\!\}})_{\{\!\{p \mapsto x\}\!\}} = a$$

$$x.g :: C_{\{\!\{x \mapsto y.f\}\!\}} = y.f.g :: C$$

$$y.f.g :: C_{\{\!\{y.f \mapsto x\}\!\}} = x.g :: C$$

$$y.f.g :: C_{\{\!\{f.g \mapsto x\}\!\}} \neq y.x :: C$$

First-order Model

$$\frac{}{a \vdash a}\ \text{(C-Ident)}$$

$$\forall c : \mathit{Constraint}.\ [c] \vdash c$$

$$\frac{\overline{a} \vdash p.\mathbf{cls} \equiv C}{\overline{a} \vdash p :: C}\ \text{(C-Class)}$$

$$\forall \overline{a} : \mathit{List}[\mathit{Constraint}], p : \mathit{Path}, C : \mathit{String}.$$
$$\overline{a} \vdash p.\mathbf{cls} \equiv C \rightarrow \overline{a} \vdash p :: C$$

$$\frac{\overline{a} \vdash a_{\{\!\{x \mapsto p\}\!\}} \qquad \overline{a} \vdash p' \equiv p}{\overline{a} \vdash a_{\{\!\{x \mapsto p'\}\!\}}}$$

$$\forall \overline{a} : \mathit{List}[\mathit{Constraint}], a_2 : \mathit{Constraint},$$
$$x : \mathit{String}, p_1, p_2 : \mathit{Path}.$$
$$\mathbf{let}\ a := a_{2\{\!\{p_2 \mapsto x\}\!\}}\ \mathbf{in}$$
$$\mathbf{let}\ a_1 := a_{\{\!\{x \mapsto p_1\}\!\}}$$
$$\mathbf{in}\ \overline{a} \vdash p_2 \equiv p_1 \wedge \overline{a} \vdash a_1$$
$$\rightarrow \overline{a} \vdash a_2$$

Inference Rules and Entailment

$$\frac{\text{premise}}{\text{conclusion}}$$

$$\overline{a} \vdash a$$

$$premise \rightarrow conclusion$$

$$\overline{a} \rightarrow a$$

Permutation and weakening

$$[a, b, c, d] \vdash a$$
$$[b, c, d, a] \vdash a$$
$$[a] \vdash a$$

$$a \wedge b \wedge c \wedge d \to a$$

Substitution?

- Model substitution as an relation between paths?
- Populate this relation from the programs context
- Paths are representet as constants

$$subst : Path \times X \times Path \times Path \mapsto \mathbb{B}$$
$$subst(x.g.h, x, y.f, y.f.g.h)$$

x substituted with $y.f$ in $x.g.h$ is $y.f.g.h$